A Bayesian Model for Developing an Optimal Mix of Defined Contribution and Defined Benefit Plans

Armand Yambao, FSA, MAAA, EA

Presented at The Great Controversy: Current Pension Actuarial Practice in Light of Financial Economics Symposium
Sponsored by the Society of Actuaries

Vancouver

June 2003

(note: The quotations appearing in this monograph are exact, except where capitalization and punctuation were changed in keeping with modern style and grammar guidelines.)

Abstract

During the prosperous economic times of the 1990s, employees preferred defined contribution (DC) plans, where high investment returns were directly added to their accounts. On the other hand, many employers who sponsored defined benefit (DB) plans enjoyed contribution holidays and pension income from the accounting of their pension plans. However, during the tough economy in recent years, employers were surprised by the large required cash contributions for their DB plans. Also, employees suddenly realized that the benefit from their DC accounts might not be sufficient for their retirement.

Employers are usually faced with the issue of whether retirement benefits provided to employees should be through a DB plan, a DC plan or a combination of both. Several objectives may affect the decision-making process for the employer; a common one is to have predictability in cash contributions for retirement plans. From the employee's perspective, an objective is to have a sufficient retirement benefit. This paper will focus on these two objectives. A Bayesian model is used to develop an optimal mix of DB and DC plans that achieves both of these objectives. In the process of developing the model, a valuation methodology is presented. This methodology covers the determination of cash contributions for the retirement plans.

1. Background

Employers are usually faced with the problem of determining the best way to provide retirement benefits to their employees. The factors that usually affect their decision include, but are definitely not limited to: budgetary constraints, administrative capabilities, ease of communication, employee appreciation and company culture.

Before deciding on the best program to provide retirement benefits, it is essential to understand the basic characteristics of both defined benefit (DB) and defined contribution (DC) plans. The fundamental difference between these two types of retirement plans is the party that bears the investment risk. In DB plans, the investment risk is borne by the employer; in DC plans, the employee bears the investment risk. Thus, the decision-making process to determine the appropriate mix of DC or DB plans can be viewed as determining how much investment risk can be borne by either party.

From the employer's perspective, the inherent investment risk in DB plans is usually felt when the plan assets experience losses. As a consequence, more cash contributions than previously expected will be required. However, gains in the DB plan assets will sometimes provide contribution holidays.

One of the most attractive features of DC plans to employers is predictability of required cash contributions. Employees feel the investment risk in DC plans when their plan assets perform worse than expected and, consequently, provide less retirement income. On the flip side, asset gains that are more than the employees expected provide a more wealthy retirement income.

Thus, if employers do not want to bear the investment risk for the retirement plans, a DC plan will be their choice. However, employees may prefer DB plans in order to avoid the investment risk from DC plans.

This paper presents a Bayesian model that will help both employers and employees articulate how much investment risk they are willing to bear. After identifying the risk tolerance of both groups, a range of optimal mixes of DC and DB plans will be identified.

2. Methodology and Assumptions

The theoretical framework for the model in the succeeding sections is based on the discussion of the Bayesian model by Lee (1997). In particular, Chapter 2 of his book describes the posterior distribution from a normal prior and likelihood.

The valuation methodology presented in this paper can be compared to the *individual aggregate method* as described in Aitken (1996). However, the U.S. funding rules for determining minimum-required and maximum-deductible contributions for qualified DB plans are ignored.

The simulation of the random variables in the illustrative example is based only on hypothetical data on equity returns, fixed income returns, inflation and other economic variables. More sophisticated models may be used to simulate these variables. The main purpose of the example is to show the applications of the Bayesian model that is developed in this paper. The main applications are: (1) designing a retirement plan that meets both employers' and employees' objectives and (2) determining the cash contribution for the DB plan.

In developing the illustrative examples, the following simplifying assumptions are made:

- Retirement age is 65.
- There are no preretirement decrements.
- There are no other benefits other than the retirement benefit.

3. Discussion

3.1 Employer's Objective

The investment risk of DB plans is manifested in the volatility of the required cash contributions. If the employer only provides a DC plan, the obligation that it faces for a particular employee is to contribute a fixed percentage of pay each year. When the employee retires, the benefits are fully funded and the employer does not have to make any additional cash contribution. Suppose that the employer made exactly the same amount of cash contributions in lieu of a DB plan that is promised for the same employee. When the employee retires, the amount in the account balance is not necessarily the same as the value of the benefit. The discrepancy between these two amounts can be viewed as a random variable. For the purposes of the Bayesian model that will be developed in this paper, this random variable would be referred to as the "deficiency."

