# Financial Economics and the Retirement Plan Design Model

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*(note: The quotations appearing in this monograph are exact, except where capitalization and punctuation were changed in keeping with modern style and grammar guidelines.)*

## Abstract

Actuaries are frequently called upon to assist employers in redesigning their retirement programs and to help them understand the implications of different designs on participants. This led to the birth of the "traditional" retirement plan design model, whereby the actuary compares the "expected replacement ratio" calculated using certain assumptions for future salary increases and investment return.

While examining the expected replacement ratio is useful in illustrating the implications of a plan design on participants, this process is incomplete in its treatment of risk. Rather, it makes the assumption that the participant does not place any value, positive or negative, on risk. Unfortunately, this premise violates a basic principle of financial economics, that a risk-averse investor assigns a lower value to an otherwise identical financial arrangement as the risk of the arrangement increases.

A fundamental financial theory is that rational investors maximize *utility*, rather than simply expected value. A retirement plan should not be evaluated differently than other financial arrangements and, therefore, its value should be measured through a utility function.

This analysis suggests a method of incorporating utility theory into the retirement plan design model, thus keeping the model more in accord with the basic principles of financial economics.


## 1. Introduction/Background

In designing a retirement plan, it is essential that the employer define an adequate target level of benefits to provide. This is true for many reasons. The employee faces the risk that over his career he will not accumulate enough retirement savings to maintain a desired standard of living after retirement. To help manage this risk, employees rely on some or all of their retirement income to be provided from a retirement plan sponsored by their employer. Logically, from the employee's perspective, the adequacy of an employer-sponsored retirement plan lies in its ability to help employees reach their retirement income goal. Furthermore, this "goal" has often been defined as achieving a stream of income that would allow for a certain standard of living after retirement measured as the "replacement ratio" or "income replacement ratio" (McGill et al. 2002, p. 373). Specifically, this ratio is equal to the percentage of preretirement income provided for by postretirement income. Thus, one can say that a retirement plan is "adequate" if it allows the employee to achieve a certain replacement ratio.

At the same time, the employer has an objective of managing costs. The overall cost of employee benefits is quite substantial, making it absolutely essential that the employer structure its program in the most efficient way possible (Allen 2002, p. 38). This delicate balance has put great reliance on actuaries to design retirement plans with the ultimate precision. Too modest a benefit formula and the employees will be left with a plan that is inadequate. On the other hand, too rich a benefit formula may jeopardize the employer's ability to control costs and stay competitive.

While achieving an income replacement level is most efficient with a final-pay defined benefit (DB) plan (a retirement plan that explicitly defines the benefit to be a certain percentage of final salary), a number of factors, such as increased administration expense and lower employee appreciation, have led employers to prefer defined contribution (DC) plans in recent years. Furthermore, there are a number of advantages for an employer to offer a DC plan where the account balance is held entirely in company stock. However, this comes at the expense of the "participant" (employee who is eligible to receive benefits from a retirement plan), who is penalized with the transfer of risk.

In evaluating these various designs, a model that calculates the replacement ratio[1] that will be provided from the plan is needed. In addition, since the participant's retirement date can be many years away, the model will need to incorporate future investment returns as well as the length of his or her career. For a DB plan, determining the replacement ratio involves projecting an employee's salary to the date of retirement and applying the benefit formula. For a DC plan, it involves projecting the employee's account balance to the date of retirement (using projected

[1] Actuaries commonly target *replacement ratio* rather than absolute level of wealth at retirement when designing benefits. An adequate retirement plan is one that facilitates retirement; therefore, the benefit level should be compared against final salary, since participants would need to "replace" a certain level of income to retire.

salaries), calculating the level income derived from the account balance and then determining the ratio.

In general, employers seek to provide a retirement benefit that, together with a social program benefit, provides an adequate overall replacement ratio (Rosenbloom 2001, p. 579). In addition, employers usually assume that employees should meet some of their own needs. Plan design models are used with these considerations in mind and define a target replacement ratio that is based on the premise that the employer's plan is only responsible for a portion of the employee's total postretirement income. In other words, while participants need to reach a certain replacement ratio in aggregate, the amount to be provided by their employer is only one piece of the pie. Thus, the "target threshold" replacement ratio an employer may use to determine whether the plan it offers is adequate is usually lower than the total replacement ratio that a participant will need to reach in order to maintain a desired standard of living after retirement.

The "traditional model" used for designing a retirement plan is defined as one that determines the *adequacy* of a retirement plan design on the basis of whether or not the *replacement ratio calculated using actuarial assumptions* exceeds a certain target threshold. This process involves the following steps: First, the actuary selects assumptions for expected investment return and salary increases. Next, the projected benefit at retirement is determined using projected salaries, and (in the case of a DC plan) the *expected* account balance.[2] The annual benefit (in the case of a DC plan, the participant's account balance must first be converted into an annual annuity) is then divided by the expected final salary (usually the salary immediately preceding retirement) to determine the replacement ratio at retirement age.

However, since future investment returns, and even the length of an employee's career are random variables, what the traditional model is actually calculating is the *expected replacement ratio*. Determining adequacy solely from the expected replacement ratio strays away from the methodology of measuring value taught by financial economics. A retirement plan, like any other financial arrangement, should be evaluated based on the income level that is expected to be realized, the risk associated

[2] "Expected account balance" means the amount the participant will have in his or her retirement account, assuming actual future investment returns equal expected future investment returns.

with that expected amount and the risk aversion of the participant. Therefore, a particular plan design should be considered adequate if, and only if, it meets both the expected value *and* the risk constraints. Financial models that quantify expected value, risk and risk aversion do so through a utility function.

Suppose we treat the replacement ratio as a random variable. Furthermore, suppose we can identify the level of risk aversion of a participant based on his age. Since the replacement ratio will have an expected value and a standard deviation (risk) and we know the level of risk aversion of the participant, we can use a utility function to measure the value of the plan to the participant.

The use of a utility function in financial economics involves hypothesizing that an investor assigns a utility value or score to various portfolios based on the expected return and the risk of those portfolios (Bodie, Kane, and Marcus 2002, p. 151). He then chooses the portfolio with the greatest utility value. If we apply the same principle to retirement plans, we are hypothesizing that participants "score" various plan designs based on the expected replacement ratio *and* the replacement ratio *risk* of those designs. Hence, only plan designs where the *utility value* (rather than the expected replacement ratio alone) for a participant exceeds a target level would be considered adequate. As the results will indicate, this method for evaluating adequacy can have a significant effect on plan design.

Another role of traditional plan design models has been to illustrate the "winners" and "losers" under various designs. It is quite common that older, longer-service workers fare better under a final-pay DB plan while younger, shorter-service workers fare better under a DC plan. Therefore, the extent of a plan change, specifically a plan redesign, can leave certain participants better or worse off, depending on (among other factors) their age and length of service.

In addition to determining adequacy, the traditional model has been used to indicate which groups of employees would fare better (winners) or worse (losers) under alternative plan designs. As with the measure of adequacy, winners and losers are determined solely by the difference in expected replacement ratio. As a result, in a scenario where a DC plan replaces a DB plan, yet leaves the expected replacement ratio unchanged, the participant would neither be a winner or a loser.[3] This result is inconsistent with the teachings of financial economics: A financial arrangement with the same expected value and greater standard deviation is (to a risk-averse investor) less valuable.[4]

The model suggested in this analysis quantifies winners and losers under different retirement plan designs by the same methodology by which it determines adequacy: Participants whose *utility value* associated with the retirement plan shows an increase are winners, while those whose utility value exhibits a decrease are losers.

Two other results from the traditional model and their appropriateness in light of financial economics are addressed in this analysis as well. The traditional model quantifies the dollar amount of a winner or loser by measuring the difference in retirement income at the age of retirement. From a financial economics perspective, the loss (or gain) to a retirement plan participant resulting from a plan change should be measured at the time of the change. This analysis suggests alternative ways of measuring that amount. Finally, the *break-even age*—the age at which a DB plan becomes more valuable than the DC plan—is illustrated differently than in the traditional model. In determining the break-even age, the traditional model compares the replacement ratio derived from the accumulated benefits at each age under each plan. This analysis suggests, instead, that the utility value of each plan be calculated at each age and then compared.

Some of the primary risks a participant faces in a retirement plan can be summarized as follows:

- *Investment risk*: the risk that a participant's account won't realize anticipated investment gains over the course of his career.

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[3] Often the *presence* of risk is indicated by illustrating that different benefit values will result under different economic scenarios. However, the traditional model makes no attempt to place a value on the risk and alter the benefit formula appropriately.

[4] The replacement ratio under a DB plan is only risk free under certain circumstances (benefits insured by the federal government, salary increases with certainty, no preretirement decrements, etc.) This analysis initially assumes such a replacement ratio is completely risk free and then relaxes that assumption by incorporating withdrawal into the model.

- *Decrement risk*: the risk that a participant will not complete his career with one employer and the sum of the benefits from each of his plans will result in insufficient retirement income.

- *Salary growth risk*: the risk that an employee's salary will increase at a greater or lower rate than the plan sponsor had anticipated when designing the plan.

- *Mortality risk*: the risk that a retiree will outlive his retirement income.

In the traditional model, these are risks for which the actuary formulates deterministic assumptions (expected investment return, career length, salary scale and annuity conversion basis), thus ignoring the standard deviation of the replacement ratio. This analysis will include the first two risks (investment and decrement) in calculating the standard deviation of the replacement ratio and use a utility function to incorporate that measure into adequacy.

## 2. Methodology

The model presented herein determines the adequacy of a plan design by using a utility function that incorporates both the expected replacement ratio and the standard deviation of the replacement ratio measured at the *current* age of the employee. In choosing a utility function, certain assumptions must be made about the participants. We assume that the participants are risk-averse (i.e., they place a negative value on risk) and that they prefer a greater replacement ratio to a smaller one. In addition, risk aversion is assumed to be positively correlated with age.[5] For the purposes of the analyses presented herein, I make use of the following utility function that is consistent with these assumptions. The attained age utility value (AAUV) that a participant is assumed to assign to a particular plan design is calculated as follows:[6]

[5] This assumption is made on the basis that participants are less willing to accept replacement ratio risk as they get older because they have less time to recover if the investment results are poor. It is not made because participants anticipate "time-diversification"—an advantage of holding a riskier investment over a longer time horizon – —which had been argued to be a fallacy (see Kritzman 1992).

[6] The term AAUV indicates that the utility is measured at the current age of the participant, not at the retirement age.

$AAUV = E(RR) - A \bullet [Stdev(RR)]^2$,

- $E(RR)$ = the expected replacement ratio.
- $Stdev(RR)$ = the standard deviation of the replacement ratio.
- $A$ = a risk-aversion factor, representing the level of risk aversion of the plan participant. All participants are assumed to be risk-averse ($A > 0$). In addition, this factor is assumed to increase with age. Complete factors are provided in the appendix.

