IV

The Perfume of the Premium. . . or Pricing Insurance Derivatives

by John Finn and Morton Lane

Q. "What is the right price for underwriting an insurance risk?"

A. "It is where the perfume of the premium overcomes the pong of the peril."

This old saw sums up what savvy traders and underwriters know instinctively about markets: that there is no "right" price for a piece of insurance. There is simply the transacted market price, which is high enough to bring forth sellers and low enough to induce buyers. Supply and demand determine price in all markets, including sophisticated risk-transfer markets such as futures, options, and insurance.

That said, any set of transacted insurance prices (premiums) contain within them an implicit assessment of the underlying risks (perils). Our objective in this paper is to make those *implicit* risk assessments *explicit.* We do this by examining the prices paid for catastrophic futures and options (CATs)* at the Chicago Board of Trade (CBOT) and by deriving their corresponding "Implied Loss Distributions."

This original approach allows us to (1) generate theoretical prices for different CAT covers, (2) establish cheapness and dearness among the alternatives, and (3) compare the prices paid for "CAT layers" with certain types of insurance "event" covers. To illustrate our approach, we have extracted and compared third quarter eastern implied loss distributions (ILDs) for 1993, 1994, and 1995 and examined the behavior of the 1994 ILD as that hurricane season progressed.

*The analysis contained in "The Perfume of the Premium" is based upon the CBOT's ISO contracts which have been replaced with contracts based on indices provided by Property Claims Services (PCS). Accordingly, Sedgwick Lane Financial has updated and extended this analysis in a paper titled "The Perfume of the Premium II."

As of yet, there is no uniform or generally accepted method of analyzing CATs. CBOT traders approach CATs as conventional financial options. Underwriters approach CATs as a particular form of catastrophic insurance. The two approaches need to be reconciled. We begin, then, with a brief discussion of insurance versus options.

Insurance versus Options

A buyer of insurance purchases the right to be reimbursed by the insurance writer for specified losses over and above the deductible or retention. For this right, he pays a premium.

A buyer of call options purchases the right to buy the underlying instrument from the seller (writer) to capture its increased value above the strike price. For this right, he pays a premium.

Clearly the two instruments are very similar. Indeed, some have referred to insurance contracts as options with fuzzy strikes and fuzzy pay-offs. Some of the fuzziness comes from a very important principle of insurance: indemnity. Insurance pays off only if the insured has an insurable interest *and* suffers a loss. Options holders, by contrast, do not need to have an "optionable" interest to buy the option and are paid off based on the value of the underlying instrument independently of losses or gains suffered elsewhere.

For a contract to qualify as insurance, the purchaser cannot experience a net gain. This is not true of CATs. CATs are not insurance: they are an insurance surrogate. Buyers of CAT options will experience a recovery that depends on the size of industry catastrophic losses (as determined by the Insurance Service Office [ISO]) whether they experience a loss themselves. Thus, while recoveries from insurance are insured specific, recoveries from CATs are industry specific. A conventional financial option, then, is like insurance without the principle of indemnity, and conversely, an insurance cover without the principle of indemnity is like a conventional financial option.

Pricing Insurance

Textbooks on insurance refer to pricing insurance, and more particularly reinsurance, by one of four basic methods: experience rating (rate-on-line, payback), comparable cover, Pareto, or benchmark theory. None is very precise, and nearly all are based on a rearview mirror approach which involves looking at an insured's past loss experience and assuming that it will continue into the future. A gross-up factor is included in the pricing structure, and it is assumed that this will produce profitable underwriting over time.

Catastrophic reinsurance is more difficult to price because of the low frequency and high severity of catastrophic losses. The history of these occurrences is sparse. Nevertheless, the above approaches are applied to loss records stretching back many years to gain a better sense of the insurer's risk and the appropriate pricing. For example, events which have occurred, or are likely to occur, every five years must be priced in excess of a 20% rate-on-line to be profitable.

