RECORD OF SOCIETY OF ACTUARIES
1993 VOL. 19 NO. 4B

USE OF CASH-FLOW TESTING IN PRODUCT DEVELOPMENT

Instructor: DAVID N. BECKER

- Life product pricing
- Annuity product pricing
- What's expected by ASB standards?
- What's being done?

MR. DAVID N. BECKER: The charts are the major part of this session and are reproduced here. My presentation is excerpted from one given at the Investment Section Seminar in May 1993. However, for the written record all the charts from the Investment Section Seminar will be used. The introductory charts will provide useful background for individuals not fully conversant in the issues of cash-flow testing.

Although global information on cash-flow testing is provided in these opening charts, there are sources that provide more depth of detail, especially with regard to basic mechanics. Examples of such sources are: the NAIC Model Actuarial Opinion and Memorandum Regulation that discusses reserve and asset adequacy, the basis of which is cash-flow testing; *Actuarial Standards of Practice (ASOP) 7,* "Performing Cash-Flow Testing for Insurers;" *ASOP 14,* "When To Do Cash-Flow Testing for Life and Health Insurance Companies;" *ASOP 22,* "Statutory Statements of Opinion Based on Asset Adequacy Analysis by Appointed Actuaries for Life and Health Insurers;" the *Valuation Actuary Handbook*; the many *Proceedings of the Valuation Actuary Symposium*; other Society of Actuaries past seminars by both the Product Development Section and the Financial Reporting Section; the *Record* of past Society of Actuaries meetings; the *Transactions*; and the Society Special Interest Section newsletters, especially those from the Product Development Section (*Product Development News*), Financial Reporting Section (*The Financial Reporter*), and the Investment Section (*Risk and Rewards*).

The entire presentation consists of five parts. First, I will give a general background of the problem of interest rate risk and its driving forces, the response of the industry, SOA and regulators, assumptions needed for cash-flow testing, why do it, pros and cons of deterministic versus stochastic testing, and economic quantities measured in cash-flow testing.

Second, I will present a brief introduction of the technique of market-value analysis (MVA). This methodology adopts the innovations of duration/convexity analysis and option-pricing theory from fixed-income investments, and applies these concepts to insurance liabilities. For those not fully acquainted with duration and convexity, two charts are presented that define and graphically illustrate these two quantities. While this is a major advance in quantification of interest rate risk and opens the door to its management, it does not totally capture all the implications of interest rate risk on the intrinsic value of the insurance enterprise. These limitations are discussed in the last part of this section. Further information on these topics can be found in past issues of the *Proceedings of the Valuation Actuary Symposium*.

Third, I'll present the research I have done to apply modern finance theory to determine the net worth of the enterprise in a way that directly demonstrates the financial impact of embedded options in assets and liabilities, and their interaction on the net worth of the enterprise. It views the insurance enterprise as a whole, not as two parts, assets and liabilities. I call the tool resulting from this research the option-adjusted value of distributable earnings or OAVDE.

It is important to note that OAVDE is not a solution to the problem of interest rate risk, but a tool to evaluate the magnitude of interest rate risk and the impact of alternative management actions on the insurance enterprise's exposure to interest rate risk. Such an impact is measured either in terms of the return on equity (net worth) employed when the amount of equity is prespecified, or in terms of the net worth of an enterprise when the return is prespecified. This part concludes with the definition of a STRATEGY being the input to the OAVDE tool.

In the fourth part, I'll present three different applications of the OAVDE tool to real, although simplified, problems. The examples are simplified so that it is easier for the reader to understand the exact source of the risk (due to embedded options) and its financial impact on the enterprise. Single-premium deferred annuities are used because they have the least moving parts, like costs of insurance, etc., that can further complicate the example. Extending these applications to universal life is straightforward.

In the first example different ways of expressing the results of OAVDE analysis are given. This is done because there can be useful information gained from examining different measures. It is shown that different measures result in the same decisions. A problem may arise when many strategies are examined in determining trade-offs among them. Two methods are illustrated to assist in simplifying the decision-making process in this case.

In the fifth and final part, I'll provide a description of different uses of the OAVDE tool. In essence, OAVDE is a pricing/valuation tool. This part shows how the tool can be applied to answer different questions that are of key interest to management.

It is recommended that the reader, even if he or she is familiar with the basic concepts in parts one and two, briefly review them because some information is utilized explicitly in later parts.

For introductory purposes the term *cash-flow testing* is used to mean the process by which the projection of statutory income statements and balance sheets are used to make an assessment of reserve and asset adequacy.

To perform cash-flow testing one needs a cash-flow engine -- the computer software that will enable the computation of the above quantities. The engine is fueled with descriptions of the characteristics of the assets and liabilities, the interest crediting strategy, the investment strategy and a disinvestment strategy, i.e., the mechanism for handling negative cash flows. The engine computes various cash items, e.g., premiums, investment income, death benefits, health benefits, annuity benefits, surrender benefits, commissions, insurance expenses and fees, federal income tax, etc. It also computes accrual items such as statutory reserves, tax reserves, deficiency reserves, interest maintenance reserve, and asset valuation reserve. From these quantities it is possible to create the statutory income statement and balance sheet. Other measures may also be derived from the above items.

Although the presence of embedded options in the assets and liabilities makes cash-flow testing more complicated, such presence makes the testing more important to perform. For example, there is uncertainty in asset cash flows due to the presences of call features, prepayments, acceleration of sinking funds, etc. For liabilities there is uncertainty due to surrender at account value less surrender charge, loan activity, premium suspension, partial withdrawals, benefit responsive features in GICs, etc.

ISSUES
- Reinvestment
- Disintermediation
- Liquidity
- Default
- Concentration

EMBEDDED OPTIONS – ASSETS
- Sinking-fund accelerations
- Mortgage prepayments
  - Pass-throughs
  - CMOs
- Implied put on debt (asset default)
  - Bonds
  - Commercial mortgages
- Callable bonds
- Puttable bonds

EMBEDDED OPTIONS - LIABILITIES
- Crediting rate guarantees
- Cash surrender at book value
- Flexible premiums, "dump-in" provisions and windows
- Policy loans
  - Regular loans
  - "Wash" loans
- Bailouts
- Partial withdrawals
- Benefit-responsive options
- Transfer option

CONCERN
- Company management
- Regulators
- Professional societies
- Rating agencies

RESPONSES
- *Internal company studies*
- Valuation actuary movement

- Committee on Valuation and Related Areas (COVARA)
- Valuation Actuary Symposiums
- New York Regulation 126
- Amended Standard Valuation Law
  - Appointed Actuary
  - Opinion & Memorandum

CASH-FLOW TESTING

- Who does it?
- What is it?
- Why do it

WHO DOES CASH-FLOW TESTING?

- Valuation actuaries
- Asset/liability managers
- Pricing actuaries

WHAT IS CASH-FLOW TESTING?

The projection and measurement of:

- asset and liability cash flows
- balance sheet items
- statutory gains from operations and
- surplus

ASSUMPTIONS IN CASH-FLOW TESTING

- Existing liability portfolio
  - product designs
  - policyholder behavior
  - liability management behavior
- Existing asset portfolio
  - borrower behavior
  - portfolio management behavior
- Economic environment
- Experience
- New business

WHY DO CASH-FLOW TESTING?

- Understand the nature and magnitude of the risks the company is assuming.
- Demonstrate the adequacy of the statutory reserves and related assets to provide for policyholder benefits and related expenses to company management, regulators, and rating agencies.
- Discover how to better manage business.

CHOICES

- Deterministic versus Stochastic Interest Rate Scenarios
- Quantity to measure

DETERMINISTIC SCENARIOS – PROS

- Project what will happen along specific path.
- Investigate alternative management actions for impact on scenario's results.

- Demonstrate adequacy in a limited way.
- Can implement subjective bias.

DETERMINISTIC SCENARIOS -- CONS
- Difficult to assign probability to a given scenario,
- Difficult to ensure diversity of future interest rate behavior,
- Difficult to prevent subjective bias in selection of scenarios,
- Unclear in assessment of adequacy
- Difficult to implement result in management of business.

STOCHASTIC SCENARIOS - PROS
- Diversity of scenarios follows from stochastic generation method.
- Scenario probability determined by stochastic generation method.
- Generates subjectively unbiased scenarios.
- Reliability is a function of the DISTRIBUTION of results.
- Asses adequacy by looking at "tails" of distribution.
- Manage business by looking at entire distribution.

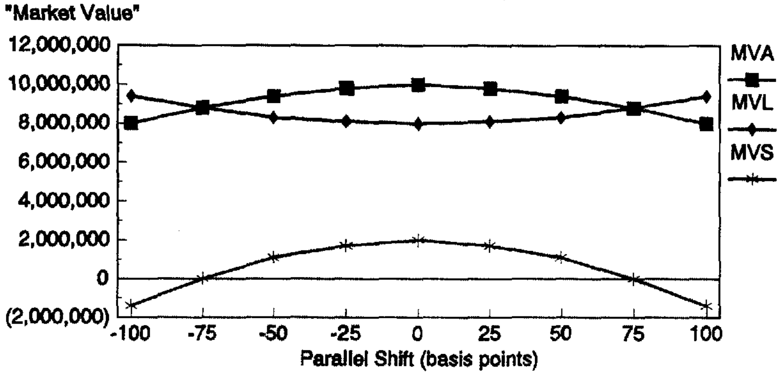
STOCHASTIC SCENARIOS - CONS
- Hard to ensure scenarios are credible and sufficiently diverse.
- Hard to determine how many are enough.
- Hard to avoid arbitrage.
- Hard to get consensus on how scenarios should be generated.
- Results may be sensitive to parameterization of stochastic generator.

