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The Role of Cash Balance and Traditional Defined Benefit Plans in Managing Retirement Risks

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Abstract

This paper considers a defined contribution (DC) plan, a cash balance plan, and traditional defined benefit (DB) plan with equal expected value at the assumed retirement age. Using utility as a measure of risk and participant preferences, the designs are compared. With this preliminary model it appears that cash balance plans are particularly well suited to minimize participant risk, particularly while employees are accumulating retirement assets.

I. Introduction

With the increasing prevalence in the United States of employer-provided retirement benefits being provided in account-based plans, such as DC plans and cash balance plans, greater attention is being focused on the risks that participants bear when the bulk of their retirement savings is in balances as opposed to life annuities. This is of particular importance, given that many baby boomers are choosing to self-annuitize instead of purchase commercial life annuities. Furthermore, given the opportunity, retirees will often elect a lump-sum payment from a DB pension plan. There is concern that there will begin to be an increase in the elderly poor and that Social Security will not provide enough of a safety net.

The purpose of this paper is to discuss how including a DB plan or cash balance plan in an employer's retirement program can positively contribute to a participant's ability to manage these retirement risks during both the pre-retirement accumulation phase and the post-retirement draw-down phase. This paper is divided into the following sections:

- Section 2 is a discussion of retirement risks and annuity availability.
- Section 3 considers the pre-retirement accumulation phase of an employee's career.
- Section 4 considers the post-retirement draw-down period of a participant's lifetime.

2. Discussion

This section discusses the risks that participants face during retirement, the individual annuity market, and how annuities from qualified plans differ from commercially available annuities.

2.1 Retirement Risks

During the accumulation phase and during actual retirement, participants face a number of risks with regard to their income. DC plans, cash balance plans, and traditional DB plans all respond differently to these risks (Bodie 1990). The risks include:

- *Replacement rate risk*. Upon retirement, there is a risk that a participant will be unable to sustain his or her standard of living. Traditional DB plans based upon final average pay provide good insurance against this as long as the participant's pay is increasing at least as much as inflation. DC plans and cash balance plans provide some protection against this as long as the interest rate credited or investment earnings exceed inflation.
- *Annuity rate risk*. The underlying interest rate that insurers use to calculate annuity premiums depends on prevailing interest rates when an individual retires. If interest rates are high, then a cash balance will be converted to a larger annuity amount. Conversely, if interest rates are low, then a cash balance will be converted to a smaller pension. DB pension plans are not subject to this risk if it is assumed that the benefit will be taken as an annuity because the

benefit is defined in terms of an annuity already. If it is assumed that the benefit will be taken as a lump sum, then traditional defined benefits react in opposition to interest rates: The lump sum is smaller when interest rates are high and larger when interest rates are low.

- *Longevity risk*. Retirees face the risk that they will outlive their savings. Life annuities offer protection against longevity risk, but DC accounts do not.
- *Investment risk*. Depending on investment experience, a retiree's income will vary from year to year because of the volatility of the investments. For benefits taken as an annuity, this is not a risk. For benefits that participants keep invested, this is a risk.
- *Inflation risk*. Purchasing power is eroded over time because of inflation. This is a risk for all plans. If a participant's retirement income is entirely provided by a fixed annuity, then the participant will lose substantial purchasing power over time. This is one of the reasons given for the unpopularity of fixed annuities.
- *Contingent outlay risk*. A retiree might have a large unexpected expense. For benefits taken as an annuity, this is a risk. Once an annuity is elected, the amount cannot be varied to cover unexpected expenses. To the extent that retirement money is left in an account, a participant can withdraw amounts necessary to cover these expenses.
- *Default risk*. There is a risk that an annuity provider will no longer be able to pay the annuity at some point in the future. Because we have assumed that the DC plan is always taken as a lump-sum payment, the DC plan is not subject to this risk. For the traditional DB plan and the cash balance plan, this risk depends on the financial strength of the employer, the annuity provider chosen by the employer, and the degree to which the benefit is insured. If the employer continues to pay the benefit, the benefit is insured by the PBGC, but only up to a certain limit. If the employer purchases a commercial annuity, it will be insured in some degree by the applicable state guaranty association, but again this may not be complete protection.
- *Self-management risk*. As participants age, they face the risk that they will no longer be able to manage their own financial affairs. Annuities provide some protection against this because they offer a fixed income without having to worry about investing. A reliable

financial advisor or family member can also help protect against this risk.