Catastrophic losses are often assumed to conform to a particular statistical distribution, such as the Pareto, compound-Poisson, or gamma distribution. The shape of the distribution is then fitted to the historical record of catastrophic losses and remains relatively fixed into the future for pricing purposes. While this approach results in consistent pricing of premiums, it does not take into account changes in factors affecting the magnitude of catastrophic losses, changes in market perceptions about catastrophe frequencies, or changes in the supply and demand of risk capital.

Our approach reverses this process. It takes traded premiums and works backwards to derive the statistical distribution which best explains these prices. This process is familiar to options traders.

Pricing Options

In 1968, Fischer Black and Myron Scholes designed a closed-form model for pricing financial options. Their model figuratively rocked the financial world. Exchanges exclusively dedicated to trading options were set up in the U.S., so that now options are traded on a wide variety of underlying securities in the over-the-counter market and on nearly every futures and securities exchange throughout the world. These days, cheap, fast computing has made the closed-form model less important, but it still lies at the core of most options pricing analyses.

Volatility is the key ingredient for the Black-Scholes model; it measures how variable the option's underlying instrument will be over a specific period of time. In the early days of options trading, traders used past prices of the underlying instrument to calculate an "historical volatility" which they plugged into the Black-Scholes model to derive the option's theoretical price.

Naturally, not everybody views history the same way. The option price traded in the market often was and is different from the price derived from a particular historical analysis of prices.

As options markets evolved, participants developed a shorthand way of pricing options that relied on the "invertibility" of the Black-Scholes formula. Portfolio managers and traders could enter the market price of a particular option into the model which would then calculate the volatility "implied" by the market. This implied volatility could then be compared directly with historical volatility to gain a sense of the cheapness or dearness of the option.

Note that neither historical nor implied volatility is the correct volatility. The only correct volatility is that which actually transpires, namely "realized volatility."

A diagram illustrating this relationship is as follows:

Historical Volatility	Implied Volatility	Realized Volatility
Based on the past	Based on current market prices	To be determined in the future (will determine if option is profitable)

So it is with our analysis of insurance pricing.

Experience Rating	Implied Loss Distribution	Realized Value of Insurance
Based on past losses	Based on current market prices	To be determined by future loss

The missing ingredients for inferring the market's underlying assumption of catastrophic losses are price transparency and standardization. CAT contracts provide these ingredients, allowing us to derive the implied loss distribution (ILD) and thereby making implicit loss assumptions explicit.

Implied Loss Distributions

Conventional option pricing models assume that percentage changes in the price of the underlying instrument are normally distributed around the mean, which is the same as saying that the prices themselves are log-normally distributed around the current price. Based on this distribution assumption, the price of a call option is simply the discounted expected value of all outcomes above the strike price. For discrete price changes, the generalized formula is:

$$\text{options price} = PV \text{ of the sum of the } [\text{value}_i \times \text{probability}_i]$$

for all i above the strike price where the $value_i$ is the *ith* price minus the strike price, and the $probability_i$ is the appropriate log-normally distributed probability of that outcome.

For catastrophic losses, which are characterized by a low frequency and high severity, neither the normal nor the log-normal distribution is appropriate. Neither has a sufficiently long tail measure to take account of the small but significant probability of huge losses, such as those caused by Hurricane Andrew. Its insured losses were eleven times the annual Florida premium—hardly an outcome which can be ignored. Gamma distributions do have sufficiently long tails, provided their parameters are appropriately set. We make the assumption that aggregate CAT losses are distributed according to a gamma distribution.

Once we have fitted a gamma distribution to *current market* prices, we can examine the traded price of CAT options and combinations, such as spreads, butterflies, and condors and compare them with their theoretical values. As with conventional options, the theoretical price of CATs is the discounted expected value of the option assuming the various levels of loss are gamma distributed. The difference between the market price and theoretical price is a measure of cheapness or dearness.

If the market is efficient and if the assumption of a gamma distribution is reasonable, the sum of these cheapness and dearness measures will be minimal.

Lane Financial uses a proprietary algorithm to search the family of gamma distributions and select the one which best explains all transacted prices and all bids and offers currently outstanding. That distribution is referred to as the ILD.