Clearly, the deficiency for DC plans is always zero. For DB plans, we can express the employer's concern regarding the volatility of cash contributions as the deviation of the random variable "deficiency" from zero. The employer's risk tolerance can be expressed by its willingness to accept a maximum probability that the average deficiency would exceed a threshold amount. That is,

D = average deficiency
and
$\text{Prob}(D > D_{\max}) < k,$ (1)
where:

- $D_{\max}$ = maximum threshold deficiency
- k = maximum probability or chance that the deficiency would exceed $D_{\max}$.

Assume that the employer puts aside a fixed percent (p%) of pay every year into a retirement account. The goal is to determine what portion (DB_{alloc}%) of the fixed rate (p%) of contribution can be set aside to fund a DB plan that would still satisfy the maximum risk tolerance of the employer as shown in the above constraint. Clearly, an allocation of DB_{alloc}% = 0% (no DB plan) would satisfy the employer's objective.

3.2 Employee's Objective

From the employee's perspective, the concern regarding DC plans stems from the uncertainty of the retirement benefit that it can provide. When the employee retires, a DB plan provides a predetermined amount of benefit. However, the benefit under a DC plan is uncertain. The employee's concern at retirement age can be expressed as the uncertainty in the value of the total retirement benefit. The value of the total retirement benefit can be viewed as a random variable, and its mean will be referred to as:

B = average present value of total retirement benefit at retirement age.

The employee's risk tolerance can be expressed as the maximum probability that B would not meet a minimum threshold amount. That is,

$\text{Prob}\ (B < B_{\min}) < k,$ (2)
where:

- $B_{\min}$ = minimum total retirement benefit.
- k = maximum probability or chance that the retirement benefit would be less than $B_{\min}$.

Both the employees' objective and employers' objective, discussed in the previous section, restrict the allocation of the contribution towards a DB plan (DB_{alloc}%).

4. Results

4.1 The Bayesian Model

Let X_i denote the individual deficiency random variable for participant *i*. X_i is the discrepancy between the value of benefit and asset balance at retirement age. Assume that X_i follows a normal distribution with mean *D*. Also, assume that the mean *D* is a random variable with a normal prior distribution. That is,

$$D \sim N\,(\,D_{mean},\, Var_D\,), \tag{3}$$

where:

- D_{mean} = mean; this is dependent on the design of the retirement plan.
- Var_D = variance; this is dependent on the target asset allocation, which determines the inherent investment risk on the plan assets.

Using Bayesian statistics, the posterior distribution of *D* is also normal:

$$P(D \mid X_i) \propto P(X_i \mid D) \bullet P(D) \tag{4}$$

The posterior distribution of *D* can be used to determine the range of allocation to the DB plan that will satisfy the employer's objective from statement (1).

The deficiency for each participant *i* can be expressed as:

$$X_i = PVB_{db} - (DB_{alloc}\%) \bullet \sum_j (p\% \bullet Sal_j \bullet R_{dbj}), \tag{5}$$

where:

- PVB_{db} = presentvalue of the DB plan benefit at retirement age.
- DB_{alloc}% = allocation of the fixed *p*% of pay contribution for the funding of the DB plan benefit.

- Sal_j = salary at age j.
- R_{dbj} = asset return from age j to retirement age.
- j ranges from the later of hire age or age at plan inception to retirement age.

For simplicity, the formula for X_i above assumes that there is a single retirement age and no preretirement decrements. Also, the concept of the deficiency is measured at the single retirement age.

A similar approach can be taken with the random variable for the total value of the retirement benefit. Let Y_i denote the total value of the retirement benefit for participant i (sum of DB and DC benefit at retirement age). Assume that Y_i follows a normal distribution with mean B. Also, assume that the mean B is a random variable with a normal prior distribution. That is,

$$B \sim N\ (\ B_{mean}\ ,\ Var_B\ ), \qquad (6)$$

where:

- B_{mean} = mean; this is dependent on the design of the retirement plans.
- Var_B = variance; this is dependent on the target asset allocation which determines the inherent investment risk on the plan assets.

Using Bayesian statistics, the posterior distribution of B is also normal:

$$P(B \mid Y_i) \propto P(Y_i \mid B) \bullet P(B). \qquad (7)$$

The posterior distribution of B can be used to determine the range of allocation for the DB plan that will satisfy the employee's objective from statement (2).

The total retirement benefit at retirement age for each participant i can be expressed as:

$$Y_i = PVB_{db} + (1 - DB_{alloc}\%) \bullet \sum_j (p\% \bullet Sal_j \bullet R_{dcj}), \qquad (8)$$

where:

- PVB_{db} = present value of the DB plan benefit at retirement age.
- $DB_{alloc}\%$ = allocation of the fixed $p\%$ of pay contribution for the funding of

the DB plan benefit.

- Sal_j = salary at age j.
- R_{dcj} = asset return from age j to retirement age.
- j ranges from the later of hire age or age at plan inception to retirement age.

