II

Defining Asset-Liability Management

The ideas of ALM can be traced back to the seminal work of Redington (1952) in which he suggested that there should be an equal and parallel treatment of assets and liabilities in actuarial valuation. This led him to the concept of *duration* and to the introduction of the technique known as *immunization.* These have become the main tools of ALM. These concepts, the development of ALM, and related issues are discussed in this chapter. To understand Redington's framework, consider a block of insurance business as a set of cash flows:

A_t asset cash flows during the t-th year of business, which consist of investment income and maturing capital;

L_t liabilities cash flow during the t-th year of business, consisting of policy claims, policy surrenders, expenses, and premium income (the last item has the effect of lessening liabilities cash flow).

Net cash flow during the year t is then $N_t = A_t - L_t$. A company becomes insolvent on an *economic basis* if any N_t becomes negative. Management, therefore, operates under the constraint of the net cash flow being positive in every period. Additionally, the present value of the stream of net cash flows represents the economic value of the business, i.e., *economic surplus*. The goal of management, however, is to maximize the economic value of the business, given the cash flow constraints and level of risk chosen, and dictated by regulation. The coordination of asset and cash flows becomes a central issue of insurance firm management.

Valuation of assets, liabilities, and net cash flows is a problem of exactly the same nature as the pricing of securities addressed in Chapter 1. However, in the case of an insurance firm in the United States, its accounting practices are additionally constrained by prescribed accounting regimes. First, the use of statutory accounting principles (SAP) is required by state insurance law. Then, for a publicly held firm, the use of generally accepted accounting principles (GAAP) is required by the Securities and Exchange Commission (although some mutual companies have voluntarily prepared GAAP statements for practical reasons, such as management information or benchmark comparison). Finally, the Internal Revenue Service requires tax-basis accounting.

However, these regulatory accounting constraints do not address the pricing of securities embedded in the insurance business in the way economic and financial theory prescribe them to do so. They fail to recognize fully the following issues:

- Changes in present value of cash flows due to changes in interest rates.
- Options granted or held by the insurance firm in its assets and liabilities portfolios.
- Long-term results of surrenders (lost future profits) and new sales (additional profits).
- Anticipated future profits on future business.
- Long-term value (positive or negative) of the existing distribution system.
- Actual market values of assets held in the investment portfolio.

These shortfalls have lead to the development of *economic value analyses* in insurance firm management, now in use at many insurance enterprises. The two most important of these are the *value-added* and *return-on-equity* methods. These are derivations of the net present value and internal rate of return methods used in financial analysis, and represent a step toward full application of the theoretical economic method of valuation of both assets and liabilities as securities.

Furthermore, the National Association of Insurance Commissioners (NAIC) has recently taken two decisive steps toward better representation of the common nature of assets and liabilities of insurers (Black and Skipper 1994). The 1990 Amendments to the Standard Valuation Law not only place the legal responsibility for valuation of liabilities on the appointed actuary, but also require asset adequacy analysis, in effect creating an integrated asset-liability framework of the firm, which is the main theme of this work. NAIC has also developed a risk-based capital (RBC) formula establishing target surplus amounts that are required above reserve requirements. The required amounts of capital are determined from four major factors related to four major categories of risk facing an insurance enterprise (Tullis and Polkinghorn 1992, Morgan Stanley & Co. 1993, and Society of Actuaries 1979):

- C-1: Asset quality and payment default risk.
- C-2: Insurance pricing risk.
- C-3: Interest rate risk, often generalized as ALM risk.
- C-4: Miscellaneous business risks.

Morgan Stanley & Co. (1993) provides the numerical formulas for these components. The actual RBC is then determined as:

$$RBC = (C-4) + \sqrt{(C-2)^2 + ((C-1) + (C-3))^2}. \quad (2.1)$$

The ratio of the insurer's adjusted capital (statutory capital and surplus, asset valuation reserve, plus any voluntary reserves and half of the policyholder dividend liability) is divided by the RBC to determine the RBC ratio used by the regulators to determine a company's capital adequacy.

As this framework implies, ALM has been traditionally associated with the interest rate risk, that is, the C-3 risk. This was, indeed, the original idea of Redington (1952). There are two major kinds of risks related to interest rates:

- Future positive cash flows may have to be reinvested at a time of high capital asset valuations, which express themselves most visibly in relatively low interest rates. This is most often referred to as *reinvestment risk.*
- Future negative cash flows may require liquidation of capital assets from the portfolio at low values, which are most visibly expressed by high interest rates. This is commonly referred to as *disinvestment risk,* or *price risk.*

The C-3 risk is further influenced by any dependencies existing between interest rates and the cash flows of assets and liabilities. Such dependencies are created by options embedded in assets and in liabilities. Examples of options embedded in assets include calls on callable corporate bonds (as well as puts on putable bonds), warrants, mortgage prepayment rights, caps and floors on floating rate bonds and mortgages, and convertibility features of bonds and preferred stocks. Examples of options embedded in liabilities are: dividend distributions, life insurance policy loans, cash value surrender, single premium deferred annuity tax-free exchange, and guarantees of interest rates, such as minimum and period rate guarantees.

