Financial Economics and Actuarial Practice

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(note: The quotations appearing in this monograph are exact, except where capitalization and punctuation were changed in keeping with modern style and grammar guidelines.)

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Abstract

Starting in the U.K. and continuing through the U.S. and Canadian actuarial professions, proponents of financial economics have been forcefully promoting a review of traditional actuarial practices and training. In particular, the financial theories first proposed by Modigliani and Miller and subsequently developed by others have been used to highlight serious weaknesses in typical actuarial thinking. In summary, it is claimed that much actuarial advice wrongly specifies value, that guidelines and standards need radical revision and that traditional actuarial intuition suffers in comparison to newer modes of thought adopted by other professions.

This paper examines concepts from both financial economics and actuarial science as applied to defined benefit schemes using a simple discounted cash-flow framework as a reference point. The general finding is that many standard modes of actuarial thought are, in fact, indefensible when examined with the tools and techniques of financial economics. The call for revision of actuarial training and practices is credible and necessary.

However, the paper also touches on areas where a heavy-handed application of finance theory could be misguided due to limitations in the simple financial economic models presented. It concludes that financial economics should be carefully integrated into actuarial thought, rather than appended to existing actuarial theory or inserted as a wholesale replacement.

1. Introduction

Financial economics has been applied to traditional actuarial problems for a long time. Sharpe (1976), Black (1980) and Tepper (1981) all present results from applying financial economics to pension funding that should have been of note to the actuarial profession. However, the ideas of financial economics have largely failed to engage actuaries until recently. Starting in the U.K. and continuing through the U.S. and Canadian actuarial professions, proponents of financial economics have been forcefully promoting a review of traditional actuarial practices and training. In particular, the financial theories first proposed by Modigliani and Miller (1958) and Miller and Modigliani (1961) and subsequently developed by others have been used to highlight serious weaknesses in typical actuarial thinking. In summary, it is claimed that much actuarial advice wrongly specifies value, that guidelines and standards need radical revision and that traditional actuarial intuition suffers in comparison to newer modes of thought adopted by other professions. While the focus of debate until now has been defined benefit (DB) pension advice, all areas of actuarial endeavor will eventually be taken to account.

The early responses to these challenges have been swift and far reaching. For example, Exley (2002) in commentary about FRS 17, a U.K. accounting requirement that mandates DB liabilities be valued using a corporate bond discount rate, states, "both the accountants and the actuaries seem truly to have been overtaken by events already. Indeed, FRS17 now seems out of date before it has even been implemented in full" (p. 2) Hershey (2003) gives FAS 87 a life expectancy of less than three years. Globally, major sections of actuarial education and training have been or are being rewritten with the tenets of financial economics at their core.

A survey of the literature does not find much consensus between the two sides of the debate. Proponents of financial economics seldom praise traditional actuarial methods and advocates of traditional approaches find little to commend in financial economic theory or practice. But are modern finance theory and actuarial science so incompatible? Has actuarial intuition and judgment been hopelessly out-of-touch and misplaced? Or should financial economics be resisted as a passing fashion?

This paper attempts to make sense of this schism. It examines concepts from both financial economics and actuarial science, as they apply to DB schemes, using a simple discounted cash-flow framework as a reference point. The general finding is that many standard modes of actuarial thought are, in fact, indefensible when examined with the tools and techniques of financial economics. The call for revision of actuarial training and practices is credible and necessary.

However, the paper also touches on areas where a heavy-handed application of finance theory may lead to inconsistencies or the creation of spurious models. It concludes that financial economics should be carefully integrated into actuarial thought rather than appended to existing actuarial theory or inserted as a wholesale replacement.

The term, "financial economics," as used in this paper, should be distinguished from modern portfolio theory and asset pricing models, such as the capital asset pricing model and the efficient market hypothesis, which form a slightly dated subset of financial economic theory. Despite being showcased in actuarial training as well as finance textbooks everywhere, these specifications are not central to modern financial economic arguments. In this sense, financial economics is also referred to in the literature as corporate finance, modern financial economics, (modern) finance theory, neoclassical economics or as postmodern financial economics.

This paper concentrates on the Modigliani-Miller stream of financial economics and does not canvas the classical theories or the option-pricing theories of Black-Merton-Scholes, except where they relate to the core themes of the paper. For an excellent survey on the span of financial economics see Whelan, Bowie, and Hibbert (2002).

Section 2 presents a discounted cash-flow (DCF) framework that is used through the paper to compare the relative approaches of actuaries and financial economics. Section 3 applies the basic concepts from financial economics to common valuation issues in DB schemes and contrasts these with traditional actuarial concepts. Section 4 briefly considers possible limitations of applying finance theory to actuarial practice. Section 5 outlines some preliminary conclusions about what actuaries should do to integrate financial economic theory into actuarial practice.

2. DCF Framework

A framework is needed to objectively assess various financial economic or actuarial techniques. As a starting point and following both actuarial and financial economic tradition, the value of any asset or liability can be modeled in terms of cash flows and discount rates:

$$(1) \quad V_a^0 = CF_a^1 (1+i_a^1)^{-1} + CF_a^2 (1+i_a^2)^{-2} + \ldots + CF_a^t (1+i_a^t)^{-t} + \ldots$$
$$= \sum_{j=1}^{\infty} CF_a^j (1+i_a^j)^{-j},$$

where,

V_a^t is the market value of asset a at time t.
CF_a^t is the cash flow generated by asset a at time t
i_a^t is the discount rate applied to the cash flows generated by asset a as at time t

Much of the gap between traditional actuarial methods and financial economics can be explained as due to differing treatment and emphasis on the various terms contained in the above equation.