The exact calculation of the replacement ratio is as follows:

$$RR = \frac{\text{Annual total income provided by retirement benefits}}{\text{Salary in the year immediately preceding retirement}}$$

This utility function is similar in structure to ones used in evaluating the trade-off between risk and return in the selection of investments. In such functions, utility is a linear function of mean and variance of investment returns and is used to rank portfolios by assigning higher scores to those portfolios with better risk/return profiles. In other words, portfolios receive higher scores for higher expected investment returns and lower scores for higher volatility of investment returns (Bodie, Kane, and Marcus 2002, p. 151). In addition, the degree to which volatility detracts from the value of the portfolio depends on a risk-aversion parameter, which may vary for different investors. If we extrapolate this methodology to the evaluation of retirement plans (from the participant's perspective), we are hypothesizing that participants assign higher scores to plan designs with a higher expected replacement ratio and lower scores to plan designs with higher volatility of the replacement ratio. In addition, there is a risk-aversion parameter that determines the extent to which the participant places a negative value on risk (as measured by the standard deviation of the replacement ratio). Hence, we extrapolate the "two-moment" methodology used in utility functions that compares different investment portfolios in order to compare different retirement plan designs. As a result, we have a plan design model that is more consistent with the some of the basic techniques and principles used in financial economics models. At the very minimum, it is an improvement over the traditional model, which ignores the value of risk altogether.

One of the properties of this utility function is that each additional unit of risk (measured in units of standard deviation of the replacement ratio) requires a greater unit of expected return in order to maintain the participant's utility value. Thus, as the plan design becomes more risky, the amount of expected value must increase at a greater rate for the utility to remain the same.

This analysis illustrates the use of this model on five different types of retirement plans:[7]

- A final-average-pay DB plan.
- A money-purchase plan, where allocation is a fixed percent of salary held in a diversified fund.
- A profit-sharing plan, where investment is held in a diversified fund.
- A money-purchase plan, where investment is solely in employer stock
- A profit-sharing plan, where investment is held solely in employer stock

Since all of these formulas depend on a participant's salary, an assumption as to the distribution of future compensation increases is needed. For the purposes of this analysis, salaries are assumed to increase with certainty, at a rate of 4.5 percent per year even after the employee has terminated service with the employer.[8]

For DC plans, an assumption is needed for the distribution of investment returns over the participant's working years. For investment accounts held in the diversified fund, the one-year investment return follows a discrete probability distribution that has an expected return of 7 percent and a standard deviation of 10 percent. For accounts held in company stock, the expected return and standard deviation are 7.50 percent and 20 percent, respectively. The complete distributions are provided in the appendix. (See Tables A1 and A2.)

[7] The analysis assumes that the only difference between a money-purchase plan and a profit-sharing plan is that the employer can exercise discretion in the allocation amount under a profit-sharing plan. In actual practice, there are other significant differences that the employer would want to consider in choosing a design. These differences are ignored for the purposes of this analysis.

[8] Salary growth risk is another risk present in retirement plans. Because of this assumption, there is no salary risk present in the model. In practice, the salary growth should vary by scenario as well, increasing the standard deviation of the replacement ratio and, thus, applying the techniques of the model more accurately.

For profit-sharing plans, an assumption is needed for the distribution of allocation amounts. In a pure profit-sharing plan, the amount allocated to the employee depends on a discretionary election made by the employer. Since the allocation amount is unknown, it can also be modeled as a random variable. This analysis assumes that the allocation is a random variable with an expected value equal to the amount the company *expects* to allocate in a given year. Further details of this distribution are provided in the appendix. (See Table A3.)

This analysis illustrates the use of the model both with and without the use of preretirement decrements. The model uses decrements to calculate the probability of a participant's career being of a certain length. The preretirement decrements decrease by age and end at age 55. Also, for the purposes of this analysis, the decrements are assumed to be independent of the benefit formula. The complete table of decrements is listed in the appendix.

All participants are assumed to retire at age 65. All account balances are assumed to be converted to an annuity at retirement age by dividing the account balance by 10.[9]

In determining the values for the utility function, 5,000 random investment return scenarios were generated using a standard sampling procedure. Each scenario calculates the investment returns over the employee's lifetime using a random number generator and the appropriate probability distribution.[10] The account balance at retirement is then calculated using these returns. In addition, where withdrawal decrements apply, the model calculates the length of the participant's career using a random number generator and the probability distribution resulting from the use of the withdrawal decrements. There is no correlation assumed between investment returns and length of career.

[9] By assuming that the account balance is annuitized at retirement age at a predetermined factor, postretirement mortality risk is identical under both DC and DB plans. Since both plans provide benefits in the same form (a life annuity), the replacement ratios can be compared with one another.

[10] In cases where the employee terminates, the employee's account is still assumed to follow the distribution of returns until retirement age.

For DC plans, in scenarios where the participant terminates employment, the replacement ratio is calculated by projecting the final account balance out to retirement age using the array of investment returns in that scenario. For DB plans, the benefit at retirement is calculated based on the salary and service earned as of the date of termination (i.e., no pay indexing). In all scenarios, the employee's salary is assumed to increase at 4.5 percent per year even after he has terminated employment. Vesting, for all purposes, is immediate.

The replacement ratio is calculated for each scenario and tabulated. The expected replacement ratio (E(*RR*)) and the standard deviation of the replacement ratio (*Stdev*(*RR*)) are determined as follows:

$E(RR) = \Sigma RR_i / 5000$ $\quad$ $Stdev(RR) = (\Sigma(RR_i - E(RR))^2 / 4999)$^.5,
where *i* is the scenario number, varying from 1 to 5,000.

When using the model without preretirement decrements, the replacement ratio under each scenario is identical for the DB plan. In this case, $Stdev(RR) = 0$ and $AAUV = E(RR)$.

References to the traditional model use identical assumptions where applicable. The expected investment return and salary increases would be identical to the ones shown above.

By using this utility function, certain relationships about the participant's preferences are implied. First, the participant's decision is unrelated to his current wealth, or the type of assets he possesses outside of his employer-sponsored retirement plan. If the participant's personal savings were highly correlated to the assets he or she would potentially own in a retirement plan, it would increase the amount of risk he or she would carry on his or her *total* retirement income (including his or her own portion). The utility function used in this analysis decreases the utility value at an increasing rate as risk increases. Therefore, the effect of the risk in the retirement plan would be more significant.

Also, it is assumed that the participant measures risk independent of the *distribution* of the replacement ratio. Hence, the participant uses the same utility function regardless of the distribution of the replacement ratio. In addition, the selection of the risk-aversion parameter and investment returns presented here was done arbitrarily, based on the author's judgment.

This analysis is undertaken merely to suggest an alternative methodology. If this type of model were to be used in actual practice, it would require "fitting" the 'A' parameter by observing empirical evidence of participant choices regarding plan designs. In addition, it would require precise assumptions for the distribution of investment returns from a valid economic model.

One limitation of this utility function is that it may not be viable for excessively generous benefit formulas. For plan designs where the standard deviation of the replacement ratio is so great, relative to the expected replacement ratio, increasing the benefit formula beyond a certain point will cause the utility value to begin decreasing. This would imply that less is preferred to more, which is obviously unreasonable. Given a particular plan type, a participant would always prefer a more generous benefit formula to a less generous formula.

When using this type of model, one should be mindful in evaluating plans with *excessively* generous provisions. For example, based on the assumptions of this analysis, the AAUV model cannot be used for employer stock allocations in excess of 15 percent per year. For other plan types (those with a lower standard deviation of the replacement ratio), the model should develop reasonable results using any feasible benefit level.

Several analyses have used utility theory to compare the adequacy of retirement plans (see Nowiejski 2003), yet utility theory isn't used in actual practices. While the primary conclusions developed in this analysis are identical to others (utility theory is used to illustrate that equating the expected replacement ratio results in a DC plan that is less favorable than a DB plan), some of the details of this analysis differ. The following is a summary of the important differences:

- Rather than comparing the expected utility value of the accumulated value of benefits at retirement age, this analysis assumes participants "score" the plan design at their current age based on the expectation and volatility of their replacement ratio. This change assumes that participants place relatively greater value on plan designs that automatically provide income relative to final salary as compared to plan designs that provide similar absolute income, but with no explicit relationship to final salary.

- This analysis incorporates withdrawal before retirement age and is, therefore, able to illustrate the preference for DC plans (or other front-loaded formulas) of younger participants.

- Only the preretirement (accumulation) phase is considered. The postretirement or ("drop-down") phase is ignored by the simplifying assumption that all participants annuitize at retirement age using a known, predetermined conversion factor.

- This analysis is specifically framed in a way to illuminate the weaknesses of the traditional plan design model in light of financial economics. Retirement consultants, one hopes, will gain insight on the shortcomings of the traditional model, and what could result in a significant effect on plan design. It is not intended to suggest a model that can be utilized in current practice without any adjustments. Practical applications using a utility-based analysis should calibrate the model such that the economic and demographic inputs (salary scale, investment return, annuity conversion, withdrawal) are appropriately correlated to each other in each scenario. Again, see Nowiejski (2003) for a good example of this.

## 3. Discussion

### 3.1 Adequacy

The most effective illustration of the use of a utility-based plan design model can be seen when it used alongside the traditional model in the design of a retirement plan. Consider the following scenario: An employer is considering providing a retirement plan for its employees. The actuary determines, based on appropriate life cycle models, that employees need 40 percent of their final salary from the employer's retirement plan. (This amount, combined with social program benefits and the employee's savings, will provide complete retirement income. As indicated earlier, a carveout design like this is quite common.) The employer hires the actuary to determine the appropriate formula parameters to provide the specified retirement income in each one of five plan designs listed below:

- Final-average-pay DB plan.
- Money-purchase DC plan.
- Profit-sharing plan.
- Money-purchase plan held entirely in company stock.
- Profit-sharing plan held entirely in employer stock.

The plan is designed around a participant hired at age 35 with a salary of $50,000, and it is assumed he will work his full career (there are no preretirement decrements) with the employer. The results from the traditional model are illustrated in Table 1:

**Table 1**
**Calculation of Adequacy Using the Traditional Plan Design Model**

| Plan Type | Design Parameter | Benefit at Normal Retirement Age | Final Salary | Replacement Ratio |
|---|---|---|---|---|
| Final-Average-Pay DB | 1.39% of Final Three-Year Average Salary | $71,681 Annual Life Annuity | $179,202 | 40% |
| Money-Purchase | 9.27% of Salary Allocated Per Year | $716,807 Account Balance | 179,202 | 40 |
| Profit-Sharing | Expect Annual Allocations of 9.27% | $716,807 Account Balance | 179,202 | 40 |
| Money-Purchase: Employer Stock | 8.59% of Salary Allocated Per Year | $716,807 in Stock | 179,202 | 40 |
| Profit-Sharing: Employer Stock | Expect Annual Allocations of 8.59% | $716,807 in Stock | 179,202 | 40 |

The accrual rate for the DB plan is equal to the accrual rate that is needed for the annuity amount to reach 40 percent of projected final salary. For the DC plans, the annual allocation is an amount that, assuming the expected investment return is realized, would accumulate to an account balance that, when converted to an annuity, would provide the desired 40 percent replacement ratio. Since each one of these designs provides an expected replacement ratio equal to 40 percent, the traditional model would deem any one of these plan designs adequate for the sample employee. However, implying that the employee is indifferent between these designs is an unreasonable assertion in light of certain fundamental principles of financial economics.