The Bayesian model presented above assumes that the prior distribution of D and B are normally distributed. Also, it is assumed that the likelihood of X_i's and Y_i's follow a normal distribution. If these conditions are not satisfied, the posterior distribution of D and B may still be estimated by data augmentation techniques discussed by Lee (1997).

The main purpose of using Bayesian statistics is to be able to interpret D and B as random variables. Under classical statistics, D and B would be considered fixed parameters, and it would not make sense to have a probability distribution for these parameters.

4.2 A Valuation Methodology

Suppose that a combination of DB and DC retirement plans has been designed that satisfies both the employers' and employees' objectives from statements (1) and (2). From the retirement plan design, a fixed percentage of payroll ($DB_{alloc}\% \bullet p\%$) will be set aside each year for the funding of the DB plan. Each year, a stochastic valuation will be performed to simulate the posterior distribution of the average deficiency (D). If there is a significant probability that the average deficiency (D) is less than a tolerable amount, a contribution holiday to fund the DB plan may be possible. On the other hand, if there is a significant probability that the average deficiency would exceed acceptable levels, additional contributions may be made to fund the DB plan.

Suppose that the valuation is done when the participant i, who was hired at age h, is currently age v. The deficiency for this particular participant i at the valuation date would be:

$$X_{iv} = PVB_{db} - (DB_{alloc}\%) \bullet \sum_{j} (p\% \bullet Sal_j \bullet R_{dbj}) - Assets_{iv} \bullet R_{dbv}, \qquad (9)$$

where:

- PVB_{db} = present value of the DB plan benefit at retirement age.
- DB_{alloc}% = allocation of the fixed percentage p% of pay contribution for the funding of the DB plan benefit.
- Sal_j = salary at age j.
- R_{dbj} = asset return from age j to retirement age.
- $Assets_{iv}$ = value of DB assets allocated for participant i at the valuation date; this is basically the accumulation of previous contributions for participant i.
- j ranges from the current age v to retirement age

The posterior distribution of the average deficiency D at the valuation date v is determined by simulation from the following distribution:

$$P(D \mid X_{iv}) \propto P(X_{iv} \mid D) \bullet P(D). \tag{10}$$

To determine if a contribution holiday can be justified for the current year, statement (9) for determining X_{iv} can be adjusted by making the index j run from age v + 1 to retirement age. If the resulting posterior distribution of D results in a significant probability that the average deficiency is less than a tolerable amount, then a contribution holiday may be justified.

On the other hand, if the posterior distribution of D indicates that there is a significant probability that the average deficiency is more than a tolerable level, an additional contribution may be made to improve funding level. The criteria for determining “significant probability” or “tolerable level of deficiency” will depend on the employer’s risk tolerance.

Essentially, the funding method is “individual aggregate” because the DB plan assets are allocated to each participant in order to assess the funding level of the plan. The major difference between this new approach and the traditional approach using the individual aggregate method is that the normal cost in the traditional approach is recalculated each year. Under the new approach, the fixed percent of pay (DB_{alloc}% • p%) is determined at the plan design stage, and the current contribution requirement is predetermined. Adjustments in the current contribution requirement may be necessary when there are significant deviations in the expected distribution of the average deficiency D, as discussed previously.

A “risky” asset allocation of the DB plan assets will be reflected in the volatility of the average deficiency. For instance, suppose D_{eq} is the average deficiency random variable if the assets are invested in 100 percent equities, whereas D_{bond} is the average deficiency random variable if the assets are invested in 100 percent bonds. The median for D_{eq} will be lower than the median for D_{bond}, but the 90th percentile for these two random variables may be close in value or even be the same. This reflects the greater volatility in the average deficiency for plans that are more heavily invested in equities. Thus, if two identical plans have the same amount of assets and the only difference is that one is more heavily invested in equities, then the valuation methodology presented here may not necessarily recommend a different amount for the current year's cash contribution. Under the current funding rules, a plan that is more heavily invested in equities can justify a higher funding interest rate to discount the liabilities and generate a lower cost for the plan.

Another advantage of the valuation methodology presented here is that it shows off the potential gains from investing more in equities by making the comparisons based on the median of D. At the same time, it does not hide the possible volatility of outcomes. Note that investing in a more conservative asset mix does not necessarily minimize the volatility of the average deficiency, but rather, the investment of the plan assets should match the nature of the plan's liabilities. For instance, the volatility of the average deficiency for a cash balance plan that credits interest based on equity index returns would be minimized if the assets were invested in similar equity indices.

4.3 Example: Plan Design Stage

Suppose that an employer's typical new hire is 35 years old with $30,000 annual salary. The employer is planning to sponsor a retirement plan and willing to contribute 6 percent of the payroll each year. The goal is to determine how much of this contribution should be split between a DC and a DB plan while meeting the following objectives:

1. Employer's goal: The probability that the average deficiency would exceed $100,000 is at most 1 percent.
2. Employee's goal: The probability that the average value of the retirement benefit would not exceed $200,000 is at most 1 percent.

