Pension Funds: A Capital Adequacy Test With A Soft Mismatch Cushion

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Presented at The Great Controversy: Current Pension Actuarial Practice in Light of Financial Economics Symposium
Sponsored by the Society of Actuaries

Vancouver

June 2003

(note: The quotations appearing in this monograph are exact, except where capitalization and punctuation were changed in keeping with modern style and grammar guidelines.)

Abstract

Suppose that, after a downturn in the financial markets, pension funds have funding ratios in the neighborhood of 100 percent, all cushion reserves having been more or less exhausted. Further suppose that one cannot reasonably count on any substantial amount of additional funding through increased contributions, at least in the short to medium term. If financial market conditions improve, what level should the funding ratios reach before any assets can be released, at least from a solvency perspective? For a typical pension fund in the Netherlands, the answer appears to be around 160 percent, at least on the basis of information available at the end of 2002. The minimum solvency cushion proposed in this article fluctuates as driven by market conditions (primarily equity prices, the term structures of nominal and real interest rates, inflation) between a floor of zero and an upper boundary. This is in contrast to methods based on minimum risk-based capital ratios and value-at-risk methods commonly used in the banking industry.

1. Introduction: Conceptual Framework

This article proposes a method for determining a soft minimum solvency cushion for a funded defined benefit (DB) pension plan. It builds on several previously developed ideas. Euverman (1997) introduced a solvency cushion for an equity portfolio on the basis of the history of an equity index. It was designed to have the property of sympathetic motion in relation to the ups and downs of the market. This concept is generalized here to the funding index and developed further. The focus on a liability-adjusted return rate derived from the funding ratio is natural and not a new, of course; for example, see Leibowitz, Bader, and Kogelman (1996). In 2002, Watson Wyatt Brans & Co. introduced the methodology described here to its clients; a detailed technical description was published in a brochure authored by van Gaalen (2003). Speed et al. (2003) independently published a note on the relationship between pension assets and liabilities, in which they develop a "liability benchmark portfolio" similar to the liability index used here. The general structure of the capital adequacy test was adapted from PVK (2001). The wisdom of saving during "seven years of plenty" to prepare for "seven years of famine"—seven presumably being a random variable—has been well understood since Joseph interpreted Pharaoh's dream; see Genesis 41 in The Old Testament (1996).

1.1 The Pension Promise

1.1.1 Type of Pension Plan

Throughout this article, the pension promise is understood to be a DB plan offered by an employer to its employees. In such a plan, the benefits are related to salary and years of service, typically payable in the form of life annuities of various types. The plan is assumed to be funded by means of a pension fund.

1.1.2 Governing Documents

The pension plan amounts to a contract, ideally a complete one. Given the legal environment, including government supervision and enforcement, this contract is laid down in a set of official documents such as those listed below, reflecting standard practice in the Netherlands:

- *Employment contract.* The pension promise is an integral part of the terms of employment.

- *Plan rules.* These should include all plan provisions, such as rules for eligibility; formulae for all types of benefits provided (including accrued, attainable and contingent benefits), definitions of credited service and covered salary, the standard retirement age, rules for early and deferred retirement, vesting rules, pension indexation and other benefit adjustments after termination of service, rules for benefit commutations and the participants' own contributions.

- *Trust deed.* This document contains the fund's bylaws or articles of association. It must include procedures for dealing with a deficit, such as an across-the-board benefit reduction under certain circumstances, and rules applicable in case of a termination of the plan or the fund, covering aspects such as ownership of any surplus assets in that situation.

- *Funding covenant.* This contract between the employer and the pension fund describes the funding method and the contribution formula in detail. The contribution formula determines the amounts to be contributed annually to the pension fund by the employer and the employees. Contributions may be positive or negative.

- *Technical memorandum.* This document describes the fund's investment strategy as well as its official set of actuarial methods and assumptions, to be used for implementing the contribution formula, calculating benefit commutations based on actuarial equivalence and the like. These official methods and assumptions need not the same as those used for the capital adequacy test discussed here.

1.2 Legal Aspects

This section reflects some important legal aspects, taken from Dutch pension law.

Status of the pension fund

The pension fund must be a legal entity separate from the employer, governed by a board of trustees of which at least 50 percent are employee representatives. All trustees must protect the interests of all parties concerned in a fair and equitable manner.

Basic funding requirement

All accrued pensions must be fully funded, independently of the employer's future business risk. This basic rule applies at all times and covers the benefits of all participants—active employees, retirees and other pensioners with benefits in payment as well as former employees with deferred vested rights. Fully funded pensions are not necessarily guaranteed pensions; see Section 1.5.5.

Implementation, enforcement and supervisory authority

All documents listed in Section 1.1, which together define the pension promise, require government approval. Moreover, the government implements and enforces the basic funding requirement by means of a capital adequacy test. Implementation and enforcement are delegated to a supervisory authority, a government agency.

1.3 Terminology and Notation

Assets

Assets are investments that have been segregated and restricted to secure and provide benefits under the plan. The assets are owned by the pension fund, although negative contributions may be possible under the funding covenant. Debts to third parties are treated as negative assets. The value of the assets at time t is denoted by $A(t)$.

Liabilities

The liabilities of the plan are the accrued benefits, including pensions already in payment, deferred vested pensions and all pensions that have been accrued by the active participants on the basis of their service up to the valuation date. The value of the liabilities at time t is denoted by $L(t)$. Neither the fund's nor the employer's credit risk is taken into account in the determination of this amount (also see Section 1.5.5).

Funding ratio

The funding ratio at time t, denoted by $FR(t)$, is defined as the ratio of the assets to the liabilities at that time; that is, $FR(t) = A(t) / L(t)$.

Surplus ratio

The surplus ratio at time t, expressed as a proportion of the liabilities and denoted by $SR(t)$, is defined as the difference between the funding ratio and 100 percent; in symbols: $SR(t) = FR(t) - 100$ percent. A deficit is interpreted as a negative surplus. Some or all of the surplus may be restricted due to the capital adequacy test. Moreover, the surplus, being a part of the plan assets, is owned by the pension fund.