Trivially, there are differences in terminology and jargon. For example, if the asset is an equity, then the *i*'s are usually referred to as the expected return. If the market value being calculated is a liability the *i*'s would be referred to as discount rates.

The equation can be rearranged so that it appears to be different for different objects and objectives. Bond cash flows (coupons) are the same through time so that the *i*'s can be grouped into an annuity expression together with a terminal part. Equity cash flows can be expressed as a base amount increasing by a growth factor. Reinvestment in new but similar assets can be assumed or not. Values can be turned into returns so that return expectations appear on the left-hand side of the equation.

Individual terms can be expressed and thought about in a variety of mathematical ways. For example, cash flows can be thought of as:

- Constant, for example, a zero-coupon bond with no default-risk, that liability cash flows are known with certainty;
- Simple functions of each other, for example, equity cash flows are assumed to grow at a constant rate through time;
- Stochastic, for example, zero-coupon bond with default risk, equity cash flows being uncertain, liabilities being not precisely known and so on;
- Risk-adjusted via assessments of utilities or other methods; or
- Naturally occurring (coupons and dividends) or due to trading activities.

A complete enumeration of the various ways to treat equation (1) is a task outside the ambit of this paper. This paper restricts itself to a few interesting points of difference between actuarial and financial economic methodologies and model construction ideas.

3. Applying Financial Economic Concepts to DB Schemes

3.1 No-Arbitrage

The cornerstone of financial economics has become the principle of no-arbitrage. Financial models should strive to be free of situations where an individual could simply and easily earn a profit without risk (an arbitrage). If an arbitrage can be constructed from the model then it is wrongly specified.

One seemingly trivial example is constraining models so that one dollar of equities is valued the same as one dollar of bonds. Bader (2001) uses this relationship to construct a zero-valued swap that is short one dollar of bonds and long one dollar of equities—the Bader swap. We can write this constraint in terms of equation (1) (assuming a constant discount rate for both bonds and equities) as:

$$\begin{aligned} V^0_{baderswap} &= V^0_{equities} - V^0_{bonds} \\ &= \sum_{j=0}^{\infty} CF^j_{equities}(1+i_{equities})^{-j} - \sum_{j=0}^{n} CF^j_{bonds}(1+i_{bonds})^{-j} \\ &= 0 \end{aligned}$$

Reworking the example contained in Gold (2002), imagine a Bader swap that consisted of the following assets:

- \$1 worth of short 15-year zero-coupon bonds yielding 6 percent.

- $1 worth of long equities with dividends reinvested, with an expected return of 8 percent per annum and with an expected standard deviation of return of 16 percent per annum.

A common actuarial approach to a wide variety of problems is to discount projected cash flows. Assuming that the Bader swap is held for *n* years we would calculate the expected cash flow for both bonds and equities and thus for the entire portfolio:

$$\begin{aligned} V^{o}_{\text{equities}} &= 1 = CF^{n}_{\text{equities}} \, (1+i_{equities})^{-n} \\ &\Rightarrow \; CF^{n}_{\text{equities}} = (1+i_{equities})^{n} \\ V^{o}_{\text{bonds}} &= 1 = CF^{n}_{\text{bonds}} \, (1+i_{bonds})^{-n} \\ &\Rightarrow \; CF^{n}_{\text{bonds}} = (1+i_{bonds})^{n} \end{aligned}$$

$$\begin{aligned} CF^{n}_{\text{baderswap}} &= CF^{n}_{\text{equities}} - CF^{n}_{\text{bonds}} \\ &= (1+i_{equities})^{n} - (1+i_{bonds})^{n}. \end{aligned}$$

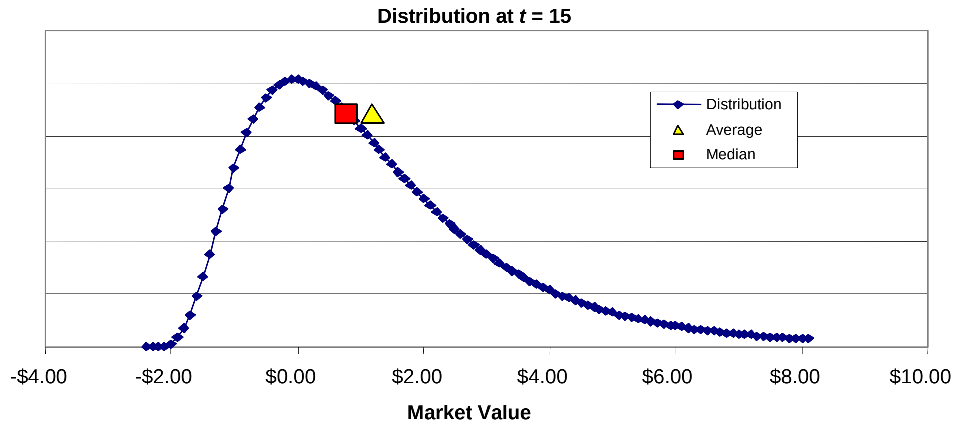
We would then substitute this expected cash flow into equation (1) and apply a discount rate in order to determine the value of the swap. Assuming that the Bader swap discount rate chosen is the equity discount rate:

$$\begin{aligned} AV^{0}_{\text{baderswap}} &= CF_{\text{baderswap}} \, * \, (1+\mathrm{i}_{\text{baderswap}})^{-15} \\ &= \left[(1+\mathrm{i}_{\text{equities}})^{n} - (1+\mathrm{i}_{\text{bonds}})^{n}\right] / (1+\mathrm{i}_{\text{equities}})^{n} \\ &= 1 - \left[(1+\mathrm{i}_{\text{bonds}})^{n} / (1+\mathrm{i}_{\text{equities}})^{n}\right] \end{aligned}$$