Like implied volatility for conventional options, ILDs are not based on history or extrapolated expectations. An ILD is simply the distribution which best explains current market prices. It does not look at all possible distributions or nonstandard distributions (although it could). It confines itself to gamma distributions in the same way that Black-Scholes implied volatility confines itself to log-normal distributions with a mean of zero.

ILDs for 1993, 1994, and 1995

Given the methodology described above, Lane Financial has derived ILDs for third quarter eastern CATs (where most of the trading has taken place) using midseason 1993 prices, early season 1994 prices, and the current preseason 1995 prices. These distributions are shown in Figure 1. There are significant differences between the years, which can best be seen by comparing the implied probability of loss at a high level of attachment with the implied probability of loss at a lower attachment.

1993 was the first and "thinnest" year of CAT trading. Spread trading (simultaneously buying and selling options) did not begin until August 1993, midway through the hurricane season. The prices that did trade, however, implied that the perceived probability of high aggregate ISO losses occurring was low relative to the perceived probability of lower levels of aggregate losses. In 1994, in contrast, the market perceived and/or feared that large events were more probable (or what is the same thing, demanded and paid very high prices for covers with high attachment points) relative to lower down events. Preseason 1995 prices (trading as of February 1995) have struck a balance between the preceding years.

One way to see these interyear differences is to compare the theoretical value of a lower struck CAT call with that of a higher struck CAT call. Since CAT futures and options are capped at a 200% loss ratio, we can standardize the comparisons using rates-on-line, which is the ratio of premium to exposure. Table 1 compares the theoretical ROLs of the 50 and 150 calls for each of the three years traded.

Figure 1
Implied Loss Distributions (Third Quarter Eastern)