Minimum funding ratio, restricted assets and nonrestricted assets

Suppose $MFR(t)$ denotes a given minimum funding ratio at time t, expressed as a proportion of the liabilities. Then the restricted assets are

$$RA(t) = L(t) \times \min \{ FR(t), MFR(t) \},$$

and the nonrestricted assets are

$$NRA(t) = \max \{ A(t) - RA(t), 0 \}.$$

1.4.1 Capital Adequacy Test

The general structure of the capital adequacy test as described below is based on the "Principles for a Financial Assessment Framework" published by the Pensions & Insurance Supervisory Authority of the Netherlands; see PVK (2001). The definitions of the three subtests have been modified somewhat to suit the purposes of this article. The assumed consequences of failing to pass the capital adequacy test, discussed in Section 1.4.2, are more or less in accordance with the Dutch system.

1.4.1 Broad Outline

A capital adequacy test must be performed at the end of each year. It consists of three components: a minimum test, which covers immediate plan termination; a solvency test, which is a minimum test with a horizon which is strictly speaking one-year but in effect multiyear; and a continuity test, which is an assessment of the fund's long-term prospects.

1.4.1.1 Minimum Test

The minimum test is simply a snapshot intended to ensure that the participants' accrued benefits are covered by sufficient assets in the hypothetical case of immediate discontinuance. Accordingly, its scope is limited, as follows:

1. Any future measures to be taken in case of financial emergency are excluded from consideration.

2. All future contributions and all future pension accruals are excluded from consideration.

3. A full settlement of all liabilities cannot be deferred and will take place on the valuation date.

The first restriction prevents any advance recognition from being given to the option of somehow reducing the liabilities or increasing the assets in case of distress. In view of the second restriction, the minimum test is based on a closed-group valuation, taking into account the assets accumulated in the fund and the liabilities accrued under the plan up to the valuation date. The third restriction implies that the mismatch issue (see Section 1.5.6) is irrelevant here.

Implementation

At the valuation date t, the liabilities must be fully covered by the assets:

$$FR(t) \geq 100\%.$$

1.4.1.2 Solvency Test

The solvency test is somewhat broader in scope than the minimum test:

1. any future measures to be taken in case of financial emergency are excluded from consideration;

2. all future contributions and all future pension accruals are excluded from consideration; and

3. a full settlement of all liabilities may have to take place at any time during the year immediately following the valuation date; if this does not happen, then the fund's financial position should be sufficiently strong so that there are good prospects that the capital adequacy test and, in particular, the solvency test will be passed again as it is repeated one year later.

In comparison with the minimum test, the first two restrictions are identical. Consequently, as in the case of the minimum test, a closed-group valuation is used on the basis of the assets and liabilities at the valuation date, but now it cannot be assumed that there will be an immediate settlement. Instead, there is a one-year horizon, in accordance with PVK (2001). Moreover, the legal requirement that all accrued pensions must always be fully funded independently of the employer's future business risk implies that the solvency test at the end of the upcoming year must be anticipated. Thus, the third requirement has been strengthened accordingly, and one could argue that the horizon of the solvency test is actually multiyear, comprising the upcoming year plus the horizon of the next year's solvency test.

In the special case in which the assets perfectly match the liabilities in all respects, all sources of financial and actuarial risk being taken into account, there is no mismatch risk; otherwise, a mismatch or solvency cushion will be required to pass the solvency test (also see Section 1.5.6).

If the pension fund is comparable to a futures market in which all positions are constantly marked to market, the assets may be viewed as collateral for securing the liabilities. However, it is important to understand that the solvency test is to be performed under the assumption that margin calls to obtain additional collateral are not allowed.

Implementation

At the valuation date t, the minimum test must be satisfied and there must be an adequate nonnegative solvency cushion in the fund:

$$FR(t) \geq MFR(t) = 100\% + \kappa(t),$$

where $MFR(t)$ and $\kappa(t)$ denote the minimum funding ratio and minimum solvency cushion, respectively, both as a proportion of the liabilities.

1.4.1.3 Continuity Test

Restrictions defining the scope of the continuity test are:

1. any future measures to be taken in case of financial emergency are excluded from consideration;

2. all future contributions and all future pension accruals, both determined in accordance with the pension plan, are taken into account in an open-group model; and

3. a full settlement of the liabilities may be required at any time in the future.

Accordingly, the following aspects, which are not critically examined in the first two subtests, all fall within the scope of the continuity test:

- Funding system: contribution formula, funding targets.

- Investment strategy: asset allocation, asset/liability management, risk insurance and derivatives policy.

- Long-term prospects in open-group model.

Implementation

The implementation of the long-term continuity test is not explored in this article; instead, it is assumed that a suitable funding system is in place, that an appropriate investment strategy has been adopted and that the fund's long-term prospects are satisfactory, at least as long as its minimum and solvency positions are examined from year to year.

1.4.2 What Happens When a Fund Fails to Pass the Test

Any capital adequacy test would be pointless if a failure to satisfy its requirements would have no serious consequences. Thus, some assumptions are necessary.

General information requirement

In any case, the fund is required to submit a report at the end of each year to the supervisory authority, the participants of the plan and the employer. This includes the results of applying the capital adequacy test. Consequently, all parties directly concerned will be kept informed from year to year.

Failure to pass the continuity test

If a fund fails to pass the continuity test, then structural improvements are called for, such as a change in the contribution formula, the asset allocation or the provisions of the plan, such as the indexation clause. Alternatively, a substantial amount of additional funding may be required. None of this will be explored here, as the continuity test is beyond the scope of this article.

Failure to pass the minimum test and the solvency test

If a plan fails to pass both the minimum test and the solvency test, then an emergency must be declared and special measures must be taken. The emergency plan must be drawn up in consultation with the supervisory authority, which will monitor the plan closely. It should normally include one or more of the following measures:

- *Capital restoration plan.* The plan sponsor agrees to deposit additional contributions within an agreed time schedule. A portion of these contributions may be chargeable to the participants.

- *Change in asset allocation.* It may be possible to reduce the minimum solvency cushion and hence the minimum funding ratio by modifying the asset allocation.

- *Across-the-board reduction of benefits.* The funding ratio may be increased immediately to any desired level by an across-the-board benefit reduction.