Redington's solution to the management of interest rate risk called for *immunization* from the interest rate risk. It was rooted in a very basic idea from elementary calculus. If $f(x)$ is a function of a variable x, and if the derivative $f'(x)$ exists, then we have the following approximate identity:

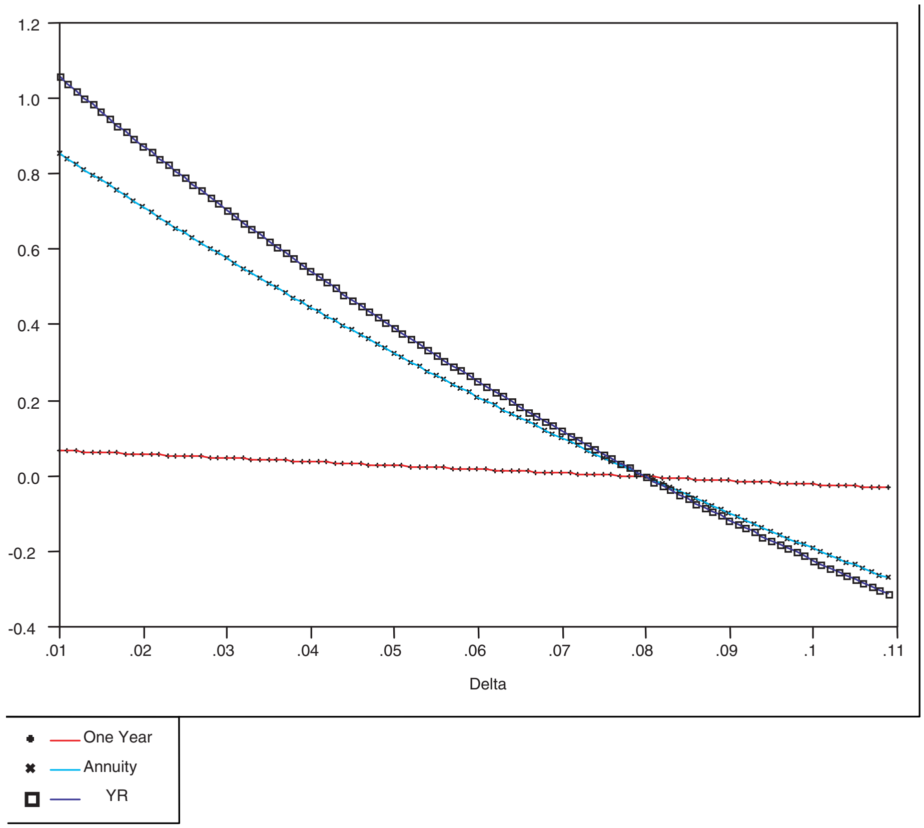
$$f(x + \Delta x) \approx f(x) + f'(x)\Delta x. \quad (2.2)$$

If the derivative equals zero, a small change in x, denoted here by Δx, results in almost no change in the value of the function. If we denote the market value of assets of the firm by $A(i)$, where i denotes the effective annual rate of interest, and the market value of liabilities as $L(i)$, then the surplus of the enterprise $S(i)$ equals $A(i) - L(i)$. If we apply the reasoning presented above to the function $S(i)$, then we should manage assets and liabilities in such a way that $A'(i) = L'(i)$. This would result in the enterprise being immunized from small changes in interest rates; in other words, the value of the surplus would not change given a small change in interest rates.

How does one implement such a strategy? Let P be the price of a security producing cash flows in the future, i be the effective annual rate of interest, and δ be the corresponding force of interest. For the sake of simplicity, assume temporarily that there is only one effective annual interest rate, regardless of maturity. This assumption will be relaxed later.