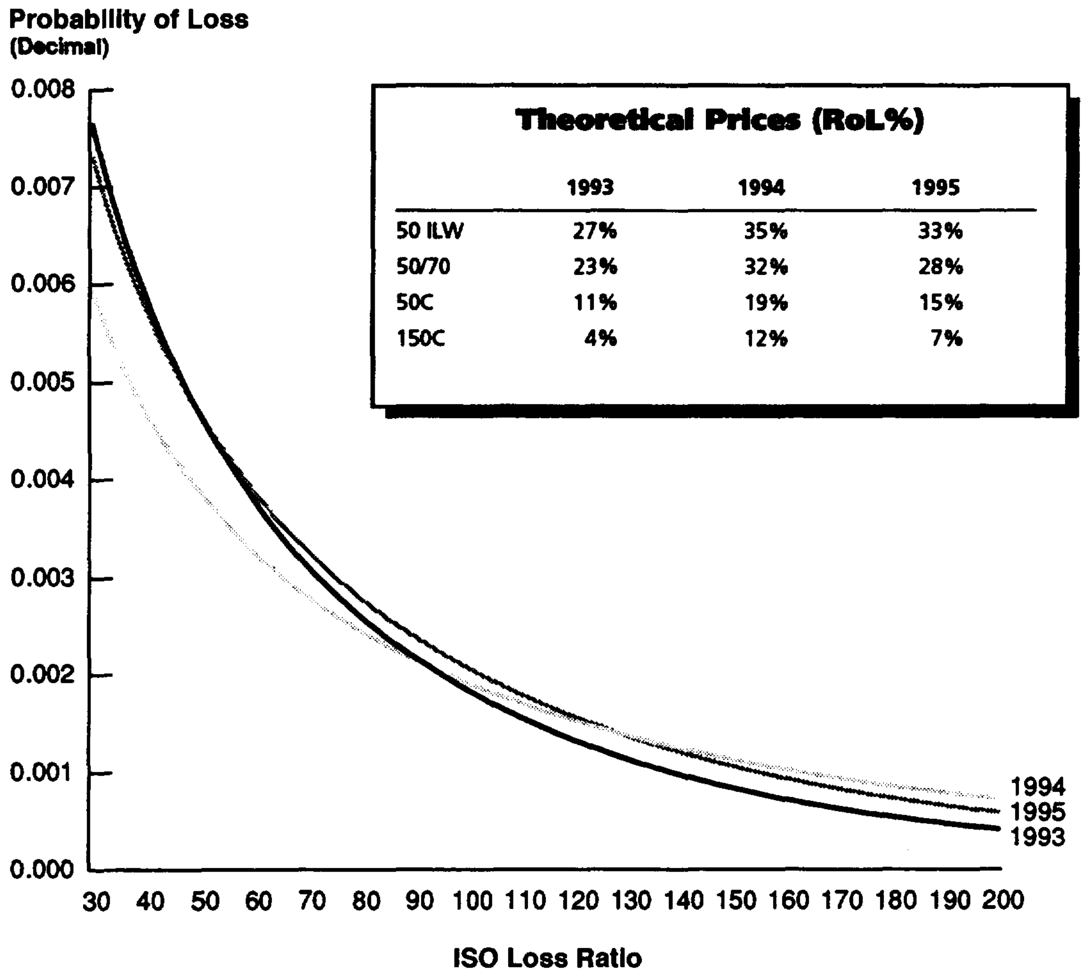


Table 1
Theoretical Prices (ROL%)

	1993	1994	1995
50 Call	11%	19%	15%
150 Call	4	12	7

Now the difference in the relative market prices becomes apparent. Prices have changed dramatically during the past few years as supply and demand in the reinsurance market and at the CBOT have changed.

Franchise Covers versus Layers

In Table 1, a buyer would only receive full recovery from his or her CAT cover if third quarter eastern aggregate loss ratios equalled or exceeded 200%. One can think of CAT covers as proportionate covers, but more specifically, they are proportionate by industry loss, not insured loss. For example, in 1995, $1 of full cover attaching at a 50% ISO loss ratio and achieving full recovery at 200% ISO would cost 15¢.

Most trading in CAT covers has actually occurred in the 50/70 call spread. In this spread, recovery attaches at 50% ISO and is fully received above 70% ISO. Naturally, since this type of cover pays out in full sooner, it is more expensive. In 1995, the 50/70's theoretical rate-on-line is approximately 29%.

Press clippings over the past two years reveal a great deal of debate about whether the price for this layer is too expensive (and therefore should not be bought) or too cheap (and therefore should not be sold). Some of these comments are self-serving excuses for inaction. Beyond this, however, comparing the 50/70 directly with the underlying cash market is difficult. What exactly is being compared? Traditional catastrophic reinsurance programs are reinsured, not industry, specific. Industry loss warranties (a.k.a. franchise covers or original market loss warranties) usually pay out in full once the industry trigger is reached, not proportionately over a layer of industry events. Also, most ILWs are not covers for aggregate losses; they are single-event covers with reinstatement provisions. Finally, not all policies cover the same causes of loss or reporting times as the CBOT contract.

Lane Financial's theoretical model cannot resolve all these differences but can make a comparison between covers which pay out fully at a trigger level and those which pay out proportionately over a layer. For example, assume an ILW is based on a 50% ISO trigger, covers third quarter east coast (ISO) aggregate losses, and has no reinstatement provisions. The theoretical price of this warranty cover is shown in comparison with the 50/70 layer in Table 2.

TABLE 2
THEORETICAL PRICES (ROL%)

	1993	1994	1995
50% ILW	27%	35%	33%
50/70 Layer	23	32	28

What the table shows is that instant-pay warrants should be priced some 20% or more higher than the equivalent-attachment 20-point CAT layers. (It is implicitly assumed that with the ILW the reinsured will always have losses sufficient to make the claim. To the extent that this is not true, the ILW price should be discounted).

A buyer of the above 1995 50% ILW at a 35% ROL should be willing to pay up to 29% for the 50/70 CAT layer. Similarly, sellers prepared to sell the ILW should be indifferent between selling it at 36% and selling the 50/70 layer at 29%. In 1994, the seller would have been indifferent between a 50% ILW at 40% ROL and the 50/70 layer at 32% ROL.

With the above discussion, we have started an explanation of the prices at which different covers should trade. In a future paper, we will look at the implied cost of reinstatements versus aggregate cover.