- *Transfer entire plan to insurance company.* In this case, a full settlement takes place, which may require additional cash or an across-the-board benefit reduction.

As indicated in Table 1, each of these measures tends to have advantages as well as disadvantages.

TABLE 1
Comparison of Emergency Measures

	Capital Restoration Plan	Change in Asset Allocation	Across-the-Board Reduction Of Benefits	Transfer to Insurance Company
Advantages	+ Consistent with the spirit of a funded DB plan + Immediate or gradual improvement of funding ratio	+ Quick and easy, as long as financial markets are functioning smoothly	+ Immediate improvement of funding ratio	+ Mismatch risk (see Section 1.5.6) disappears + Any surplus may be recovered to some extent
Disadvantages	- May be costly - Negative influence on employer's own solvency position and cost of capital	- May be inconsistent with the desired asset allocation - Timing may be suboptimal; dynamic asset allocation may be risky - No improvement of current funding ratio	- Inconsistent with the spirit of a funded DB plan	- An insurance company may not be the optimal funding vehicle - Loss of control - May require additional funding or an across-the-board benefit reduction

1.5 Some Valuation Principles and Risk Management Concepts

This section outlines some general principles and concepts underlying the valuations of the assets and liabilities of funded pension plans as well as the assessment of their capital requirements.

1.5.1 Logical Consistency

It is taken for granted that the valuation methods used with respect to the plan's assets and liabilities must be logically consistent. This is necessary to avoid hidden reserves and implicit, if not opaque, smoothing methods. Without such a basis, it is not clear how any systematic, transparent analysis could proceed.

1.5.2 Consistency With Market Prices

Moreover, assets and liabilities must be marked to market whenever possible. While the inclusion of this requirement is not necessarily compelling in a general setting, it appears natural against the background of a competitive market economy.

1.5.3 Fair Value

To the extent that the set of prices implied by the previous consistency requirements is incomplete, it is assumed that it has been extended to form a complete pricing system. The resulting prices are called fair values. In this article, no distinction is made between "fair value" and "(current) market value".

1.5.4 The Role of Smoothing

While implicit smoothing is ruled out by the use of current market values, nothing prevents explicit smoothing mechanisms from being incorporated into, for example, the plan's contribution formula or the formulation of the capital adequacy test.

1.5.5 The Distinction Between *Ex Ante* and *Ex Post*

Various embedded options may be exercised if certain events occur. By means of the default option, the employer's liabilities may be reduced, including any contributions owed to the pension fund. If the fund itself experiences a deficit—for example, due to unfavorable developments in the financial markets—then its liabilities may have to be reduced in an emergency plan. Accordingly, the full-funding requirement means that contributions must be paid in accordance with an approved funding covenant as pensions are accrued, and that the capital adequacy test must be passed. It does not mean that payment of the pensions must be 100 percent guaranteed. Throughout this article, it is understood that such flexibility, while obviously relevant after the fact or *ex post*, cannot be taken into account beforehand or *ex ante* to reduce the liabilities.

1.5.6 Risk Management

Mismatch risk

If all outgoing future cash flows constituting the plan's liabilities are exactly and with complete certainty matched by future incoming cash flows generated by its assets, then there is no mismatch risk. If such a match cannot be realized, then a shortfall may occur in the future. Apart from any current shortfall, this is called mismatch risk.

Mismatch or minimum solvency cushion

If the assets are equal to the liabilities, so that $FR(t) = 100\%$ and there is no mismatch risk, then there is a perfect match. Otherwise, given the asset allocation, the mismatch risk depends on the surplus ratio $SR(t)$. With some additional notation it would be easy to show rigorously that the surplus has a negative influence on the mismatch risk, other things equal. Accordingly, the surplus will considered as a mismatch risk cushion.

2. The Problem With Risk-Based Capital and Value at Risk

For a commercial bank, solvency is obviously a matter of life and death; rumors about an impending insolvency could easily alarm the market and cause a panic and a disastrous run on the bank. Moreover, one bank failure might lead to another, cause severe losses to the taxpayers and even disrupt the money market. Consequently, it stands to reason that a bank is required to maintain adequate risk-based capital ratios as well as continuously monitor its value-at-risk measures, and quickly take corrective action if those ratios fall to unacceptably low levels or if the probability of entering the danger zone exceeds a low threshold value. When that happens, a speedy adjustment of the bank's long and short positions is necessary, and a recapitalization may be triggered, for example, to obtain more equity capital.

A pension fund needs to protect its solvency position, too, but for rather different reasons: not so much to prevent a panic, let alone a run on the pension fund, as to safeguard the interests of the plan participants. This is why the law—at least, under the assumptions made here—requires a sufficient amount of assets to be deposited as collateral in a pension fund that is legally separate from the employer.

But the cost of giving pension plan participants full protection from the ups and downs of the market may well be prohibitive and interfere with the pension fund's special functions. After all, it can be argued, for example, on macroeconomic grounds, that converting assets—ultimately not simply financial securities, but tangible capital investments in a variety of real-world projects—into the cash flows that constitute pension annuities is the main task of pension funds.

From such a macroeconomic capital allocation perspective, it appears that this task cannot be carried out if pension funds are barred from having a substantial mismatch between their assets and liabilities. After all, they can neither easily renegotiate their net liability positions nor quickly obtain additional capital, except perhaps if sponsored by comparatively large employers. Consequently, a case can be made that the solvency protection regime applicable to pension funds must not be so strict as to interfere with their particular role in the economy by forcing them to keep their exposure to mismatch risk low.

However, rather than attempting to prove the general validity of the argument that pension funds should be allowed to incur substantial mismatch risk, this article takes for granted that solvency protection systems designed for commercial banks and based on risk-based capital ratios and value-at-risk measures, while providing potentially useful information, are not appropriate for pension funds, at least as far as the determination of their minimum solvency requirements is concerned.

The volatility $\sigma(t)$ of the funding index $\phi(t)$ defined in chapter 4 can be used in simple value-at-risk calculations. For example, the minimum funding ratio could be set equal to

$$100\% + k \times \sigma(t),$$

k being a suitable constant.