2.2 Insuring Against These Risks

Life annuities provide insurance against several of these risks (longevity, investment, and self-management risk). It is intuitively appealing to combine a life annuity to protect against these risks and also maintain an investment account to deal with other types of retirement risks (inflation and contingent outlay expense). However, the demand for commercially available individual annuities is very low. Some of the reasons cited for this are:

- Commercial individual annuities are too expensive and, thus, provide too low of a rate of return compared with other potential investments. Much of the expense comes from adverse selection, but some of it comes from marketing, taxes, reserves, etc. (Mitchell et al. 1999).
- Individuals are afraid of inflation diminishing the purchasing power of fixed annuities similar to what happened in the high-inflation environment of the late 1970s (Steigerwald 2000).
- Individuals have a bequest motive to leave a portion of their estate to their heirs. A life annuity leaves nothing for the heirs. Additionally, individuals are concerned with premature death and the insurance company receiving a windfall.
- Financial advisors have a distaste for fixed annuities.

2.3 Employer Provided Annuities

From the participant's perspective, the economics of purchasing a commercial annuity or electing an annuity from a DB plan or a cash balance plan are extremely different. Therefore, a participant who would not purchase a commercial annuity may elect an annuity offered by a qualified pension plan. Generally, the economic differences between electing an annuity from a DB plan (traditional or cash balance) and purchasing a commercial annuity include the following:

- In a DB plan, the benefit is defined in terms of an annuity. Therefore, participants can plan and rely on receiving a known or estimable annuity amount in the future. In a cash balance plan, the annuity is typically determined from the accumulated balance and

prevailing interest rates. Therefore, if a participant desires an annuity, planning with a cash balance plan is more difficult.

- In a qualified pension plan, a participant generally only has one opportunity to elect an annuity or a lump sum. The participant cannot optimize value by electing a lump sum, and then choosing an annuity from the pension plan at some point in the future. Of course, a participant could elect a lump sum from the pension plan and then purchase a commercial annuity at some point in the future.
- Annuity values in qualified pension plans may include early retirement or optional form conversion subsidies that make them actuarially more valuable than lump-sum payments. Alternatively, annuities from a pension plan may include the possibility of either ad hoc or automatic cost-of-living increases that may be forfeited if the participant elects a lump sum.
- Conversely, a traditional DB plan may determine its lump sums in a manner that provides for more than the legally required minimum lump sum. The plan may determine the lump sums using an interest rate or mortality assumption that is favorable to participants.
- Some types of commercial annuities allow individuals to choose investments and earn rates of returns based upon those investments. This is a very attractive option to participants.
- While cost-of-living increases may be included in a qualified pension plan, they are generally tied to a measure such as the Consumer Price Index or are a fixed percentage of the benefit. Additionally, qualified pension plans sometimes are amended to include ad hoc cost-of-living increases for past retirees.
- In a DB plan, the participant does not pay any commissions or expense loads that may be charged by a commercial annuity provider. Perhaps, more importantly, large qualified plans may be able to avoid the adverse selection that is present in the individual annuity market by limiting lump-sum availability.
- The assumptions used to convert a balance to an annuity are different from those used by commercial annuity providers. In converting a cash balance into an annuity and a traditional DB annuity to a lump sum, the pension plan sponsor has some latitude, but must conform to requirements of the Internal Revenue Code (IRC).

- An employer may either pay an amount from the plan's assets on a monthly basis or purchase a commercial annuity on behalf of the participant. Once the participant begins receiving an annuity, the participant cannot change his or her election just because the employer decides to purchase a commercial annuity. Additionally, if the employer purchases a commercial annuity, the participant is insulated from the difference in the expected and actual cost of the annuity.