The Effect of Time

The passage of time is an important factor in trading conventional financial options and can best be seen when the price of the underlying remains steady. These option prices do not decline linearly; they actually "decay" more rapidly as expiration approaches. What about insurance? What is the value of an insurance policy during its last few months if there have been no claims? Does its value decline linearly?

Third quarter eastern CAT trading over the 1994 hurricane season has provided the first insight into the effect of the passage of time on the value of insurance options as shown in Table 3. Because the 1994 season was the quietest season since 1925, it is an ideal period to gauge the effect of the passage of time. Figure 2 shows how the market assessed the risk (in probability terms) for the final three months, two months, and two weeks of the loss period. Corresponding points from the ILDs reveal that, as with financial options, prices seem to decline at an accelerating rate.

TABLE 3
THEORETICAL LEVELS
(ROLs)

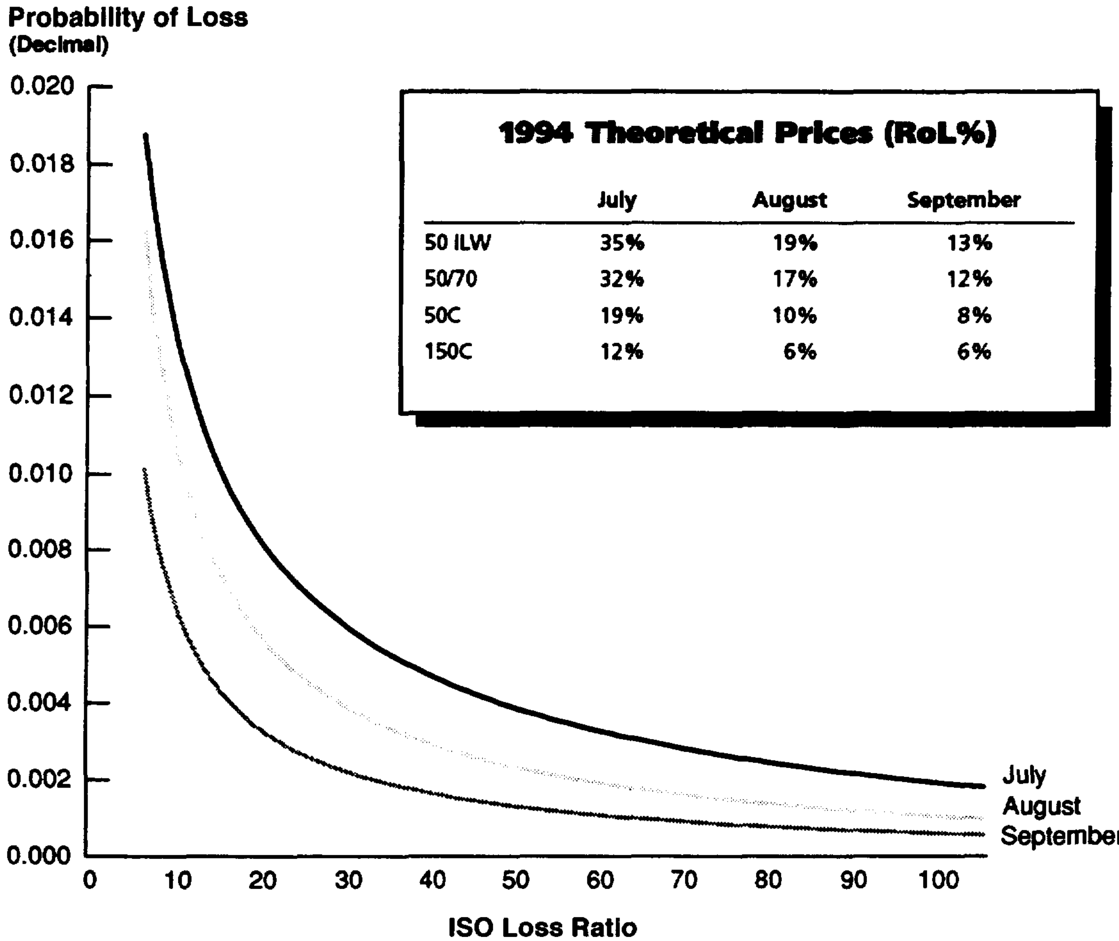
	July	Aug	Sept
50/70:	32%	17%	12%

In active years, end-of season prices are likely to be highly volatile just like financial options.