Consider the difference between the first design and the fifth design. A profit-sharing plan, where the investment is held entirely in company stock puts the employee at the most risk with respect to his or her replacement ratio (the standard deviation is the highest). In contrast, a DB plan would provide the same replacement ratio with certainty. Clearly if these two financial arrangements were sold in the market, the first plan would cost considerably more. The difference in cost would be the extra risk borne by the participant under the profit-sharing/employer stock design. "Financial economics teaches that the value of risk is measured by the price necessary to dispose of it" (Bader and Gold 2003, p. 31). The difference in cost would be a risk premium demanded by the fifth plan owner. The use of the traditional model assumes that the plan participant doesn't assess a similar risk *penalty* onto a plan design. If it were assessed, as it is in the models typically used to evaluate other financial arrangements, this penalty would negatively affect the adequacy of the design.

The model presented in this analysis incorporates the risk penalty into the measure of adequacy. Consider Table 2, which illustrates the same five designs, only instead, the adequacy standard is determined based on whether the *utility value* exceeds 40 percent (assuming a risk-aversion factor of $A = 1$):

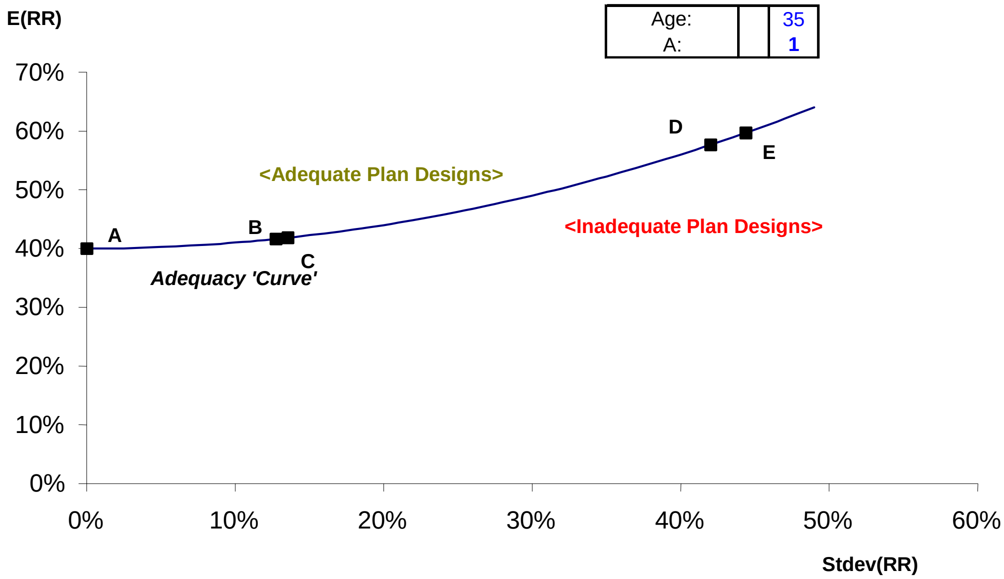
**Table 2**
**Calculation of Adequacy Using a Utility Model**

| **Plan Type** | **Design Parameter** | **E(*RR*)** | ***Stdev*(*RR*)** | ***AAUV*(*A* = 1)** |
|---|---|---|---|---|
| Final-average-pay DB | 1.39% of Final Three-Year Average Salary | 40.00% | 0.00% | 40.00% |
| Money-Purchase | 9.64% of Salary Allocated Per Year | 41.63 | 12.75 | 40.00 |
| Profit-Sharing | Expect to Allocate 9.69% | 41.84 | 13.56 | 40.00 |
| Money-Purchase: Employer Stock | 12.43% of Salary Allocated Per Year | 57.65 | 42.01 | 40.00 |
| Profit-Sharing: Employer Stock | Expect to Allocate 12.87% | 59.70 | 44.38 | 40.00 |

Here the results are significantly different. To maintain an adequacy standard based on a utility level of 40 percent, plans that have a standard deviation of the replacement ratio (*Stdev*(*RR*)) greater than zero need to provide greater allocations to compensate for the additional risk. The results and contrast between this model and the traditional model are illustrated further in Figure 1.

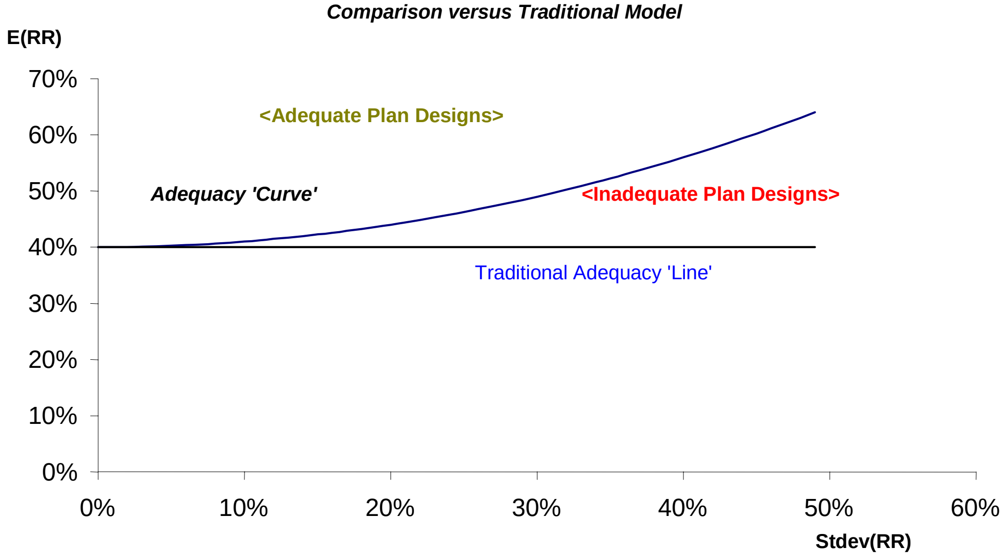
## Figure 1
## Measurement of Retirement Plan Adequacy

*Adequacy Standard: AAUV = 40%*




| Point | A | B | C | D | E |
|---|---|---|---|---|---|
| Type | Pension | Money Purchase | Profit Sharing | Employer Stock | Profit Sharing Employer Stock |
| E(RR) | 40.0% | 41.62% | 41.84% | 57.65% | 59.70% |
| Std(RR) | 0.00% | 12.75% | 13.56% | 42.01% | 44.38% |
| Normal Cost | 9.27% | 9.64% | 9.69% | 12.43% | 12.87% |

***Comparison versus Traditional Model***



The first result to notice is that the adequacy threshold is not a horizontal line, but rather a convex curve. Plan designs that lie on the curve (or at any point above the curve) would meet the adequacy standard, whereas plan designs that fall below the curve would not meet the utility-based adequacy standard. While the designs along a particular curve do not provide the same expected replacement ratio, the combination of expected replacement ratio and standard deviation of the replacement ratio is such that the participant would have the same utility value.

Each curve can then be thought of as an "indifference curve," since a participant would be indifferent between plan designs that provide the same utility. It is also important to note that no DC plan designed exclusively by the traditional model (equating the expected replacement ratio) would meet the adequacy standard. This is best illustrated in the second graph, where the traditional model results plot along a horizontal line that lies below the adequacy curve.

The costs illustrated in Figure 1 are calculated according to the conventional ways of illustrating retirement plan cost as a percent of payroll. The normal cost for the DC plans is simply the annual allocation to the account. The normal cost shown for the DB plan is calculated using the *entry age normal* (as a percent of compensation) funding method and a valuation rate of 7 percent[11] As indicated in the graph, the *reported cost* of the pension plan is the least expensive of the five designs. Since funding the DB plan can capture the risk premium before it is actually earned (it can discount liabilities at the expected return on assets without recognizing the cost of investment risk), it will always be able to provide the highest level of utility at the lowest reported cost. An employer providing the same utility value with a DC plan will have to recognize the replacement ratio risk and increase the contribution amount, therefore reporting a greater cost.

At this point, the model does not incorporate the probability of not reaching retirement age. Historically, this has always posed a difficult decision for plan sponsors, the degree of protection for workers who terminate service before retirement age. For some employers, this is not a consideration. In this case, the model would be run exactly as illustrated

[11] The calculation of the normal cost does not use any preretirement decrements and is based on the employer choosing a valuation rate of 7 percent based on the anticipated investment returns of the underlying assets.

and the following relationship will hold: *A DB plan converted to a DC plan, whereby the benefit formula is determined by equating the expected replacement ratio, will always result in a plan that is less adequate, regardless of the age of the participant.*

The switch to a profit-sharing plan also increases the standard deviation of the replacement ratio. The traditional model ignores this risk by assuming the employer always contributes the expected contribution amount. This example shows that the results are, in fact, not very significant. The additional allocation required so that the participant maintains his or her utility value for a profit-sharing plan is only slightly higher (9.69 percent) than the allocation required for the money-purchase plan (9.64 percent).[12]

### 3.2 Winners and Losers

Perhaps an even greater responsibility of a plan design model than determining adequacy is illustrating the concept of winners and losers. Since certain workers will inherently fare better under certain plan designs and fare worse under others, it is important for plan sponsors to contemplate this predicament when modifying their retirement program. Also, since few employers are implementing a plan where none exists, and because reducing benefits for older workers has the potential for much negative publicity, the analysis of winners and losers is often more valued than that of adequacy. The traditional model illustrates the concept of winners of losers in the same manner as it does adequacy: A participant who has a higher replacement ratio at normal retirement age (again, expected replacement ratio) is a winner, while one with a lower replacement ratio is a loser. The AAUV model differs in the measurement of winners and losers in the following ways:

- It measures the effect of the plan change at the time of the change.
- It determines a winner or a loser by a change in utility value.

[12] For older participants in a profit-sharing plan, the ratio of principal to investment income is greater. Therefore, the additional risk for profit-sharing plans may be more significant. The actuary should consider this development when designing profit-sharing plans for older participants.

The following example should illustrate these differences in winner-loser analysis: A participant originally hired at age 35 is now age 45 with 10 years of past service. In addition, his current salary is $50,000. His employer has a DB plan that provides a normal retirement benefit of 1.3924 percent of final three-year average salary per year of service. The employer wishes to terminate the DB plan and provide all future benefits through a money-purchase plan held in a diversified portfolio. The annual allocation to the money-purchase plan will be 13.64 percent of salary. Hence, the plan is to be designed so that this participant would neither be a winner or a loser under the traditional model as indicated in Table 3.