3. Motivation of an Alternative Solution

Under the restrictions defining the scope of the solvency test, the surplus ratio

$$SR(t) = FR(t) - 100\% = \{ A(t) - L(t) \} / L(t)$$

moves up or down depending on market variables such as the returns on the assets involved and the term structures of nominal and real interest rates, apart from inflation and actuarial factors such as mortality. All those variables and factors may be treated as exogenous. Moreover, neither additional contributions nor any future emergency measures may be taken into account. So what would be the point of calculating risk-based capital ratios and monitoring value at risk, apart from information that is up to date?

There is an alternative: rather than determining the minimum solvency margin as a scalar amount separately at each valuation date, for example, using risk-based capital ratios or value-at-risk measures, it could be made subject to the restriction that the implied solvency ratio $FR(t) / MFR(t)$ be fairly stable.

In terms of a minimum funding ratio $MFR(t)$, the definition of the solvency test and its effectively multiyear horizon suggest that, at least for a suitably long period starting at the valuation date t_1—that is, for t between t_1 and a sufficiently distant future time t_2—the following equation should hold:

$$MFR(t) = a \times FR(t) = a \times \{ A(t) / L(t) \},$$

where a is a constant between 0 and 1, and where the assets $A(t)$ and $L(t)$ evolve without any additional contributions and pension accruals. After all, if $a = 1$, then the solvency ratio $FR(t) / MFR(t)$ remains just adequate at 100 percent, and if $a < 1$, then a constant solvency cushion is maintained as a percentage of the liabilities. Note that the function $MFR(t)$ should be independent of a, which merely expresses the ratio of this function to the funding ratio of a particular fund.

On the one hand, however, the minimum test implies that

$$MFR(t) \geq 100\%,$$

so the previous equation inevitably breaks down as soon as $FR(t) < 1 / a$.

On the other hand, assume it is desired to have a suitable upper boundary $b(t)$ that the minimum funding ratio should not be permitted to exceed.

These two restrictions suggest that the functional form should be more like

$$MFR(t) = \max \{ 100\%, \min [a \times FR(t), b(t)] \}.$$

Intuitively speaking: since the assumptions of the solvency test rule out margin calls except in case of emergency, the fund's solvency position should remain essentially unchanged in the sense that $FR(t)$ remains greater than $MFR(t)$ as long as $FR(t)$ remains between 100 percent and $b(t)$.

Now suppose $FR(t_1) = 100\%$ and market conditions are improving, that is to say, $FR(t)$ is increasing for t in some interval starting at t_1. For one thing, this obviously implies that $FR(t) \geq 100\%$, so additional contributions cannot be obtained. Hence the only practical solution would be to let $MFR(t)$ be equal to $FR(t)$, at least until the upper boundary $b(t)$ is reached.

How should $b(t)$ be determined? One possibility is setting $b(t) = b_1(t) \times b_2(t)$ where $b_1(t)$ is practically constant over time and $b_2(t)$ incorporates some measure of current market volatility. In this article, this refinement is ignored, and it is assumed that $b(t)$ should change slowly.

The key observation is this: if assets are released as soon as $FR(t)$ passes $b(t)$, then it seems reasonable to require that $b(t)$ be such that the likelihood of reaching the floor of 100 percent again is sufficiently small. On the other hand, given that requirement, $b(t)$ should arguably be chosen as low as possible to prevent the amount of restricted assets from rising to excessive levels, in view of the cost of capital.

Furthermore, it also seems sensible to require that additional assets be released as long as $FR(t)$ keeps increasing beyond $b(t)$. This implies that, as soon as $FR(t)$ begins to decrease from a level higher than $b(t)$, no further assets are released, but any assets that have already been released up to that point cannot be restricted again—after all, margin calls are against the rules except in cases of emergency.

All this motivates the construction discussed in the next section.

4. Solvency Testing With a Soft Mismatch Cushion

This section illustrates a method that can be used for determining a minimum reserve to be used in solvency testing at pension funds. The main idea is that this reserve should function as a cushion that expands (or inflates) up to a certain maximum as financial market conditions improve, but contracts (or deflates) during market declines. It was inspired by, and is a generalization of, a method introduced by Euverman (1997) for calculating a separate solvency cushion for the equity portfolio held by a pension fund, which was intended to be combined with solvency reserves for other risk factors.

4.1 Definition of Asset Index

Given an asset allocation $AA = (\pi_1, \ldots, \pi_k)$, where the proportions π_i satisfy the conditions $0 \leq \pi_i \leq 1$ and $\sum\pi_i = 1$, and given reinvestment indices $I_i(t)$ at time $t \geq t_0$ for the asset types $i = 1, \ldots, k$, the asset index $\alpha(t)$ is defined as the combined reinvestment index reflecting the history of the total return on a hypothetical portfolio invested in those indices in accordance with *AA*, with frequent rebalancing. It is assumed that there are no cash flows such as contributions and benefit payments.

4.2 Definition of Liability Index

The liability index $\lambda(t)$ is defined as the reinvestment index based on a hypothetical liability-matching portfolio at time t_1 of the valuation, reflecting the history of the total return on that portfolio. In practice, $\lambda(t_1)$ is calculated first and then the history $\lambda(t)$ is determined by going backwards from t_1 to the starting time t_0 using historical return data. Pension accruals, being the liability-side analogue of contributions on the asset side, and cash flows such as benefit payments are disregarded. In the case of index-linked pensions, the matching portfolio consists of index-linked bonds, and the total return rate includes the increase of the index—for example, the rate of inflation, if the pensions are linked to the price index.

4.3 Definition of Funding Index

The funding index is defined as the quotient of the asset index and the liability index.

That is to say,

$$\phi(t) = \alpha(t) / \lambda(t).$$

From this definition it follows that the growth rate of the funding index is equal to the growth rate of the asset index minus that of the liability index.

4.4 An Example of a Minimum Solvency Cushion Formula

Given the required time series data starting at time t_0 up to the valuation date t_1, consider the historical maximum value of the funding index up to time t

$$\mu(t) = \max \{ \phi(u) : t_0 \leq u \leq t \}.$$

The relative funding index at that time, being the current funding index in proportion to this maximum, is given by

$$\rho(t) = \phi(t) / \mu(t),$$

and its minimum value over the immediately preceding year is

$$\rho_{min}(t) = \min \{ \rho(u) : t - 1 \leq u \leq t \},$$

the time parameter t being expressed in units of one year. Clearly, $\rho(t)$ and $\rho_{min}(t)$ are both between zero and one.