Using the return assumptions and time horizon in Section 3.1.3. gives the value of the Bader swap as $0.24.

One modification to the above approach is to recognize that the cash flow of the Bader swap is a stochastic variable rather than a constant value. If we assume that the cash flow for bonds is a constant value (i.e., no default risk) then the swap value after 15 years *(t = 15)* will be distributed as per the graph in Figure 1.

Figure 1
Bader Swap



Again, discounting at the equity discount rate and integrating over this distribution gives an actuarial value for the swap of $0.44. The value is greater than the discounted expected value due to the log nature of compound interest and resultant positive skew of the distribution.

So however cash flows are treated and whatever the discount rate, actuarial assessments of current value will be positive, and significantly so, in comparison to current price. And this is the central dilemma—traditional actuarial methods lead to a positive current value being assigned to the Bader swap even though the market value is zero.

It is clear that the Bader swap has an economic, market, real, theoretical and practical value of zero. The fact that actuaries can arrive at a positive value for the swap is a consequence of how we treat the terms of the DCF equation:

- Actuaries tend to add together or otherwise algebraically manipulate expected values of stochastic variables such as cash flows. When stochastic variables have different distributions (in magnitude or shape) then these are simply not additive.
- No one can deny that the distribution of cash flows arising from the Bader swap is inherently risky compared with a certain cash flow.

For example, there is about a 30 percent chance that its value will be negative even after 15 years. However, using the expected value for cash flows or integrating across a cash-flow distribution, as shown in the above example, takes no account of the risk and the need for investors to be compensated for accepting this risk.

- The use of a constant discount rate is problematic. The reality is that the discount rate itself varies through time (has a term structure) and is stochastic.

3.2 Law of One Price, Matching and the Liability Discount Rate

The law of one price contends that, if assets or liabilities have the same cash flows, then they should have the same value. If this wasn't the case, an investor could buy the higher valued asset and sell the lower valued one for an easy arbitrage profit. In the context of equation (1), assets or liabilities with the same cash flows should have the same discount rates:

$$V_a^o = \sum_{j=1}^{\infty} \mathrm{CF}^j (1+i_a^j)^{-j}$$

$$V_b^o = \sum_{j=1}^{\infty} \mathrm{CF}^j (1+i_b^j)^{-j}$$

$$V_a^o = V_b^o \text{ Implies that } \quad \mathrm{i}_a^j = \mathrm{i}_b^j \quad \text{for all j.}$$

A set of assets that have the same cash flows as a set of liabilities is said to be a match or a hedge of the liability. The law of one price implies that the liability and the matching asset should have the same value. Given the DCF framework, this means that the liability discount rates should be set equal to the discount rates implied by the matching asset. Assuming that liability cash flows are certain, the application of the law of one price leads to the use of bonds as matching assets and the use of the interest rate term structure as the liability discount rates.

So financial economics suggests that known and certain liabilities are valued as (assuming constant discount rates through time):

$$L_{FE} = \sum_{t=1}^{\infty} CF_{liabilities}^t (1+i_{bonds})^{-t}$$

In contrast, a traditional actuarial approach is to use the expected return of the assets being invested in as the discount rate for the

$$L_{Actuary} = \sum_{t=1}^{\infty} CF_{liabilities}^t (1+i_{equities})^{-t}$$

liabilities. If we assume that assets are invested 100 percent in equities this gives:

The relative difference between the two valuations is:

$$(L_{FE} - L_{Actuary}) / L_{FE} = \sum_{t=1}^{\infty} CF^{t}_{liabilities} \left[(1+i_{bonds})^{-t} - (1+i_{equities})^{-t}\right] / \sum_{t=1}^{\infty} CF^{t}_{liabilities} (1+i_{bonds})^{-t}.$$

If the liabilities are assumed to be a cash flow in n years time, we can simplify the calculation to:

$$\begin{aligned}(L_{FE} - L_{Actuary}) / L_{FE} &= CF_{liabilities} \left[(1+i_{bonds})^{-n} - (1+i_{equities})^{-n}\right] / (CF_{liabilities} (1+i_{bonds})^{-n}) \\ &= [(1+i_{bonds})^{-n} - (1+i_{equities})^{-n}] / (1+i_{bonds})^{-n} \\ &= 1 - (1+i_{equities})^{-n} / (1+i_{bonds})^{-n} \\ &= AV_{baderswap}.\end{aligned}$$

In effect then, when actuaries use equity discount rates for liabilities, they are deducting the actuarial value of the Bader swap from the economic value of the liabilities. This is functionally equivalent to adding the Bader swap to the value of the assets so that the value of equities is set equal to the market value plus the actuarial value of the Bader swap.