**Table 3**
**Winner/Loser Analysis Using the Traditional Model**

| | **Before Plan Change** | **After Plan Change** |
|---|---|---|
| 1. Years of Service | 10 | 10 |
| 2. Years of Service at Retirement | 30 | 30 |
| 3. Projected Final Salary | $115,393 | $115,393 |
| 4. Current Three-Year Average Salary | N/A | $45,816 |
| 5. Projected Three-Year Final Average Salary | $110,495 | N/A |
| 6. Annual Benefit Provided by DB Plan | $46,156 | $6,379 |
| 7. Replacement Ratio Provided by DB Plan (6. / 3.) | 40.00% | 5.53% |
| 8. Expected Account Balance to be Provided by Money-Purchase Plan | $0 | $397,776 |
| 9. Annual Income Derived From Expected Account Balance of Money-Purchase Plan (8. / 10) | $0 | $39,778 |
| 10. Replacement Ratio Provided by Money-Purchase Plan (9. / 3.) | 0.00% | 34.47% |
| 11. Total Replacement Ratio (7. + 10.) | 40.00% | 40.00% |

Since the total replacement ratio provided by the two plans is expected to be the same, the participant would be neither a winner nor a loser. This example should fully illustrate the weakness of the traditional model in light of financial economics. If the participant fully expects to complete his or her career with the employer, he or she is not indifferent between these two designs. The participant is not being compensated for the extra risk he or she bears under the new design and will be receiving less in value as measured by utility.

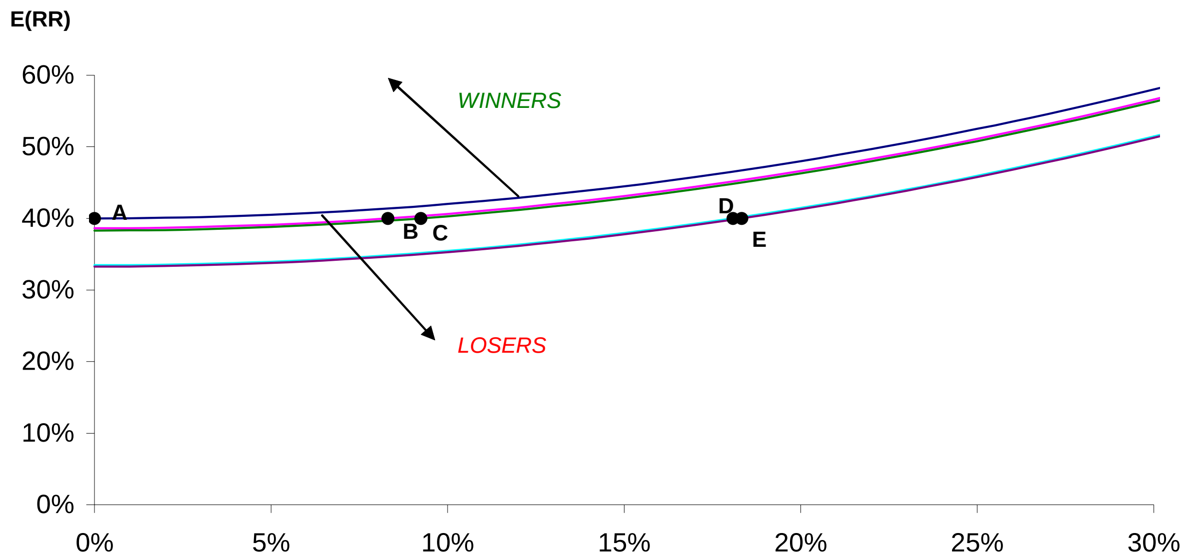
Now, consider Table 4, which illustrates the same scenario, only with additional results through the use of the AAUV model.

**Table 4**
**Winner/Loses Analysis Using a Utility Model**

| **Before Plan Change** | | **After Plan Change (*A* = 2)** | |
|---|---|---|---|
| 1. Expected Replacement Ratio | 40.0% | 1. Expected Replacement Ratio | 40.0% |
| 2. Standard Deviation of Replacement Ratio | 0.0 | 2. Standard Deviation of Replacement Ratio | 8.3 |
| 3. AAUV (1. – A x $2.^2$) | 40.0 | 3. AAUV (1. – A x $2.^2$) | 38.6 |

The AAUV model is able to assess a risk penalty and indicate that the new design is less valuable. As a result of this risk penalty, the participant will have a lower utility value and be a loser. The application of the AAUV model to a design where the pension is frozen and replaced by any of the four DC plans (money-purchase, profit-sharing, money-purchase/employer stock, profit-sharing/employer stock) is illustrated in Figure 2.

## Figure 2
## Winner/Loser Analysis Using a Utility Model




| | |
|---|---|
| Current Age: | 45 |
| Past Service: | 10 |
| Current Salary: | 50,000 |
| A: | **2** |

| | Design |
|---|---|
| **A** | No change |
| **B** | Freeze pension, money purchase going forward |
| **C** | Freeze pension, profit sharing going forward |
| **D** | Freeze pension, employer stock going forward |
| **E** | Freeze pension, ps in stock only going forward |

| **Point** | **A** | **B** | **C** | **D** | **E** |
|---|---|---|---|---|---|
| **E(RR)** | 40% | 40.0% | 40.0% | 40.0% | 40.0% |
| **std(RR)** | 0% | 8.3% | 9.2% | 18.1% | 18.3% |
| **AAUV** | 40.0% | 38.6% | 38.3% | 33.5% | 33.3% |

*Note: Current Plan: 1.392% of Final 3 year Average Salary - Defined Benefit*

The graph in Figure 2 indicates where each of the design scenarios (A-E) would plot on an axis of expected return and standard deviation of the replacement ratio. Next, a utility curve is plotted containing all of the points that have the same utility value as each of the designs. In the AAUV model, any participant who moves from one curve to another line, below the original, will be a loser. Since the parameters of each of these designs were determined by equating the expected replacement ratio only, as per the traditional model, each design leaves the participant no worse off.

However, since replacing a DB plan with a DC plan by equating the expected replacement ratio will always lower the participant's utility (same E(*RR*), higher *Stdev*(*RR*)) any one of these plan changes will result in the participant being a loser.

### 3.3 Quantifying the Dollar Amount of the Plan Change

Once winners and losers have been established, it is often desirable to quantify the dollar amounts associated with these participant gains and losses. The employer will likely be less concerned with nominal losses resulting from a plan change than significant losses. Here again, the model presented in this analysis will differ from the traditional method. In the traditional model, the gain or loss is simply the change in replacement ratio at the normal retirement age, either as a ratio or an actual dollar amount.

In the AAUV model, the change in the participant's value is equal to the investment required (positive or negative) on behalf of the participant to equate the utility after the plan change to the utility before the plan change. Thus, while the effect may first be "realized" at retirement age, the gain or loss to the participant occurs the moment the change is implemented. In addition, the cost to the participant is also a function of his or her age and risk aversion. Expressing the dollar amount in the AAUV model is accomplished this way in order to stay in accord with financial principles. (As soon as a plan change is implemented, the participant places a different value on the retirement plan.) In contrast, in the traditional model, the dollar amount is once again quantified by measuring the effect at the retirement age using the difference in expected replacement ratio.

Let's go back to the previous example (a plan change from a DB plan to a money-purchase DC plan). To maintain his or her utility at the level it was before the plan change, the participant could make a one-time allocation, or a "makeup piece," at the time of the plan change to his or her retirement plan. Since the cost to the employee is the investment required to maintain his or her AAUV, the loss to the employee is equal to the price of this makeup piece. An example of this concept is illustrated in Table 5. Here it is assumed that the employee maintains his or her prior utility value by making a one-time allocation to his retirement account in the same investment that his employer provides as an allocation.

**Table 5**
**Effect of Plan Change Using a Utility Model**

| | Before Plan Change | After Plan Change | Makeup Piece | Total |
|---|---|---|---|---|
| 1. E(*RR*) | 40.00% | 40.00% | 1.60% | 41.60% |
| 2. *Stdev* (*RR*) | 0.00% | 8.30% | .55% | 8.94%[13] |
| 3. *AAUV* | 40.00% | 38.62% | 1.59% | 40.00%[14] |
| 4. One-Time Cost | N/A | N/A | $5,091 | $5,091 |

*Notes*: Current plan: 1.392% final three-year average pay; new plan: 13.64% of salary money-purchase allocation in diversified fund; age: 45 (*A* = 2); past service: 10 years; current salary: $50,000.

To maintain the same utility the participant had before the plan change (40 percent), the participant must contribute $5,091 of his or her own money. This is the amount that should be identified as the gain or loss to the participant.

In reality, a makeup piece can be made with other investments.[15] For instance, a risk-free bond can be purchased with face value equal to the difference in utility multiplied by projected final salary and an annuity conversion factor. In the previous example, a risk-free bond that provides a guaranteed income at retirement age of 1.4 percent of final salary would keep the participant whole on a utility basis. If such a bond traded for less than $5,091, the employee could be kept whole at a lower cost. As a result,

[13] This includes the extra variance due to the correlation between the makeup piece and the DC plan.

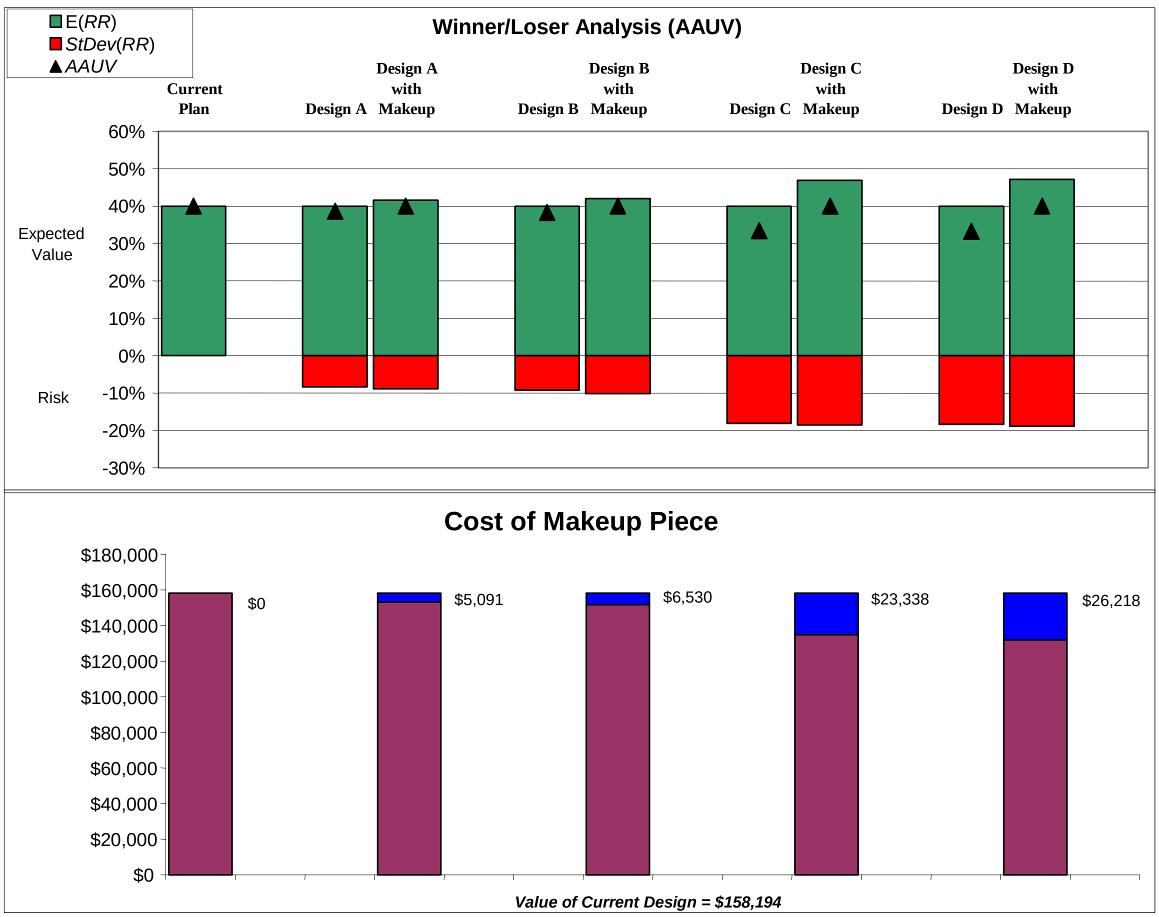
[14] The utility provided by the makeup piece evaluated alone is not equal to the additional utility it will provide when combined with the underlying plan. Since the utility function demands a greater amount of expected value for each marginal increase in risk, the utility of the underlying plan plus the makeup piece will always be less than the sum of the utility for each piece calculated separately.