For simplicity, assume that a cash balance plan is being considered by the employer. Thus, the PVB_{db} is just the accumulated account balance at retirement age. If a traditional final-average-pay plan were considered, then the simulation for the present value of benefits at retirement age (PVB_{db}) would be based on annuity purchase rates at retirement age.

The following assumptions are also made:

- Thirty percent of the assets (both DB and DC) are invested in equities and 70 percent are invested in fixed income.
- The average and standard deviation of equity returns are 7 percent and 16 percent, respectively.
- The average and standard deviation of fixed income returns are 3 percent and 3.5 percent, respectively.
- The average and standard deviation of inflation are 2.5 percent and 0.5 percent, respectively.
- The merit increase is fixed at 1.5 percent. The salary increase is inflation plus merit increase.
- The average and standard deviation of interest crediting to the cash balance are 3 percent and 0.01 percent, respectively.

To estimate the posterior distributions for *D* and *B*, simulations (2,000 trials each) were done for six possible values of DB_{alloc}% (0 percent, 20 percent, 40 percent, 60 percent, 80 percent, 100 percent). The results for the simulations on each value of DB_{alloc}% are summarized in Table 1. It was assumed that, if 6 percent of pay is contributed to funding of the cash balance plan, the cash balance pay credit is 9 percent (i.e., the pay credit is 150 percent of the cash contribution). Note that, as expected, the mean for both average deficiency *D* and average total benefit *B* goes up as the allocation to the DB plan increases. Also, as the allocation to DB plan increases, the volatility of *D* increases but the volatility of *B* decreases. Figures 1, 2 and 3 summarize the results in Table 1 and illustrate the range of values for DB_{alloc}% satisfying the objectives stated above.

If the employer's maximum tolerable *D* is more than $100,000, the maximum allocation for the DB plan will be more than 60 percent. Also, if the employer's risk tolerance is increased from 1 percent to 10 percent, the maximum allocation for the DB plan will increase from 60 percent to 80 percent.

From the employee's perspective, the optimal allocation for the DB plan will be less than 35 percent if the minimum average benefit is lower than $200,000 or the risk tolerance is more than 1 percent.

Table 1
Summary of Simulated Distribution of *D* and *B* for Certain Values of DB_{alloc}%

DB_{alloc}%	Random Variable	Mean	Standard Deviation
DB_{alloc}% = 0% DB benefit = None DC benefit = 6.0%	Average deficiency (*D*)	0	N/A
	Average retirement benefit (*B*)	$249,793	$43,674
DB_{alloc}% = 20% DB benefit = 1.8% cash balance pay credit (1.2% cash contribution) DC benefit = 4.8%	Average deficiency (*D*)	12,660	8,647
	Average retirement benefit (*B*)	262,452	35,214
DB_{alloc}% = 40% DB benefit = 3.6% cash balance pay credit (2.4% cash contribution) DC benefit = 3.6%	Average deficiency (*D*)	25,319	17,293
	Average retirement benefit (*B*)	275,112	26,873
DB_{alloc}% = 60% DB benefit = 5.4% cash balance pay credit (3.6% cash contribution) DC benefit = 2.4%	Average deficiency (*D*)	37,979	25,940
	Average retirement benefit (*B*)	287,771	18,810
DB_{alloc}% = 80% DB benefit = 7.2% cash balance pay credit (4.8% cash contribution) DC benefit = 1.2%	Average deficiency (*D*)	50,638	34,587
	Average retirement benefit (*B*)	300,431	11,619
DB_{alloc}% = 100% DB benefit = 9.0% cash balance pay credit (6.0% cash contribution) DC benefit = none	Average deficiency (*D*)	63,298	43,233
	Average retirement benefit (*B*)	313,090	8,107

