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INTRODUCTION TO DERIVATIVE PRODUCTS

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- Basic description of instruments
 - interest rate swaps, caps and floors
 - futures and options
- General pricing concepts
- Counterparty credit risk
- Applications for life insurance company portfolios

MR. MARTIN P. KLEIN: We'll talk about derivative products, define what they are, discuss how they are priced, and then cover how they can be applied to insurance company portfolios. We have an illustrious panel for you. Peter Minton is a principal in the insurance group of Morgan Stanley and he has been for two years. Prior to joining Morgan Stanley, he was a money manager at CNB Investment Counselors. Frank Sabatini is the national director of asset/liability management services at Ernst & Young. Prior to joining Ernst & Young, Frank was chief financial officer (CFO) in the pension operations area at Connecticut Mutual. I am a partner and chief actuary with Analytical Risk Management, which is a firm providing product development and asset/liability management services to insurance companies and other financial institutions. Following my remarks, where I'll be defining what derivatives are, Peter will talk about how derivatives are priced, and then Frank will discuss applications for life insurance company portfolios.

Derivative products have existed for quite some time, but generally insurance companies really haven't taken full advantage of these vehicles. With the events of the last few years, circumstances have evolved such that derivatives will become an integral tool for insurance companies. As many of you are aware, with the insolvencies of many insurance companies recently due to either credit risk or illiquidity risk, and with the generally strict treatment given credit risk by the new risk-based capital regulations, many insurance companies are shifting their focus away from credit risk and toward interest rate, or what I like to call analytical, risk. Obviously, interest rate risk must be and fortunately can be both measured and managed. With the advent of cash-flow testing, companies and regulators will become more adept at evaluating and managing interest rate risk. Derivatives will become a very important tool in managing interest rate risk, not only in new business written but also in existing business.

Let's talk more specifically about derivative products. I'll group them into three different categories. We'll first talk about futures, then options, and then finally we'll talk about interest rate swaps, caps, and floors.

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A futures contract is an agreement between a buyer (or a seller) and an established futures exchange or its clearinghouse, in which the buyer (or seller) has the obligation to take (or make) delivery of a specific amount of an item at a specific price at a specific time. If the investor has brought a futures contract, he is said to be "long" the futures. If he has sold a contract, he is said to be "short" the futures. There's a variety of items for which futures contracts are available, including commodities, stocks, and bonds. Let's talk about financial futures. We could talk about pork belly futures, but they don't have much application for many of you, with respect to your life insurance companies. Financial futures can deal with interest rate risk, stock or equity risk, or currency risk. Obviously, all three of these can have application for insurance companies, but interest rate risk is the risk that is generally the most critical for life insurance companies.

I will provide an example. There is a portfolio manager at Automatic Bank Check (ABC) Life, which is a very prolific company. Huey, the portfolio manager, sells or shorts a futures contract on 6% ten-year treasury notes for settlement in one year. The agreed-to price at settlement in one year is \$10 million. In other words, Huey must deliver 6% ten-year treasury notes in one year, at which time he will receive \$10 million.

Let's think about two possible interest rate scenarios. In the first scenario, interest rates rise and the bonds are worth \$9 million at the end of the year. Huey makes \$1 million on the futures contract. If rates fall, on the other hand, and the bonds are worth \$11 million, Huey loses \$1 million on the futures contract. Futures might be used in a speculative way. For example, if Huey thinks interest rates are going to go up, he might short futures. If interest rates do in fact go up, he wins big. If they go down, he loses. But the real application for insurance companies is the ability to use futures as a hedge. Like most life insurance companies, ABC Life has many bonds on its balance sheet. On a market-value basis, bonds go down in value as interest rates go up, and they go up in value as interest rates fall. Huey, by shorting futures, offsets these market-value swings of the bonds, because the short futures contract position performs the opposite of how the bonds perform.

Now let's move to options. An option is a contract in which the seller grants the buyer the right to purchase from, or sell to, the seller a particular instrument at a specific price (which is called the strike price) at a specified time. In comparison to futures, options involve the right to exercise and futures involve an obligation. This difference will become clearer when we get to the example on options. In return for granting this right to the buyer, the seller gets some money, usually called the option price or the option premium.

There are two basic types of options. One is a call option, in which the seller gives the buyer the right to purchase a designated instrument at the strike price. An option in which the seller grants the buyer the right to sell the designated instrument at the strike price is called a put option. There are two types of exercise rights; i.e., two ways that the purchaser of the option can exercise the option. One is called a European option, in which the buyer can exercise the option only at the expiration date. With an American option, the option can be exercised anytime during the term of the option, up to and including the expiration date. So, an American option is more valuable than a European option.