[15] This theoretical analysis is assuming the employee can contribute a makeup piece with the same tax treatment that his or her retirement plan receives. Since, under a qualified plan, he or she would be contributing pretax dollars, the participant's cost would be lower than shown above. Under U.S. law, the participant would be limited to the extent of a tax deferral at a single point in time. For the purposes of this analysis, we ignore the tax considerations and assume that no such limit exists.

*the cost to the employee is the lowest cost where the employee can maintain the AAUV that had existed under the prior plan.*

Returning to the example where a final-pay DB plan is converted to one of four DC plans, Figure 3 illustrates the dollar amount associated with the winner or loser status of the hypothetical participant.

**Figure 3**
**Effect of Plan Change Using a Utility Model**




The top graph in Figure 3 demonstrates that the value of a design is composed of both expected value (expected replacement ratio) and risk (standard deviation of replacement ratio). Without any additional contribution, the utility value where risk is present is lower than the original plan where risk is absent. Also, we see that, when the specified contribution or makeup piece is added, the utility of the new plan *plus* the makeup piece is identical to the utility under the prior plan. Therefore, the loss of value to the participant is the price of this makeup piece.

The bottom graph in Figure 3 simply illustrates the relative value of the makeup piece as compared to the value of the current design.[16] As expected, the riskier designs (from the point of view of the participant) require a greater makeup piece.

### 3.4 Introducing Withdrawal

If this model represented participant preferences exactly, there would be no preference for DC plans over DB plans. Since this is obviously not the case in real life, (younger participants greatly value the higher accruals in the early years of DC plans), we can better reflect market conditions by introducing withdrawal decrements into the model. Again, this model introduces withdrawal in a fashion different from that of the traditional model. The traditional model reflects withdrawal merely by calculating and comparing the accumulated replacement ratio at different ages. This illustration reflects the earlier accumulation, or front-loading of DC formulas. In other words, a significantly larger portion of the replacement ratio is earned in the earlier years under a DC plan as opposed to a final-pay DB plan. This would imply that the DC plan provides greater value to a participant until the accumulated replacement ratio is less than it would be under the DB plan.

In contrast, the AAUV model simply incorporates withdrawal into the basic principle defining the model. The value of a plan to a participant is a function of the expected income it will provide for retirement, the risk associated with that expected amount and the risk aversion of the participant. Introducing withdrawal decrements simply introduces another risk. (Previously, investment risk was the only risk.) The most notable effect of including withdrawal in the model is that there will now be a nonzero standard deviation of the replacement ratio under the DB plan (as well as a

[16] The value of the current design is calculated as AAUV x final salary x 10, discounted from retirement age to attained age using 5.50% as a "risk-free" discount rate. In other words, the value of a current design is assumed to be equal to the price of a zero-coupon risk-free bond (we simplify greatly and use the same discount rate for all maturities) that would mature at the participant's normal retirement age providing a risk-free benefit. This benefit would have the same utility value as the current design.

significantly lower expected replacement ratio for younger participants).[17] As a result, the utility value for younger participants will be considerably higher under the DC plan than under the DB plan. In addition, the expected replacement ratio now has different meanings under the traditional model and the AAUV model. Therefore, we refer to the traditional model output as the *replacement ratio at normal retirement age* (RR at NRA), while the expected replacement ratio is the probability weighted replacement ratio.

An example of using the AAUV model with withdrawal decrements to illustrate the effect of a plan change is indicated in Table 6.

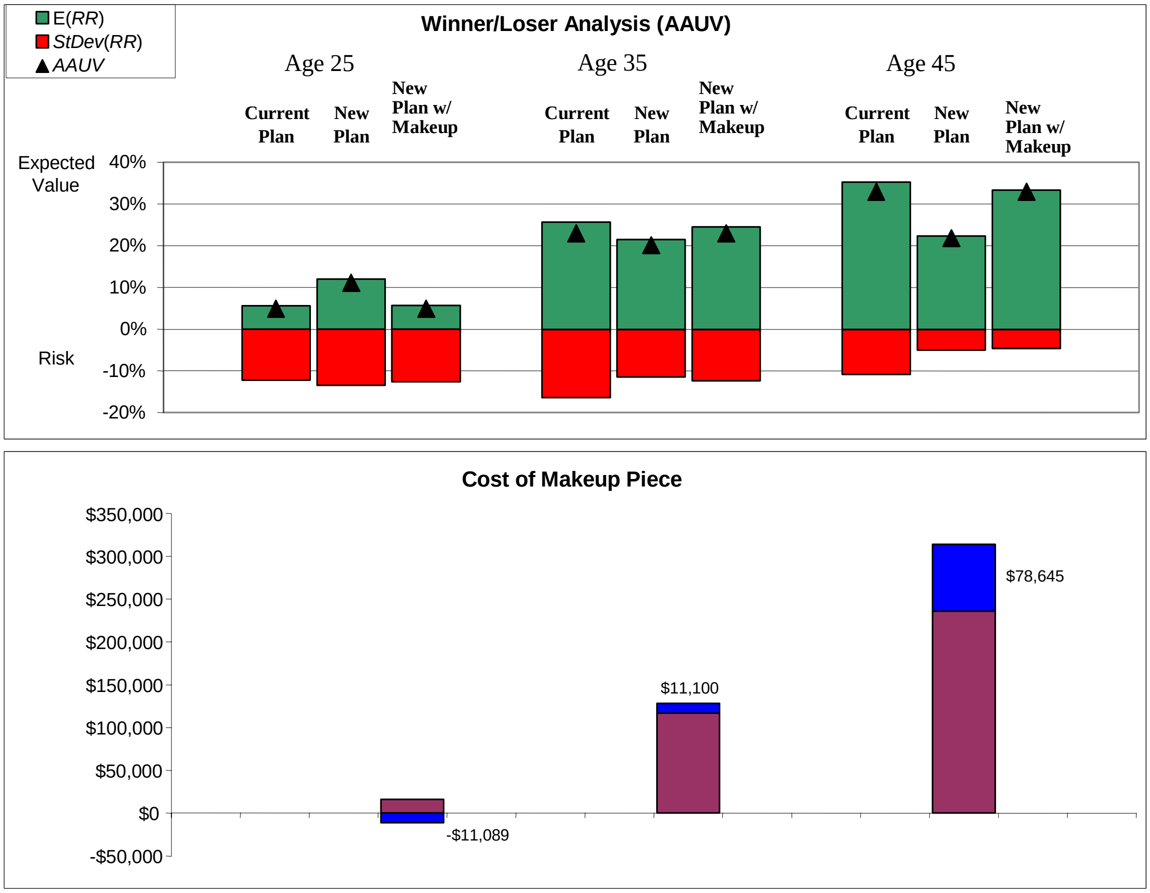
**Table 6**
**Effect of Plan Change Using a Utility Model that Incorporates Withdrawal**

| | **Age 25 (A = .5)** | | **Age 35 (A = 1)** | | **Age 45 (A = 2)** | |
|---|---|---|---|---|---|---|
| | Before Plan Change | After Plan Change | Before Plan Change | After Plan Change | Before Plan Change | After Plan Change |
| 1. *RR* at NRA | 40.0% | 40.0% | 40.0% | 29.0% | 40.0% | 24.0% |
| 2. E(*RR*) | 5.6 | 12.0 | 25.6 | 21.4 | 35.3 | 22.3 |
| 3. *Stdev*(*RR*) | 12.2 | 13.5 | 16.4 | 11.5 | 10.8 | 5.0 |
| 4. *AAUV* | 4.9 | 11.1 | 22.9 | 20.1 | 32.9 | 21.8 |

*Notes*: Current plan: 1.044% of final three-year average salary; proposed plan: freeze pension, provide ongoing 6.08% of salary allocation to money-purchase DC plan; age at hire: 25; salary at hire: $50,000.

[17] In each scenario, it is assumed that the employee's career is a given length determined with withdrawal decrements and a random number generator. The replacement ratio is then calculated based on the length of the employee's career. Since the employee's career length will not be the same in every scenario, the replacement ratio of the DB plan will not be the same in every scenario. Hence, *Stdev*(*RR*) > 0.

**Figure 4**
**Effect of Plan Change Using a Utility Model that Incorporates Withdrawal**




*Note:* Makeup pieces are shown relative to the value of the current design.

Figure 4 supports the results from the table. Each result is different than illustrated in the traditional model. Under the traditional model, the age-25 participant is indifferent between the two plans. Empirical evidence would not support this result. It is quite probable that most age-25 participants will not remain in service with the same employer until retirement age and, therefore, will not realize the increased benefit accruals in the later years of a final-pay plan. As a result, the utility provided from the DC plan will be greater than that of the DB plan. Hence, the age-25 participant benefits from the plan change. Now, since he or she is a winner, this participant can take a *negative* position in an investment if he or she wanted to keep his utility value at the same level as under the prior plan. The dollar amount of this negative position is the participant's gain. It is also important to note, that while the age-35 and age-45 participants are losers under both the traditional model and the AAUV model, the magnitude of the loss is smaller under the AAUV model.

As the traditional model illustrates, the DB plan will provide a significantly greater benefit at retirement age. However, since the DC plan is front-loaded, when we introduce withdrawal into the model, we see that the *expected* replacement ratio is only slightly higher. If the participant withdraws, the replacement ratio of the DB plan decreases much more than that of the DC plan. In addition, the standard deviation of the replacement ratio for the DB plan is greater (16.4 percent vs. 11.5 percent). As a result, the difference in utility value is considerably less than the difference in replacement ratio at retirement. Finally, we measure the loss at the participant's current age, since the effect of the plan change is realized at the current age.