3.3 The Irrelevance Principle, Stakeholders and Asset Allocation

Modigliani and Miller (1958) first introduced no-arbitrage to show that the first-order effect of the debt/equity mix for a company is irrelevant to calculations of its value. Their second paper (Miller and Modigliani 1961) proposed the same argument for dividend policy—that the first-order effect of the dividend payment policy was irrelevant to calculations of company value. They went on to argue that what is relevant are various second-order effects such as taxation, agency issues and so on. These findings have become known as the irrelevance principle or proposition.

One of the main breakthroughs in Modigliani and Miller (1958) was their success in arguing that the corporation was largely a legal fiction and that decision making should be examined at an individual or stakeholder level. Typical stakeholder categories include shareholders, management, employees, consultants and the government. Whelan, Bowie, and Hibbert (2002) provides the following examples:

"The people making the decisions about capital structure are not always the people who own the company. The company management's interests are not perfectly aligned with those of the shareholders. The company represents only a small portion of a shareholder's wealth, but is a significant player in the employees' lives. Broadly, management will prefer company growth to growth in shareholder value" (p.43).

Given the multiple interests and objectives of these stakeholder groups, it is important to identify and assess their interactions before answering questions of optimal financial decision making.

The arguments are easily applied to an asset allocation decision for a DB plan so that the first-order effect of the asset allocation decision can be seen as irrelevant. From the point of view of a shareholder, a decision to increase equity exposure in a DB scheme simply transfers equity risk to the market value of the company. If a shareholder disagrees with this decision they can simply sell equities to reverse the risk increase. From a shareholder point of view, there is, thus, no ideal asset allocation for DB plans. Instead, the asset allocation should be decided on second-order effects such as taxation, surplus ownership and agency effects.

This argument applied to the U.K. environment has seen many practitioners questioning the wisdom of holding equities in DB schemes. To choose three of many second-order effects in order to illustrate the argument style:

- With respect to taxation effects in the United Kingdom, it makes sense for shareholders to generally hold their bonds indirectly via a DB scheme (given their tax-free status) and to hold equities directly (as dividend credits are not claimable by pension schemes and due to the preferential tax treatment of equities).
- Assuming that positive surpluses can result in benefit improvements for members but that deficits need to be funded by the sponsor, then shareholders are receiving the risk of equity ownership without the full reward.
- In order to reduce risk being transferred to the corporate balance sheet, reduce frictional costs and promote transparency, DB schemes should hold the asset that best matches the liabilities. The best

matching asset is inflation-linked gilts, given the liability profile of U.K. schemes and the ready supply of these assets.

In contrast, actuaries concern themselves mostly with the first-order effect of asset allocation and see the issue from a scheme-centric point of view. The conclusion most often reached is that DB plans should hold lots of equities. The traditional arguments leading to an equity bias can be divided into two types, an investment advice argument or a matching asset argument, both of which lead to the use of the expected equity return as the discount rate of the liabilities.

The "investment advice" argument runs along the following lines:

1. Equities will probably outperform bonds in the long run.
2. DB schemes should then hold mostly equities in order to lower the costs of funding liabilities.
3. This lowering of cost should be taken into account (e.g., for determining and monitoring funding requirements).
4. All this is assisted by valuing liabilities using the asset discount rate.

The first statement can be construed as an assessment that the distribution of the Bader swap is largely positive and should be attractive to long-term business (i.e., not very risky with high expected return). Modern financial economics is silent on this—people are entitled to their opinion about what is risky to them.

The second statement implies that management and shareholders benefit from lower expected costs and members benefit from the possibility of benefit improvements; a win-win situation. In reality, there is a lot more going on. For example, this logic ignores that equity risk is simply passed through to the individual shareholders and has no extra value. Overall, the statement highlights the naiveté of a scheme-centric viewpoint that fails to acknowledge the relative interests of the various stakeholders. Having said this, the statement does not contravene the major tenets of financial economics if stated about an individual—people are entitled to and should act on their opinions about relative asset values.

The third statement is critical. It effectively means that *all* stakeholders should *act like* the value of the Bader swap is equal to the actuarial value rather than to the economic value of zero. Financial economics says that while two individuals may assess the value of an asset to be vastly different they should see that the market price is the medium of exchange and thus the only sensible value.

The last statement is a practical one. It looks anomalous if the market value of assets is adjusted by adding the actuarial value of the Bader swap so it should be deducted from the liability value instead.

The "matching asset" argument is that DB liabilities are mostly affected by salary inflation, equity returns are also affected by salary inflation and thus the best matching asset for salary-related liabilities are equities. This argument is consistent with the proposition that asset allocation first-order effects are irrelevant and that DB schemes should hold the matching asset. However, the arguments supporting the equity/salary match have been dented by research such as Exley, Mehta, and Smith (1997).

In a recent summation of their thinking, Exley, Mehta, and Smith (2002) reinforces this theme.

"There is no statistical evidence or robust economic theory to suggest that equities match salary related liabilities. While the match is not perfect, index linked bonds are the best match for such liabilities, since the link between prices and salaries has been much more stable than the link between equity dividends and salaries" (p. 2).