Although these are not precisely economic differences, two other points are worth mentioning. First, under a traditional DB plan and cash balance plan, participants' default election is generally an annuity. Both the participant and his or her spouse have to elect in writing to receive a lump-sum benefit. This may supply a small amount of bias towards annuities in qualified plans.

Second, a participant's election to receive an annuity or lump sum may be somewhat related to his or her attitude toward the former employer. If the participant left the employer on bad terms, it seems natural that the participant may have a bias towards electing a lump sum and severing his relationship with his former employer.

Because of all the differences outlined above, it is difficult to extrapolate from analysis of the individual annuity market to qualified plans directly. However, because annuities from qualified plans cannot explicitly charge for adverse selection and overhead, these annuities may be more valuable to participants than commercial annuities. Additionally, cash balance plans and DB plans can be helpful in managing pre-retirement accumulation risk.

3. Pre-retirement Accumulation

The section of the paper analyzes the expected utility at retirement age for a DC plan, cash balance plan, and a traditional DB Plan. The participant's election of an annuity or lump sum is considered in Section 4.

3.1 Plan Designs

The details of the plan design are explained for an individual currently age 40 with an assumed retirement age of 65. The plan designs considered are:

- *A DC plan.* The employer contributes 7.593% of pay for the participant each year. Each year, the participant's investment earnings are based on the returns on cash, equities, and bonds. It is assumed that the participant's account is 5% invested in cash, 60% in equities, and 35% in bonds, and earns on average 9.29%.¹ It is also assumed that the participant rebalances his or her portfolio each year and maintains the same investment mix throughout his or her career.
- *A cash balance plan.* The employer credits 9.273% of pay for the participant each year. Interest is credited to the participant's account annually based upon the GATT rate.²
- *A traditional DB plan.* The employee's life annuity benefit is equal to 1.3333% of final compensation for each year of service. For this purpose, final compensation is equal to the participant's compensation in the year preceding retirement. At retirement, the participant's annuity is converted to a lump sum at the GATT rate and the 1983 Group Annuity Mortality Table (50% male/50% female).³

¹ An interest rate of 9.29% is probably higher than many actuaries would use in a plan design study. However, to avoid biases in favor of the DC plan, it was necessary to use an expected return in selecting the equivalent designs that corresponded to the stochastic returns. *Money* magazine suggests the following allocations: (1) For individuals in their 40s and 50s—bonds/cash 20%, small-cap stocks 15%, large-cap stocks 45%, and foreign stocks 20%; (2) For individuals ages 55–65—bonds/cash 25% and large-cap stocks 75%; (3) For retired individuals—cash 5%, bonds 55%, and large-cap stocks 45% (Lee 2001). For simplicity, a single asset allocation was used.

² The preceding December's average constant maturity 30-year Treasury rate was chosen as the GATT rate to be consistent with Notice 96-8. The 30-year Treasury rates were stochastically generated. See the appendix for a further explanation of how the returns are developed. The average 30-year Treasury rate after 2,000 trials was 7.53%.

³ Thirty-year Treasury rates and the 1983 GAM Table were used to be consistent with IRC 417(e). For this application, because the assumed retirement age was 65, it was not necessary to consider differences that may arise because of early retirement subsidies being included or excluded from lump sums.

The plan designs were chosen so that the participant's expected value at retirement age is the same under all three plans if a participant earns a 9.29% return in the DC plan and the average GATT rate is 7.53%.⁴ Additional plan features included the following:

- Employer pays for the entire benefit and no employee contributions are permitted.
- Immediate 100% vesting in employer contributions.
- No withdrawals or loans are permitted until retirement.
- All employer contributions are credited on the last day of the year.
- Employer contributions, investment earning and interest credits are credited annually on the last day of the year.
- Maximum benefit and compensation limits were ignored.

3.2 Accumulation Method

Each year the compensation for the individual is calculated based on the prior year's compensation and the stochastically generated salary increase. At the end of the year, the participant's cash balance and DC account are credited with the contribution and appropriate investment earnings.