Figure 1
Average Deficiency Goal

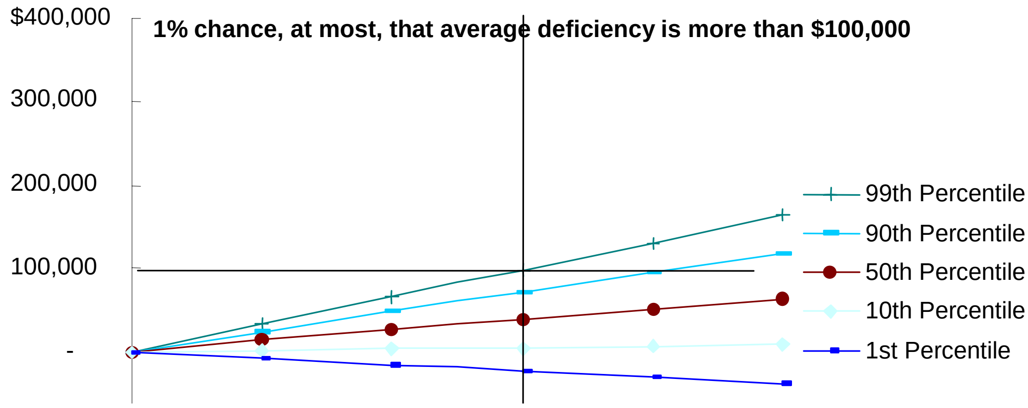
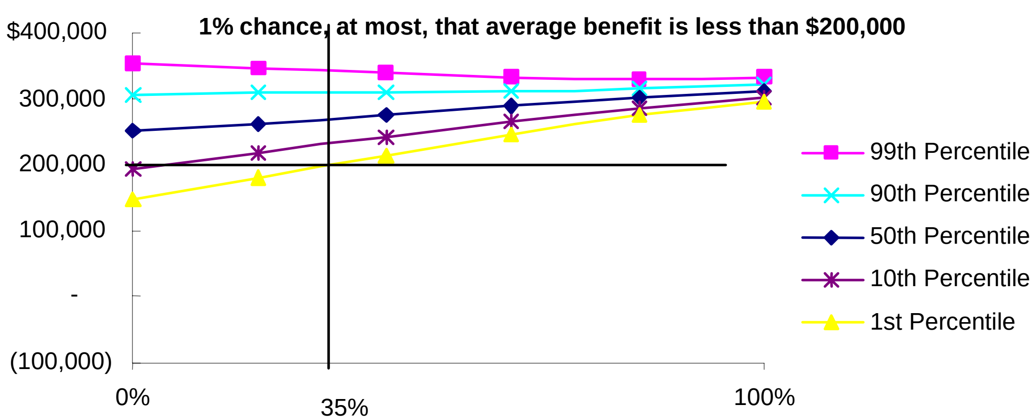
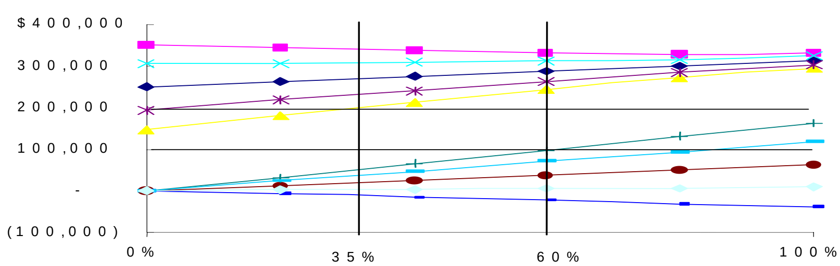


Figure 2
Average Benefit Goal



Contribution % for defined benefit is at least 35%

Figure 3
Average Deficiency and Average Benefit Goal



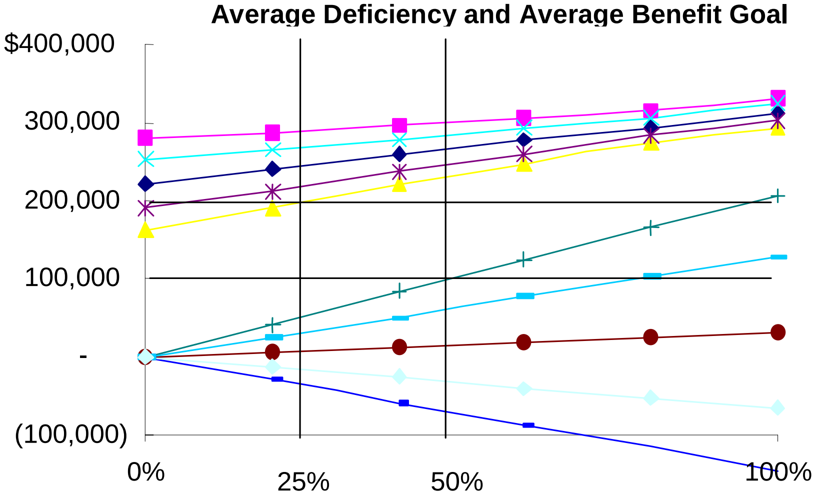
Contribution % for defined benefit is between 35% and 60%

4.4 Example: Effect of Changing Asset Allocation

In the previous example, it is assumed that 30 percent of both DB and DC assets are invested in equities. However, in general, employers—compared to individual employees—usually invest more assets in equity. If the DB asset equity allocation is increased from 30 percent to 50 percent, there will be more uncertainty on the possible values of the average deficiency. Thus, in order to still meet the employer's objective, the maximum allocation for DB plans will decrease from 60 percent to 50 percent.