WHAT TO MEASURE?
- Market values (liquidity)
- Retained earnings
  - distribution of surplus
  - distribution of present value of surplus
- Asset and liability cash-flow characteristics
- Degree of immunization of dollar amount of surplus or surplus/asset ratio
- Distributable earnings

In market-value analysis the key assumption is that the surplus, net worth or equity of an enterprise at a point in time is defined to be the present value of existing assets less the present value of existing liabilities.

MARKET VALUE OF ASSETS, LIABILITIES, AND SURPLUS
- Market value of assets (MVA)
- Market value of liabilities (MVL) computed by applying finance methods for fixed-income securities to the liability cash flows.
- Market value of surplus (MVS) computed as the difference between MVA and MVL.
- Goal is to have the graph of MVA above graph of MVL for a significant parallel shift in the yield curve (Chart 1).

CHART 1
"Market Value" Diagrams
"Market Values"
Assets, Liabilities, and Surplus



APPLICATION OF MARKET VALUE ANALYSIS

- Surplus is defined as present value of asset cash flows less present value of liability cash flows.
- Liability parameters: spread, duration and convexity.
- Asset targets: liability spread + profit spread; asset duration equal to liability duration; and convexity equal to or exceeding liability convexity.
- Goals(s): immunize profit spread, "surplus," ratio of surplus to assets, etc.
- The greater the excess of MVA over MVL for a greater range of yield curve shifts indicates lesser exposure to interest rate risk.

DURATION & CONVEXITY

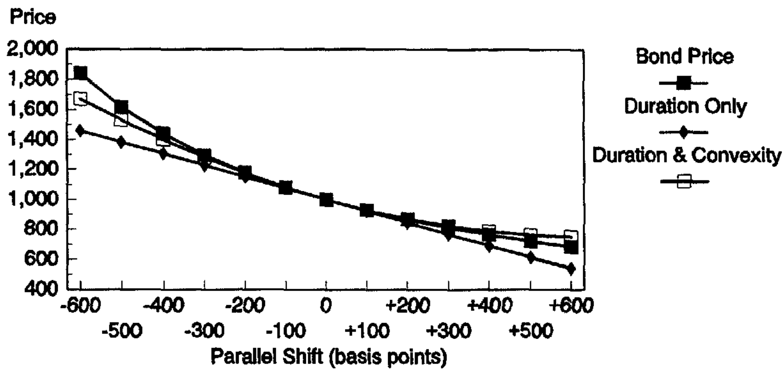
- Duration:
  - Macauley duration
  - Modified duration
  - Effective duration:
  - ▸ the negative of the relative price change of a fixed-income security for a unit change in the level of interest rates
  - ▸ the negative of the slope of the tanget line to the price curve of a fixed-income security
  - Convexity:
  - ▸ the degree of "bend" in the price curve of a fixed-income security

Several issues immediately arise. First, the appropriate definition of surplus is not necessarily as given above. Second, the definition of market value of liabilities is not clear and is an issue of some debate. Third, it focuses on the enterprise as two parts, not as an economic whole. Fourth, it may not capture all relevant information about the enterprise. These issues are actually related at a deeper level of analysis.

Despite these limitations such an analysis is an extremely valuable tool and represents a significant improvement in the ability to deal with interest rate risk.

As shown in Chart 2, definition is too restrictive and there is ambiguity in the definition of the present value of liabilities. Some of the ambiguity stems from making an incomplete analogy between fixed-income securities and insurance liabilities. There is no secondary market for insurance liabilities in the same fashion there is for assets, such as fixed-income securities, common stock. But it is in this sense that such market values are often attempted to be defined. And even in this view there is not uniformity in the definition of market value.

CHART 2
Duration & Convexity
Bond Price Estimated by Duration & Convexity



When insurance liabilities are acquired, it is other insurance enterprises that are purchasing both the liabilities and supporting assets. They are making the purchase for their benefit of the expected earnings off of the combination of the two. The resulting market value of surplus from the interpretation in the four preceding paragraphs is not what a willing buyer would pay a willing seller to acquire the liabilities and supporting assets.

Not all relevant cash flows or costs of capital are considered, such as, federal income tax, DAC tax, the cost of capital in holding target surplus (whether determined by an internal formula, based on rating agency formula or NAIC's risk-based capital (RBC), statutory reserves, deficiency reserves, interest reserve maintenance and asset valuation reserves, the timing differences caused by tax reserves.

Because market-value analysis does not consider the enterprise as a whole, it may lead to suboptimization in managing interest rate risk. To illustrate, an enterprise may engage in a hedge transaction, e.g., purchase interest rate caps to mitigate interest rate risk. Because market-value analysis does not reflect all relevant transactions, it may result in buying a hedge that actually reduces the ultimate profitability of the enterprise. In short, it is possible to buy too much interest rate insurance as well as not enough.

Lastly, if market-value analysis is applied too precisely, the success depends on accurate estimation of the policyholder exercise of liability options. One has to be comfortable that either the estimation is accurate, or that results are not sensitive to the mis-specification of the estimate. In OAVDE analysis, the goal is to identify robust strategies that optimize results over a spectrum of alternative conditions. It does not necessarily try to micromanage the solution, although it could be used in that way.

The major challenges are to identify, quantify, and manage interest rate risk. This has to be done in a way that allows management to set the price for new business, reprice an existing block of business, or decide on a price to pay for acquiring a block or an entire insurance company, or on the price to accept for a sale where the price reflects the inherent interest rate risk.

LIMITATIONS TO MARKET VALUE ANALYSIS

- Model requires adoption of internal company cash-flow-based goals.
- Model does not include all relevant cash flows.
- Incomplete analogy:
  - Insurance liability is NOT a security.
  - No secondary market exists for liabilities in same sense as for securities.
  - MVS is NOT what a "buyer" would pay a "seller."
- Method may not be "robust" in that it requires precise measurement of liability parameters for management action.
- Does not explicitly recognize:
  - different legal/regulatory/tax environments
  - cost of capital deployed as reserves and RBC
  - Federal income tax (FIT) and the timing of enterprise free cash flows (shareholder dividends)
- Many strategies may change MVA and MVL curves to a more favorable shape, i.e., indicating better immunization, but total cost of change is *not* revealed. It could lead to a costly hedge strategy.

CHALLENGES

- How to measure interest rate risk?
- How to manage interest rate risk?
- How to set the price to reflect interest rate risk for
  - new business?
  - an existing block?
  - an entire company?
- How to analyze experience?

The price of a security reflects its inherent risk. From finance theory the price or value of a security, also known as intrinsic value or fair value, is the risk-adjusted present value of free cash flows. Free cash flows are those amounts of money that can be freely transferred to the owner of the security. The owner must be able to dispose of those amounts in any way he or she desires, in order for them to be free cash flows.

For example, if the security is a bond, then in exchange for the purchase price, the owner receives the coupon income, any call premium (if the bond is called), and the maturity value. The owner is free to do anything with the income, call premium or maturity value. Any inherent interest rate risk due to the level and volatility of interest rates and the presence of any call option is reflected in the price of the bond in the marketplace.

If the security is common stock, then, in exchange for the purchase price, the owner has the free use of any dividends paid by the company. It might be argued that the owner of the stock also has access to an increase in share value which could occur if, in lieu of paying a shareholder dividend, the company reinvested in new production. However, it is shown in finance theory that the value of the firm is the same, whether the dividends are paid in cash or redeployed in new production producing a return equal to the firm's cost of capital. Thus, the benefit to the owner of the increase in share price due to redeployment of dividends into new projects equals the benefit to the owner of receiving those dividends in cash.

The other item needed from finance is option pricing theory because it will enable the appropriate reflection in price of the presence of embedded options. This can be used to determine price if a desired return is specified, or determine return if a desired price is specified.

If you can't measure risk, then you can't manage risk! Price is the measure of risk. For an insurance enterprise, OVADE is the measure of price.