3.4 Value and Actuarial Judgment

The main principles of financial economics so far explored in the paper do not depend in any way on markets being efficient. They simply require our models of the world to be arbitrage free. Stated another way, if markets or people act irrationally, that does not imply that financial professionals should create models that are also irrational.

Pemberton (1998a) sets out several ways in which the value of an asset to an individual investor may be conceived of as being different to the current market price:

- The asset could have tax advantages for an individual investor not possessed by the marginal investor. One obvious example is where deferred capital gains create a tax benefit.

- The marginal investors could be wrong about the value of the asset. For example, they could be acting on a common but incorrect set of information with which to judge value.
- There may be inefficiencies in the price formation process. For example, an arbitrage opportunity may exist but not be recognized or exploited by marginal investors.
- The marginal investor may not be the same as the average investor. For example, the risk preferences of the marginal investor could be different to the larger population.
- The risk inherent in the asset may be less for the individual investor than for the average investor due to differences in risk preferences. For example, an investor with a long time horizon may need less compensation for short-term price volatility, especially if they hold the view that prices are mean-reverting.

Financial economics is not in conflict with any of the above list of possible interpretations of value. For example, advice to a long-term investor to invest in equities because they will outperform bonds in the long-term on a risk-adjusted basis can be translated into a belief that an arbitrage opportunity exists and that this should be exploited by the investor.

What financial economics does say though is that since this value is not instantaneously realizable it should not be interpreted as a *current* value. Many of the differences between price and subjective interpretations of value occur because the conceived value will only be confirmed at a future point in time. Financial economics is about making the current value equal to the market price or economic value. It is silent as to what future value assets and liabilities may have other than constraining models such that current value equals current price at all points in time. The implications of this emphasis on current value are explored further in Section 4.2.

Actuaries are often concerned with future values more so than current values. As shown by financial economics, actuarial models tend to downplay or ignore the information contained in current or economic values.

Any resolution of financial economics and actuarial practice must firstly distinguish between current values and future values. Given this distinction, we should then ensure that models of future values do not contradict current values, for example, by creating arbitrage opportunities. As Pemberton (1998a) puts it, our models should respect market values.

Financial economics also tends to avoid solutions involving subjective judgments. One rationale for this is that allowing a subjective or judgmental approach to assessing value is prone to certain risks:

- In very general terms, people are subject to hubris and tend to overrate their ability to forecast asset prices and returns. Arguing that the value of an asset is different to the economic or market value is effectively stating that an arbitrage situation exists. If this is the belief, then actuaries should argue this explicitly rather than have perceptions of value embedded implicitly in valuation advice.

- Allowing a range of subjective interpretations of value allows the possibility that a stakeholder will take advantage of the range to further their own interests at the expense of the other stakeholders. The most common example is where management makes decisions that do not increase shareholder value. This is known as agency cost. For example, a weak valuation basis may be desired by management to lessen the accounting cost of DB funding in current accounts or otherwise disguise the DB operating result in order to maximize their bonuses.

- A variation of this problem is where actuaries who use high discount rates or who smooth values through time may be selected by management so that there are business pressures on actuaries to depart from the true economic value of the liability.

- Opaque economic contracts are more difficult to assess in terms of value and are thus open to more criticism with respect to the above problems in comparison to a transparent and objective valuation basis.

Actuaries have a poor track record in managing these risks. Again, respecting economic values would go a long way towards guarding against the dangers of using subjective judgment in assessments of value.

4. Limitations in Applying Financial Economics

Financial economics as described in this paper focuses on simple theoretical constructs and thought experiments that illustrate ways in which models may be wrongly specified. A vital step in applying financial economics is the identification of theoretical limitations: how the basic tenets may be inappropriately applied, how they can lead to inconsistencies in model specification and where solo application without more general reasoning can lead to incomplete models of reality.

This paper identifies two areas that should be of concern to actuaries (there is no doubt many others):

- Modeling liabilities where there is an imperfectly matching asset or no matching asset.
- An overemphasis on current values.

4.1 Incomplete Matching

There are two types of risk-adjustment considered in applied finance. The first is a very general principle, where economic utility function arguments conclude that people are risk-avoiders and that risk should be rewarded by higher expected return. The second is the capital asset pricing model (CAPM) and variations that go further and state that not all risks are the same. Some risks are diversifiable and are thus not rewarded by higher expected returns. Other risks are nondiversifiable and attract a risk premium.

The choice of risk adjustment has some important implications where an exactly matching asset cannot be found.

Imagine that you are the holder of an asset that produces a known series of cash flows with certainty. The value of the asset can be calculated using no-arbitrage and appropriate government bond yields. Now imagine that, for whatever reason, the central value of the cash flows stayed the same but there was now some variance around the expected value of each cash flow. If you subscribe to the view that this increase in variance needs to be compensated by an increase in expected returns, then the economic value would fall.

If you subscribe to CAPM, then a determination of the reasons behind the variance is needed. If the variance is due to systemic risk an increase in expected return would be needed. If the variance is diversifiable, then no change in value would occur. These relationships between cash-flow variance and expected return are relatively unambiguous and universally accepted, e.g. a bond with a higher default risk has a higher yield.

In contrast, imagine a holder of a *liability* that requires payment of a known series of cash flows with certainty. The value of the liability can be calculated using no-arbitrage and appropriate government bond yields. Now imagine that, for whatever reason, the central value of the cash flows stayed the same but there was now some variance around the expected value of each cash flow. What happens to the value of liabilities?