Withdrawal decrements should be more useful in winner/loser analysis than in evaluating adequacy. Employers often do not design the plan with any concern for the adequacy of participants who do not complete their careers with the employer. For this reason, plan sponsors may wish to run this type of model without withdrawal to determine the adequacy level and then use the model again with decrements to conduct winner/loser analysis.

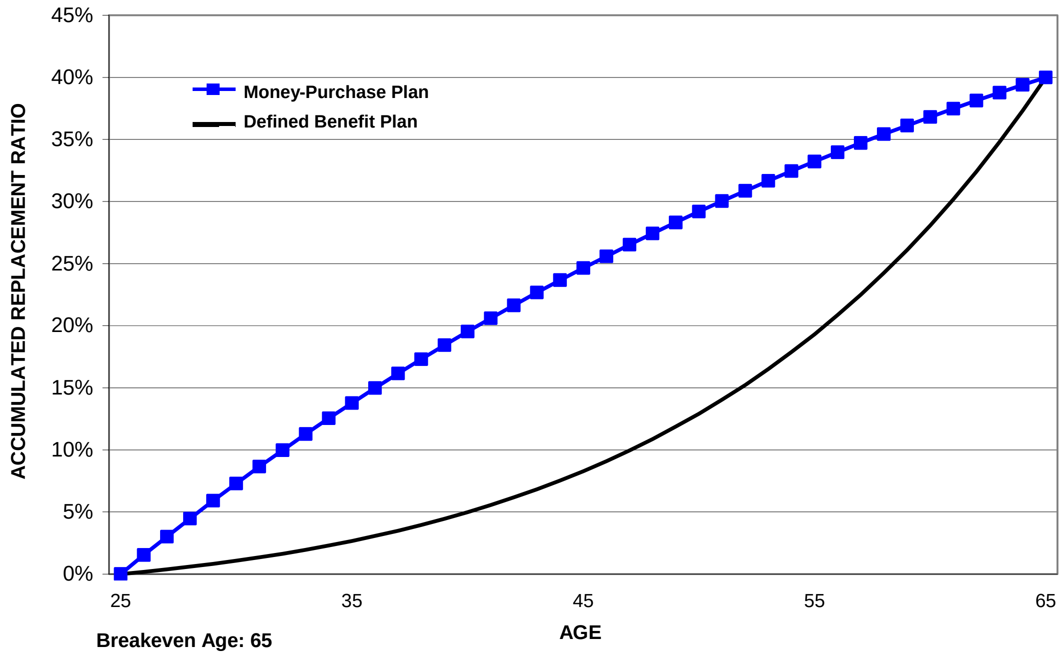
### 3.5 The Break-Even Age

Another significant difference between a utility-based model and the traditional model is in the depiction of the break-even age. When comparing the accrual patterns of two different retirement plan designs, we define the break-even age as "the age that a participant must reach (in-service) for one plan to become more valuable than the other (assuming the individual is a participant in both)." This concept is commonly used in illustrating the fact that DC plans are more valuable for younger participants than DB plans.

The traditional model illustrates the value of the plan at each age by the replacement ratio based on benefits accumulated as of the measurement date. In contrast, the AAUV model illustrates the value at each age using the utility value at each age. Figure 5 illustrates the difference between these two methodologies in comparing a final-average-pay DB plan with a money-purchase DC plan with the same replacement ratio at normal retirement age:

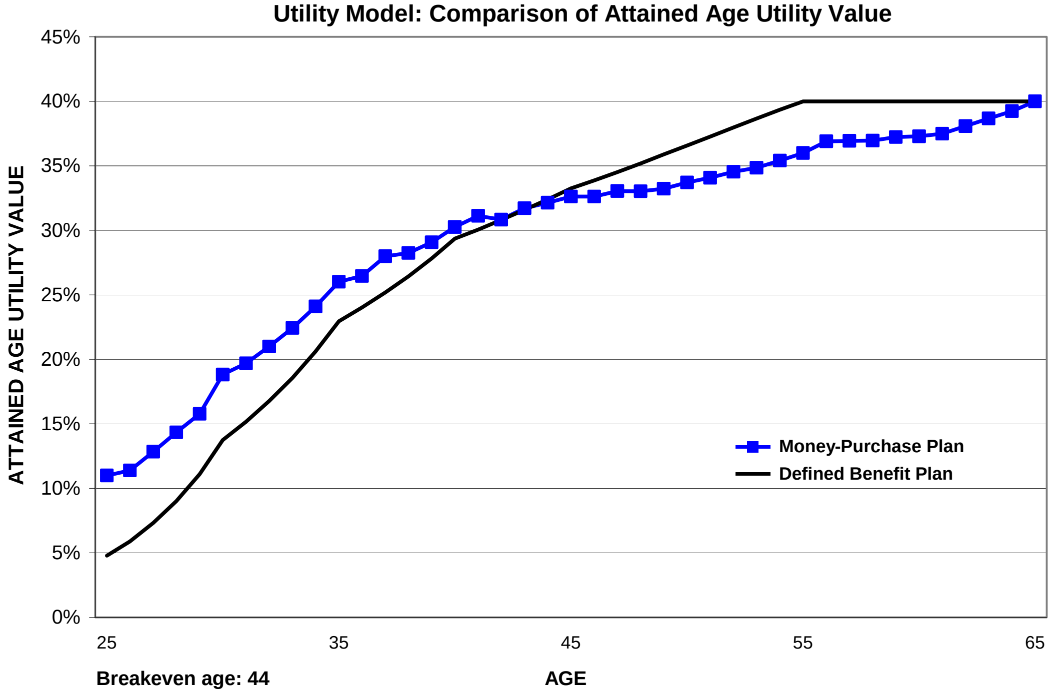
## Figure 5
## Comparison of the Break-Even Age in the Traditional Model Versus a Utility Model

**Traditional Model: Comparison of Accumulated Replacement Ratio**




**Breakeven Age: 65**

**Utility Model: Comparison of Attained Age Utility Value**




**Breakeven age: 44**

*Notes:* Defined benefit accrual rate: 1.044% final three-year average salary; money-purchase plan parameters: 6.08% of salary; investment return in money-purchase plan: 7%

The first graph in Figure 5 is the classic "accrual chart." It shows how DC plans accrue benefits at a greater rate early in a participant's career and then level off as the participant ages. In contrast, the final-pay DB plan provides most of the value in last few years and provides very little value in the early stages of a participant's career. If the plan is designed such that the expected replacement ratio is the same, the point where the accumulated replacement ratios are the same is at the normal retirement age. This point should be clear from the first graph. If the participant terminated employment at any time before the normal retirement age (and all assumptions are realized), he or she would have been better off with the DC plan. Therefore, the traditional model asserts that the participant is better off with the DC plan until he or she reaches normal retirement age.

The second graph in Figure 5 illustrates the use of the AAUV model. In this graph, the value of the plan at the attained age of a participant is equal to the utility value derived from his or her expected replacement ratio, the standard deviation of the replacement ratio (reflecting decrements) and level of risk aversion. The straight line is the utility value of the DB plan. Since the probability of an individual working until normal retirement age is very low when he or she is young, the expected replacement ratio is quite low (the high accruals in the last few years are unlikely to be realized). Thus, the DB plan provides very little utility in the younger years.

As the participant ages and earns more service, the expected replacement ratio increases and the standard deviation decreases, increasing the utility value of the plan each year.[18] When the participant works until an age where the probability of termination before retirement age is 0 (age 55 in this model), the utility value is at a maximum. It then remains at that maximum point until the participant retires. On the other hand, the DC plan provides a relatively higher utility while the participant

[18] Initially, the standard deviation increases as the expected replacement ratio increases (i.e., the participant has more to lose by terminating), but then decreases as the participant ages and becomes less likely to terminate. The data points supporting this graph are provided in Table A5 of the appendix. It is also important to note that the utility value is only an appropriate means of comparison for two plans at the same age. The same absolute utility value will have a greater dollar amount for an older participant, since he or she is closer to the normal retirement age.

is younger and then levels off as the participant ages. (In this example, we assume the expected return on investments is actually realized.)[19]

During the middle stages of his or her career, the participant experiences a minimal gain in utility. While the expected replacement ratio increases because the participant becomes less likely to terminate before retirement age, the standard deviation of the replacement ratio decreases very slightly. In addition, since the model assumes that the participant becomes more risk-averse as he or she gets older, the utility value increases very marginally.[20] The end result is that the value of the plans will cross much earlier in a participant's career. Using the assumptions of this analysis, the break-even age is 44. (See Table A5 in the appendix for the data points supporting this graph.)

This relationship also should pose a question about the adequacy of DC plans for older workers. Older employees who have had their DB pension plans redesigned as DC plans often were made "whole" by equating the expected replacement ratio. (Other older workers may have simply been cut back.) Actuaries have justified that setting the level of benefits such that the expected replacement ratio of the participant remains the same ensures fair treatment of older, longer-service workers. However, when a utility-based model is used, the value of DC plans for older workers is shown to be less than it would have been under the DB plan. This is true in light of the fact that a participant who terminates early would receive greater benefits under the DC plan. However, as the utility model indicates, in scenarios where the replacement ratio at normal retirement age is the same, the DC plan will

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[19] There must be an assumption about accumulated investment returns when performing a break-even analysis. In this example, if the DC plan experiences enormous investment returns, clearly there would be no break-even age. In contrast, if the plan's investments perform extremely poorly, the DB plan would provide greater utility and "break even" earlier than indicated in the example. When calculating the AAUV value at each age, it assumes the expected investment return has been realized on past investment returns, but future returns follow the same one-year random distribution.

[20] It is even possible that the utility of a DC plan may decrease, even though the expected return is being realized. This is because the increase in expected replacement ratio is not enough to offset the additional risk aversion. This, of course, is highly sensitive to the pattern of risk aversion. It does not mean that the plan has a lower dollar value. The same utility value provides a greater dollar value to an older participant, since he or she would be closer to the normal retirement age.

provide greater utility for younger workers and the DB plan will provide greater utility for older, longer-service workers.

## 4. Practical Example: The Switch to Employer Stock

The next example illustrates the use of the AAUV model on what would be a practical (and certainly a controversial) scenario in today's marketplace. Furthermore, this is a situation where the traditional model is useless. Suppose a plan sponsor wishes to implement a requirement that employees must maintain their retirement accounts (the portion attributed to employer contributions) entirely in the employer's stock. Such a requirement would drastically increase the standard deviation of the participant's replacement ratio, causing the utility value to decrease significantly. This would be especially true for workers with greater risk aversion. Table 7 and Figure 6 illustrate the use of the AAUV model in evaluating this plan change.[21] Separate examples are shown for participants with past service. (In these examples, the utility values reflect withdrawal and the plan change affects *future* allocations only. In other words, the accumulated account balance at the time of the plan change does not have to be converted into company stock.).