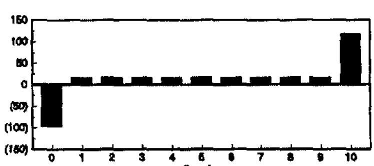
FINANCE THEORY

- Value of a security
  - intrinsic value or fair value
  - risk-adjusted present value of free cash flows
- Option pricing theory
  - methodology that properly accounts for the impact of embedded options
  - can determine price given a target return or return given a target price

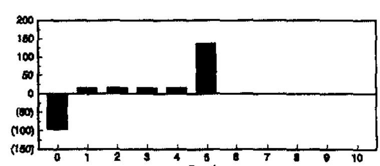
(See Chart 3)

CHART 3
Free Cash-Flow Diagrams

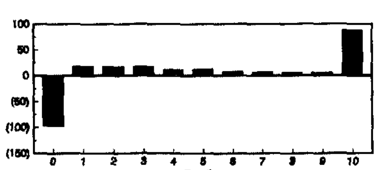
Noncallable Bond



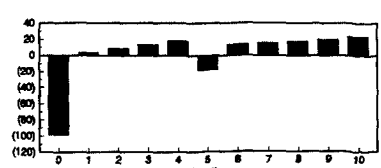
Called Bond



Common Stock



Distributable Earnings



PRICE OF A SECURITY

- For a bond
- For a stock
- For an insurance enterprise, the free cash flows to its shareholders are restricted by:
  - what can legally be paid according to state insurance department regulations
  - what can prudently be paid given sound risk-based capital requirements
  - call this quantity "distributable earnings"

If the security represents ownership in an insurance enterprise, then one must determine the free cash flows. Unlike other industries, law regulates the amounts of shareholder dividends payable. This law ties shareholder dividends to statutory accounting. This fact often causes people to turn away from the use of statutory net income as conservative statutory accounting does not reflect the true economics of the enterprise. But the fact is that it does affect the true economics. Cash flows within the insurance entity that are not capable of being paid to shareholders are not free cash flows. Hence, they cannot be used either to be paid to shareholders or to fund new business.

In addition to the limitation of statutory net income, the amount of capital that needs to be held either due to internal target surplus formulas or to satisfy external capital requirements e.g., NAIC RBC levels and rating agency requirement, should be reflected in the determination of free cash flows. The enterprise needs to hold such capital, and it must provide an adequate return to shareholders on it.

Before the introduction of the interest maintenance reserve (IMR) and the asset valuation reserve (AVR), the free cash flows would have been defined to be equal to: after-tax statutory gain (ignoring any investment income from surplus funds); plus after-tax realized capital gains and losses; plus after-tax investment income on company-held risk-based capital; less the change in such risk-based capital. With the introduction of IMR and AVR, this definition would need to be modified appropriately. But in the examples that follow, these two items will not play a part because selling of assets will not occur as part of a strategy. The free cash flows of an insurance enterprise are referred to as distributable earnings.

WHAT IS NEEDED?

- The correct economic quantity to measure to obtain price
- The correct net present value methodology to discount this quantity
- Stochastic methods from finance to assess the cost of embedded options

DISTRIBUTABLE EARNINGS

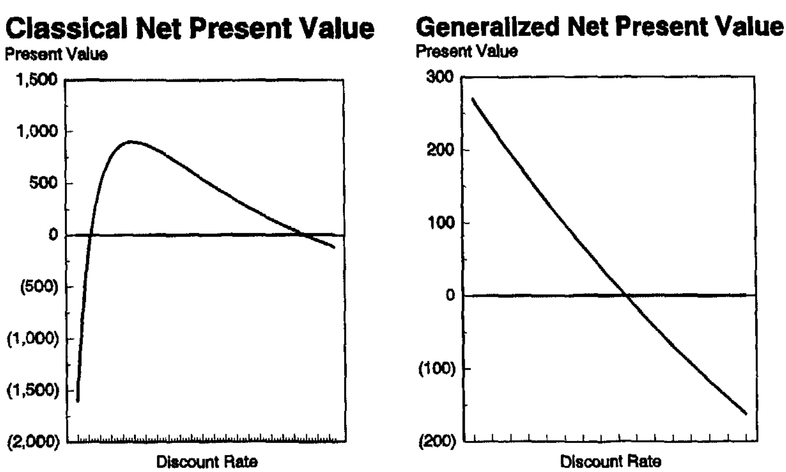
- Statutory gain, plus
- After-tax realized capital gains and losses, plus
- After-tax investment income on opening risk-based capital, less
- Change in risk-based capital.

When applying discounting to distributable earnings, one difficulty arises that does not occur, in general, with fixed-income securities or common stock. The problem, which can apply to new business, existing business or the entire company, is that it is possible for one or more future distributable earnings to be negative, or for all distributable earnings beyond some duration to be negative. Such future negative distributable earnings are possible under the adverse conditions which can occur in cash-flow testing, but they are also possible in flat and unchanging yield curve environments. They can be caused by many things, e.g., cliff surrender charges, commission and/or override bonuses, mortality and/or expense charge reversions to the policyholder, reversionary interest rate credits, etc. These situations are not limited to universal life and annuities. They can occur in disability income and long-term-care insurance as well. Their significance is found in either the resulting need for additional shareholder capital infusions at various points in time, or the withholding of prior distributable earnings to prefund the subsequent additional capital infusions. That doesn't occur with bonds, for example.

This difficulty is not new. The classic problem called The Pump Project was discussed in the finance literature of the 1950s by J. Lorie and L.J. Savage ("Three Problems in Rationing Capital" *Journal of Business* 28, no. 4, Oct. 1955, 229-239) see Chart 4. In this example, they have a project with two internal rates of return. While Lorie and Savage had given a specific analysis to this problem, there was no general framework from which to resolve it.

When these conditions occur, it is possible that the net present value algorithm will result in economically meaningless results. As it does in the case of The Pump Project. In this case the discounting in option pricing will not result in a useful result. This problem can be remedied by using the generalized net present value algorithm which will always provide a meaningful result. The generalized net present value algorithm is consistent with option pricing theory.

CHART 4
The Pump Project



GENERALIZED NET PRESENT VALUE AND GENERALIZED INTERNAL RATE OF RETURN

- *Transactions of the Society of Actuaries* (Volume XL, 1988).
- *Reinsurance Reporter*, Lincoln National Life Insurance Company (Fall/Winter, 1989)

The major advantage in using arbitrage-free paths that reprice the initial treasury curve is that the values of securities so priced will be priced consistently relative to treasury securities. Therefore, they will be priced consistently relative to one another. Option pricing results in a price that edits out the impact of the exercise of embedded options in the security. In option pricing, given the price as input, then the method solves for the option-adjusted spread or spread-to-treasuries, that would be earned after taking into account potential exercise of any embedded options. This spread can be compared to that of other securities or the buyer's standards and a decision made. If a spread is given as input, then the method solves for the price required for the buyer to realize the spread, where the price reflects the impact of the embedded options. In either case, a decision can be made relative to the potential buyer's standards.

When used with distributable earnings the price is OAVDE, and the spread is the spread earned by the investor in the insurance enterprise. The method values OAVDE consistently with respect to treasuries, as well as other fixed-income securities. As time passes and the computation is performed again, the new OAVDE will be valued consistently with respect to treasuries at that time. In this manner, the change in OAVDE between the two dates is a meaningful number (both being consistent relative to Treasury prices at the two dates) and a legitimate comparison can be made. Using nonarbitrage-free interest rate paths may not result in values that are comparable over time.

When analyzing a set of paths, it is possible to solve for a spread that discounts to the given price for each path separately. In this way one has a statistical distribution of spreads based on the probabilities of the paths. It is possible to compute the mean of such individual spreads and other statistical measures, e.g., standard deviation, skewness and kurtosis. Note that the mean of the individual spread will not necessarily equal the option-adjusted spread (OAS), because the OAS is the spread that discounts the cash flows over all paths simultaneously.

STOCHASTIC METHOD TO ASSESS COST OF EMBEDDED OPTIONS

- Average present values of cash flows computed over set of "arbitrage free" paths.
  - Will reprice Treasury securities.
  - Will satisfy put/call parity.
  - Will provide for relative valuation of securities on a consistent basis.
- Used in finance to compute option-adjusted price and/or option-adjusted spread over Treasuries of a fixed-income security that contains embedded options, e.g., call option, prepayment option.

By using arbitrage-free paths the net worth of the insurance enterprise (OVADE) is valued consistently with other securities.

COMPUTATIONS

- Option-Adjusted Spread
  - Choose a value for OAS and discount all cash flows for each path using the product of (1 + risk free rate + OAS) for that path.
  - Average all the present values using the probability weighting for each path.
  - If the average equals zero, then the OAS value is correct; if not, then repeat with new value.
- The OAS is a single spread that "solves along all paths simultaneously" in that it is the value that makes the average of the net present values for all paths equal to zero.
- Distribution of Spreads
  - For each path solve for the spread such that for the given path the net present value is zero.
  - Obtain the distribution of spreads computed over all paths and calculate:
    - ▸ mean
    - ▸ standard deviation
    - ▸ other moments as needed
- The mean spread is the average of the separate spreads for each path.
- The OAS is not necessarily equal to the mean spread.

The steps to compute OAVDE are as follows: (1) develop an arbitrage free set of interest rate paths that reprices the Treasury curve; (2) use those paths as input to a cash-flow engine that produces distributable earnings; (3) discount the distributable earnings over each path using the generalized net present value algorithm in conjunction with the one-period future rates for that path; (4) take the probability weighted generalized net present values. Note that the distributable earnings calculated along each path for each time interval are assumed to have been paid to the shareholders in cash. So the impact of actually paying cash to shareholders is also correctly reflected.

As noted, OAVDE accepts either a given spread or a given price as input, and solves for the other. Instead of using a spread-to-treasuries, it is possible to modify the formula so that a level hurdle rate, such as the enterprise's cost of capital, can be used. This is also shown in the above reference.

The following formulas are included to illustrate the algebra of the computations. The classic discounting formula will be presented first. Then the appropriate formula for discounting using the generalized net present value algorithm will be shown. The generalized net present value algorithm is consistent with the discounting in option-pricing theory.

Terms:

p index for paths.

P the total number of paths.

t index for time period.