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Again, let's go through another example. Louie is another portfolio manager who sits right next to Huey at ABC Life. Louie purchases European-style put options on 6% ten-year treasury notes that are now valued at \$10 million. With European options, he can exercise only at expiration, which we'll say is in three months. The strike price in this example is \$9.5 million. The option premium is \$100,000. In this first scenario, in which interest rates rise, the value of the bonds sinks to \$9 million. The put, which is purchased for \$100,000, is exercised, and the bonds are sold for \$9.5 million. So, although the bonds are worth only \$9 million, the put "stops" the loss for drops in value below \$9.5 million.

Let's look at another scenario in which interest rates fall. Interest rates fall such that the bonds are now worth \$11 million. What happens with the option? Nothing. In this case, Louie will not exercise the option. He has bonds that now are worth \$11 million, so he is not going to exercise his put option and sell them for \$9.5 million. Here, the put just expires worthless, and Louie is out the \$100,000 that he paid. But, he will not incur a loss beyond the \$100,000 he paid for the option premium. Compare that with futures. If interest rates had fallen, and Huey were short the futures contract, a market-value loss would be incurred on the futures contract. In the put option example, there is no such loss; an option is one-sided, but a futures contract is symmetric.

Now let's move to the area that probably has the largest application for life insurance companies – interest rate swaps, caps, and floors. An interest rate swap is an agreement between two participants or counterparties to exchange interest rate payments at periodic intervals during the term of the agreement based on a notional principal amount. This notional principal amount is never paid or received, but rather serves as a basis upon which interest rate payments are determined. Although there are several variations of interest rate swaps, as well as currency swaps and equity swaps, the typical interest rate swap involves an exchange of fixed-rate interest payments for floating-rate interest payments. Typically, the floating-rate payments are linked to a short-term floating index called London Interbank Offered Rate (LIBOR) or perhaps in some cases to T-Bills, or to the federal funds rate, or to some other such index. No premium is generally paid upon entering a swap. The floating rate in a swap for LIBOR is generally LIBOR flat, and the fixed rate is determined at the initiation of the swap, depending on market conditions, such as yield-curve shape. Swaps are used to convert interest rate payments for a financial vehicle from fixed rate to floating rate or vice versa.

Let's go through another example (Table 1). Dewey is a portfolio manager who works with Huey and Louie (no big surprise). Dewey is managing the asset portfolio backing a block of floating-rate liabilities. Let's say the liabilities have a floating credited rate of three-month LIBOR plus ten basis points. So every three months, these liabilities reset at three-month LIBOR plus ten basis points. Illiquid fixed-rate assets are in the asset portfolio that Dewey manages. Examples of such assets include private placements and commercial mortgages. Let's say these assets have a book yield of 8%. Obviously, current rates are much lower, but the book yield on these assets is 8% and they mature in five years.

Dewey is concerned that the liabilities float off of LIBOR, but the assets are fixed rate, and he cannot easily sell them. What is a portfolio manager to do? Well, he can

convert the fixed-rate assets to floating rate by entering into interest rate swaps. The notional amount of the swap equals the amount of assets in the portfolio, and the expiration date of the swap lines up with the maturity date of the assets. In this example, the swap Dewey would enter into is such that he pays fixed and receives floating, because he's trying to convert his fixed-rate securities to floating rate.

TABLE 1
Example - SWAP

Assets	
5-year fixed-rate bonds	8.00%
5-year interest rate swap:	
-- pay fixed rate	(5.70%)
-- receive floating rate	LIBOR
Net earned rate	LIBOR + 2.30%
Liabilities:	
-- credited rate	LIBOR + .10%
Gross spread	2.20%

Given current market conditions, the fixed rate that he would pay in the swap might be 5.70% for a five-year swap, and he would receive three-month LIBOR. So let's think about what Dewey is getting, net of the swap. He's getting 8% on the fixed-rate assets, but on the swap he's paying out 5.70% and he's receiving LIBOR. Therefore, net of the swap he's getting LIBOR plus 230 basis points. On the liability side, as we said, the credited rate is LIBOR plus ten basis points, so he's earning LIBOR plus 230 basis points, paying LIBOR plus ten basis points, and the difference between the two, i.e., the spread, is 220 basis points. The nice thing about this spread of 220 basis points is that it is independent of interest rates. It's locked in until the assets mature.