The 30-year Treasury rate, cost-of-living increases, and salary increases were stochastically generated for each year. Additionally, the returns on each asset class were stochastically generated (see the appendix for a further explanation of how the returns are developed).

Additionally, 1.0% was subtracted from the DC plan returns each year to reflect that investment expenses are netted against the participant's accounts. Incorporating such an assumption is appropriate because most DC plans are invested in mutual funds or similar investment/insurance products. The participant typically bears the annual recordkeeping costs through asset-based charges. Therefore, it seems appropriate to reduce the return the participant can achieve in the plan by an amount of 1.0%.

⁴ The DB design was chosen to replace 33.3% of pay with 25 years of service at retirement before offsetting for Social Security. Once this parameter was chosen, I solved for the employer contribution for the DC and cash balance plans.

When the participant reaches his or her assumed retirement age, his or her DB annuity is calculated and converted to a lump sum at the appropriate GATT rate. The annuity factors are calculated assuming one annual payment at the beginning of each year.

3.3 Choice of Utility Function

The utility function used is: $u(\text{accvalue}) = [(\text{accvalue}^{1-g} - 1)/(1-g)] * 10,000$ for $g=.5$ and 2 , where

- accvalue is equal to the sum of the amount in the qualified plan, personal savings, and an estimated Social Security benefit converted to a lump sum. This sum is divided by a scaling factor of 10,000. For the traditional DB plan, the annuity benefit is converted to lump sum based on GATT rates. The estimated Social Security benefit is converted from an annuity to a lump sum using a static annuity factor of 11.
- The Social Security benefit is estimated as follows: 90% of salary for salary up to 10% of the taxable wage base, plus 30% of salary for salary in excess of 10% of the taxable wage base but less than 50% of the taxable wage base, plus 10% of salary for salary in excess of 50% of the taxable wage base but less than 90% of the taxable wage base.
- γ is a measure of the participant's tolerance for risk. The greater the value the more risk averse a participant is.
- 10,000 is a scaling factor.

3.4 Summary of Results

The results in Table 1 are based upon 2,000 stochastic trials for an individual currently age 40 with an assumed retirement age of 65 and a current annual salary of \$50,000. A utility ranking of 1 means that the average utility was highest for the plan design and a utility ranking of 3 means that the design had the lowest utility. The results are not surprising. The expected values are approximately the same, so the plan with greatest standard deviation (the DC plan) had the smallest utility. The cash balance plan that offers fairly consistent investment earnings had a higher utility than the DB plan lump sum that is dependent upon the GATT rate upon retirement.

Table 2 expands the analysis to other ages. For each age, a plan design was chosen that produces the same expected value at age 65. The cash balance plan produces the highest utility at all ages. The DC plan produces the lowest utility except at later ages where it outperforms the DB plan. The DB plan producing a lower utility at later ages is somewhat counterintuitive but, for this purpose, it was assumed that the participant elects a lump sum. Therefore, the participant is subject to interest rate risk with regard to the lump sum.

The next section considers how participant elections regarding lump sums and annuities from DC plans, traditional DB plans, and cash balance plans affect the participant risk.

4. Draw-Down Phase

Having demonstrated in Section 3 that cash balance plans can maximize participant utility during the pre-retirement accumulation phase, this section examines what happens upon retirement as a participant lives off his pension, Social Security benefit, and retirement savings. Upon retirement from a traditional DB plan or cash balance plan, a participant must elect whether to receive an annuity or a lump sum.

4.1 Plan Designs

The plan designs described in Section 3 were used for the draw-down analysis. They were chosen so that the participant's expected value at the assumed retirement age of 65 are equal. Other plan features are similar to those described in Section 3, with the following additions necessary for the post-retirement period:

- The DC plan is always paid as a lump sum.
- The traditional DB plan may be paid as an annuity.
- The cash balance plan is paid as an annuity. The cash balance is converted to an annuity using GATT mortality and interest.
- A participant must make an election at the assumed retirement age 65.
- The participant cannot change his or her election.
- Taxes are ignored.

4.2 Accumulation Method

The participant's benefit is accumulated during his or her working years as described in Section 3. After retirement, it is assumed that the participant's personal savings earn the same rate as the DC account.