N the total number of time periods per path (the initial time period is t = 0)

j a general index.

Σ used to represent a summation.

Π used to represent the result of a series of multiplications.

$r_{p,t}$ the one period future rate for path p and time period t.

OAS option-adjusted spread.

OAY option-adjusted yield.

$tr_{p,t}$ federal income tax rate for path p and time period t.

prb_p probability for path p.

$DE_{p,t}$ distributable earnings for path p and time period t.

Classical Discounting Formula

$$\text{OVADE} = \sum_{p=1}^{P} prb_p * \left\{ \sum_{t=0}^{N-1} [DE_{p,t+1} / \prod_{j=0}^{t} (1 + r_{p,j} + \text{OAS})] \right\}$$

If there is a time zero distributable earnings amount, then it will be the same for all paths, and it is simply added to the right side of the equation.

The above formula provides the case when discounting via a spread-to-treasuries. If a constant option-adjusted yield is desired, then the formula becomes:

$$\text{OVADE} = \sum_{p=1}^{P} prb_p * \left\{ \sum_{t=0}^{N-1} [DE_{p,t+1} / (1 + \text{OAY})^{t+1}] \right\}$$

Again, if there is a time zero distributable earnings amount, then it will be the same for each path and add it to the right-hand side of the equation.

The formulas using the generalized net present value algorithm are as follows:

Let $\text{PVB}_{p,N}(i) = DE_{p,N}$ and

$\text{PVB}_{p,t}(i) = \text{PVB}_{p,t+1}(i) / (1 + r) + DE_{p,t}$

for $t = N - 1, N - 2, \ldots 1,$ or 0, if there is a time 0 distributable earnings amount;

where $r = i$ if $\text{PVB}_{p,t+1}(i) > 0,$, and $r = (1 - tr_{p,t}) * r_{p,t}$ if $\text{PVB}_{p,t+1}(i) < = 0$.

Here i is chosen to be $r_{p,t}$ + OAS, if the discounting is to be done at a spread-to-treasuries, or OAY, if a level discount rate is desired. OAY could be either the enterprise's cost of capital or other hurdle rate.

This formula can be summarized as:

$$\text{OVADE} = \sum_{p=1}^{P} prb_p * \text{PVB}_{p,1}(i),$$

if there is no time zero distributable earnings amount. If there is a time zero distributable earnings amount, then the formula becomes:

$$\text{OVADE} = \sum_{p=1}^{P} prb_p * \text{PVB}_{p,0}(i).$$

Note that in the formula above, if the prior PVB term is negative, then the discount rate r is 1 plus the after-tax, one-period, risk-free rate. This is to allow for the tax that the insurance enterprise would have to pay on positive funds held that are needed to meet a subsequent loss. This is equivalent to computing the federal income tax required, adding it to the DE for the period and then discounting by 1 plus the one-period risk-free rate. Thus it is consistent with discounting at the one-period risk-free rate in option pricing, but is computationally more efficient.

APPLICATION TO DISTRIBUTABLE EARNINGS

- Develop set of arbitrage-free interest rate paths and their probabilities.
- Measure distributable earnings along each path. Such earnings will reflect the exercise of options in both the assets and liabilities.
- Discount along each path using the generalized net present value algorithm.

THE OPTION-ADJUSTED VALUE OF DISTRIBUTABLE EARNINGS

"A Method for Option-Adjusted Pricing and Valuation of Insurance Products" *Product Development News*, Issue 30, November, 1991, pp. 1-6.

RESULTS TO ANALYZE

- Given a "price" target, then obtain the option-adjusted spread, the option-adjusted internal rate of return or yield, and the distributions of spreads and yields.
- Given a spread/yield target, then obtain the option-adjusted price and the distribution of prices.

Insurance companies have several degrees of freedom in managing their business. They can choose the liability design and management strategy (e.g., premium loads, fees, costs of insurance, etc.), the interest rate crediting strategy, the investment strategy (including any hedging), and the disinvestment strategy, i.e., how they will handle negative cash flows. (Note: these negative cash flows, i.e., asset cash flows less liability cash flows, are internal to the insurance enterprise. They are augmented by any negative cash flow that is needed to pay out distributable earnings to shareholders.)

Thus, the insurance enterprise can specify all four strategies. Use the term STRATEGY to mean the choice of one each of the specific strategies. The set of all alternative strategies can then be evaluated using OAVDE. The goal is to find the STRATEGY that is both feasible and practical in both the insurance marketplace and the investment marketplace that optimizes OAVDE.

A STRATEGY consists of:

- A product liability design and product management strategy;
- an interest crediting strategy;
- an investment strategy; and
- a disinvestment strategy.

For a STRATEGY to be *acceptable* and *successful* it must be *feasible* and *practical* in the insurance marketplace and in the investment marketplace.

In the first example, or application of OAVDE, it will be assumed that of the four strategies the liability design, the interest crediting strategy and the disinvestment strategy are fixed. The only strategy that is allowed to vary is the investment strategy.

The liability is a single-premium deferred annuity (SPDA) with a 4% guaranteed crediting rate, and a surrender charge that grades to zero over seven years. The interest crediting strategy is a portfolio rate with the renewal rate being guaranteed for each policy year at a time. The interest margin is set so that the internal rate of return on distributable earnings (where interest rates are flat and unchanging) is 15%. The disinvestment strategy is to borrow at the short rate plus a spread.

EXAMPLE 1: EVALUATE INVESTMENT STRATEGIES

- SPDA -- guaranteed interest rate of 4%,
- Use portfolio rate crediting strategy,
- Assume negative cash flows borrowed,
- Interest margin set so that with a flat yield curve the internal rate of return on distributable earnings is 15%,
- Use the April 30, 1992 yield curve, and
- Analyze five investment strategies.

Investments will be limited to noncallable bonds. There are no embedded options in the investments. Thus, any resulting financial impacts to the insurance enterprise can only result from option exercise of the policyholders exercising the option to surrender the policy for the cash surrender value. Five investment strategies will be tested. They are: invest long (25-year bonds); when positive cash is available, then purchase new assets having a duration of 4.6 years; when cash is available, then purchase a ladder, i.e., a combination of two-, three,- five,- seven, and ten-year bonds; invest so as to maintain an asset portfolio duration as close as possible to three years; invest so as to maintain an asset portfolio duration as close as possible to five years. The April 30, 1992 yield curve was chosen.

INVESTMENT STRATEGIES

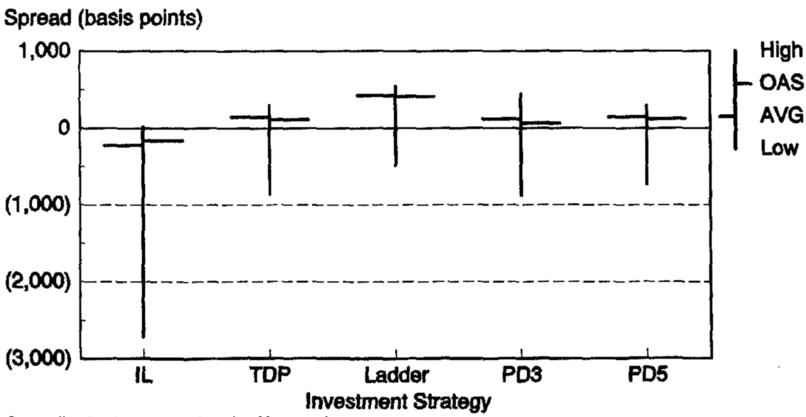
- Invest long. (IL)
- Purchase new assets with constant duration, 4.5 years. Target Duration of Purchase (TDP)
- Ladder (two, three, five, seven, and ten-year maturities). (Ladder)
- Maintain constant portfolio duration of three years. (PD3)
- Maintain constant portfolio duration of five years. (PD5)

WAYS TO SUMMARIZE RESULTS

- Graphical analysis of distribution of spreads and the OAS.
- Graphical analysis of distribution of yields and the OAY.
- Graphical analysis of the distribution of distributable earnings and the option-adjusted value of distributable earnings at discount rates of the risk-free rate (RFR), the cost of capital (12%), and the hurdle rate (15%).

The initial investment to cover the statutory strain and necessary target surplus by the insurance company is the initial distributable earnings amount. On this basis, the option-adjusted spread-to-treasuries that the insurance enterprise can earn, can be computed. Chart 5 shows the range of spreads for each investment strategy, the option-adjusted spread and the mean spread computed over all paths. Note that the OAS can differ from the mean spread as the OAS solves the equation over all paths simultaneously.

CHART 5
Option-Adjusted Spread
Distribution of Spreads for Five Strategies

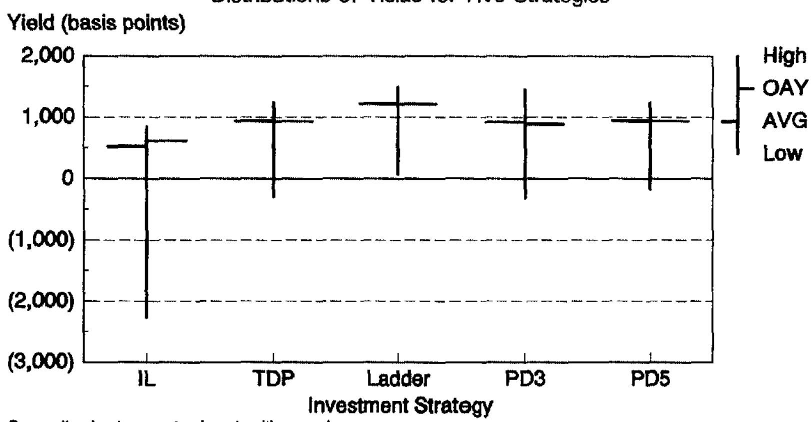


Generalized net present value algorithm used.