On the other hand, if the DC equity allocation is decreased from 30 percent to 10 percent, the uncertainty of the average retirement benefit is reduced and the employee can tolerate more allocation for the DC plan while still meeting the original objective. The minimum allocation for DB plans will decrease from 35 percent to 25 percent. Figure 4 summarizes the changes in the distributions of the average deficiency and average benefit if the asset allocation is changed.

Figure 4
Average Deficiency and Average Benefit Goal



Contribution % for defined benefit is between 25% and 50%

percent of pay to DB funding and 3 percent of pay to a DC plan) and adopts a cash balance plan that provides a 4.5 percent pay credit. The next part of the example illustrates the valuation procedure presented in this paper. For this purpose, the following assumptions are made:

1. There were 100 employees covered by the new retirement plan when it was established on Jan. 1, 2002.

2. The characteristics of the employees are as follows:

Age	Pay	Number of Employees
22	$25,000	20
28	35,000	35
33	40,000	20
36	45,000	15
40	52,000	10

3. The DB plans assets were invested in 30 percent equities, and the asset returns from 2003 to 2005 were as follows:

Year	Asset Return
2003	6.00%
2004	5.50
2005	4.05

4. The interest crediting on the cash balance accounts were as follows:

Year	Interest Crediting
2003	5.00%
2004	6.05
2005	5.93

5. The actual pay increases were 5 percent each year.

6. The cash contributions from the 2002 to 2005 plan years were made on Dec. 31 of the plan year. The amount of each contribution was 3 percent of pay.

7. All other assumptions were realized as expected. That is, there were no decrements and no new entrants.

On Jan. 1, 2006, the assets would have accumulated to $515,700. The distribution of the average deficiency *D* is evaluated on Jan. 1, 2006, by simulating the deficiency X_{iv} (as defined in statement 9) for each of the participants. Table 2 shows the participant data on that date.

Table 2
Participant Data as of Jan. 1, 2006

Age	Pay	Number of Employees	Cash Balance	$Assets_{iv}$
26	$30,388	20	$5,270	$3,466
32	42,543	35	7,378	4,852
37	48,620	20	8,432	5,545
40	54,698	15	9,486	6,238
44	63,206	10	10,962	7,209

Using the previous assumptions on asset returns, inflation, salary increase and interest crediting, simulations were used (1,000 trials each) to estimate the posterior distribution of *D* under different asset allocations. The results are summarized in Figure 5. A higher equity allocation results in a potential lower average deficiency (based on the 50th percentile). However, the potential downside risk is also greater under higher equity allocations.

If the goal to minimize the cost of the plan (as measured by average deficiency) was based on the 50th percentile, a 100 percent allocation to equity would be optimal. This is similar to the current valuation methodology because it is a single-scenario approach using best estimate (50th percentile) assumptions. However, if the goal to reduce cost was based on the 90th percentile, the optimal asset allocation for equities is only 30 percent. Figure 6 highlights the comparison of the 50th and 90th percentiles.

Figure 5
Average Deficiency Distribution
Under Certain Asset Allocation

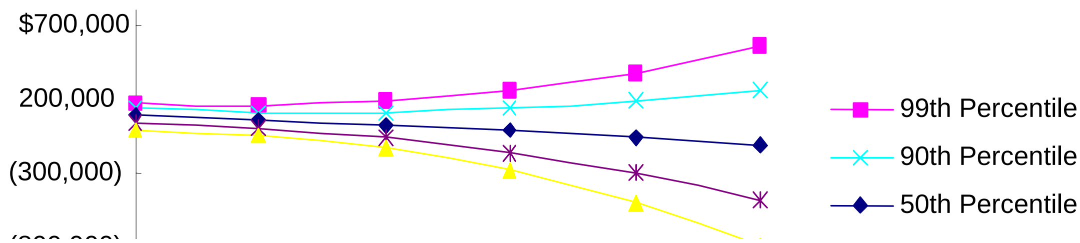
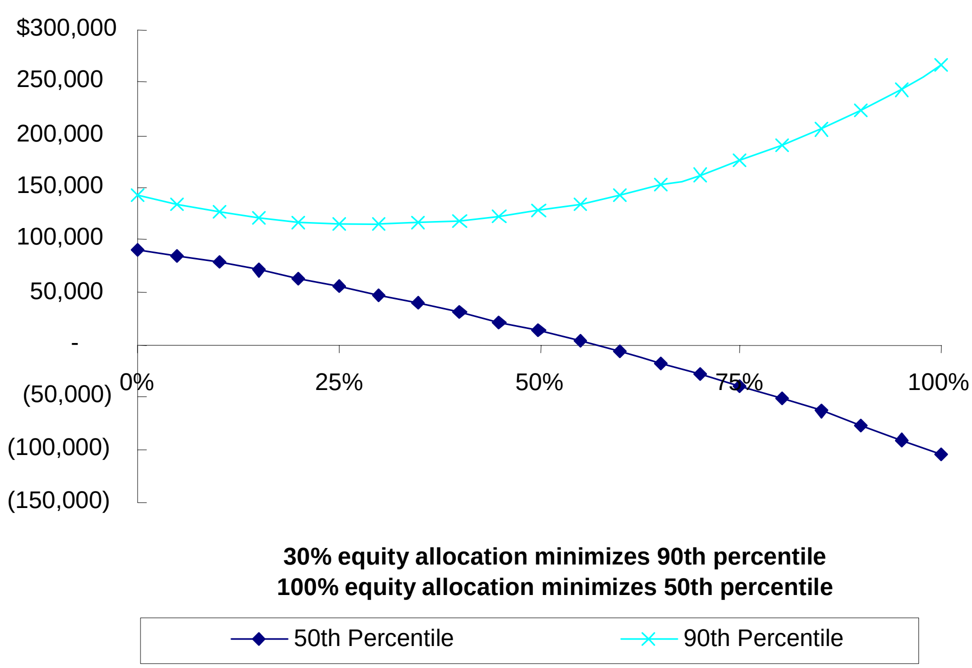