4.3 Pay-Down Method

At the beginning of each year, the participant withdraws an amount from his or her combined DC and personal savings account. The amount withdrawn is as follows:

Amount Withdrawn = $80\% * \text{salary}_{\text{ind}}$ – estimated Social Security benefit – annuity, where

- $\text{salary}_{\text{ind}}$ is the salary in the year preceding retirement indexed with cost-of-living increases for each year after retirement.
- The Social Security benefit is estimated as follows: 90% of salary for salary up to 10% of the taxable wage base, plus 30% of salary for salary in excess of 10% of the taxable wage base but less than 50% of the taxable wage base, plus 10% of salary for salary in excess of 50% of the taxable wage base but less than 90% of the taxable wage base. The Social Security benefit is indexed each year with cost-of-living increases.
- Annuity is the annual annuity amount from the cash balance plan or traditional DB plan.

Since salary is indexed with inflation, the amount needed each year is assumed to increase annually with inflation. If there is an annuity amount, the amount withdrawn increases faster than inflation because the annuity does not increase with inflation. After the annual withdrawal is made, the combined DC and personal savings account is stochastically credited with investment earnings.

The draw-down process continues until the participant dies. Balances are not allowed to become negative.

4.4 Choice of Utility Function

The utility function used was as follows:

$$U = \sum [(c_t^{1-g} - 1)/(1-g)] * 10,000 * 1.07536^{65-x} \text{ for } g = .5 \text{ and } 2,$$

where

- U is summed from age 65 until the participant's death.
- x is the current age for the summation.
- c_t represents the amount of consumption during the year. It is equal to the sum of the amount withdrawn from the combined DC and personal savings account, the qualified plan annuity, and the estimated Social Security benefit. c_t is also divided by a scaling factor of 10,000.
- γ is a measure of the participant's tolerance for risk. The greater the value the more risk averse a participant is.
- This utility function assumes that there is no bequest motive.

4.5 Summary of Results

The results in Table 3 were based on 2,000 stochastic trials for an individual currently age 40 with an assumed retirement age of 65 and a current annual salary of \$50,000. For this table, it is assumed that the participant takes his or her DB balance as an annuity.

Clear-cut conclusions cannot be drawn from the results. Whether a male prefers the DC plan depends on his level of risk aversion. The less risk-averse he is, the more likely he is to prefer the DC plan. A risk-averse male prefers the DB plan to the cash balance plan. The conversion from the cash balance to an annuity is not as favorable for males as it is for females.

A female's first choice is the cash balance plan. This is intuitively appealing because the cash balance provides a female with a relatively low-cost annuity. The cash balance is converted to an annuity using the 1983 GAM table (50% male/50% female), which is based on higher mortality rates. She is more likely to prefer the traditional DB plan to the DC plan when she is more risk-averse.

5. Conclusions

The model presented in this paper is preliminary and, as a result, may have limited practical application. However, it demonstrates that the research being conducted in the individual annuity market can easily be carried over to the qualified plan context as a method for considering the way that qualified plan designs influence participant's retirement risks.

Even with a basic model, it appears that cash balance plans, in particular, can be extremely helpful to participants in managing both pre-retirement and post-retirement risks. In pre-retirement, cash balance plans provided employees with the highest utility ranking. Because cash balances offer greater insurance against retirement risks than do DC plans, much could be done for the public good if legislation was passed that further legitimized cash balance plans.

Additionally, the insurance value of cash balance plans could be enhanced if these plans were allowed to convert account balances to annuities at a guaranteed rate. Currently, to avoid whipsaw problems, it is very common for plans to use the GATT mortality and interest rate. This diminishes one of the advantages of DB plans: The participant is no longer able to plan for his or her retirement with a guaranteed annuity amount. Instead, the participant has a fairly predictable cash balance.

Because it is often in the participant's best interest to elect an annuity, it would be good policy for cash balance plans to be able to offer easily for predictable annuities. Because this would subject the plan to some degree of anti-selection from participants electing annuities from the plan when interest rates were high, there is already some disincentive for plans to offer this option to participants. Therefore, the legislative obstacles should be removed.