Financial economics tells us that we can use the same logic as above if we have a matching asset. The liabilities would move in line with the matching asset and would decrease in value (or stay the same if you invoke CAPM and the variance is diversifiable).

If no matching asset exists we must proceed differently. From the point of view of the counterparty to the liability (i.e., the asset holder), the value may be less as a higher discount rate may be necessary to compensate for the cash-flow uncertainty. For the liability holder it seems to be more complicated.

In very general terms, the uncertainty of the cash flows makes for a worse liability than before. Thus the liability increases in value rather than decreases as it does for the asset side. A common actuarial approach is to split the liabilities into two parts (see, for example, IAAust Discount Rate Task Force 2001):

1. The certain portion which represents cash flows with the same expected value but zero variance. This portion would have a value equal to the certain cash-flow situation. The discount rate applying to the certain cash flows would also be that of the matching asset.
2. An uncertain portion representing cash flows with an expected value of zero and a variance in cash-flow amounts. This portion could be valued in a variety of ways (e.g., using option pricing technology), all of which would lead to a positive value.

Exley, Mehta, and Smith (1997) state:

> "In a simple world, the value of the defined benefit promises to employees is the same as the cost of the same promises to the shareholder of the sponsoring company, provided values and costs are measured in an economically consistent fashion … . Thus—we can only reduce the cost of pension benefits to companies by reducing their value to employees" (p. 1).

A simple world in this context is one where all liabilities have exactly matching assets.

In summary, the application of financial economics says that an increase in liability volatility decreases the value of liabilities if a matching asset exists but increases (or possibly leaves unchanged) the value if a matching asset cannot be found.

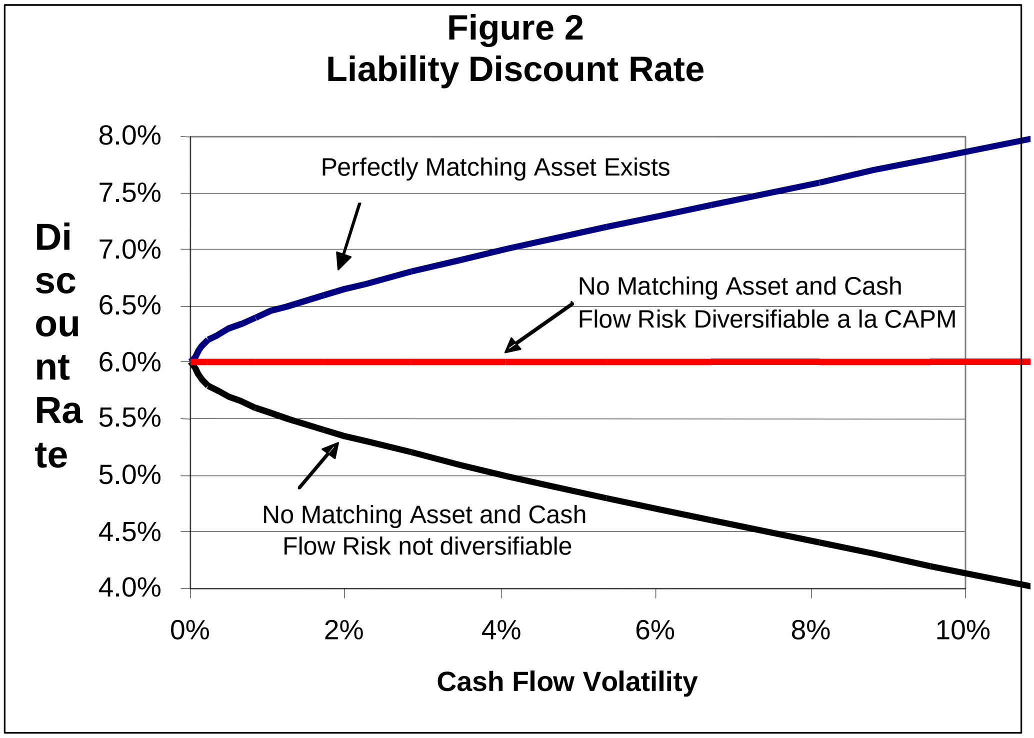
This discontinuity extends to where matching is imperfect. Finance theory says that if cash flows match *exactly* then so should market values and discount rates. To prevent simple arbitrage anomalies, the value of a liability should be exactly equal to the matching asset. But is a liability valued using the same discount rate of an asset that *almost* matches or should a slightly higher value be placed on the liability in recognition of the slightly higher risk faced by the liability holder? If yes, then the discount rate needs to be *lower* that the discount rate of the almost matching asset rather than higher, or a separate reserve is required.

A similar problem occurs when the liability volatility is unrelated to any available asset. Then, a CAPM-type argument would suggest that for DB schemes:

- The liability volatility is diversifiable by the shareholder and, thus, can be ignored. In this case, the appropriate discount rate is related to the assets that match the expected cash flows.
- The liability volatility is probably not diversifiable in the hands of the members as pension benefits are likely to represent a large portion of their wealth and salary-linked assets are not available. For example, retrenchment may represent a downside risk for the member (that pension assets are worth less than the expected value) that is highly correlated with the downside risk that future wealth is also low. Compensation for this volatility, *in the hands of the members*, means that the economic value of the liabilities is less than the economic value placed on the liabilities by the shareholder.