Let's keep this example in mind as we now talk about interest rate caps and floors, which also can be used to hedge this particular kind of risk. For a premium that's paid to the seller, the purchaser of a cap or floor buys protection (or insurance, in the parlance of actuaries) against rising or falling interest rates, respectively. An interest rate cap is an agreement in which the seller, in return for a premium, reimburses the buyer for increases in a particular interest rate index above a predetermined level, which is generally called the strike yield, based on a notional principal amount. This strike yield serves as the "deductible" for this "insurance" contract. A higher strike yield on an interest rate cap is a bigger deductible, so the premium on a cap with a higher strike yield will be lower. An interest rate floor works very much like a cap in reverse. An interest rate floor reimburses the buyer for decreases (as opposed to increases, as for a cap) in a particular interest rate index below a predetermined level, the strike yield, based on the notional principal amount.

Let's recall our friend Dewey, who has illiquid fixed-rate assets backing floating-rate liabilities. With interest rate swaps, he could enter into a swap to convert these fixed-rate assets to floating rate. Another approach might be to buy interest rate caps. Dewey could purchase interest rate caps that expire in five years, with LIBOR being

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the hedging index. Payments under the cap would be determined and made at the end of each quarter, should LIBOR exceed the strike yield at the end of the quarter. The last time I looked, LIBOR was 3.25%, which is relatively low. One thing Dewey might do is buy an interest rate cap with a strike yield of LIBOR at 3.25%, basically an at-the-money cap. For any increases in LIBOR above rates, the cap would pay off. That's going to be a very expensive insurance policy. It's like buying an auto policy with zero deductible. Even if you get into a little fender bender, that insurance policy has to pay off. Dewey doesn't want to pay a big premium and doesn't really need full protection, so he buys an out-of-the-money cap with a strike yield of LIBOR at 6%. So when LIBOR goes above 6%, the cap pays off.

Look at Table 2 and consider the two things that can happen. Either LIBOR can stay at levels of 6% or lower each quarter, or it can rise above 6%. For levels of LIBOR below 6%, let's look at what happens. The fixed-rate assets yield 8%, of course. There's an option premium that's paid for the cap. Usually these premiums are paid up front, but for accounting purposes, the cost is spread out over the life of the option in this example. Here, the annual premium that's booked is 75 basis points. As long as LIBOR stays below 6%, Dewey is going to earn 8% on the assets, pay 75 basis points for the cap, so net of the cap premium he's earning 7.25%. The liabilities are indexed to LIBOR plus ten basis points, which is 3.35%. The gross spread, that is the earned rate minus the credited rate, is 715 basis points minus LIBOR. Remember LIBOR is 3.25%, so that's a spread currently of close to 4%.

TABLE 2
Example - CAP

Assets	LIBOR < 6%	LIBOR > 6%
5-year fixed-rate bonds	8.00%	8.00%
5-year cap (strike @ 6%):		
- pay premium annualized	(.75)	(.75)
- receive	0	LIBOR - 6%
Net earning rate	7.25%	LIBOR + 1.25%
Liabilities:		
- credited rate	LIBOR + .10%	LIBOR + .10%
Gross spread	7.15% - LIBOR	1.15%

But what if LIBOR rises above 6%? LIBOR was well above 6% just a few years ago. This is where the value of the hedge comes in. Again, the fixed-rate assets yield 8%, the cost of the hedge is 75 basis points, but now the cap pays off as LIBOR goes above 6%. So net of the cap, Dewey is getting LIBOR plus 125 basis points, the credited rate is LIBOR plus ten, and that leaves a spread of 115 basis points. So the worst case gross spread as rates go up is 115 basis points. For rate increases above 6%, Dewey is hedged, so he's indifferent to rate rises above that level.

Peter will now talk about how derivatives are priced, and also discuss counterparty credit risk.

MR. PETER A. MINTON: I'm not going to attempt to get too in-depth, but these are very deep subjects and probably deserve more time than can be devoted here. The basic idea behind pricing options and swaps and caps and floors is to first create a set of future rates or underlying prices through a methodology, which is based upon market volatility. This can be done by using either observed volatility or volatility obtained from the options market and then applying some arbitrage-free principles. Then, the payoffs at any given epoch (either through a lattice or through the paths of rates) are determined. And finally, the present value of those future payoffs is calculated. The various different models follow this approach, each in a slightly different fashion.

In a Heath-Jarrow-Morton model, in which a path of the yield curve is described, the payoff at any given point for swaps is the differential between the fixed rate and a level of the floating-rate index at that time period. If a swap is a fixed-for-floating swap, paying fixed at 6% and receiving floating at 3.25%, then the first cash flow through the path will be that differential times the notional amount. One aspect of a swap is that it is not a principal bearing instrument. It is a notional type of contract in which no principal actually changes hands, and payments are made in a net form. So, therefore, for a swap with \$100 of notional amount paying 6% and receiving 3.25% at the first point along the path, the (annualized) payment to the swap counterparty would be \$2.75. So for swaps, a set of paths is created in which the fixed-to-floating components are compared to determine what will be paid or received. Caps and floors are somewhat similar. A cap with a 6% LIBOR strike will not pay at the first point along the path, because the rate is at 3.25%. It is not until LIBOR is up above 6% on the path that the cap begins pay. The converse is true with a floor. For options, the value at any given epoch is the maximum of the value of exercise at that point and the present value of the future exercise of the option. We'll soon revisit what that means.