It is quite clear that P is a function of i (or δ), and this is an inverse relationship if the cash flows are positive. If we are interested in the rate of change of P, it is easy to notice that the magnitude of P affects its rate of change. In the simplest, and well-studied, case of exponential growth, the rate of change of P with respect to δ is proportional to P. In view of this relationship, it might be more appropriate to consider the value of a unit of P, or better yet, $\ln P$. In illustrating the intuitive appeal of that approach, assume the function $\ln P$ equals zero when the security has the unit value, that is, sells at par. In this work, we will assume for some illustrations that all securities are at par when the force of interest δ equals 8%, or 0.08. Figure 2 shows the relationship of $\ln P$ to for a unit

FIGURE 2
Sensitivity of Various Instruments to the Force of Interest



of each of the following securities: a one-year pure discount bond, a 30-year fixed coupon bond (such as the U.S. Government 30-year bond), and a 30-year fixed annuity certain.

In addition to the expected inverse relationship, notice the following. Because of the 10:1 ratio in units on the axes, it may not be immediately clear that the one-year bond graph is a straight line of slope negative 1. But it is. We can also see that the other two curves are not straight lines, and that their tangents' slopes are much steeper than the slope of the one-year bond's line (with all of them being negative). The opposite (we use the opposite to make the quantity positive) of the slope of the tangent to the curve of the graph of $\ln P = F(\delta)$ with F denoting the functional relationship of $\ln P$ to δ, has acquired a special significance in mathematical finance. It is called the *duration* (or *effective duration,* but we will use the term "duration") of P. Mathematically speaking, it represents the logarithmic derivative of price with respect to the force of interest. Duration will be denoted by D, or $D(P)$, if it is necessary to identify the security whose duration is being discussed. Observe that, if i is the effective annual rate of interest equivalent to δ, then:

$$\frac{dF}{di} = \frac{dF}{d\delta}\frac{d\delta}{di} = \frac{dF}{d\delta}\frac{1}{1+i},$$
$$\frac{d^2F}{di^2} = \frac{1}{(1+i)^2}\left(\frac{d^2F}{d\delta^2}\right) + \frac{1}{(1+i)^2}\left(-\frac{dF}{d\delta}\right). \tag{2.3}$$

(2.3) is, in fact, true for any function F. Furthermore, if the security produces certain cash flows CF_t at various times t, $t \geq 0$ in the future (i.e., is a form of a riskless bond), we have:

$$P = \sum_{t \geq 0} CF_t(1+i)^{-1}, \tag{2.4}$$

so that the price of this security is merely a polynomial in $v = (1+i)^{-1}$, and therefore its derivative is the sum of derivatives of the terms

$$\frac{dP}{di} = \sum_{t \geq 0} CF_t(-t)(1 + i)^{-t} \quad (2.5)$$

and

$$\frac{d(\ln P)}{di} = \frac{dP/di}{P} = -\frac{1}{1+i}\frac{\sum_{t \geq 0} tCF_t(1+i)^{-t}}{\sum_{t \geq 0} CF_t(1+i)^{-t}}, \quad (2.6)$$

as well as

$$\frac{d(\ln P)}{d\delta} = -\frac{\sum_{t \geq 0} tCF_t(1+i)^{-t}}{\sum_{t \geq 0} CF_t(1+i)^{-t}}. \quad (2.7)$$

The last formula introduces naturally one of the very first concepts of ALM: the *Macaulay duration* (first appearing in the work of Macaulay, 1938 and later independently reintroduced by Samuelson, 1945 and Redington, 1952). It can be defined as the weighted average time to maturity of cash flows of the security:

$$\text{Macaulay duration} = \sum_{t \geq 0} \frac{CF_t(1+i)^{-t}}{\sum_{t \geq 0} CF_t(1+i)^{-t}}. \quad (2.8)$$

From the analysis presented above it follows immediately that for a security with certain cash flows:

$$\text{Macaulay duration} = -\frac{d(\ln P)}{d\delta} = -\frac{dP/d\delta}{P} \quad (2.9)$$

$$\frac{1}{1+i}\cdot \text{Macaulay duration} = -\frac{d(\ln P)}{di} = -\frac{dP/di}{P}. \quad (2.10)$$

The last expression is commonly referred to as *modified* (in reference to securities with cash flows independent of interest rates), or *effective* (in reference to securities with cash flows dependent on interest rates, i.e., securities with embedded options) *duration,* and is what will be known as *duration* throughout this work. It represents the instantaneous rate of change of the natural logarithm of the price with respect to the effective annual interest rate, with the minus sign adjusting for the inverse relationship. It effectively tells us what portion of the value of the security will be lost as a result of a small, instantaneous unit increase in interest rates, or what portion would be gained as a result of an instantaneous unit decrease in interest rates. We can also see that if, instead of effective annual interest rate, we utilize force of interest, then there is no need for dividing by $(1 + i)$ to obtain duration from Macaulay duration. Vanderhoof (1972) and Fabozzi (1993) give examples of calculations of Macaulay duration and effective duration of various financial instruments. Some of the results are:

- The duration of a perpetuity (irredeemable bond, e.g., perpetual preferred stock with a fixed dividend) immediate equals its present value.
- The duration of an n-year annuity certain (e.g., a mortgage without prepayments) equals:

$$\frac{(Ia)_{\overline{n}|}}{\ddot{a}_{\overline{n}|}}. \quad (2.11)$$

- The duration of a bond redeemable at par with the coupon rate equal to i equals $a_{\overline{n}|}$.
- The duration of a perpetuity growing at a rate g (the classical constant growth model of common stock) equals:

$$\frac{1}{i-g}. \quad (2.12)$$

Note that Equation (2.12) is derived under the assumption that g is not a function of i, and its applicability in practice is debatable (there are no stocks that grow at the same rate forever). The duration of equities is further discussed in Chapter 5.

At this point, we should ask ourselves how duration can be used for immunization as proposed by Redington. Again, our purpose as an insurance enterprise is to immunize the surplus of the firm, with $S(i) = A(i) - L(i)$. What exactly is the meaning of that statement? Do we want to protect the actual dollar value of the surplus or the ratio of the surplus to assets, that is, the firm's capital ratio? Both interpretations are valuable and meaningful.

Regarding the question of protecting the dollar value of the surplus, suppose we want to minimize the change in the absolute level of surplus under a small change in interest rates. This means that

$$\Delta S \approx 0 \text{ for } \Delta i \approx 0. \quad (2.13)$$

This does imply that

$$S'(i) = A'(i) - L'(i) \approx 0. \quad (2.14)$$

But

$$-P'(i) = P(i) \cdot D \quad (2.15)$$

for any security. The quantity $-P'(i) = P(i) \cdot D$ is called the *dollar duration* of a security and represents a dollar change in the value of the security in response to a very small unit change in the interest rate. The strategy of immunizing the dollar value of the surplus calls for setting the dollar duration of the asset portfolio equal to the dollar duration of the liabilities portfolio.

One can, however, hardly imagine a successful insurance enterprise following solely that strategy. Why? Because the regulatory constraints on the company's surplus are typically expressed in terms of its capital ratio, not actual surplus level. Furthermore, this strategy calls for the company remaining immune to market disallocations of wealth, while its customers remain exposed. This is not a strong marketing point. Finally, this approach does not address the key goal of the enterprise: to maximize the value of the firm given the regulatory and solvency constraints.

The second, and apparently more reasonable, approach to classical immunization, calls for preserving a company's capital ratio, or surplus ratio, that is, the ratio $S(i)/A(i)$. Let k be the initial value of capital ratio of the firm. Note that immunization of the capital ratio k is equivalent to immunization of the ratio of liabilities to assets $L(i)/A(i) = 1 - k$, or equivalently, the natural logarithm of that expression $\ln(1 - k)$. Under an infinitesimal change of the interest rate, this implies that we want to set:

$$\frac{d \ln(A(i))}{di} = \frac{d \ln(L(i))}{di}. \tag{2.16}$$

Thus, the strategy of immunization of surplus ratio calls for setting the duration of assets equal to the duration of liabilities. This is the most common approach to immunization. One can argue that it was this idea, or more precisely its strengths and weaknesses, that gave rise to all of the modern techniques of ALM, which will be discussed further in Chapter 3.

Existing ALM Techniques

Van der Meer and Smink (1993) give an extensive overview of existing ALM techniques. Let us review their proposed classification of asset-liability management methodologies in view of the proposed integrated approach to financial intermediation. Later parts of this work will discuss why all of those methods needed to be developed.

The first group of techniques, as proposed by Van der Meer and Smink (1993), are static methods, such as:

- cash flow payment calendar,
- gap analysis,
- segmentation, and
- cash flow matching.

These methods are generally rooted in the traditional "spread" perspective of financial intermediation. In that perspective, intermediaries do not create a new "private issue derivative," but rather provide access to existing markets (this still creates a new security, but merely by pooling of resources). In particular, the value of these methods rests on the assumption of the deterministic (static) nature of cash flows, that is, their complete predictability. Clearly, such approaches have very limited use for an insurance enterprise. They can, however, be useful for a pension plan, structured settlement, or other payout security, if its cash flows are fully or nearly fully predictable.

The *cash flow payment calendar* method presents a schedule of a firm's positive and negative cash flows and, thus, provides a tool for detecting dangerous imbalances in such flows. *Gap analysis* (Clifford 1981) is used by banking firms. The gap is defined as the balance sheet value difference between fixed and variable rate assets and liabilities. If the gap is nonzero, this implies interest rate risk exposure. In its pure form, the method is of little value, but if it is refined to account separately for various maturities "buckets," it becomes helpful. *Segmentation* (Attwood and Ohman 1984) is a technique used by insurance firms. Its essence lies in segmenting the firm's liabilities portfolio into portions, each of which receives a separate asset portfolio designed to mirror the structure of liabilities.