Let $\theta_p(t)$ denote the lower quantile of order p of the empirical distribution of the values $\rho_{min}(u)$ with $t_0 + 1 \leq u \leq t$, so that $100p$ percent of those values are less than or equal to that quantile.

Consequently, there is a probability of p that a particular year selected at random from $[t_0, t_0 + 1]$, $[t_0 + 1, t_0 + 2]$, . . . , $[t_1 - 1, t_1]$—according to a uniform distribution—has the property that the lowest relative funding index measured during that year is less than $\theta_p(t_1)$.

That is to say, if the empirical distribution of $\rho_{min}(t)$ is viewed as a probability distribution, then the observed proportion p becomes the corresponding probability of the occurrence of a reduction worse than the reduction factor $\theta_p(t_1)$. If p is small, say 5 percent, this reduction factor represents a presumably disastrous decline of the funding index.

Accordingly, consider the minimum cushion, expressed as a proportion of the liabilities, that is defined by the formula

$$\kappa(t) = \max \{ [\rho(t) - \theta_p(t)] / \theta_p(t)], 0 \}$$

and the corresponding minimum funding ratio given by

$$MFR(t) = 100\% + \kappa(t) = \max \{ \rho(t) / \theta_p(t), 100\% \}.$$

Suppose the funding index $\varphi(t)$ reaches a historical maximum at time t_2 after t_1. Then $\rho(t_2) = 1$ and $MFR(t_2) = 1 / \theta_p(t_2)$. Moreover, if the actual funding ratio $FR(t_2)$ is equal to that minimum and the disastrous decline represented by the reduction factor $\theta_p(t_2)$ is applied, then the remaining funding ratio is exactly 100 percent. This is based on the assumption that there are no pension accruals, contributions and other cash flows after t_1.

Now consider n future years $[t_2, t_2 + 1]$, $[t_2 + 1, t_2 + 2]$, . . . , $[t_2 + n - 1, t_2 + n]$ and pick one of these years, say $[T - 1, T]$, at random, again according to a uniform distribution.

Then the probability that $\rho_{min}(T)$ is less than or equal to $\theta_p(T)$ can be approximated by p, at least if n is sufficiently large. This approximation is reasonable if, for lack of anything better, it must be assumed, with as little reliance on philosophical induction as possible, that $\theta_p(t)$ is more or less stationary over time. It may be argued that such a procedure need only be justified as a policy rule, without pretending to prove a statistical law. Accordingly, the approximate nature of the parameters involved may be ignored, as is done below.

Since $\mu(T) \geq \mu(t_2) = \varphi(t_2)$, it follows that the probability is at most p that at some time u during that random future year $[T - 1, T]$, the ratio $\varphi(u) / \varphi(t_2)$ will be less than or equal to $\theta_p(T)$. But unfortunately, $\theta_p(T)$ is not known at time t_2. Instead, $\theta_p(t_2)$ is used—and one can argue that this choice, too, need only be justified as a reasonable candidate to be incorporated in a policy rule, rather than as an optimal or at least satisfactory estimation procedure in some sophisticated statistical sense.

The upshot is that, if $FR(t_2) = MFR(t_2)$, then there is at most probability p that $FR(u)$ will be less than 100 percent at some time u during the random future year

$[T-1, T]$. This implies that the expected value of the proportion of the years between t_2 and $t_2 + n$ in which the funding ratio is less than 100 percent at some point is also at most p.

Suppose that, after time t_2, assets are being released if and while $FR(t)$ is increasing and greater than $MFR(t_2)$, but not due to simultaneous changes in $MFR(t)$. This may be incorporated by freezing the funding index $\varphi(t)$ so that it becomes constant during such periods. Note that this modification of $\varphi(t)$ does not affect the functions $\rho(t)$, $\rho_{min}(t)$, $\theta_p(t)$, $\kappa(t)$ and $MFR(t)$. However, the inequality $\mu(T) \geq \mu(t_2)$ becomes the equality $\mu(T) = \mu(t_2)$. This implies that, if $FR(t_2) = MFR(t_2)$, the probability is equal to p, rather than at most p, that $FR(u)$ will be less than 100 percent at some time u during the random future year $[T-1, T]$. And it also follows that, the expected value of the proportion of the years between t_2 and $t_2 + n$ in which the funding ratio is less than 100 percent at some point is also equal to p, rather than at most p.

4.5 Numerical Example

Table 2 lists minimum funding ratios $MFR(t)$ calculated in accordance with the formula proposed in Section 4.4 (with $p = 5\%$) and based on stylized time series data from the end of 1955 onwards; more details are given in Appendices A and B.

TABLE 2
Minimum Funding Ratios *MFR*(*t*)
for Various Asset Allocations

Time *t* (End of Year)	Asset Allocation: 100% Index-Linked Bonds	Asset Allocation: 50% Equities 50% Regular Bonds	Asset Allocation: 50% Equities 50% Index-Linked Bonds
1993	100%	143%	122%
1994	100	124	114
1995	100	121	112
1996	100	120	116
1997	100	125	126
1998	100	124	127
1999	100	158	140
2000	100	142	126
2001	100	128	115
2002	100	100	100

If the asset portfolio consists of *100 percent index-linked bonds*, then no mismatch cushion is required and the minimum funding ratio is 100 percent. This is based on the assumption that there is a perfect match between a subset of the assets and the liabilities, and that actuarial risk factors such as mortality may be ignored or treated separately.

If the asset allocation is *50 percent equities and 50 percent regular (i.e., non-index-linked) bonds*, as in a typical Dutch pension fund, then the minimum funding ratio was 143 percent at the end of 1993, and 158 percent at the end of 1999, for example.

The relative increase by (158 – 143) / 143 = 10 percent can be explained as follows:

- The funding index $\phi(t)$ increased by 17 percent.
- The historical maximum funding index $\mu(t)$ increased by 4 percent; this represents a release of assets from the minimum solvency margin.
- Hence, the relative funding index $\rho(t) = \phi(t) / \mu(t)$ increased by approximately 17% – 4% = 13%.
- However, the floor $\theta_{5\%}(t)$ increased by 3 percent.
- Consequently, in comparison with this floor, the increase was roughly 13% – 3% = 10%.