The graphs indicate that for each investment strategy, the distribution of spreads about the mean is negatively skewed (more lower than higher than the mean) and has high positive kurtosis (more data points far away from the mean than would happen in a normal distribution and data points clustering about the mean with higher probability than in a normal distribution). Results may tend to cluster about the mean, but they deviate with higher probability than expected for a normal distribution and they tend to sharply deviate to low values. That means that there is far more downside risk than there is upside potential. Another way of saying this is that the pricing results using a flat and unchanging yield curve is probably an optimistic assessment. The graph shows that the ladder investment strategy has the highest OAS with the smallest range and the invest long strategy has the lowest OAS (negative, in fact) and the largest range, or plenty of downside risk. Of the five strategies, the ladder appears to be the best.

Chart 6 shows the results on an option-adjusted yield basis, i.e., where a level discount rate is used instead of a fixed spread to the one-period future rate. Again, the ladder strategy would result in the highest option-adjusted yield. Note the similarity of the outcomes and the ranges for each investment strategy here when compared to the OAS graph (Chart 5). This option-adjusted yield is about 12%. This is 3% less than the static pricing result of 15%. If the insurance enterprise had a "go/nogo" criteria of an option-adjusted yield no less than 12%, then the ladder would be chosen as the only acceptable investment strategy.

CHART 6
Option-Adjusted Yield
Distributions of Yields for Five Strategies



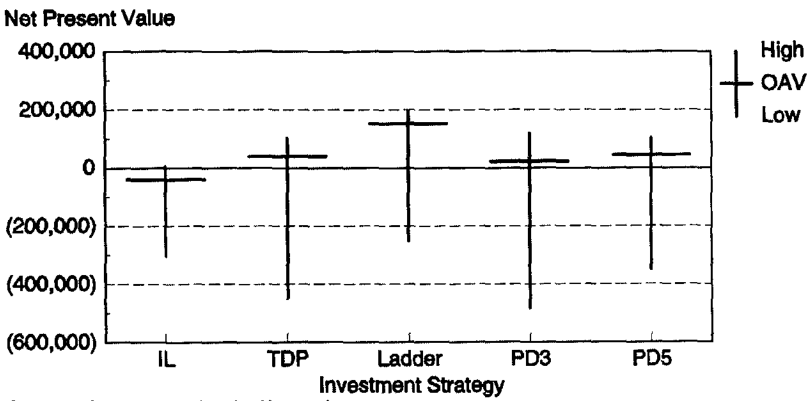
Generalized net present value algorithm used.

Charts 7 and 8 show the OAVDE that results when computed assuming zero spread over Treasuries (Chart 7) and a constant yield of 12% (Chart 8). Note that in this case, only the distributable earnings after the initial investment are considered. The OAVDE indicates the investment that should be made assuming the desire of a given OAS, in this case zero, or a given level yield. In each chart the option-adjusted value of distributable earnings is shown and the range of present values over the set of paths used. Here higher is better. Both graphs would result in choosing the ladder investment strategy. Note that the option-adjusted value for the ladder strategy in chart 8 is just slightly greater than zero. This is consistent with the fact that the option-adjusted yield is slightly greater than 12%.

Note that the ladder has the greatest minimum net present value of distributable earnings over all paths in charts 7 and 8. When analyzing results and making decisions it is important to think not only in terms of means or OAS and OAVDE, but in terms of the statistical distributions of outcomes. It is straightforward to prepare both probability density function graphs and cumulative distribution function graphs for each investment strategy. One can analyze the percentage of paths for which outcomes are less than a prespecified comfort level. Additional insight can be gained by overlaying these graphs on the same graph.

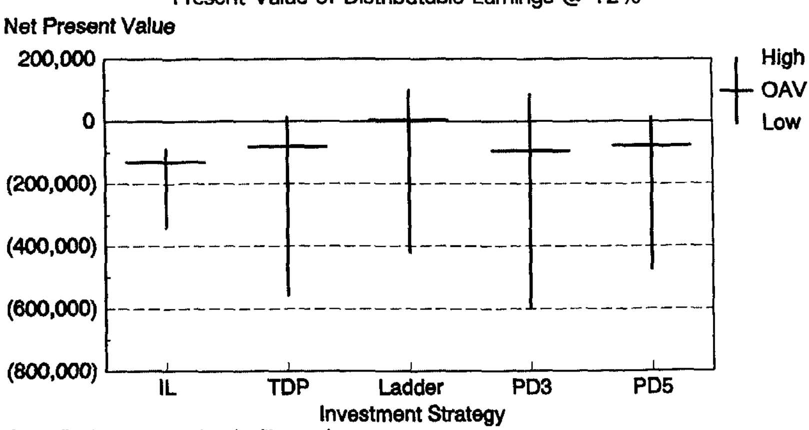
If a great many strategies have been tested instead of only these five (where only the investment strategy was allowed to vary), then even these graphs might have been less useful in decision making. This is because the statistical distribution is important in addition to the OAS or OAVDE values. Two methods may be employed to alleviate some of this difficulty.

CHART 7
Option-Adjusted Value of Distributable Earnings
Present Value of Distributable Earnings @ RFR



Generalized net present value algorithm used.

CHART 8
Option-Adjusted Value of Distributable Earnings
Present Value of Distributable Earnings @ 12%



Generalized net present value algorithm used.

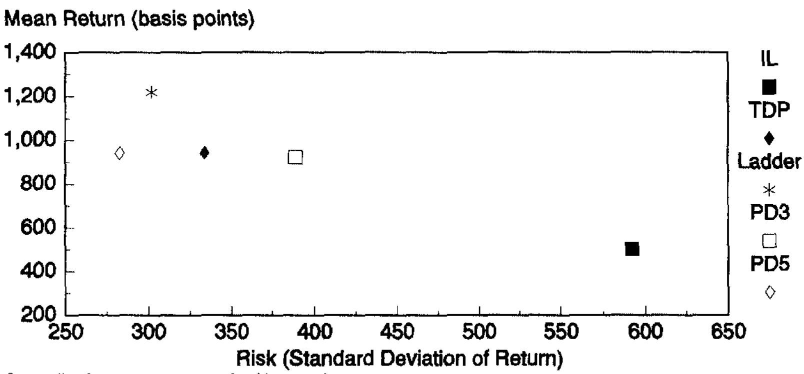
First, mean/variance diagrams can be used. Here, the mean return or spread is plotted on the vertical axis and the standard deviation is plotted on the horizontal axis. This is the Markowitz efficient frontier approach (chart 9). Using this method the goal is to have a compromise between the mean return and risk, i.e., the standard deviation of return. In chart 9 it is clear that the ladder investment strategy is superior in these terms. By the way, check out the invest long strategy!

MEAN/VARIANCE DIAGRAMS

- Plot return (mean) versus risk (standard deviation).
- Markowitz' Efficient Frontier.
- Uses only first two moments of distribution (due to normal distribution assumption).
- Must estimate trade off between risk and return.

It is interesting to note that the portfolio duration 5 strategy is superior to the portfolio duration 3 strategy. This may seem counterintuitive because an asset strategy whose duration is shorter is normally thought to be superior to one whose duration is larger for SPDAs. But an analysis of the results shows this not to be the case here.

CHART 9
Risk/Return Diagram
Mean/Variance Criteria
Markowitz Efficient Frontier



Generalized net present value algorithm used.

Recall that the crediting strategy is based on the portfolio earned rate. If one invests slightly longer, then one will credit slightly more interest. When the path-by-path results of these two investment strategies were investigated, it was discovered that in the declining interest scenarios the results for the duration 5 strategy were clearly superior to those for the duration 3 strategy. This was to be expected. When the rising scenarios were examined, the duration 5 results for the mildly increasing scenarios were still superior to those for duration 3, apparently due to the portfolio rate crediting method. In the steeply increasing scenarios, the duration 3 results were superior to the duration 5 results, as was expected. But in the mean and option-adjusted value, then duration 5 was superior to the 3.

Do not be tempted to extrapolate these results to general principles. The results are a function of all of the assumptions. Investments were limited to noncallable bonds. Note that the 4/30/92 yield curve was steeply increasing.

There is some risk in using the mean/variance diagram. It is due to the fact that it only uses the first two moments of the distribution and inherently assumes the distribution is normal. As noted above, the distribution is highly nonnormal. Thus one should exercise caution. Another difficulty is that one must estimate the tradeoff between risk and return. How much additional risk are you willing to assume to raise your expected return?

A second method to address the problem of comparing results of many strategies is to use the concept of risk-adjusted value (RAV) or risk certainty equivalent. This was developed by John M. Cozzolino and appeared in the *Sloan Management Review*, Spring 1979. The method uses an exponential utility function and requires an estimate of the company's or product manager's aversion to risk. But once done, the method linearly rank orders all strategies. The best can then be easily identified. The method also has the advantage of using all the moments of the probability distribution. Thus no important information is omitted. Chart 10 shows the rank ordering for each of three discount rates: the risk-free rate (one period future rates), 12%, and 15%. No matter which discounting basis is chosen, the maximum RAV occurs for the ladder strategy.