Figure 6
Comparison of 50th and 90th Percentiles of Average Deficiency



deficiency, an extra $100,000 contribution may be made to bring the assets up to $615,700. The additional assets may be allocated to individual participants by using the current percentage allocation of the assets. For instance, $672 of assets could be added to the assets of participants who are age 26. This is obtained by dividing $3,466 (current asset allocated to those age 26) by $515,700 (current total asset) and multiplying this by the $100,000 contribution.

Figure 7 illustrates the effect of an extra contribution on the 50th percentile under different asset allocations. For a 30-percent equity allocation, the average deficiency decreases by $6,555. If the equity allocation is higher, say 80 percent, the effect of an extra contribution on the average deficiency is more significant (decrease by $11,440). However, the effect of an extra contribution on the 90th percentile of average deficiency is less significant at higher equity allocations. As illustrated in Figure 8, the decrease in average deficiency under a 30-percent and 80-percent equity allocations are $5,143 and $452, respectively. This indicates that, if the decision to make a contribution is based on the 90th percentile, the extra contribution is worthwhile only if the asset allocation to equity is not significant.

Figure 7
50th Percentile Of Average Deficiency

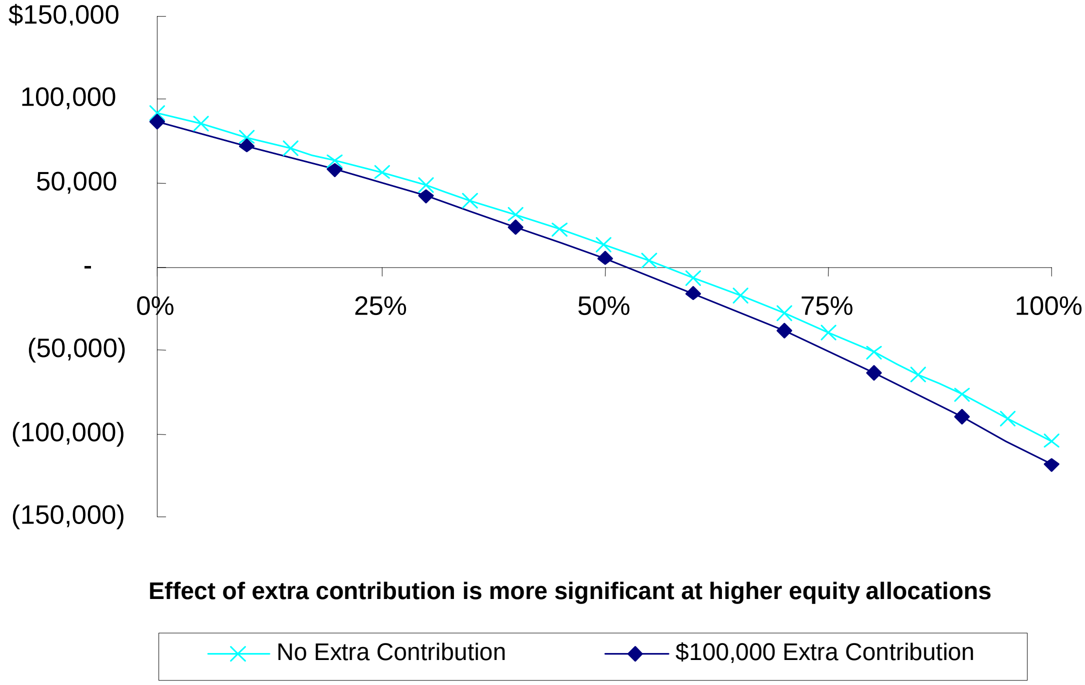
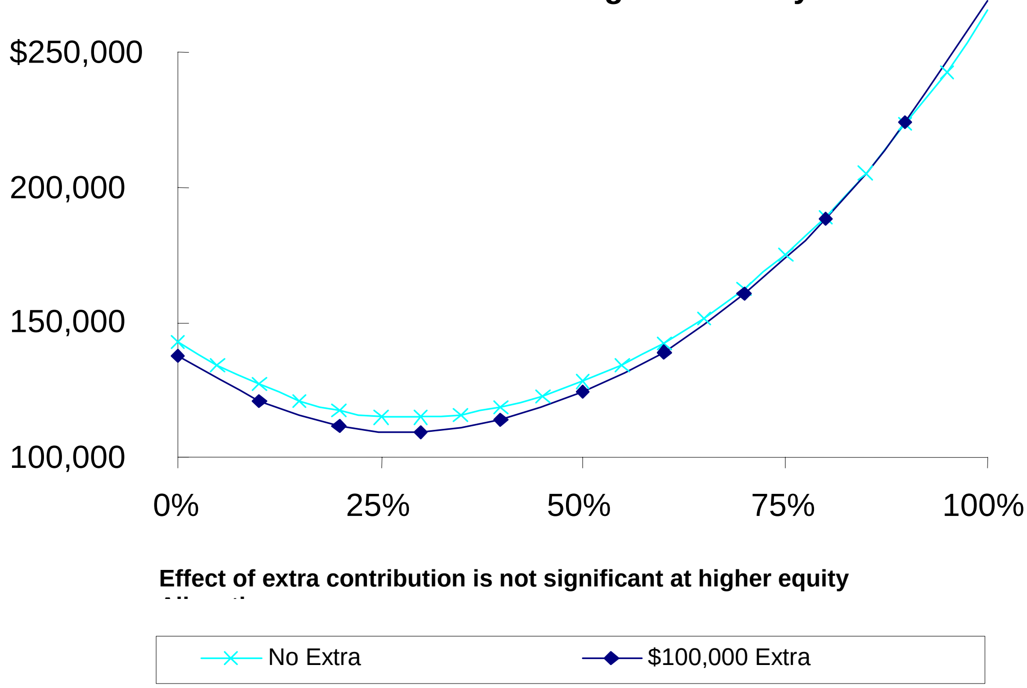


Figure 8
90th Percentile Of Average Deficiency



5. Conclusions

The Bayesian model presented here has two main applications. It is a tool for designing a total retirement package that takes into account the risk tolerance of both the employer and the employee. The employee's objective of meeting the retirement benefit goal can also be viewed as an employer's objective to provide more meaningful benefits. The Bayesian model also provides a tool for making decisions on how much contribution can be made toward the funding of the DB plan. In this aspect, the focus is the risk tolerance of the employer. The main advantages of this valuation methodology are: (1) It shows off the potential gains of investing in more equities, and (2) it highlights the risk involved in equity investments.