One interpretation of this is that the DB contract is destroying value compared with direct compensation.

The relationship between the liability discount rate selected, cash-flow volatility and the existence of a matching asset is presented in the Figure 2.



This is not the interpretation of Exley, Mehta, and Smith (1997), who state:

> "Under the market framework, the suitability of a hedge portfolio, for constructing values, is assessed on a symmetric basis. The possibility of the hedge fund assets outperforming the liabilities needs to be given equal weight to the possibility of underperformance. Such a convention is necessary to ensure that a promised cash flow stream is valued consistently by both parties. By the same token, it is inappropriate to insert arbitrary 'margins for prudence' in a market value calculation. Prudent for members may be imprudent for current shareholders but at the same time prudent for potential investors" (p. 22).

The problem with this logic is that the necessity of consistent valuation is a feature of the financial economic model rather than a feature of reality.

4.2 Overemphasis on Current Values

The focus of financial economics is very much on the creation of static models that model current or near-term values. Modigliani and Miller (1958) set the tone for a focus in financial economics on market values above all else. They suggested that all tests of value can be replaced by one test: "Will the project, as financed, raise the market value of the firm's shares?" They went on to note that "such a test is entirely independent of the tastes of the current owners, since market prices will reflect not only their preferences but those of all potential owners as well" (p. 264).

Whelan et al (2002) correctly state that the Modigliani-Miller propositions were groundbreaking, in part because they demonstrated that "focusing too narrowly on any one corporate financial feature could result in unintentional destruction of value" (p. 39). This statement could well be applied also to the narrow focus on current price.

This focus on current value is now a widespread condition that can easily lead to irrational behavior. As Keynes (1936) says, "Human nature desires quick results, there is a peculiar zest in making money quickly, and remoter gains are discounted by the average man at a very high rate" (Chapter 12 section 5 sub-point 4 paragraph 5). To quote a more recent investor, Buffett (1997), about the dangers of concentrating on current value or wealth:

> "If you plan to eat hamburgers throughout your life and are not a cattle producer, should you wish for higher or lower prices for beef? Likewise, if you are going to buy a car from time to time but are not an auto manufacturer, should you prefer higher or lower car prices? These questions, of course, answer themselves.
>
> "But now for the final exam: If you expect to be a net saver during the next five years, should you hope for a higher or lower stock market during that period? Many investors get this one wrong. Even though they are going to be net buyers of stocks for many years to come, they are elated when stock prices rise and depressed when they fall. In effect, they rejoice because prices have risen for the "hamburgers" they will soon be buying. This reaction makes no sense. Only those who will be sellers of equities in the near future should be happy at seeing stocks rise. Prospective purchasers should much prefer sinking prices".

In contrast to financial economics, a concern with future values is a key actuarial endeavor. Actuaries should remain focused on long-term objectives; we are guardians of the long-term view and short-term considerations are less important, largely illusory and can often be harmful to long term objectives. Much actuarial practice stems from this belief: Smooth asset values to discount recent market value movements; use the long-term expected cost as the economic cost so as to reduce the emphasis on short-term variations; act and account as if long-term expectations are certain. The fact that many of these practices are questionable in the light of finance theory does not mean that a long-term focus is misguided.

4.3 Modeling a Portfolio of Assets Rather Than an Asset

One implication of the emphasis on current value by financial economics is that static, snapshot-type models are preferred over dynamic, ongoing-concern type models. As an example, Chapman, Gordon, and Speed (2001) favor a "defined accrued benefit method" for valuation of pension liability where there is no account taken of future salary increases. The rationale is that salary increases, although likely are not a contractual obligation and, thus, shouldn't affect a balance sheet. If asking the question—"What is the current value of the assets and liabilities?"—this approach seems fair enough to argue.

However, if the question being asked is—"What does the pension fund look like through time (i.e., as a going concern)?"—then what is "likely" is a better criteria than what is "contractual" in selecting the features to be modeled. In this case, salary increases should be taken into account.

A going-concern-type question will also mean that we need to think about a dynamic portfolio of assets that matches our dynamic view of the liabilities. And, in general, the expected return on a single asset or liability is not the same as the expected return on an ongoing portfolio of assets.

For example, investing in a 10-year bond is not the same as investing in a portfolio of 10-year bonds on an ongoing basis. Imagine a portfolio with a mandate to hold 10-year zero-coupon bonds. At the start of the year, the portfolio manager purchases bonds with a duration of 10.5 years and holds it for a year at which time they have a duration of 9.5 years. These bonds are then sold and 10.5-year bonds are again purchased.

Using equation (1) and assuming that the expected term structure is invariant, the expected return each year will be equal to the change in price of the bond (ignoring the finer points of compound interest):

$$r_{bonds} = (\prod_{j=1}^{9}(1+i_{bonds}^{j})^{-1} - \prod_{j=1}^{10}(1+i_{bonds}^{j})^{-1})/(\prod_{j=1}^{10}(1+i_{bonds}^{j})^{-1} = i_{bonds}^{10} \quad ;$$

i^{10} is commonly known as the forward rate at year 10, so the bond portfolio expected return is equal to the forward rate rather than the yield.