Analysis using either utility theory or other risk measures should be increasingly incorporated into retirement plan discussions as actuaries increase their profile with regard to the important issues surrounding post-retirement risks.

Appendix

The following is a description of the stochastic iterations performed in this analysis. First, 30-year Treasury rates were generated. Other returns then were generated stochastically based on their correlation with the 30-year Treasury rates. After all of the returns were generated, the personal savings and age at death were generated.

Development of 30-Year Treasury Rates

Constant, fixed-rate 30-year Treasury rates were needed to convert participant annuities into lump sums and cash balances into annuities.⁵ It was assumed for these purposes that the preceding average December 30-year Treasury rate would be used.⁶

Thirty-year Treasury rates were generated monthly and assumed to be mean reverting over a 36-month period with a mean of 7.5%. Looking at the period from January 1981 through December 2000,⁷ this produces a random error with a mean of 7.29×10^{-6} (assumed to be 0 for the stochastic iterations) and a standard deviation of 0.3%.

Although the 30-year Treasury rates were generated monthly, only the December Treasuries were used to develop other returns. Thus, the formula for the 30-year Treasury is as follows:

$$i_{mon}(m) = i_{mon}(m-1) + (m_t - i_{mon}(m-1))/36 + e * CorrectionFactor$$

and

$i(t) = i_{mon}(m)$ when m corresponds to December of year t , where

- $i_{mon}(m)$ represents the average constant, fixed maturity 30-year Treasury rate for month m .
- $i(t)$ represents the average December constant, fixed 30-year Treasury rate for year t .

⁵ IRC § 417(e)(3)(A)(ii)(II). IRS Notice 96-8.

⁶ IRC Regulation 1.417(e)-(4).

⁷ Federal Reserve Board Release H.15, released Feb. 26, 2001 (<http://www.federalreserve.gov/releases>).

- μ_i is equal to 7.5%.
- ε is the random error and is equal to a random draw converted to a $N(0,1)$ distribution multiplied by σ_i . σ_i represents the standard deviation of the error using the least squares formula for the period January 1985–December 2000. σ_i is equal to 0.3%.
- Correction factor is 1 unless $i(t-1)$ was less than 4.5% or greater than 13.5%. If $i(t-1)$ was less than 4.5% or greater than 13.5%, then Correction Factor is equal to -1 or 1 as necessary to force mean reversion.
- $i_{\text{mon}}(0)$ was assumed to be equal to 7.5%.

After 2,000 iterations for a period of 80 years, the average 30-year Treasury generated was 7.53%.

Development of Cash Returns

After developing the 30-year returns, the next step was to develop the cash returns. Cash returns were necessary because it was assumed that cash equivalents were one of the investment options for the DC plan. The cash return was assumed to equal the 90-day Treasury rate.

Using the least squares method based on the period January 1985–December 2000,⁸ the cash rate was assumed to be related to the 30-year Treasury as follows:

$$\text{Cash}(t) = i(t) * .861 - .00745 + e,$$

where

- $\text{Cash}(t)$ is the cash return for year t .⁹
- $i(t)$ represents the average December constant, fixed 30-year Treasury rate for year t .
- ε is the random error and is equal to a random draw converted to a $N(0,1)$ distribution multiplied by σ_i . σ_i is equal to 0.3%.

⁸ Federal Reserve Board Release H.15, released Feb. 26, 2001.

(<http://www.federalreserve.gov/releases/H15>). The least squares method was applied to monthly data.

⁹ This method of determining cash returns assumes that the cash rate for the entire year is chosen in December of each year. It probably would be more accurate to determine the rate more often than annually. However, this simplification is unlikely to produce significant error in the model.

After 2,000 iterations for an 80-year period, the average cash return generated was 5.73%.