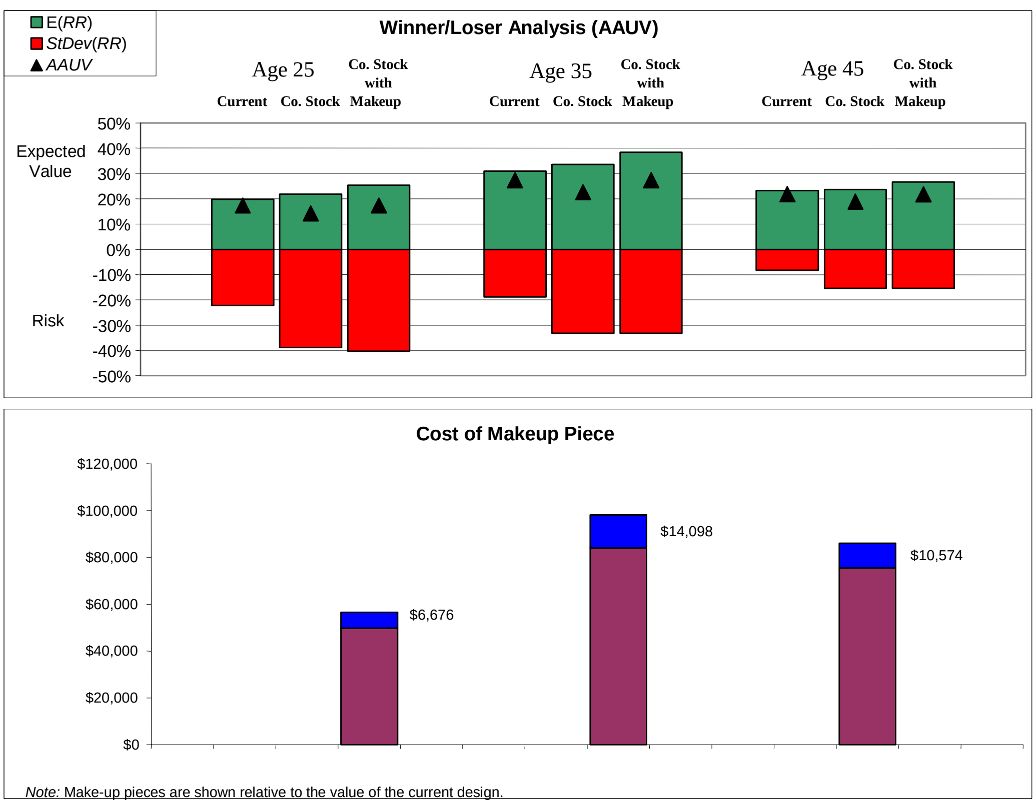
[21] These results assume that the stock is held until normal retirement age. This may be unreasonable in light of current practice where the participant may diversify after leaving the company. If the participant were able to diversify after leaving, than the standard deviation would be lower and the utility value would be higher than illustrated in this example.

## Table 7
## Utility Effect of Allocating Employer Stock in a Defined Contribution Plan

| | Age 25 (*A* = .5) Past Service: 0 Years | | Age 35 (*A* = 1) Past Service: 0 Years | | Age 45 (*A* = 2) Past Service: 0 Years | |
|---|---|---|---|---|---|---|
| | Diversified | Company Stock | Diversified | Company Stock | Diversified | Company Stock |
| 1. E(*RR*) | 19.74% | 21.77% | 30.89% | 33.60% | 23.11% | 23.62% |
| 2. *Stdev*(*RR*) | 22.20% | 38.87% | 18.93% | 33.20% | 8.20% | 15.50% |
| 3. *AAUV* | 17.27% | 14.22% | 27.31% | 22.60% | 21.76% | 18.82% |
| Makeup Piece | N/A | $6,676 | N/A | $14,098 | N/A | $10,574 |

*Notes*: Current plan: 10% money-purchase, in a diversified fund; proposed plan: 10% money-purchase, entirely in company stock.

## Figure 6
## Utility Effect of Allocating Employer Stock in a Defined Contribution Plan



Since the employer stock provides a comparable replacement ratio at retirement, the traditional model would not indicate this plan change as one that leaves the participant with less in value. However, when we use a utility-based model, it illustrates that there is a decrease in value.

Table 8 illustrates the results of the model when used where the participants have past service. In this example, since the participants have accumulated a significant portion of their retirement savings in a DC plan, the ramifications of future asset allocations become even more significant. Under the assumptions set forth in this model (increasing risk aversion with age and increasing marginal disutility of risk), a participant who has accumulated a large portion of his or her savings in a diversified fund would be looking to continue to diversify his investments or invest future allocations in risk-free assets.

Thus, requiring that all future allocations be made in a *single* stock has a relatively greater impact in a situation where the participant already has developed a portfolio that is highly correlated to stock.[22] In other words, since the participant already has developed a fairly high standard deviation of his replacement ratio, each *additional* unit of risk requires a greater amount of expected value in order for the participant's utility to remain unchanged. Therefore, workers with past service are affected more significantly than workers with no past service.

This analysis indicates another major difference between the traditional model and a utility-based model. In the traditional model, both the investment choice (aside from its expected return) and the composition of the individual's accumulated plan benefits are irrelevant in evaluating the value of a retirement plan. This example indicates that, when we incorporate utility into the model, both of these issues are, in fact, relevant because they affect the standard deviation of the replacement ratio. Furthermore, this effect can be quantified as shown in Figure 7.

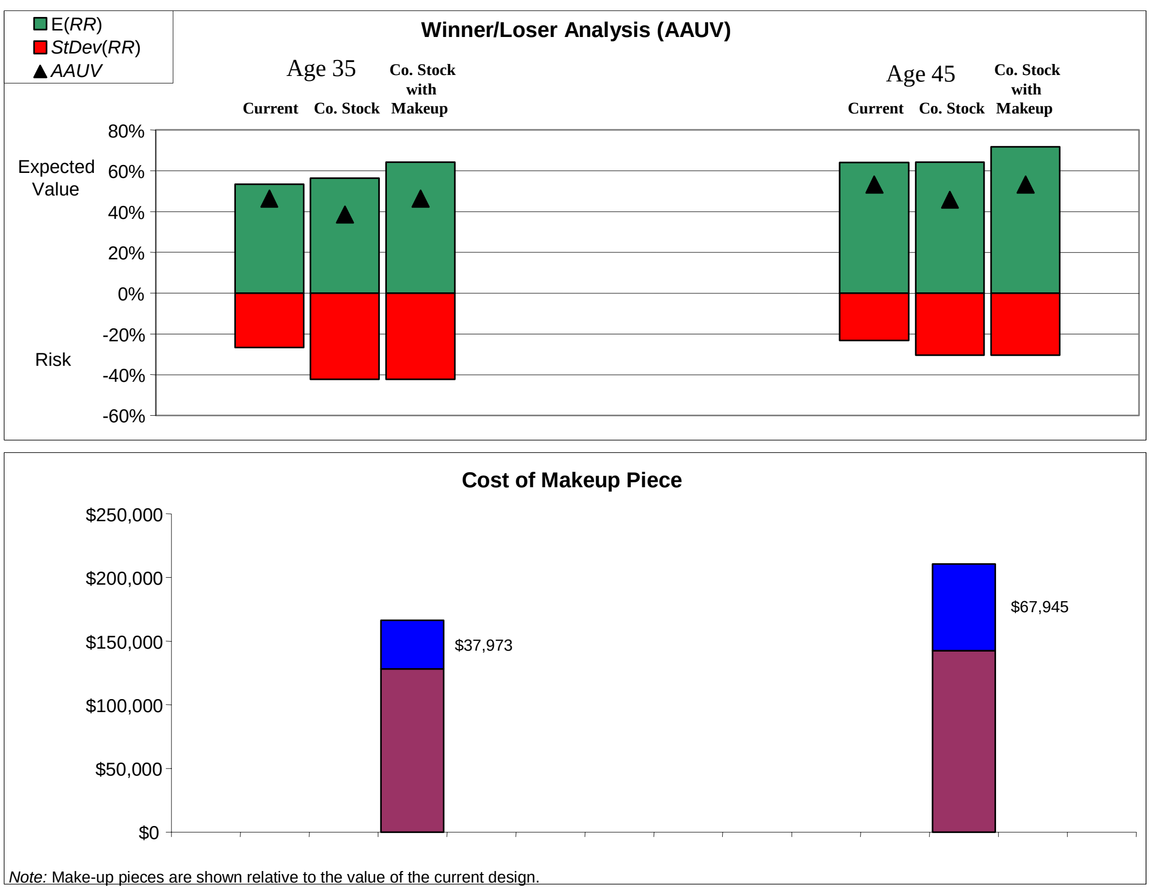
[22] In this analysis, the employer's stock is assumed to be highly correlated to the diversified fund. To the extent that stock serves as a hedge to the diversified fund, the effect of the plan change would be less disadvantageous (perhaps even advantageous) to the participant.

## Table 8
## Effect of Allocating Employer Stock to Participants with Past Service

| | **Age 35 ($A = 1$) Past Service: 10 Years** | | **Age 45 ($A = 2$) Past Service: 20 Years** | |
|---|---|---|---|---|
| | Before Plan Change | After Plan Change | Before Plan Change | After Plan Change |
| 1. E(*RR*) | 53.31% | 56.33% | 63.96% | 64.25% |
| 2. *Stdev*(*RR*) | 26.57% | 42.33% | 23.15% | 30.40% |
| 3. *AAUV* | 46.25% | 38.41% | 53.24% | 45.77% |
| Makeup Piece | N/A | $37,973 | N/A | $67,945 |

*Notes*: Current plan: 10% money-purchase, in a diversified fund; proposed plan: 10% money-purchase, entirely in company stock.

## Figure 7
## Effect of Allocating Employer Stock to Participants with Past Service



The requirement of employer stock to be held as the only investment option has been a widely publicized topic as of late. Corporate failures have left participants with far less assets in their retirement account than needed. From a risk-adjusted standpoint, the participant is clearly worse off. However, it is important to consider that employers have an incentive to provide DC allocations entirely in their own stock (tax-deductibility of dividends, keeping the company in friendly hands, etc.). This incentive has some financial gain to the employer. If this gain were passed directly to employees in the form of increased allocations, the participant could possibly be better off than had the employer chose to allocate less, but in a less risky investment.

The traditional model is completely useless in measuring this effect on employees. Clearly, the additional allocation will provide a greater expected replacement ratio. Therefore, if analyzed with the traditional model, the plan change would appear indisputably advantageous. However, with the allocation entirely in stock, there is considerably extra risk. A utility-based model would be able to measure this trade-off correctly. In other words, it would be able to determine which plan design has the greater value, a smaller allocation with less risk or a larger allocation with greater risk.

## 5. Conclusion

Suppose two individuals were offered two different financial arrangements. The first individual was offered an amount payable at a specified date in the future with certainty. The second individual was offered a payment at the same time in the future and could *expect* to receive the same amount as the first individual; however the second individual could receive more or less depending on the outcome of unknown future events. To assert that both of these individuals are equally well off is to dismiss the overwhelming evidence observed in financial markets that individuals attach a negative value to risk (Bader and Gold 2003, p. 31).