Cheapness and Dearness

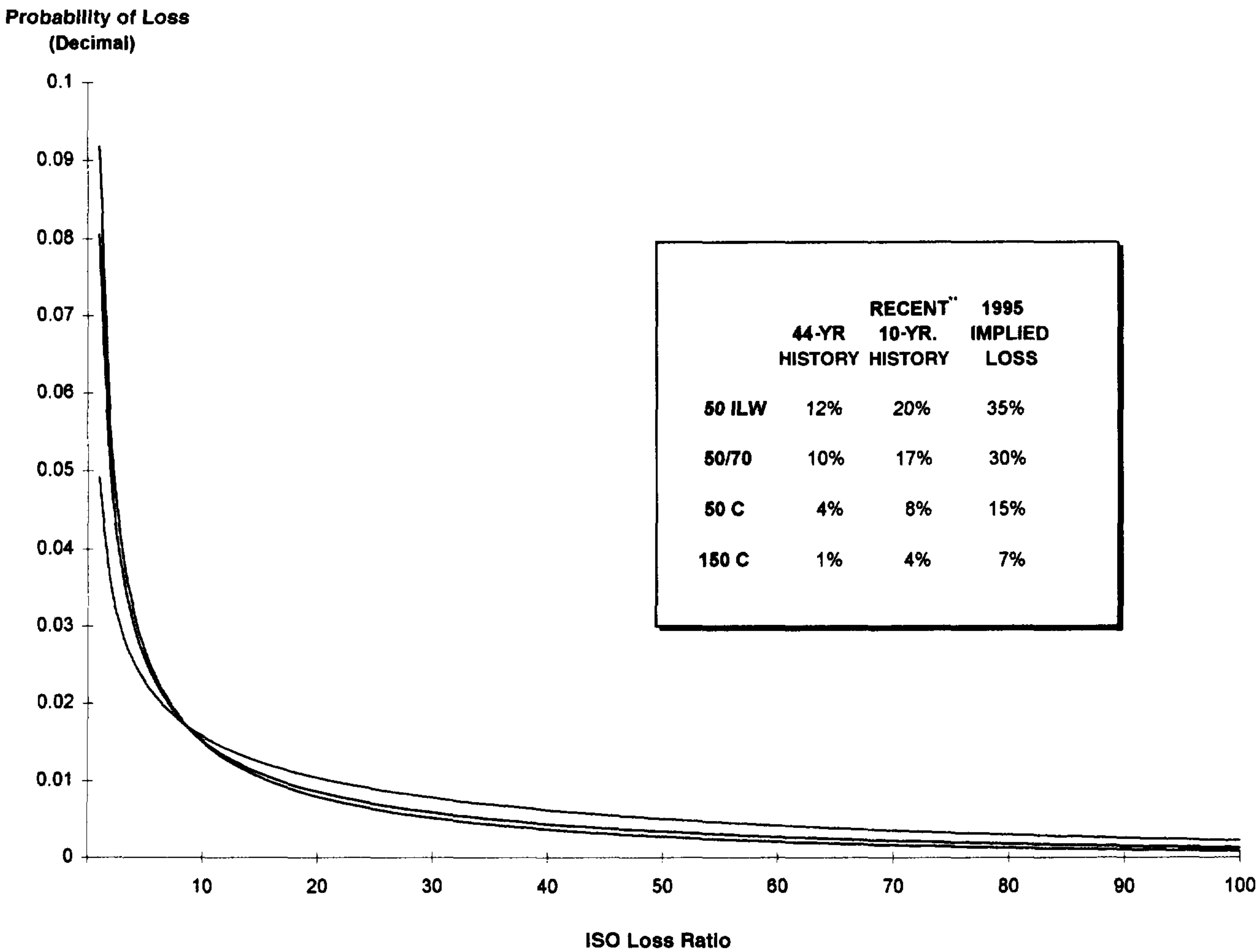
This paper has mainly dealt with using implied loss distributions derived from existing market prices to value insurance derivatives. As research on these instruments (and their new cousins—PCS, CATs and crop insurance contracts) develops, more theory and analyses will occur. For the present, however, a theoretical model produces a most important framework within which to trade. Certainly Lane Financial has made successful use of these models for its own