Increase in Cost of Living

The annual increase in cost of living was needed to develop salary increases. The annual increase in cost of living was assumed to be equal to the increase in the Consumer Price Index (CPI) for “Urban Consumers U.S. City Average.” The CPI was assumed to have a mean of 3.5% with a standard deviation of 1.75%. Additionally, it was assumed that the CPI was correlated to the annual change in 30-year Treasury, with a correlation factor of 0.35.¹⁰ Thus, the formula for the CPI was as follows:

$$\text{CPI}(t) = \mu_{\text{CPI}} + (\rho * \sigma_{\text{CPI}} / \sigma_{\Delta i}) * (\Delta i - \mu_{\Delta i}) + \epsilon ,$$

where

- $\text{CPI}(t)$ represents the percentage increase in the CPI during year t .
- μ_{CPI} is the mean annual increase in the CPI. μ_{CPI} was assumed to be 3.5%.
- ρ is the correlation factor between the annual CPI and the change in the 30-year Treasury rate. ρ is assumed to be 0.35.
- σ_{CPI} is the standard deviation of the CPI. It was assumed to be 1.75%.
- $\sigma_{\Delta i}$ is the standard deviation for the annual change in interest rate. It was assumed to be 0.36% based on the period 1981–1999.
- Δi is equal to $i(t) - i(t-1)$.
- $\mu_{\Delta i}$ is the mean annual change in average December 30-year Treasury rate. This is assumed to be 0.
- ϵ is the random error and is equal to a random draw converted to a $N(0,1)$ distribution multiplied by σ_{CPI} .

After 2,000 iterations for an eighty-year period, the average cost-of-living increase generated was 3.5%.

¹⁰ Based upon the period 1981–1999.

Development of Salary Increases

Salary increases are a function of both cost-of-living increases and productivity/merit gains. Productivity gains are usually highest earlier in a person's career. Therefore, the salary increase was determined as follows:

$$\text{Salary}(t) = \text{CPI}(t) + \text{Productivity}(\text{age}) + \varepsilon,$$

Where

- $\text{Salary}(t)$ represents the increase in salary investments during year t .
- $\text{CPI}(t)$ is the increase in CPI during year t .
- Age is equal to the participant's age at time t .
- Productivity (age) is the assumed general and age-specific productivity/merit portion of the salary increase. The assumed increases were: 4% (ages 20–29), 3% (ages 30–39), 2% (ages 40–49), 1% (ages 50–59), and 0.5% (age 60 and older).
- ε is the random error and is equal to a random draw converted to a $N(0,1)$ distribution multiplied by σ_{SAL} . σ_{SAL} is equal to 1%.

Development of Bond Returns

Bond returns were needed because it was assumed that a bond fund was offered as a potential investment in the DC plan. Bond returns were assumed to have a mean of 8% and a standard deviation of 9%.¹¹ Bond returns were assumed to be correlated to the change in the 30-year Treasury rate each year, with a correlation factor of -0.85 .¹² Thus, the formula for the return on bonds was determined as follows:

$$\text{Bond}_t = m_{\text{BOND}} + (r * s_{\text{BOND}} / s_{\Delta i}) + e,$$

where

- $\text{Bond}(t)$ represents the return on bond investments during year t .

¹¹ These represent total returns including interest payments, capital gains and losses, and defaults. The bond fund is assumed to have a composition of bonds similar to the Lehman Brothers Aggregate Bond Index.

¹² Based upon the period 1981–1999.

- μ_{BOND} is the mean annual return on bonds. μ_{BOND} was assumed to be 8%.
- ρ is the correlation factor between the annual return on the bond and the change in 30-year Treasury rate. Bond returns were assumed to be correlated to the change in the average December 30-year rate each year, with a correlation factor of -0.85 .
- σ_{BOND} is the standard deviation bond returns. It was assumed to be 9%.
- $\sigma_{\Delta i}$ is the standard deviation for the annual change in 30-year Treasury rate. It was assumed to be 1.36% based on the period 1981–1999.
- Δi is equal to $i(t) - i(t-1)$.
- $m_{\Delta i}$ is the mean annual change in the 30-year Treasury rate. $\mu_{\Delta i}$ is assumed to be 0.
- ϵ is the random error and is equal to a random draw converted to a $N(0,1)$ distribution multiplied by σ_{BOND} .