Cash flow matching is a technique designed to eliminate the financial intermediary altogether. Under this approach, the scheduled negative cash flows produced by the liabilities are projected and then a portfolio of assets producing the same cash flows is purchased. Clearly, the assets at hand must be sufficient for such a purchase. Furthermore, a perfect and complete projection of cash flows is not always possible. Note that if even such a complete projection is possible, there is the problem of finding the least costly portfolio satisfying the conditions.

Algorithms for efficient cash flow matching have been developed by Kocherlakota, Rosenbloom, and Shiu (1988, 1990). When cash flow matching is discussed, it is commonly assumed that the securities used in the process are zero-coupon risk-free government bonds. Use of such bonds simplifies the process, because any combination of cash flows can be obtained from them as long as the maturities desired are available (zeros maturing shortly before the times when cash is needed can also be used by utilizing cash holdings for the remaining period). But, as already noted in the discussion on arbitrage pricing models, securities producing cash flows form a vector space.

Zero-coupon risk-free bonds form a basis for the subspace of that space consisting of securities producing a finite number of deterministic cash flows, that is, cash flows independent of the future states of the world. It is well known from the theory of linear algebra that a basis of a vector space is not unique, and other bases can be obtained from any given one through a linear transformation. Therefore, as long as the cash flows are deterministic and finite, any set of deterministic securities that forms a basis can be used to form a cash flow matching portfolio. But life is not always as simple as linear algebra. If solutions involve negative investments in some securities, (i.e., short sales), they may not be allowed for insurance firms. Practical solutions must address such limitations.

Furthermore, if higher-yield, fixed, illiquid income securities, such as private placement bonds and mortgages, can be assured to have risk-free cash flows and produce a matching portfolio, one could utilize those securities, hopefully at lesser cost because of their lack of liquidity. One more important observation about cash flow matching is that if the matching portfolio of assets is designed correctly, this is the ultimate buy and hold strategy, which does not incur any more costs.

If future cash flows of liabilities are uncertain, this method has a natural theoretical extension. Let S_t be the collection of the future states of the world at time t. A security that pays a unit (e.g., $1) exactly at time t, exactly when $S \in S_t$ occurs, and 0 otherwise, is called an *Arrow-Debreu security.* The matter may get complicated if securities cash flows depend on the past (which is true for home mortgages, e.g.), in which case the elements of the set S_t must include in their definition a description of the path of events leading to their occurrence at time t. Such approach is naturally used in multi-period models of a market. In either case, for a security with a finite number of cash flows in a world with a finite number of states at each time, the full set of Arrow-Debreu securities clearly forms a vector space basis. If there existed a perfect market for Arrow-Debreu securities, a cash flow match could be constructed for a contingent security. In absence of a perfect market for Arrow-Debreu securities, financial intermediaries provide a limited one. What else is a $1 one-year term life insurance policy than a private issue Arrow-Debreu security?

In addition to these basic static strategies, one could use the same method for varying future scenarios of interest rates or other variables, or other assumptions, in effect at least partially accounting for dependencies between the economic variables, assets, and liabilities. This approach is called *multiscenario analysis,* and in the United States it is the basis for the New York state Regulation 126 requirement of cash flow testing of an insurance firm. This type of analysis undoubtedly provides a deeper insight into the nature of the risks faced by the firm. But it does not provide information about the true market-related value of liabilities, and the economic value of the firm, especially since the scenarios might be picked by a biased observer (this is definitely the case with the Regulation 126, which prescribes extreme scenarios of a rather unrealistic nature, which may have been realized in the United States only a couple of times).

The second group of asset-liability strategies are dynamic in nature and relate to the integrated approach we have been advocating here. They all attempt to account for the relationship between assets and liabilities and other factors influencing the balance of the two. Van der Meer and Smink (1993) classified dynamic methods as value driven and return driven:

- Value-driven methods can be either passive:
  - immunization,
  - model dependent immunization,
  - key rate immunization
- or active:
  - contingent immunization,
  - portfolio insurance,
  - constant proportion portfolio insurance.
- Return-driven methods can be subdivided into two categories:
  - spread management, and
  - required rate of return.

Value-Driven Methods

The value-driven strategies are all descendants of the classical idea of Redington (1952): to protect the company surplus from interest rate risk.