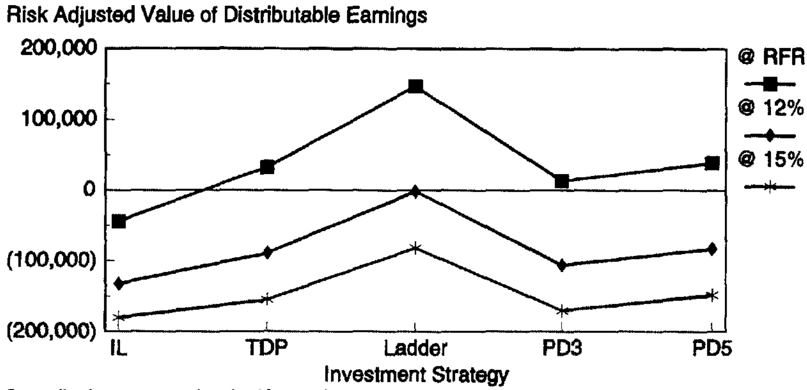
RISK ADJUSTED VALUE

- Risk certainty equivalent
- Based on Utility Theory
- Need estimate of company's aversion to risk
- Uses ALL moments of distribution
- RAV linearly ranks all strategies. Therefore, there is no need to make intuitive trade off.

A natural question to ask is to what degree did the favorable results attributable to the ladder strategy depend on the fact that the initial yield curve was steeply increasing? Additional tests were done with a shallow positively sloped yield curve, a shallow negatively sloped yield curve and a steep negatively sloped yield curve.

For the two shallow-sloped yield curves the ladder was again the best investment strategy. For the steeply negatively sloped yield curve, the ladder was the second best strategy, but it was a close second. Interestingly, the portfolio duration 3 strategy was the worst in this case. Thus for the given liability design, interest crediting strategy and disinvestment strategy the ladder investment strategy was robust over a spectrum of initial yield curves. These results also persisted if the initial yield curve was parallel shifted. Note again that if the allowable universe of assets were extended to include sinking funds, callable bonds, mortgages, mortgage-backed securities, CMOs, etc., then far more combinations of asset allocation and maturity would have been possible and tests would have to be made to determine the optimal investment strategy. Here the risk-adjusted value method would be of assistance in assessing the results.

CHART 10
Risk-Adjusted Value
Risk Certainty Equivalent



Generalized net present value algorithm used.
Risk aversion factor equal to 1/1,000,000.

The second example focuses on evaluation of liability designs. This asks the question of what does it cost to provide additional liability options. In this example, the crediting strategy is assumed to be portfolio; the investment strategy is assumed to be the ladder; and the disinvestment strategy again borrow.

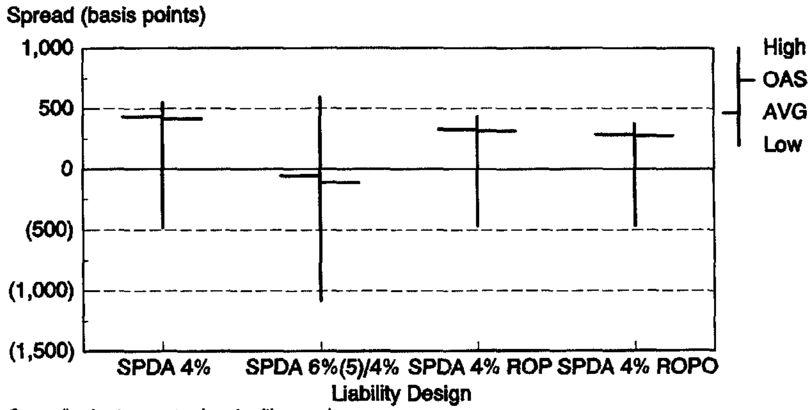
There are four liability designs considered. First is the bare bones annuity design of the first example with the same interest margin. Second, the bare bones annuity is enhanced with a guaranteed minimum crediting rate of 6% for the first five years and 4% thereafter. This too has the same interest margin as in the first example. Third, a return of premium feature is attached to the bare bones annuity whereby the owner may surrender his or her contract at any time and receive at least the premium paid. Note that in this case the interest margin is increased to reflect the cost of holding additional statutory reserves and additional risk-based capital (which is a function of the reserve). This does not represent the exercise cost of the option granted; it merely restores the company to its static profitability of 15% on distributable earnings to compensate them for holding larger reserves and risk-based capital. The additional margin required is about 10-14 basis points. Fourth, the return-of-premium feature is attached to the annuity, but the company uses the original margin, i.e., it absorbs the cost of holding the higher reserves and risk-based capital.

EXAMPLE 2: EVALUATE LIABILITY DESIGNS

- SPDA with guaranteed interest rate of 4% (SPDA 4%)
- SPDA with guaranteed interest rate of 6% for five years and 4% thereafter (SPDA 6% (5)/4%)
- SPDA (4%) with return of premium guarantee and interest margin adjusted for cost of guarantee due to extra reserve required. (ROP)
- SPDA (4%) with return of premium guarantee and original margin. (ROPO)

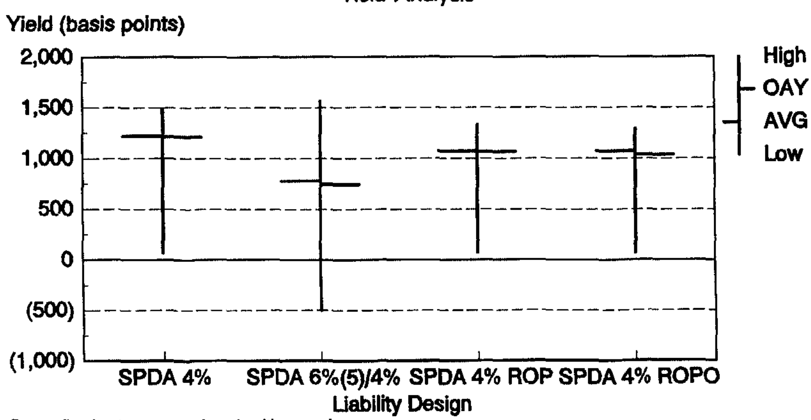
Charts 11 and 12 demonstrate the distributions of spreads, option-adjusted spreads and option-adjusted yields obtained for each liability design. Again the distributions tend toward negative skewness (more downside risk than upside potential) and positive kurtosis (more data clustered about the mean than in a normal but also more outliers than a normal distribution would have). The bare bones annuity has an OAS of just less than 500 basis points over treasuries and an option-adjusted yield (OAY) of just over 12%, as before.

CHART 11
Comparison of Liability Designs
Spread Analysis



Generalized net present value algorithm used.

CHART 12
Comparison of Liability Designs
Yield Analysis



Generalized net present value algorithm used.

The cost in lost profitability of the temporary interest guarantee (6%/4%) is enormous. The OAS is negative and the OAY is less than 8%. Clearly the temporary guarantee is a very costly option to grant. The return of premium annuity has a nearly 2% less OAY and the return of premium annuity with the original margin has an OAY slightly less due to its lower margin.

Chart 13 shows the results on a RAV basis. They are consistent with the OAS and OAY results.

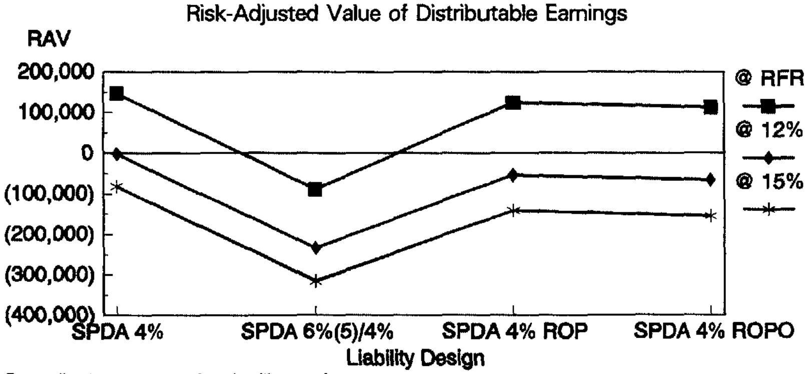
In practice, one would have to broaden the range of strategies in order to determine if it was possible to offer any of the added liability features and still maintain the desired OAY of 12%.

Chart 14 offers an additional analysis for this example. Here the OAVDE, discounted at the risk-free rate for illustration purposes, is shown for the base yield curve and parallel shifts of +/- 25, 50, and 75 basis points. (Note that nonparallel shifts can and should be tested as well.) The change in OAVDE is relatively small for the bare bones annuity as the yield curve shifts. The results for both return-of-premium designs are relatively similar and slightly below the base case.

Recall that the OAVDE value represents the probability-weighted present value of future distributable earnings across a fan of arbitrage-free paths. These paths are based on the initial yield curve and reprice the then current treasury curve. When a shift in the curve is made, then an entirely new set of arbitrage free paths, each with its own probability, is generated; an entirely new set of distributable earnings is computed for each path; and, their option-adjusted present value is determined.