FIGURE 2
IMPLIED LOSS DISTRIBUTIONS (1994 THIRD QUARTER EASTERN)



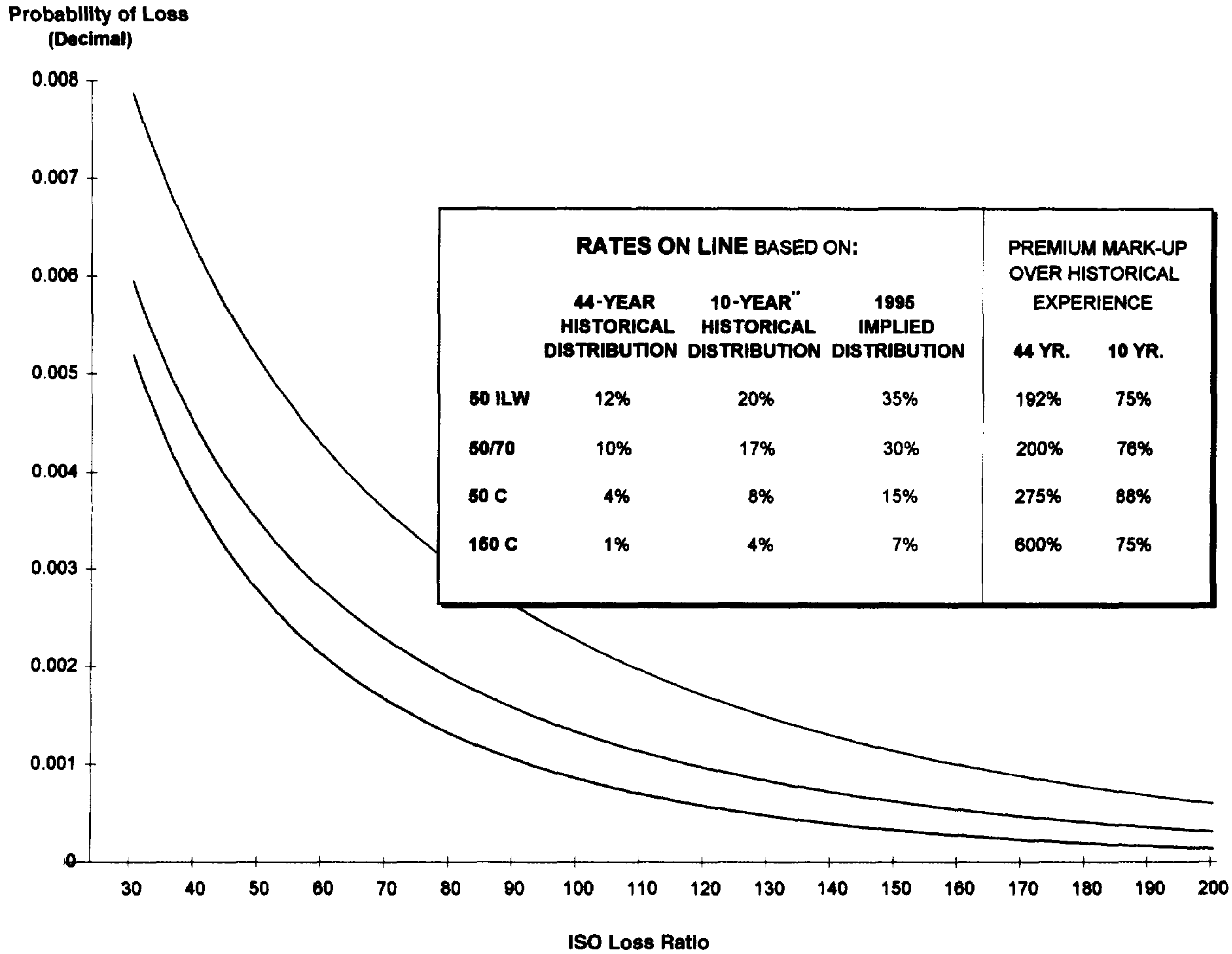
	July	August	September
50 ILW	35%	19%	13%
50/70	32%	17%	12%
50C	19%	10%	8%
150C	12%	6%	6%