In a dynamic model of the world, actuarial intuitions would say that the same concept should apply to liabilities: If the liability is an ongoing one and duration is roughly constant (i.e., in a steady state), then the appropriate rate if we are assuming a constant discount rate, should be the forward rate rather than the yield of the matching asset. On the other hand, if the liability cash flows are fixed and the liability is being run down (or we are valuing it as though it is), then the most appropriate discount rate is the yield of the matching asset.

4.4 Using Return Statistics as a Proxy for Risk

A long-standing convenience in finance theory is to assume that higher expected return is a reward for higher return volatility (or various alternative risk measurements based on return).

In a DCF framework, volatility of return can be caused by either volatility of cash-flow terms or volatility of discount rate terms. To assess the distribution of returns and use this as a proxy for risk, while convenient, means that differences in investor risk preferences to cash flow and discount rate volatility are ignored.

For example, while investors will need to assess the cash-flow risk inherent in the purchase of a two-year bond (i.e., the default risk), whether they need to consider discount rate risk will depend on their intended holding period. If the intent is to hold the bond till maturity, they would be indifferent to discount rate risk.

Yet again, this is a case where the focus on current value statistics obscures the real risk preferences of the investor—although the current value of the two-year bond at the end of the first year will be effected by discount rate movements, the future value at the end of the second year will be unchanged.

The extension of this example to equities is quite natural. Changes in equity discount rates will cause very real changes in current values but changes to future values will be less. Decreases to current values caused by an increase in the equity discount rate will be in part compensated by higher future returns, and vice versa. Depending on the time horizon, future values may well be higher given higher discount rates and current value movements may be inversely correlated with future value movements.

Cash-flow volatility may affect values in a very different way to discount rate volatility. Changes in near-term cash flows would affect current and future values in a roughly equivalent way. Changes in far-off cash flows may not affect current value much due to heavy discounting but substantially affect future values.

In summary, the statistical analysis of past returns (i.e., changes to current values) may not be a reliable guide to the statistics of future values. For example, evidence that equity returns are uncorrelated with salary inflation may shed little light on whether future equity values are correlated with future DB liabilities.

There is also the problem of choosing distribution models that are easy to use and otherwise fit in with existing theoretical solutions. The prime example is using arithmetic mean returns rather than geometric means. According to Exley, Mehta, and Smith (1997):

"The constraints to make sure that a bundle of cash flows are priced consistently with the sum of the values taken separately also appear clumsy in a geometric mean context. For this reason, academics typically favor the use of arithmetic means" (p. 15).

As Fitzherbert (2001) points out, this avoidance of theoretical clumsiness has unfortunately also led to large tracts of spurious empirical research.

5. What Should Actuaries Do?

5.1 Revision of Practice

This paper has highlighted several modes of thinking which should be reviewed if not jettisoned from actuarial practice. Actuaries should be trained out of adding together expected values and trained into understanding that a positive expected value is not the same as a positive value (market-based or otherwise).

We should recognize and enunciate the various stakeholders in actuarial models and their interests and seek to analyze the interactions between them. In particular, we should strive to become experts in agency costs and how to avoid them. With respect to DB schemes, more transparent valuation bases should contribute to the avoidance of agency costs.

We should acknowledge the inevitable moral hazards and interest conflicts that arise from the application of actuarial judgment and seek ways to limit their adverse effects. We should especially pay attention to the use of "funding" arguments that may create very real arbitrage opportunities for one stakeholder over another.

Current market prices and economic values should be respected. It should be realized that the issue of what is current value is owned by other professions. We should respect their views and strive to link our long-term models and ideas with accounting practices, current market prices and economic realities rather than ignore discontinuities. Within this context we should communicate our case for the importance of long-term value considerations.

The idea that equities are the best assets for long-term liability holders should be argued from within the context of the irrelevancy principle and the law of one price. For example, all stakeholders benefit from equity holdings on a risk-adjusted basis and agree to the usefulness of equity risk in the DB system. It should be made clear that the argument is that an arbitrage opportunity exists and should be exploited by a long-term liability holder. The argument should also be made explicit rather than implicit in the selection of a discount rate. As Gordon (1999) says, "the actuary is confusing valuation and investment advice" (p. 4).

The practice of using the discount rates of the underlying assets for valuing liabilities should no longer be considered good practice. In general, actuaries should be indifferent to the investment strategy of the fund except where giving investment advice.

5.2 Integration of Financial Economics

It is striking how easily financial economics can be combined with actuarial thinking to create more robust solutions to common actuarial problems. Even the most powerful arguments against financial economic *practices* are best constructed in the light of financial economic *theory*. Financial economics has tested some core generalizations built into actuarial practice and found them wanting. Despite its destructive effects, modern finance theory may also turn out to be a cornerstone of actuarial theory.

In my opinion, the main liabilities of financial economics include:

- Ignoring problems where no-arbitrage conditions cannot be applied or where subjectivity is unavoidable. One obvious example highlighted by this paper is the selection of a discount rate where no matching asset can be found.
- Creating snapshot, current value models rather than dynamic, future value ones. This leads to a narrow focus on the short-term impacts of financial decision making at the expense of long-term considerations.
- The use of return statistics as a proxy for risk.

Traditional actuarial strengths are almost a perfect "match" for these liabilities. As a result, the combination of financial economics and traditional actuarial thought could well create more robust financial models than each has in isolation.

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