There are three basic methods for pricing options and swaps: the Black-Scholes model, an arbitrage-free binomial lattice model, and a multi factor, arbitrage-free stochastic process, which is, in essence, a Heath-Jarrow-Morton model. The Black-Scholes model is only used for pricing options on futures or equity options. Commodities would fall under this as well, but for these purposes, it is probably a decent simplification to say that the only time a Black-Scholes model is used in the fixed-income market is when pricing options on futures. The binomial lattice model can be used for pricing swaps of most types, caps, and floors. The lattice generally is not used for futures or equity options, but can be used for options on futures, over-the-counter (OTC) options. In the arbitrage-free stochastic process, all the same types of things can be done as in the binomial lattice model.

The process behind Black-Scholes is that of a closed-form solution for valuing. It was initially conceived for European-style options on noncoupon-paying instruments. The formula for the call option is presented in Table 3. First, an adjustment for coupon-paying instruments is made by setting the current price equal to the current price of the instrument minus the present value of the coupons paid during the option period and minus the change in the accrued income for that same period. This determines what the equality is between owning the bond and owning cash and an option on the bond, which is why there is an adjustment for the coupons paid and the accrued income for fixed-income securities. Second, it is important to discuss the risk-free

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rate. If this is a five-year option, for example, the risk-free rate is not the short rate, but the five-year treasury rate. There are numerous proofs as to why this is true.

TABLE 3
Discussion of the Models
Black-Scholes

- The Process
 - Black-Scholes is a closed-form solution for valuing European-style options on noncoupon-paying instruments.
 - The formulation for a call option is:

$$\text{Call} = SN_{(x)} - Ke^{-rt}N(x - \sigma\sqrt{t}) \quad , \quad \text{where } x = \frac{\ln(S/Ke^{-rt}) + 1/2\sigma\sqrt{t}}{\sigma\sqrt{t}} .$$

S = current price.
K = strike price.
σ = price volatility.
r = risk-free rate.
t = time to expiration.

- For coupon-paying instruments: $S = S\text{-}PV(\text{coupons paid}) - \Delta i$.

The following disclaimer is for Tables 3 and 4. This memorandum is based on or derived from information generally available to the public from sources believed to be reliable. No representation is made that it is accurate or complete. Certain assumptions may have been made in this analysis which have resulted in any returns detailed herein. No representation is made that any returns indicated will be achieved. Changes to the assumptions may have a material impact on any returns detailed. Past performance is not necessarily indicative of future results. Price and availability are subject to change without notice. The foregoing has been prepared solely for informational purposes and is not an offer to buy or sell or a solicitation of an offer to buy or sell any security or instrument or to participate in any particular trading strategy. Morgan Stanley & Co. Incorporated and others associated with it may have positions in, and may effect transaction in, securities and instruments of issuers mentioned herein and may also perform or seek to perform investment banking services for the issuer of such securities and instruments. Additional information is available on request. To Our Readers Worldwide: In addition, please note that this publication has been issued by Morgan Stanley & Co. Incorporated and approved by Morgan Stanley International, a member of the Securities and Futures Authority, and Morgan Stanley Japan Ltd. We recommend that investors obtain the advice of their Morgan Stanley International or Morgan Stanley Japan Ltd. representative about the investments concerned. NOT FOR DISTRIBUTION TO PRIVATE CUSTOMERS AS DEFINED BY THE U.K. SECURITIES AND FUTURES AUTHORITY.

Some of the strengths of Black-Scholes are that it is easy to implement, it is very fast, and it is the predominant model for options on equities and bond futures. One weakness of this model is that it really doesn't value or model the pull to par properly. The pull to par is where a bond, bought either at a discount or premium, is priced at par at maturity. Black-Scholes does not capture the effect in which the value will be pulled to par as you move closer and closer to maturity. Another weakness is that the ability to use something other than a constant, risk-free rate is also problematic. It cannot be used to value swaps, caps, or floors. In addition, the adjustments must be made for coupon-paying instruments.