After 2,000 iterations for an 80-year period, the average bond return generated was 8%.

Development of Equity Returns

Equity returns were needed because it was assumed that an equity fund was offered as a potential investment in the DC plan. Equity returns were assumed to have a mean of 12% and a standard deviation of 15%.¹³ Equity returns were assumed to be correlated to the change in the 30-year Treasury rate each year, with a correlation factor of -0.55 .¹⁴ Thus, the formula for the change in interest rate was determined as follows:

$$\text{Equity}(t) = m_{\text{EQUITY}} + (r * s_{\text{EQUITY}} / s_{\Delta i}) + e, \text{ where}$$

- $\text{Equity}(t)$ represents the return on equity investments during year t .
- μ_{EQUITY} is the mean annual return on equities. μ_{EQUITY} was assumed to be 12%.
- r is the correlation factor between the annual return on equity and the change in 30-year Treasury rate. Equity returns were assumed

¹³ These represent total returns including both capital appreciation and dividends.

¹⁴ Based upon the period 1981–1999.

to be correlated to the change in 30-year rate each year with a correlation factor of -0.55 .

- s_{EQUITY} is the standard deviation of the return on equities. It was assumed to be 15%.
- $s_{\Delta i}$ is the standard deviation for the annual change in 30-year Treasury rate. It was assumed to be 1.36% based on the period 1981–1999.
- Δi is equal to $i(t) - i(t-1)$
- $m_{\Delta i}$ is the mean annual change in the 30-year Treasury rate. $\mu_{\Delta i}$ is assumed to be 0.
- ϵ is the random error and is equal to a random draw converted to a $N(0,1)$ distribution multiplied by μ_{EQUITY} .

After 2,000 iterations for an 80-year time period, the average equity return generated was 12.02%.

Personal Savings

Target personal savings as a percentage of pay at retirement age was assumed to vary by current age to reflect variations in the employer-provided benefit based on service. The savings assumptions used were: 6.67% (age 30), 20% (age 40), 33.33% (age 50), and 46.67% (age 60).

Personal savings were assumed to have a standard deviation of 10%, and the actual amount of personal savings was determined by summing the target personal savings and a random error, where the random error is equal to a random draw converted to a $N(0,1)$ distribution multiplied by 10%.

Once the personal savings percentage was generated, the percentage was multiplied by salary and an annuity factor of 8.5 to convert the personal savings into a lump-sum amount.

Age at Death

Age at death was stochastically generated based on the RP2000 healthy annuity tables using Projection Table AA. Using this mortality table, a cumulative probability distribution was constructed. For each iteration, a random draw (uniformly distributed over the range 0 to 1) was generated, and

then the age at death corresponding to the random draw based on the cumulative probability distribution was found.

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Table 1
**Comparison of Pre-retirement Accumulation for DC Plan,
Cash Balance Plan, and DB Plan for Current Age 40**

	Defined Contribution Plan	Cash Balance Plan	Defined Benefit Plan
Standard deviation	\$ 156,170	\$51,250	\$ 67,207
Utility ranking with $\gamma=.5$	3	1	2
Utility ranking with $\gamma=2$	3	1	2

Table 2
**Comparison of Pre-Retirement Accumulation for DC Plan,
Cash Balance Plan and Defined Benefit Plan for Multiple Ages**

Starting age	g	Defined Contribution Plan	Cash Balance Plan	Defined Benefit Plan
30	.5	3	1	2
30	2.0	3	1	2
40	.5	3	1	2
40	2.0	3	1	2
50	.5	3	1	2
50	2.0	3	1	2
60	.5	2	1	3
60	2.0	2	1	3

Table 3
**Comparison of Draw-Down Phase for a DC Plan,
Cash Balance Plan and DB Plan for Current Age 40**

Utility Ranking	Defined Contribution Plan	Cash Balance Plan	Defined Benefit Plan
$\gamma=.5$, male	1	2	3
$\gamma=2$, male	3	2	1
$\gamma=.5$, female	2	1	3
$\gamma=2$, female	3	1	2