The model also may be used to explain part of the popularity of DC plans. If the employee's objective were ignored, a 100 percent allocation to DC plans would always satisfy the employer's objective. Also, a higher risk tolerance of the employee would include $DB_{alloc}\% = 0\%$ in the optimal solution set. DC plans have increased in popularity because DB plans are more complicated and expensive to administer. The extra expense for DB plans can be taken into account in this model by allocating a certain part of the DB_{alloc}% toward payment of the extra expense load. This would result to either higher values of D or lower values of B.

As illustrated in the example, the model at the valuation stage may come up with different optimal asset allocations, depending on the objectives and risk tolerance of the employer. If the 50th percentile is used as the criterion to minimize plan costs, the valuation methodology that was presented here will provide the same recommended asset allocation as the current methodology. An advantage of the methodology presented here is that it also shows the risk or worst-case scenarios (90th percentile) involved in equity investment. It was also illustrated that the decision to make an extra contribution will depend on the asset allocation and whether the plan costs are measured using the median results (50th percentile) or worst-case scenarios (90th percentile).

Note that D is defined as the mean of the individual deficiency. This is basically the average amount that the employer has to contribute so that the DB benefit at retirement age would be fully funded. It can also be viewed as an extra one-time payment to the employee at retirement age. Thus, the maximum tolerable value for D can be expressed as a percentage of salary at retirement that the employer is willing to pay.

References

Aitken, W.H. 1996. *Pension funding and valuation*. 2nd ed. Winsted, CT: Actex Publications.

Lee, Peter M. 1997. *Bayesian statistics: An introduction* 2nd ed. New York: John Wiley & Sons.