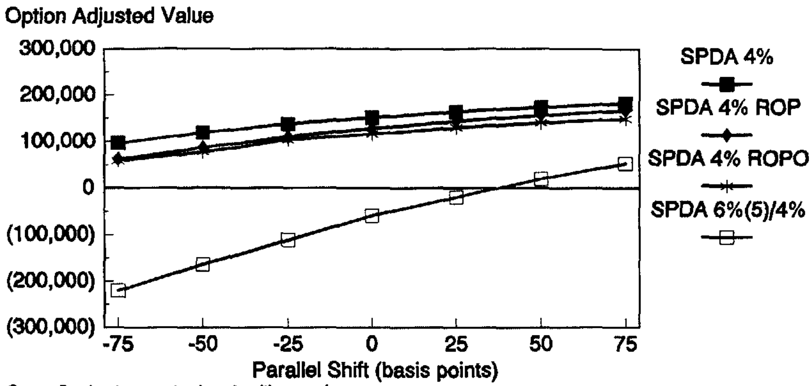
Chart 14 shows the results of seven OAVDE computations; the first based on the initial yield curve and six more based on the parallel shifts of the yield curve described above. The change in the level of OAVDE represents the sensitivity of the OAVDE or option-adjusted net worth of the enterprise to the given instantaneous shifts in the yield curve.

CHART 13
Comparison of Liability Designs
Risk-Adjusted Value of Distributable Earnings



Generalized net present value algorithm used.
Risk Aversion Factor 1/1,000,000

CHART 14
Comparison of Liability Designs
Option-Adjusted Value of Distributable Earnings
@ Risk-Free Rate



Generalized net present value algorithm used.

The graph of the temporary guarantee annuity is materially below the others, indicative of its significant cost in profitability to the company. The graph of this case, however, has a steep upward slope. This reflects the fact that if interest rates rise, the value of the temporary guarantee drops (therefore larger OAVDE) as it is less likely that the insurance enterprise will have to pay off on it. But as rates fall, the value of the temporary guarantee becomes much more costly as the probability of pay off increases (thus lower OAVDE).

Chart 15 shows a market value of liabilities (at zero spread-to-treasuries) graph for these liability designs. Here the higher the line, the greater the amount the company needs to hold to fund the liability. For this case, the liability cash flows are discounted at zero spread-to-treasuries. As rates fall, the cost to the company of the temporary guarantee annuity rises and vice versa. These results are consistent with those for OAVDE.

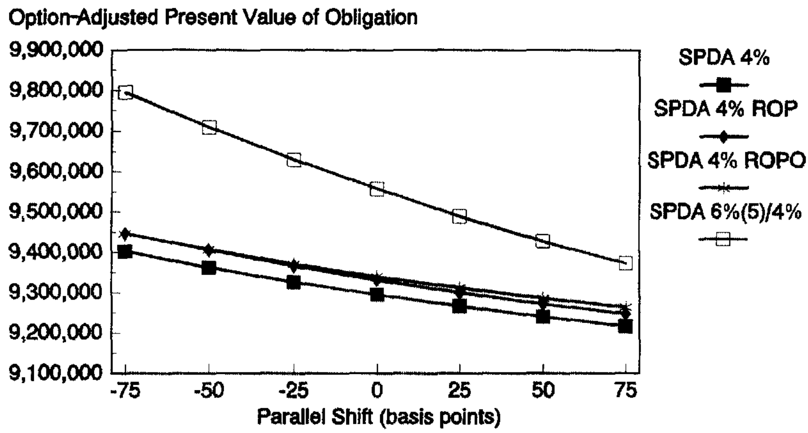
When using market-value analysis to examine hedge strategies, it was noted that it is possible to overhedge. This can be avoided by the following approach. First identify strategies that produce acceptable market-value diagrams. Choose the strategy that optimizes OAVDE. Yield-curve changes should be considered. One could also optimize OAVDE directly as the value of the firm is the ultimate quantity to hedge.

EVALUATION OF HEDGE STRATEGIES

- Identify strategies that result in acceptable cash-flow management as determined by MVA/MVL analysis.
- For each strategy, compute the option-adjusted value of distributable earnings (OAVDE) and determine how it changes for shifts in the yield curve.
- Choose the strategy with the best OAVDE results. (risk-adjusted values, etc.)

The third example is an application of OAVDE analysis to the evaluation of hedge/management action alternatives. In this case there is an existing block of single-premium deferred annuities, similar in structure to those of the first example. These annuities credit a portfolio rate of interest. The assets supporting this existing block are a combination of noncallable bonds, sinking funds, mortgages, mortgage-backed securities and CMOs. As will be seen, the block has exposure to interest rate risk.

CHART 15
Comparison of Liability Designs
Market Value of Liabilities



EXAMPLE 3: EVALUATE HEDGE/MANAGEMENT ACTION

- SPDA – 4% interest guarantee.
- Alternatives:
  - Base Case – Do nothing.
  - Option 1 – Buy caps as insurance for company.
  - Option 2 – Buy caps as insurance for policyholders and company.
  - Option 3 – Self insure.
  - Option 4 – Buy caps as part of regular investment program.

There are five courses of action that management is considering. The first, or base case, is do nothing, i.e., maintain the status quo, make no attempt to hedge the interest rate risk.

The second, option 1, is buy interest rate caps as "insurance" for the company. The idea is that since the company sold the put option to the annuity policyholder, they will hedge that risk by buying interest rate caps. The caps are designed with strike levels that will only pay off for a run up in interest rates of 200 to 300 basis points. This is stop-loss insurance. The SPDA's prior interest margin will be increased by the amount needed to repay the cost of purchasing the caps at the company's cost of capital. If the caps pay off, then the income goes to the benefit of the company.

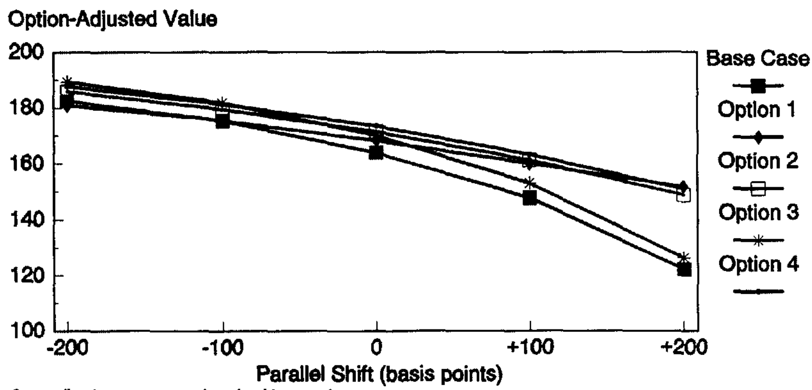
The third, option 2, is buy interest rate caps as insurance for both the policyholders and the company. The same cap structure is used and the interest margin increased the same amount as in option 1. The caps are included in the portfolio, and if they pay off, then the amount of cap income is part of the portfolio investment income. The company takes its margin from the net earned rate on the portfolio.

The fourth, option 3, is self insure. Here the company increases the interest margin by the amount needed to pay for the cap program, but does not actually buy the caps. Instead, it allows the extra margin to be paid to the shareholders via increased distributable earnings.

The fifth, option 4, is not to treat caps as "insurance" at all, but instead treat caps as part of the normal investment program. In this case the cap program is established by selling existing assets to fund their purchase and no change is made to the interest margin.

Chart 16 presents the OAVDE analysis (net worth) for all five alternatives for the initial yield curve and parallel shifts of +/- 100 and 200 basis points. Here the OAVDE results are discounted at 12%, the enterprise's cost of capital.

CHART 16
Option-Adjusted Value of Distributable Earnings
@ 12%



Generalized net present value algorithm used.

A static appraisal of this block at the cost of capital produces a value of about $185 million. The OAVDE of the base case at the cost of capital produces a value of about $165 million. Thus the hidden cost of the embedded options in the assets and liabilities and in their interaction is worth about $20 million. A buyer who utilized only static appraisals would overpay significantly for this block if he or she continued to manage it along base-case lines.

If interest rates fall there is a mild increase in the OAVDE of the block. But if interest rates rise, then there is a material decline. Option 3, self insure, provides a slightly higher value than the base case across all shifts, but this is due to taking a higher margin. The risk exposure is still there.

Options 1, 2 and 4, however, exhibit much better risk characteristics if interest rates rise. They also show a better OAVDE result for the zero basis-point shift than the base case. This is due to the increased utility of the caps if interest rates rise from the level at the zero basis-point shift. Results for all three are similar if rates fall. Options 1 and 2 appear to offer the best results. It is seen that Option 2 is superior to Option 1 for shifts of less than 200 basis points. But for shifts of 200 basis points or more, option 1 is superior.

It might be thought that option 2 is superior because if the caps pay off, then the income is shared with the policyholders which prevents lapse. In fact, this is not the case. Recall that the crediting strategy is a portfolio crediting strategy. In option 2 the caps become part of the portfolio. Consider what the portfolio initially looks like under options 1 and 2. In option 1, the portfolio has no caps. The caps belong to the shareholders. In option 2 the portfolio initially looks just like option 1 but it includes the caps. That means that the book value of the caps is included in the book value of the portfolio and the amortization of the cap book value appears as negative investment income in the portfolio.