As long as the floor $\theta_{5\%}(t)$ tends to move slowly, the bulk of the change in the minimum funding ratio, $MFR(t)$, from year to year is explained by the autonomous change in the funding index—that is to say, by changes in market conditions—apart from the fact that assets may be released from time to time. On the other hand, emergency measures, such as additional contributions, are required when a declining funding index would tend to pull $MFR(t)$ below 100 percent.

This system permits the solvency reserve to be exhausted at the end of 2002, primarily due to a decline of the funding index (or the market) that was disastrous by historical standards; the relative funding index $\rho(t)$ dropped from 1.000 at the end of 1999 to 0.590 at the end of 2002. Since this is below the floor $\theta_{5\%}(t)$, which was at 0.624, an emergency plan is needed if the actual funding ratio $FR(t)$ is equal to $\mathrm{MFR}(t)$.

At any rate, no assets can be released before $\rho(t) = 1$; at that point,

$$MFR(t) = 1 / 0.624 = 160\%$$

if $\theta_{5\%}(t)$ remains at 0.624.

If the asset allocation is *50 percent equities and 50 percent index-linked bonds*, then the minimum funding ratio $MFR(t)$ tends to be significantly lower, but only on average, because of imperfect synchronicity. For example, in the years shown in Table 2, the two averages are 128 percent (with 50 percent in equities and 50 percent in regular bonds) and 120 percent (with 50 percent in equities and 50 percent in index-linked bonds). In this case, $\theta_{5\%}(t) = 0.714$ at the end of 2002; if $\theta_{5\%}(t)$ remains at that level, no assets can be released until

$$MFR(t) = 1 / 0.714 = 140\%.$$

5. Conclusion and Final Comments

A method for designing a solvency test with a soft mismatch cushion has been described and illustrated with numerical examples. This cushion has the property that it fluctuates as driven by market conditions between a floor of zero and an upper boundary. This is in contrast to the maintenance of risk-based capital ratios and constant value-at-risk monitoring, which are standard practice at other financial institutions such as commercial banks. Reasons have been given why this should be so, given the special role of pension funds in the economy. To increase transparency, the solvency protection system proposed here could be made an integral part of the pension promise.

5.1 One-Scenario Approach

As Samuelson (1994) has observed, there is only one history of capitalism. Thus, if the history of the funding index is a particular realization or sample path of an underlying stochastic process, it appears to be a weak basis for estimating the probability distribution of that stochastic process and the parameters involved. Moreover, it is far from clear that its distribution would be stationary, with an invariant parametric form and stable parameters, given the obvious significance of unique events and developments, and in view of the lack of regularity in the sample path itself. Without any further theorizing, this article takes the position that producing quasi-experimental data in the form of additional histories by means of a random scenario generator would not result in an improvement over the admittedly naïve probabilities and expected values derived here.

5.2 Efficient Markets and Market Sentiments

According to some, equity markets may be microefficient without being macroefficient. For example, see Samuelson (1994). This means that mispricing is possible in the sense that equities may be overpriced in periods of “irrational exuberance" and underpriced when the prevailing market sentiment is unduly depressed; also see Shiller (2000). This article assumes that the markets are sufficiently microefficient, but takes no position as to whether they are macroefficient or rational. Similarly, no assumption is made about mean reversion and the risk premium as far as the long-term return on equities is concerned. Note that the soft minimum solvency cushion proposed here has the desirable property that it tends to inflate in a wave of optimism and deflate in a wave of pessimism, other things equal.

5.3 Building Up and Maintaining Additional Reserves: Current Risk Assessments

The solvency test is only a minimum requirement. The method of the soft mismatch cushion described here is not intended to suggest that, during what may appear to be bear markets—that is, when the relative funding ratio $\rho(t)$ is low in comparison with the current floor value $\theta_p(t)$ — an improvement is more likely than a further decline, or that a decline to levels significantly below that floor is not to be expected. Neither is it intended to deny the validity and usefulness of current value-at-risk information.

Accordingly, there may be good reasons for building up and maintaining additional reserves for extra safety, possibly taking into account current risk assessments such as value-at-risk measures. All this should be examined in the continuity test, the design of which is beyond the scope of article.

5.4 Extension to Funding System

An obvious candidate for a funding system (or contribution formula) is such that $FR(t)$ would be more or less maintained at the level of $(1 + c) \times MFR(t)$ for some constant c, for example, $c = 10\%$. Given the additional margin, the resulting contribution could easily be smoothed to produce a fairly stable rate, at least as long as $MFR(t)$ remains above 100 percent.

5.5 Problem Issues

Several practical problems are encountered in the implementation of the formulae suggested in Section 4.4 and illustrated in Section 4.5:

- The results depend on some arbitrary parameters, such as the starting date t_0 of the economic time series (here Dec. 31, 1955).

- There is only limited information about the history of real interest rates, necessitating the use of stylized estimates.

As already indicated in Section 5.3, the risk that the solvency reserve will be completely exhausted—as $\rho(t)$ crashes through the floor $\theta_p(t)$—should be taken into account in the continuity test, for example on the basis of current risk assessments such as value-at-risk measures.