Passive Strategies

Immunization, also called *standard immunization,* requires matching either dollar durations or durations of assets and liabilities. It should be noted that the method assumes that a company's assets exceed its liabilities. There are several problems with the duration matching strategy, one of which is that straightforward matching of durations may actually lead to an increase in the firm's interest rate risk, a topic that will be covered in Chapter 3. In addition:

- The method assumes perfect knowledge of future cash flows A_t and L_t, whereas, in reality, the cash flows may be hard to predict exactly, and determination of their exact timing might be even more difficult. Furthermore, cash flows might not be independent of interest rates, as assets held may be called or prepaid while liabilities issued may have features allowing customers to put them back to the company.

- There could be a risk of default by the issuers of assets, which makes their cash flows uncertain. This is precisely the C-1 risk, but in fact it is very closely related to the asset-liability profile of the company, as default risk can be expressed in the following terms: The issuer of a risky bond preserves the right to put the assets backing the bonds (either directly backing it in a mortgage bond or indirectly in a debenture) to the bondholder.
- The change in interest rates a company experiences may be large in relation to the infinitesimal amount assumed in theory. One should, therefore, use higher-order derivatives and a further extension of the Taylor power series expansion of the function $\ln S(i)$ for a better approximation of changes in the surplus level. This leads to better utilization of the concept of convexity, which will be discussed in Chapter 3. Furthermore, there is not just one interest rate for all maturities, but rather a full term structure of interest rates that should be examined. This, in turn, leads to the concepts of *partial durations,* or *key rate durations,* which will also be discussed later.
- Over time, and as interest rates change, duration changes its value. The change of duration with the passage of time is referred to as *duration drift.* Both of these factors can, and usually do, affect assets and liabilities differently, which makes it necessary to rebalance the asset-liability portfolio. In practice, this results in additional monitoring and transaction costs, rendering classical immunization less attractive.

The other immunization strategies address some of the weaknesses of classical immunization. They all rely on a more realistic approach to the notion of interest rate. In the existing capital markets, a risk-free future cash flow is not discounted at the same effective annual rate of interest regardless of the term of the cash flow. Mathematically speaking, this means that the force of interest δ is a function of time $\delta = \delta(t)$, the function representing the *term structure of interest rates* (Fabozzi 1993). When speaking of the term structure, one must carefully distinguish between the following concepts:

- The *spot rate* for time t, representing the effective annual interest rate for a zero-coupon bond issued now and maturing at time t.
- The *coupon yield,* representing the annual coupon rate for a par bond issued now and maturing at time t.
- The *forward rate* (interest rate, or instantaneous force of interest, for forward purchases of bonds), which, for the force of interest, is best explained by presenting its mathematical relationship to the spot rate: If δ_t is the t-period spot force of interest, and ϕ_t is the forward force of interest at time t, then

$$e^{t\delta_t} = e\int_0^t \phi_s ds. \tag{2.17}$$

Effectively, forward force of interest ϕ_s, $s < t$, represents the path of instantaneous forces of interest which leads to the compounded force of interest rate δ_t over the period $[0, t]$. Either the set of spot forces of interest $\{\delta_t\}$, or the set of forward forces of interest $\{\phi_t\}$, or the set of corresponding effective spot or forward interest rates, is referred to as the *yield curve,* and is the functional representation of the term structure of interest rates.

There are many theories explaining empirically observed yield curves, but the main ones (Fabozzi 1993; see also Gwartney and Stroup 1995) can be summarized as follows:

- The *expectations theory* holds that currently observed forward rates represent market participants' expectations of the future instantaneous spot rates at the corresponding time.
- The *liquidity preference theory* holds that forward rates for the future times exceed actual market participants' expectations of future instantaneous spot rates by the amount of *liquidity premium* compensating market participants for additional risk of holding fixed income instruments for longer periods of time.
- The *preferred habitat* theory states that various market participants have various maturity preferences for cash flows they are to receive and bid up prices of the future cash flows (causing the corresponding spot rates to fall) that are more desirable to them.

  A yield curve is termed *arbitrage-free* if the forward rates given by it, that is,

$$\phi_t = t\frac{d\delta_t}{dt} + \delta_t \tag{2.18}$$

  are positive for all values of t. Negative values of forward rates would lead to riskless arbitrage opportunities.