FIGURE 3
HISTORICAL LOSS EXPERIENCE VERSUS IMPLIED LOSS DISTRIBUTION EARLY MAY, 1995



	44-YR HISTORY	RECENT** 10-YR. HISTORY	1995 IMPLIED LOSS
50 ILW	12%	20%	35%
50/70	10%	17%	30%
50 C	4%	8%	15%
150 C	1%	4%	7%

Figure 4
Historical Loss Experience versus 1995 Implied Loss Distribution Early May, 1995



	RATES ON LINE BASED ON:			PREMIUM MARK-UP OVER HISTORICAL EXPERIENCE	
	44-YEAR HISTORICAL DISTRIBUTION	10-YEAR** HISTORICAL DISTRIBUTION	1995 IMPLIED DISTRIBUTION	44 YR.	10 YR.
50 ILW	12%	20%	35%	192%	75%
50/70	10%	17%	30%	200%	76%
50 C	4%	8%	15%	275%	88%
150 C	1%	4%	7%	600%	75%

*Based on PCS data prior to 1989 and ISO data since 1989, back-trended at 10% per year by the CBOT. **1983 through 1992 proprietary trading and market making. Using Table 4 as an illustration, consider these prices and their theoretical values as of February 15, 1995:

TABLE 4
ACTUAL VERSUS THEORETICAL PRICES
EASTERN CATASTROPHE CONTRACTS
THIRD QUARTER 1995

Instrument	Bid/Ask	Theoretical Price
Future		47.7
Calls		
100	8.5/12.0	10.0
150	4.0/ 5.5	3.6
190	0.2/ 0.9	0.6
Call Spreads		
35/55	6.2/ 7.5	7.1
45/65	6.0/ 6.5	6.1
50/70	5.3/ 5.5	5.7
60/80	4.0/ 5.5	5.0
100/120		3.0
140/160	1.5/ 2.0	1.9
160/180		1.5
Butterflies		
20/55/90	7.0/12.5	5.5
50/100/150	/ 6.5	5.5
Condors		
40/60 100/120	1.5/	3.5
50/60 70/80	0.4/	0.7

Clearly, the theoretical model shows a number of interesting things. First, it indicates that the futures theoretical price is 46.8 (i.e., an ROL of 23.4%).

Second, of the outright calls, the 150 call seems particularly overbid at 4.0 (ROL of 8%) versus its theoretical price of 3.5. It would make a good sale.

Third, of the call spreads, the 45/65 is well bid (with its bid close to its theoretical prices), and the 50/70 is well offered at 5.5 (it is two ticks below its theoretical price). The 140/160 also provides a good offer at 2.0.

Fourth, the 20/55/90 butterfly was at one time bid at 7.0. According to the model, it should have sold because its theoretical value was only 5.1.

In the exotica corner, of the two condors where bids have been shown, the 50/60 and 70/80 are reasonably priced to theory, whereas the 40/60 and 100/120 are cheaply bid.

Finally, we have shown two prices for the 100/120 and 160/180 where no market has presently been shown. The well-prepared market-maker (who accepted the analysis proposed here) would be prepared to bid or offer around 3.1 and 1.5, respectively.

Concluding Remarks

In this paper we have discussed the differences and similarities between insurance and options pricing, derived implied loss distributions from market prices transacted at the CBOT, and used these ILDs to contribute to the debate on whether traded contracts (surrogate insurance) are cheaper or more expensive than real insurance. We have also shown how the ILDs can be used to identify cheap and expensive alternatives. Although this paper has covered a lot of ground, it has barely scratched the surface of the possibilities that lie ahead.