Appendix A

TABLE A1
Example 1

Time: end of year	Asset index	Liability index	Funding index	Maximum funding index up to t, based on monthly data	Relative funding index, as proportion of current maximum	Lower 5% quantile of $\rho_{min}(t)$	Minimum solvency cushion in relation to liabilities	Minimum funding ratio, excluding solvency cushion	Minimum funding ratio, including solvency cushion
t	$\alpha(t)$	$\lambda(t)$	$\varphi(t)$	$\mu(t)$	$\rho(t)$	$\theta_{5\%}(t)$	$\kappa(t)$	M	$MFR(t)$
1955	1.000	1.000	1.000	1.000	1.000	1.000	0%	100%	100%
1956	1.011	1.044	0.968	1.000	0.968	0.982	0%	100%	100%
1957	1.019	1.140	0.894	1.000	0.894	0.891	0%	100%	100%
1958	1.208	1.189	1.016	1.016	1.000	0.890	12%	100%	112%
1959	1.415	1.239	1.142	1.142	1.000	0.890	12%	100%	112%
1960	1.521	1.292	1.178	1.178	1.000	0.890	12%	100%	112%
1961	1.640	1.346	1.219	1.219	1.000	0.890	12%	100%	112%
1962	1.605	1.424	1.127	1.219	0.925	0.890	4%	100%	104%
1963	1.724	1.517	1.137	1.219	0.932	0.890	5%	100%	105%
1964	1.819	1.649	1.103	1.219	0.905	0.890	2%	100%	102%
1965	1.861	1.775	1.049	1.219	0.860	0.890	0%	100%	100%
1966	1.840	1.931	0.953	1.219	0.782	0.824	0%	100%	100%
1967	2.120	2.043	1.038	1.219	0.851	0.782	9%	100%	109%
1968	2.338	2.172	1.076	1.219	0.883	0.782	13%	100%	113%
1969	2.406	2.386	1.008	1.219	0.827	0.782	6%	100%	106%
1970	2.508	2.534	0.990	1.219	0.812	0.782	4%	100%	104%
1971	2.723	2.793	0.975	1.219	0.800	0.740	8%	100%	108%
1972	3.080	3.086	0.998	1.219	0.819	0.740	11%	100%	111%
1973	2.654	3.416	0.777	1.219	0.637	0.740	0%	100%	100%
1974	2.348	3.525	0.666	1.219	0.547	0.637	0%	100%	100%
1975	2.941	3.679	0.799	1.219	0.656	0.542	21%	100%	121%
1976	3.220	3.793	0.849	1.219	0.696	0.542	28%	100%	128%
1977	3.286	3.844	0.855	1.219	0.701	0.543	29%	100%	129%
1978	3.379	3.831	0.882	1.219	0.724	0.546	33%	100%	133%
1979	3.549	3.865	0.918	1.219	0.753	0.552	37%	100%	137%
1980	4.330	4.375	0.990	1.219	0.812	0.560	45%	100%	145%
1981	5.039	4.981	1.012	1.219	0.830	0.571	45%	100%	145%
1982	6.002	5.601	1.072	1.219	0.879	0.579	52%	100%	152%
1983	7.586	6.125	1.239	1.239	1.000	0.579	73%	100%	173%
1984	8.966	6.679	1.343	1.343	1.000	0.585	71%	100%	171%
1985	10.042	7.294	1.377	1.411	0.976	0.593	65%	100%	165%
1986	11.038	7.765	1.421	1.477	0.962	0.595	62%	100%	162%
1987	11.206	8.130	1.378	1.611	0.855	0.598	43%	100%	143%
1988	13.390	8.699	1.539	1.611	0.955	0.601	59%	100%	159%
1989	14.573	9.444	1.543	1.643	0.939	0.603	56%	100%	156%
1990	12.636	10.347	1.221	1.643	0.743	0.605	23%	100%	123%
1991	14.831	11.350	1.307	1.643	0.795	0.609	31%	100%	131%
1992	16.034	12.442	1.289	1.643	0.784	0.613	28%	100%	128%
1993	19.773	13.593	1.455	1.643	0.885	0.617	43%	100%	143%
1994	18.800	14.842	1.267	1.643	0.771	0.621	24%	100%	124%
1995	21.585	17.359	1.243	1.643	0.757	0.624	21%	100%	121%
1996	24.781	20.092	1.233	1.643	0.750	0.624	20%	100%	120%
1997	30.005	23.282	1.289	1.643	0.784	0.626	25%	100%	125%
1998	34.390	26.771	1.285	1.643	0.782	0.631	24%	100%	124%
1999	41.116	24.125	1.704	1.704	1.000	0.633	58%	100%	158%
2000	41.196	25.539	1.613	1.791	0.901	0.633	42%	100%	142%
2001	39.804	27.412	1.452	1.791	0.811	0.633	28%	100%	128%
2002	34.695	32.820	1.057	1.791	0.590	0.624	0%	100%	100%

Note: Pensions linked to price index asset allocation: 50 percent equities and 50 percent regular (non-index-linked) bonds.