Key rate immunization was developed independently by Ho (1990) and Reitano (1990, 1991a, 1991b). It replaces the functional relationship $P = P(i)$, which is the basis of duration analysis and classical immunization by a function of several variables:

$$P = P(i_1, i_2, \ldots, i_n), \qquad (2.19)$$

where $i_1, i_2, \ldots, i_n$ are appropriately chosen effective annual spot rates of interest that affect the value of P. The main reason for the analysis of the term structure of interest rates is the common empirical observation that interest rates for various maturities do not always change in exactly the same magnitude and direction—that is, yield curve shifts are not always parallel. This means that, instead of observing just one change in the interest rate Δ_i, we may indeed have differing $\Delta_{i1}, \Delta i_2, \ldots, \Delta i_n$. Instead of setting the derivatives, or logarithmic derivatives, of $A(i)$ and $L(i)$ equal, key rate immunization sets the entire corresponding sets of partial derivatives of $A(i_1, i_2, \ldots, i_n)$ and $L(i_1, i_2, \ldots, i_n)$ equal to each other.

A further refinement of the classical immunization theory is offered by *model dependent immunization.* With this approach, a theory (usually a stochastic process) describing the evolution of the yield curve over time is the starting point. The process governing the evolution of the yield curve is dependent on one or more parameters. By setting sensitivities of assets and liabilities of the firm with respect to those parameters equal to each other, we may obtain immunization of the surplus from changes in those parameters, with results similar to those in classical or key rate immunization. The methodology for this is presented by Cox, Ingersoll, and Ross (1985) and Jarrow and Morton (1992). Hull (1993) provides an extensive overview of the models of the yield curve process.

The prior three ALM strategies are all represented as *passive* because of their common goal of preserving the existing level of surplus.

Active Strategies

Active strategies aim to guarantee a minimum acceptable level of surplus while providing room for active asset portfolio management in hopes of achieving higher asset returns.

Contingent immunization was developed by Leibowitz and Weinberger (1982, 1983). Under this approach, if assets exceed the current amount needed to pay the liabilities, the asset portfolio is actively managed in hopes of achieving outperformance. If the value of the portfolio declines to the amount needed for immunization, the active management strategy is abandoned in favor of immunization.

Portfolio insurance is a method of ALM based on the strategy of replicating an option on a capital asset with a dynamically adjusted portfolio of cash and a capital asset (Leland and Rubinstein 1981). Recall the definition of a European call and a European put on a capital asset S, as stated in Chapter 1. Assume here that the asset S does not provide any dividend or other form of income. Let c be the price of a European call, and p be the price of a European put. Let the options have the exercise price of X in the prescribed time T in the future. Assume for simplicity that the capital asset does not produce income between now and T (this can be achieved by reinvesting income in additional units of the capital asset). Consider a portfolio of one unit of capital asset S, and one put p, on that unit of capital asset. If we are only concerned with the terminal value of the portfolio, at the time T, this portfolio will be worth S if $S \geq X$, and X if $S < X$. However, if we instead consider the portfolio consisting of a call c on one unit of capital asset, and cash earning interest at a risk-free rate amounting to the present value of the exercise price X, then this portfolio will have exactly the same value at the time

$$c + PV(X) = S + p. \qquad (2.20)$$

This identity is referred to as the *put-call parity* (Hull 1993). It shows the relationship of a portfolio of cash and capital asset to a portfolio of call and put, as we also have $S - PV(X) = c - p$.

Now, consider the following strategy. For the period of time from now until time T, if the price of the capital asset exceeds $PV(X)$, hold the capital asset. Any time the price reaches $PV(X)$ (the value of which changes over time) from above, sell the asset and hold cash. However, if the price again reaches $PV(X)$ back, this time from below, buy the capital asset back. This strategy results in an assurance of holding the capital asset at the time T if $S \geq X$ at that time, and holding X in cash if $S < X$. The payoff of this strategy is therefore identical to that of the portfolio of the capital asset and a put, or cash (risk-free asset) in the amount of $PV(X)$ and a call. This *replication* of an option is precisely the basis of the simplest form of portfolio insurance.

A variation on that theme is offered by the *constant proportion portfolio insurance* (Black and Jones, 1987). Under this strategy, part of the asset portfolio, called the *reserve account,* is invested in the risk free asset, while the rest in managed in an active fashion. However, the proportion of the account actively managed remains constant over time. There are also other variations of this strategy.

Return-Driven Methods

Return-driven dynamic strategies concentrate on the returns earned by the enterprise.

Spread Management

Spread management focuses on maintaining a yield spread between assets and liabilities. If returns of assets and liabilities are fixed, this is merely a restatement of the classical banker's job paradigm. However, in any realistic analysis of the asset and liabilities portfolios, their returns are affected by the varying maturities of cash flows, credit risk, and options embedded in assets and liabilities. This has lead to the development of such tools as *option-adjusted spread* (Herskowitz 1989) and *spread duration* (Leibowitz, Krasker, and Nozari 1989).