The portfolio yield for option 1 will be higher than for option 2. When the caps are not paying off, the numerator of the portfolio yield for option 2 is lower than for option 1 as there will be the negative investment income due to the amortization of the book value of the caps. Also, the denominator of the portfolio yield, or book value of assets, of the option 2 portfolio will be greater than for option 1 as the book value will include the book value of the caps. Thus option 2 has a lower yield than option 1. Since the interest margin is the same, then the company is actually taking more effective spread in option 2 than in option 1 for those scenarios where the caps are not paying off.

If the yield curve shifts upward by 200 basis points and the OAVDE is calculated, then a great many of the paths will have the caps paying off. In those cases more interest will be credited to policyholders under option 2 than option 1. That means less earnings to the company; and so option 2 is less attractive to the shareholders if rates rise 200 or more basis points. Overall, option 1 seems to provide the best choice.

Chart 17 presents the results of the five courses of action from the point of view of market-value analysis. Here, the market value of surplus equals the market value of assets less the market value of liabilities. For simplicity, the market value of liabilities is computed at a zero spread-to-treasuries. If a different rate were used, then the primary change would be the absolute level of the curves, not the relative positions or change in shape. Overall, option 1 appears to be the best.

Chart 18 shows the comparison of the range of results for all paths for the zero basis-point shift, i.e., initial yield curve, for all courses of action. Note again the tendency for the distributions to have negative skewness and positive kurtosis. The distributions for option 1 and option 2 are very similar. Option 2 has a slightly higher value than option 1 at the zero basis point shift. This, of course, diminishes as rates rise.

CHART 17
Example 3: Summary
Option-Adjusted Value of Surplus

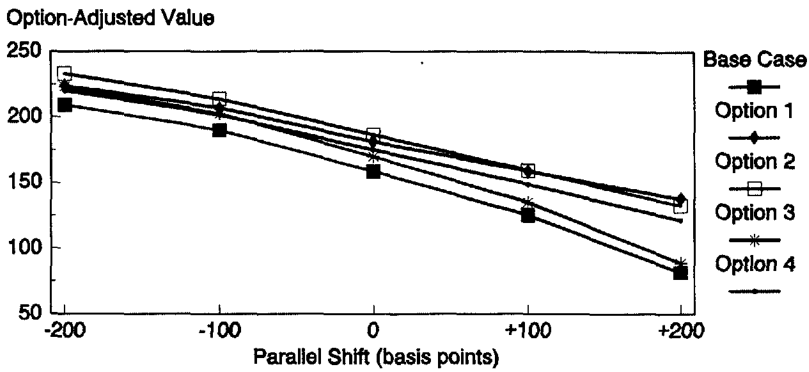
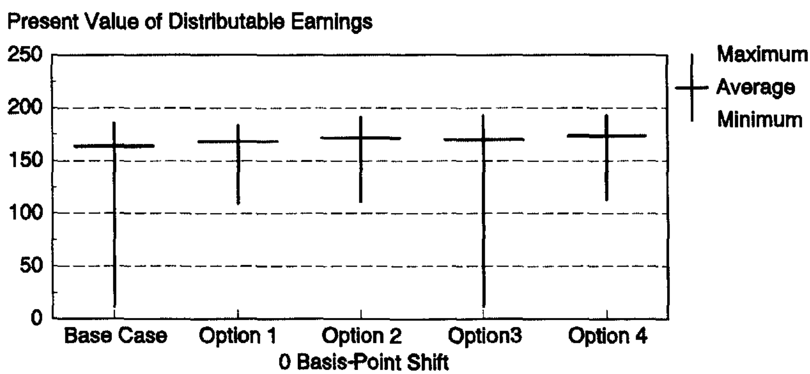


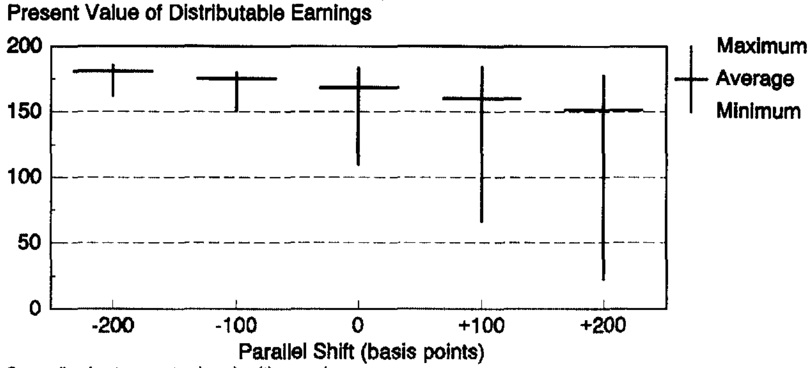
CHART 18
Distribution of Present Values
Example 3



Generalized net present value algorithm used.

Chart 19 shows the range of present values and the OAVDE for all paths tested for option 1 at the initial yield curve and the shifts of +/- 100 and 200 basis points. As you would expect, the range shrinks as rates fall and widens as rates rise. But note how the OAVDE only falls modestly as the rates rise.

CHART 19
Distribution of Present Values
Example 3
Option 1



Generalized net present value algorithm used.

In the prior charts, the OAVDE and market value of surplus have been presented for various changes in the level of interest rates. The reader can see what happens by examining the graph itself. A numerical display of these results may also be shown. Duration captures the linear change in the curve at a given point and convexity captures the degree of bend of the curve away from approximating straight line. Other things equal, the best hedge will have the lowest duration and greatest convexity among the alternatives.

Chart 20 shows the duration and convexity of the OAVDE and surplus curves for all five alternatives. Option 1 has the lowest duration and the highest convexity for both OAVDE and surplus.

CHART 20
Example 3: Duration & Convexity at 0 Basis-Point Shift

O A V D E
@ 12%

	Duration	Convexity
Base Case	8.4	(280.6)
Option 1	4.6	(63.6)
Option 2	5.3	(129.0)
Option 3	8.4	(308.2)
Option 4	5.2	(138.7)

O A V of Surplus

	Duration	Convexity
Base Case	20.3	(113.1)
Option 1	13.3	191.3
Option 2	14.5	69.3
Option 3	19.8	(104.4)
Option 4	15.1	45.8

Generalized net present value algorithm used.

Chart 21 presents the same information for OAVDE and surplus for *a* +100 basis-point shift. Again option 1 is the best and by a wider margin. This is not unexpected as option 1 exerts itself as rates rise.

The fact that the alternative course of action that optimizes OAVDE also optimizes surplus may not be so surprising. The action that best hedges the value of the firm, OAVDE, which is based on future profitability, should perform well for hedging surplus. Intuitively, if you have hedged profitability, then you have hedged solvency, which is what this use of surplus is really getting at.

CHART 21
Example 3: Duration & Convexity at +100 Basis Point Shift

O A V D E
@ 12%

	Duration	Convexity
Base Case	12.7	(645.6)
Option 1	4.9	(8.4)
Option 2	6.6	(142.6)
Option 3	12.9	(639.3)
Option 4	6.6	(138.9)

Generalized net present value algorithm used.

O A V of Surplus

	Duration	Convexity
Base Case	24.1	(819.2)
Option 1	11.9	107.2
Option 2	14.4	(56.9)
Option 3	23.8	(857.7)
Option 4	15.2	(65.3)

Below is a brief summary of applications of OAVDE and several of those applications.

USES OF OAVDE

- Risk identification and quantification:
  - interest rate risk
  - other stochastic risks
- Identification of profit optimization/risk minimization strategies
- Measurement of total financial performance
- Risk-based capital quantification
- Capital allocation
- Option-adjusted gross premium valuation
- Option-adjusted appraisal values
- Option-adjusted value-added analyses

OPTION-ADJUSTED GROSS-PREMIUM VALUATION

- The option-adjusted net present value of all benefits and expenses (excluding federal income tax) less premiums computed at:
  - a spread over Treasuries equal to that of the asset portfolio, or
  - a spread over Treasuries commensurate with the risk of default of the insurance enterprise.
- Uses:
  - assess loss recognition on an option-adjusted basis; and
  - serve as a floor for the net GAAP liability on a market-value basis.

OPTION-ADJUSTED APPRAISAL VALUE ANALYSIS (OAAV)

- Option-adjusted appraisal value equals:
  - market value of assets supporting the excess of surplus and items in the nature of surplus over risk-based capital supporting the liabilities and related assets; plus
  - option-adjusted value of distributable earnings on existing business; plus
  - option-adjusted value of distributable earnings on future business.
- Can examine change in OAAV from period to period.

OPTION- ADJUSTED VALUE ADDED ANALYSIS (OAVA)

- Option-adjusted value added in excess of cost of capital (or other hurdle rate) equals:
  - Option-adjusted value of distributable earnings as of the end of the period less option-adjusted value of distributable earnings as of the beginning of the period; plus
  - distributable earnings for the period; less
  - cost of capital (or other hurdle rate) multiplied by opening present value of distributable earnings.

ANALYSIS OF OAAV AND OAVA BY SOURCE

- External environment
  - change in level and shape of yield curve,
  - change in volatility of interest rates
- Internal environment
  - change in investment/reinvestment strategy,
  - change in crediting rate strategy
- New business
- Existing business
- Shareholder dividends paid