Table A2
Example 2

Time: end of year	Asset index	Liability index	Funding index	Maximum funding index up to t, based on monthly data	Relative funding index, as proportion of current maximum	Lower 5% quantile of $\rho_{min}(t)$	Minimum solvency cushion in relation to liabilities	Minimum funding ratio, excluding solvency cushion	Minimum funding ratio, including solvency cushion
t	$\alpha(t)$	$\lambda(t)$	$\varphi(t)$	$\mu(t)$	$\rho(t)$	$\theta_{5\%}(t)$	$\kappa(t)$	M	$MFR(t)$
1955	1.000	1.000	1.000	1.000	1.000	1.000	0%	100%	100%
1956	1.037	1.044	0.993	1.000	0.993	0.993	0%	100%	100%
1957	1.054	1.140	0.924	1.000	0.924	0.931	0%	100%	100%
1958	1.248	1.189	1.050	1.050	1.000	0.924	8%	100%	108%
1959	1.462	1.239	1.180	1.180	1.000	0.924	8%	100%	108%
1960	1.559	1.292	1.207	1.207	1.000	0.924	8%	100%	108%
1961	1.684	1.346	1.251	1.251	1.000	0.924	8%	100%	108%
1962	1.663	1.424	1.168	1.251	0.934	0.924	1%	100%	101%
1963	1.842	1.517	1.214	1.251	0.971	0.924	5%	100%	105%
1964	1.997	1.649	1.211	1.251	0.968	0.924	5%	100%	105%
1965	2.122	1.775	1.195	1.251	0.956	0.924	3%	100%	103%
1966	2.111	1.931	1.093	1.251	0.874	0.920	0%	100%	100%
1967	2.430	2.043	1.190	1.251	0.951	0.874	9%	100%	109%
1968	2.743	2.172	1.263	1.263	1.000	0.874	14%	100%	114%
1969	2.894	2.386	1.213	1.263	0.960	0.874	10%	100%	110%
1970	2.962	2.534	1.169	1.263	0.926	0.874	6%	100%	106%
1971	3.231	2.793	1.157	1.263	0.916	0.856	7%	100%	107%
1972	3.762	3.086	1.219	1.263	0.965	0.856	13%	100%	113%
1973	3.438	3.416	1.006	1.263	0.797	0.856	0%	100%	100%
1974	2.868	3.525	0.814	1.263	0.644	0.793	0%	100%	100%
1975	3.533	3.679	0.960	1.263	0.761	0.651	17%	100%	117%
1976	3.683	3.793	0.971	1.263	0.769	0.651	18%	100%	118%
1977	3.605	3.844	0.938	1.263	0.743	0.651	14%	100%	114%
1978	3.661	3.831	0.955	1.263	0.757	0.651	16%	100%	116%
1979	3.834	3.865	0.992	1.263	0.786	0.663	18%	100%	118%
1980	4.897	4.375	1.119	1.263	0.887	0.681	30%	100%	130%
1981	5.544	4.981	1.113	1.263	0.881	0.684	29%	100%	129%
1982	6.425	5.601	1.147	1.263	0.908	0.687	32%	100%	132%
1983	8.057	6.125	1.315	1.315	1.000	0.694	44%	100%	144%
1984	9.361	6.679	1.402	1.402	1.000	0.700	43%	100%	143%
1985	10.306	7.294	1.413	1.475	0.958	0.706	36%	100%	136%
1986	11.345	7.765	1.461	1.516	0.964	0.709	36%	100%	136%
1987	11.411	8.130	1.404	1.648	0.852	0.711	20%	100%	120%
1988	13.977	8.699	1.607	1.648	0.975	0.713	37%	100%	137%
1989	15.435	9.444	1.634	1.723	0.949	0.714	33%	100%	133%
1990	13.987	10.347	1.352	1.723	0.785	0.714	10%	100%	110%
1991	16.146	11.350	1.423	1.723	0.826	0.714	16%	100%	116%
1992	17.050	12.442	1.370	1.723	0.795	0.714	11%	100%	111%
1993	20.476	13.593	1.506	1.723	0.874	0.714	22%	100%	122%
1994	20.823	14.842	1.403	1.723	0.814	0.714	14%	100%	114%
1995	23.878	17.359	1.376	1.723	0.798	0.714	12%	100%	112%
1996	28.560	20.092	1.421	1.723	0.825	0.714	16%	100%	116%
1997	36.071	23.282	1.549	1.723	0.899	0.714	26%	100%	126%
1998	41.817	26.771	1.562	1.723	0.907	0.714	27%	100%	127%
1999	48.339	24.125	2.004	2.004	1.000	0.714	40%	100%	140%
2000	48.144	25.539	1.885	2.095	0.900	0.714	26%	100%	126%
2001	47.040	27.412	1.716	2.095	0.819	0.714	15%	100%	115%
2002	42.948	32.820	1.309	2.095	0.625	0.714	0%	100%	100%

Note: Pensions linked to price index asset allocation: 50 percent equities and 50 percent index-linked bonds.

Appendix B

TABLE B1
STYLIZED DATA AND ASSUMPTIONS USED IN THE EXAMPLES

Asset Allocation = *AA*	π_1 = 50% in equities and π_2 = 50% in regular or index-linked bonds, as indicated; rebalanced at the beginning of each month
Equity Index = $I_1(t)$	Reinvestment index, as follows: ▪ from 1970 onwards: MSCI world index ("monthly gross index"); monthly data in U.S. dollars (USD) ▪ before 1970: unofficial MSCI world index ("price index") obtained from Global Financial Data Inc., combined with an estimated dividend yield of 3.5% per annum; annual data, interpolated geometrically to estimate monthly data, in U.S. dollars (USD) ▪ all amounts converted from U.S. dollars (USD) to Dutch guilders (NLG) and euros (EUR) on the basis of current exchange rates; EUR 1 = NLG 2.20371
Bond Index = $I_2(t)$	For regular bonds: reinvestment index for government bond portfolio, uniformly distributed over zero-coupon bonds with durations of 1, . . . , 10 years, rebalanced at the beginning of each month and based on the stylized nominal yield rates described below For index-linked bonds: identical to liability index
Asset Index = $\alpha(t)$	Combination of equity and bond index, based on assumed asset allocation
Liability Index = $\lambda(t)$	Reinvestment index for government bond portfolio, uniformly distributed over zero-coupon bonds with durations of 1, . . . , 30 years, rebalanced at the beginning of each month and based on the stylized real yield rates described below, plus the effect of inflation, since the pensions are linked to the price index.
Stylized Nominal Yield Rates	▪ until 1990: capital market yield rate ("kapitaalmarktrente") from CBS (Netherlands statistics office); constant during each calendar year; this is not a term structure, but a single rate ▪ from 1990 onwards: term structure of market yield rates on zero-coupon government bonds, estimated and published by the Bundesbank (German central bank); monthly data

Stylized Real Interest Rates	Strongly simplified with respect to the period before the introduction of the euro, on the basis of a small number of point estimates (annual rates): ▪ until 1974: constant at 2.5% ▪ from 1974 until 1980: linearly increasing from 2½% to 6% ▪ from 1980 until 1990: linearly decreasing from 6% to 5½% ▪ from 1990 until 1995: linearly decreasing from 5½% to 5% ▪ from 1995 until 1999: linearly decreasing from 5% to 2½% From 1999 onwards: estimated real yield rate based on long-term index-linked bonds issued by France (end of 1999: 3½%; mid 2000: 3¾%; end of 2000: 3½%; mid 2001: 3¾%; end of 2001: 3½%; mid 2002: 3½%; end of 2002: 2¾%; monthly data estimated on the basis of linear interpolation
Inflation	Annual rates from CBS (Netherlands statistics office); inflation is assumed to be constant during each calendar year ▪ until 1976: "prijsindexcijfer gezinsconsumtie werknemersgezinnen" ▪ from 1976 until 1991: "prijsindexcijfer gezinsconsumptie werknemersgezinnen beneden ziekenfondsgrens met verlaagde weging" ▪ from 1991 onwards: "totaal consumentenprijsindex, alle huishoudens, afgeleid"
Starting Date = t_0	Dec. 31, 1955

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