The concept of option-adjusted spread is of particular importance in the perspective on ALM. To understand it, let us begin with the simpler, traditional concept of a bond spread. If a bond is issued by a risky company (i.e., an enterprise that cannot completely guarantee a return of principal and interest when due), it must carry a higher yield to maturity than an otherwise identical risk-free bond would. This difference in yields is referred to as the *spread* of the security. One can generalize it for yields on zero-coupon bonds to various maturities, producing spreads for the entire yield curve for this particular bond issuer. But, as pointed out by Merton (1974), credit risk of a corporate bond has a relatively straightforward interpretation in the language of options.

For simplicity, let us assume that there is only one bond issuer and one bondholder. During the life on the bond, its issuer can go into default and put the assets of the firm to the bondholder. Therefore, at any time during such bond life, the bondholder holds the present value of future cash flows of the bond, but also has written (i.e., created for the bond issuer) an option to sell (i.e., a put) the assets of the firm for that present value of future cash flows. Further simplifying the problem, let us now assume that the bond is a zero-coupon bond with value of X at maturity, and that option exercise (i.e., bankruptcy) can only occur at maturity. If S is the value of the assets of the bond issuer and we denote by p the value of the European put on these assets, with the exercise date at bond maturity, then the bondholder has $PV(X) - p$, or equivalently, $S - c$; that is, the bondholder holds the assets of the issuer, but also has given the issuer the right to repurchase them at the maturity value of the bond at bond maturity. The credit spread represents a payment for the price of the option, resulting in a higher discounting rate for which new present value $PV_{\text{new}}(X) = PV(X) - p$. This simple concept is generalized to a spread compensating for any asset or liabilities options, thus producing the option-adjusted spread over corresponding risk-free yields.

Note that Ross, Westerfield, and Jaffe (1993) attribute the discovery of the put-call parity to Russell Sage, a late 19th century financier who made loans with yields in excess of usury law limits. He did this by purchasing a mortgaged asset from the borrower and issuing the right to repurchase, while structuring the exercise price at the level creating the desired yield. Knoll (1994) provides a fascinating discussion of legal aspects of put-call parity.

Required Rate of Return Analysis

Required rate of return analysis is a strategy formulated by Miller, Rajan, and Shimpi (1989). This method starts with deriving a required rate of return on the existing liabilities portfolio, and then creating an asset portfolio with the objective of achieving such a return. The selection may be contingent on the scenarios of the future. This approach is generalized by *risk-return analysis,* echoing the Markowitz efficient frontier mentioned in Chapter 1. Wise (1984a,b), Wilkie (1984), and Leibowitz and Langeteig (1991) analyze return on surplus in relation to risks of surplus, and in the context of the asset-liability portfolio. We will return to these ideas later.

When reviewing the various ALM methodologies, we can clearly see their evolution from the traditional vision of financial intermediation to the integrated perspective. The most elementary techniques can be traced to the following very simple one-period model. Let an intermediary issue a bullet one-year liability L earning a risk-free rate of return of i. This intermediary must invest capital S in this business, and the resulting asset portfolio of value $A = L + S$ is invested in a one-year bullet security earning $i + s$. (At this point, one might ask what the risk of that security is, given that it earns premium over i, but ignore that for a moment.) Then the intermediary earns $sA + iC$ on an investment on $C,$ resulting in a one-period return of

$$i + s\frac{A}{S} = i + \frac{s}{k}, \tag{2.21}$$

where k is the capital ratio S/A of the intermediary.

This simplistic paradigm can have very simple, yet significant, complications added to it. First, the issuance of the liability and the purchase of the asset are not simultaneous. If the spread s narrows or becomes negative between those two events, the profitability of the intermediary suffers or even becomes negative. This effect is partly captured by the duration measure, and by its extensions. Now factor in some measure of the C-1 credit risk. The intermediary issues a risk-free note while purchasing a risky note and offering a put option to the risky note issuer. Let p denote that put value. Then the net position of the intermediary is valued at $(1 + k)L - L - p = S - p$. In effect, the intermediary invests its surplus in a risk-free asset and writes a put to the risky note issuer. The ‘‘net business’’ is not that of intermediation but of option writing, in line with the key proposition of Chapter 1. Therein lies the derivative.

This paradigm, of course, is subject to much further complication, because of the following:

- Maturities, assets, and liabilities vary, and their cash flows are not simultaneous.
- Liabilities have options such as return guarantees and provisions for funds additions and withdrawals.
- Asset portfolio contains various asset classes with various options and risk profiles.
- The intermediary is run on a going-concern basis—that is, new business is continually issued.
- Provisions for expenses, especially sales commissions, must be made.
- Shocks of capital markets are not as simple as one-time interest rate or spread change.

Next, we will consider some complications of this basic paradigm.