Unfortunately, many participants in retirement plans have had their plans redesigned in ignorance of this principle. The traditional model used to redesign retirement plans by equating the expected replacement ratio at a specified retirement age will design a plan that leaves the participant with less in value. Financial economics teaches that risk has a negative value that can be observed by the difference in prices between risky and risk-free investments in the market. Therefore, designing a plan in this fashion neglects this cost of risk and will leave participants worse off.

In light of the shortcomings of the traditional model, this analysis presents a utility-based model that can incorporate some of the risk inherent in a plan design. The model can evaluate the adequacy of a design incorporating its expected outcome, the standard deviation of the outcome and the level of risk aversion of the participant. This is, after all, how other financial arrangements are valued, and this analysis should carry over into evaluating the adequacy of retirement plan designs.

Using a utility-based model generates the following relationships, some of which are inconsistent with the results of the traditional model:

- A DB plan that is converted to a DC plan by equating the expected replacement ratio results in a plan that is *less adequate* for a full-career employee. The resultant plan has the same expected replacement ratio and a greater standard deviation of the replacement ratio and, therefore, will have a lower utility value.

- A DB plan that is converted to a DC plan where the investment is held entirely in the employer's stock by equating the expected replacement ratio results in a plan that is *significantly less adequate* for a full-career employee. The resultant plan has the same expected replacement ratio and a far greater standard deviation and, therefore, will have a significantly lower utility value.

- A DC plan where the investment is held entirely in the employer's stock (without the ability to diversify) is *less adequate* for a full-service career employee than is a plan that holds the investment in a less risky, diversified fund. This is assuming that these two plans have the same expected replacement ratio. This is in contrast to the traditional model, where the two plans are illustrated as equally adequate. In addition, a plan change whereby allocations in employer stock become mandatory is also a cutback in value. To the extent that the employee's accumulated benefits are highly correlated to stock, the plan change can be a significant cutback in value.

- Assuming increasing risk aversion with age, DC plans are less adequate for older, longer-service participants than illustrated by the traditional model. In turn, the addition of withdrawal decrements illustrates that

DC plans are more desirable for younger participants than the traditional model illustrates. This relationship may question the adequacy of DC plans for older, long-service workers, even if they were participants early enough in their career for their expected replacement ratio to meet a certain target threshold.

- An employer who offers both a DC plan and a DB plan can protect a wider range of plan participants than one who provides only one type of plan. Even if the replacement ratio at normal retirement age is the same, the utility value of DC plans is greater for younger workers and the utility value of DB plans is greater for older, long-service workers.

Perhaps even more important than determining the cost of a retirement plan is justifying that the plan itself was designed adequately. As retirement plans are financial arrangements, it is important that techniques used to measure their value are consistent with the teachings of financial economics.

While the traditional model has a logical approach, it is incomplete in its treatment of risk. To incorporate risk into plan design properly, a utility-based model such as the one presented in this analysis is required. I recommend the use of this type of model when assisting clients in designing a retirement plan. While this analysis is slightly more involved than the traditional model, it ensures proper treatment of workers and a measure of adequacy that is more in accord with the fundamental principles of financial economics.

## Appendix

### Table A1
### Distribution of Annual Investment Return for the Diversified Fund

| Return | Probability |
| --- | --- |
| – 15.0% | 5.0% |
| – 10.0 | 5.0 |
| – 5.0 | 7.5 |
| 0.0 | 10.0 |
| 5.0 | 7.5 |
| 7.5 | 30.0 |
| 15.0 | 20.0 |
| 20.0 | 15.0 |

### Table A2
### Distribution of Annual Investment Return for Employer Stock

| Return | Probability |
| --- | --- |
| – 40.0% | 2.5% |
| – 25.0 | 10.0 |
| – 10.0 | 15.0 |
| – 5.0 | 5.0 |
| 0.0 | 5.0 |
| 5.0 | 5.0 |
| 10.0 | 15.0 |
| 15.0 | 5.0 |
| 20.0 | 10.0 |
| 25.0 | 10.0 |
| 30.0 | 15.0 |
| 50.0 | 2.5 |

**Table A3**
**Distribution of Profit-Sharing Allocation**

| Allocation | Probability |
|---|---|
| 0% | 10% |
| 50% of the target amount | 10 |
| Target amount | 50 |
| 150% of the target amount | 30 |

**Table A4**
**Withdrawal Decrements**

| $X$ | $q_x^{(w)}$ |
|---|---|
| 25 | 20.0% |
| 26 | 20.0 |
| 27 | 20.0 |
| 28 | 20.0 |
| 29 | 20.0 |
| 30 | 10.0 |
| 31 | 10.0 |
| 32 | 10.0 |
| 33 | 10.0 |
| 34 | 10.0 |
| 35 | 5.0 |
| 36 | 5.0 |
| 37 | 5.0 |
| 38 | 5.0 |
| 39 | 5.0 |
| 40 | 2.5 |
| 41 | 2.5 |
| 42 | 2.5 |
| 43 | 2.5 |
| 44 | 2.5 |
| 45 | 2.0 |
| 46 | 2.0 |
| 47 | 2.0 |
| 48 | 2.0 |
| 49 | 2.0 |
| 50 | 2.0 |
| 51 | 2.0 |
| 52 | 2.0 |
| 53 | 2.0 |
| 54 | 2.0 |
| 55+ | 0 |

**Table A5**
**Data Supporting Break-Even Graph**

| | | Defined Contribution Plan | | | Defined Benefit Plan | | |
|---|---|---|---|---|---|---|---|
| Age | A | E(RR) | Stdev(RR) | AAUV | E(RR) | Stdev(RR) | AAUV |
| 25 | 0.50 | 0.1200 | 0.1349 | 0.1100 | 0.0559 | 0.1220 | 0.0477 |
| 26 | 0.55 | 0.1264 | 0.1444 | 0.1139 | 0.0694 | 0.1330 | 0.0588 |
| 27 | 0.60 | 0.1428 | 0.1543 | 0.1285 | 0.0858 | 0.1442 | 0.0733 |
| 28 | 0.65 | 0.1609 | 0.1642 | 0.1434 | 0.1057 | 0.1549 | 0.0901 |
| 29 | 0.70 | 0.1762 | 0.1628 | 0.1577 | 0.1299 | 0.1645 | 0.1110 |
| 30 | 0.75 | 0.2099 | 0.1700 | 0.1882 | 0.1596 | 0.1715 | 0.1375 |
| 31 | 0.80 | 0.2196 | 0.1685 | 0.1969 | 0.1758 | 0.1734 | 0.1517 |
| 32 | 0.85 | 0.2350 | 0.1715 | 0.2100 | 0.1934 | 0.1741 | 0.1677 |
| 33 | 0.90 | 0.2501 | 0.1691 | 0.2244 | 0.2126 | 0.1731 | 0.1857 |
| 34 | 0.95 | 0.2668 | 0.1648 | 0.2410 | 0.2336 | 0.1700 | 0.2061 |
| 35 | 1.00 | 0.2863 | 0.1615 | 0.2602 | 0.2564 | 0.1640 | 0.2295 |
| 36 | 1.10 | 0.2932 | 0.1612 | 0.2646 | 0.2682 | 0.1598 | 0.2402 |
| 37 | 1.20 | 0.3110 | 0.1609 | 0.2799 | 0.2804 | 0.1546 | 0.2518 |
| 38 | 1.30 | 0.3148 | 0.1578 | 0.2824 | 0.2930 | 0.1483 | 0.2644 |
| 39 | 1.40 | 0.3243 | 0.1546 | 0.2908 | 0.3060 | 0.1407 | 0.2783 |
| 40 | 1.50 | 0.3377 | 0.1531 | 0.3025 | 0.3194 | 0.1314 | 0.2935 |
| 41 | 1.60 | 0.3462 | 0.1478 | 0.3112 | 0.3260 | 0.1261 | 0.3006 |
| 42 | 1.70 | 0.3443 | 0.1455 | 0.3084 | 0.3328 | 0.1205 | 0.3081 |
| 43 | 1.80 | 0.3529 | 0.1409 | 0.3172 | 0.3395 | 0.1144 | 0.3159 |
| 44 | 1.90 | 0.3591 | 0.1408 | 0.3214 | 0.3461 | 0.1079 | 0.3240 |
| 45 | 2.00 | 0.3658 | 0.1408 | 0.3261 | 0.3528 | 0.1008 | 0.3325 |
| 46 | 2.15 | 0.3658 | 0.1357 | 0.3262 | 0.3581 | 0.0948 | 0.3387 |
| 47 | 2.30 | 0.3699 | 0.1310 | 0.3304 | 0.3632 | 0.0885 | 0.3452 |
| 48 | 2.45 | 0.3717 | 0.1300 | 0.3303 | 0.3683 | 0.0818 | 0.3519 |
| 49 | 2.60 | 0.3755 | 0.1290 | 0.3323 | 0.3733 | 0.0747 | 0.3588 |
| 50 | 2.75 | 0.3799 | 0.1247 | 0.3371 | 0.3782 | 0.0672 | 0.3658 |
| 51 | 2.90 | 0.3848 | 0.1231 | 0.3409 | 0.3829 | 0.0591 | 0.3728 |
| 52 | 3.05 | 0.3878 | 0.1178 | 0.3454 | 0.3875 | 0.0502 | 0.3798 |
| 53 | 3.20 | 0.3906 | 0.1147 | 0.3485 | 0.3919 | 0.0402 | 0.3867 |
| 54 | 3.35 | 0.3939 | 0.1090 | 0.3541 | 0.3960 | 0.0277 | 0.3935 |
| 55 | 3.50 | 0.4000 | 0.1070 | 0.3599 | 0.4000 | 0.0000 | 0.4000 |
| 56 | 3.75 | 0.4000 | 0.0908 | 0.3691 | 0.4000 | 0.0000 | 0.4000 |
| 57 | 4.00 | 0.4000 | 0.0876 | 0.3693 | 0.4000 | 0.0000 | 0.4000 |
| 58 | 4.25 | 0.4000 | 0.0847 | 0.3695 | 0.4000 | 0.0000 | 0.4000 |
| 59 | 4.50 | 0.4000 | 0.0785 | 0.3723 | 0.4000 | 0.0000 | 0.4000 |
| 60 | 4.75 | 0.4000 | 0.0757 | 0.3728 | 0.4000 | 0.0000 | 0.4000 |
| 61 | 5.00 | 0.4000 | 0.0707 | 0.3750 | 0.4000 | 0.0000 | 0.4000 |
| 62 | 5.25 | 0.4000 | 0.0605 | 0.3808 | 0.4000 | 0.0000 | 0.4000 |
| 63 | 5.50 | 0.4000 | 0.0491 | 0.3867 | 0.4000 | 0.0000 | 0.4000 |
| 64 | 5.75 | 0.4000 | 0.0362 | 0.3925 | 0.4000 | 0.0000 | 0.4000 |
| 65 | 6.00 | 0.4000 | 0.0000 | 0.4000 | 0.4000 | 0.0000 | 0.4000 |

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