The next type of model is a binomial lattice in which a lattice of rates is created. This process describes short rates by creating a lattice while moving forward through time. Based upon yield curve, the next two nodes will be placed such that the distance between the up and the down nodes are determined by volatility. The arbitrage-free characteristic of this lattice is such that a treasury instrument placed on this lattice at any point through the lattice returns observed price. The important feature of the arbitrage-free model is that it isn't possible to get an arbitrage out of this lattice. In

valuing an option, the option is the maximum at any one of these nodes, of the value if it is exercised at that point, and the present value of the future exercise of the option. If a bond is placed onto the lattice and backed through time, the price of that instrument can be determined. Therefore, if the call value or the option value is known, the intrinsic value of calling that option can be calculated. For example, by backtracking to a point in which the value of the underlying bond is \$101 based upon the path of rates, and if the bond can be called at par, the intrinsic value becomes \$1. But, if the discounted future value of calling that bond proves to be a better call point, then the discounted present value of waiting to call that option may be greater than that \$1 of intrinsic value, and so the value at that node is the maximum of the two values, and then we continue to walk back through the lattice. Based upon the previous description of options, it is obvious how to model swaps and caps and floors. Given that rates are known at any known point or node within this lattice, the term structure of rates implied by this lattice at any node is also known. It is possible to walk through the lattice and actually describe at each given point the amount paid or received on either a swap or cap or floor. Upon generating this lattice, payments at each node are determined, and then the discounted present value of those payments back through time is calculated.

What are the pluses and the minuses of this process? The strengths of these kinds of binomial lattice models are that they are arbitrage free at all points, have relatively good long-term behavior of rates, capture the pull to par, can price options and options of coupon-paying instruments, can use a term structure of volatility, and can also value swaps, caps, and floors. One weakness is that it is more time-consuming to actually do the valuation. It is certainly going to take longer to implement, because the implementation of this type of model requires some thought and some actual coding. Also, it is not densely populated at early periods. The lattice of rates was created by starting with today's rate and determining two possibilities for rates, one period hence. Therefore, in the early periods, the population of observed rates is somewhat thinly populated, and that can be a problem for short-term options. Finally, it is only a one-factor model of rates.

The final type of model is a Heath-Jarrow-Morton model (Table 4), under which a set of arbitrage-free paths is created. So, rather than creating a lattice in which two nodes emanate from one node throughout an evergrowing lattice, this model generates arbitrage-free paths of rates. With 1,000 or 500 or 100 distinct paths of rates at each point in time, an arbitrage-free term structure is implied. By using this approach, pricing assets with interest-contingent cash flows gets away from some of the problems of the lattice model, which moves from one point into two possible points. It becomes very simple in concept to again show how a swap pays off by walking forward through time, pulling out what each payment will be along each path, and determining present values under those paths to determine the valuation. Walking every single path that is created in the arbitrage-free set of paths and discounting the present values of the flows gets back to the price of this asset. An American option can be valued in a path model even though it can be exercised at any time; i.e., it contains a maximum function. The valuation problem is one of determining when it is in fact optimal to exercise the option, which is accomplished with backward induction by using a shelf of paths.

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TABLE 4
 Heath-Jarrow-Morton Model (Econometrica 1992)
 Discrete-Time Formulation of Continuous-State Stochastic Diffusion Process

$$\begin{array}{c} \left[\begin{array}{c} \text{Term} \\ \text{Structure} \\ \text{at} \\ \text{Epoch } t + 1 \end{array} \right] \\ \text{Vector} \end{array} = \begin{array}{c} \left[\begin{array}{c} \text{Process} \\ \text{Drift} \\ \text{at} \\ \text{Epoch } t \end{array} \right] \\ \text{Vector} \end{array} + \begin{array}{c} \left[\begin{array}{c} \text{Factor} \\ \text{Structure} \\ \text{of} \\ \text{Volatility} \end{array} \right] \\ \text{Matrix} \end{array} \times \begin{array}{c} \left[\begin{array}{c} \text{Stochastic} \\ \text{Factors} \\ \text{at} \\ \text{Epoch } t \end{array} \right] \\ \text{Vector} \end{array}$$

One strength of the Heath-Jarrow-Morton model is that the valuation of complex types of options that are purely path dependent is possible in this type of a model. This is the type of model to use for valuing mortgage options, because the prepayment factor is built in, based upon an assumption as to where rates have gone prior to now and, therefore, triggers some amount of prepayments.

The final point on valuing options and swaps is that there is a strong component of the shape of the forward curve in all of these models. In both the binomial lattice and the Heath-Jarrow-Morton model, if the volatility is zero, the forward curve would be the path of rates traveled. In valuing options and swaps and caps and floors, if the forward curve implies that rates are most likely to rise, the valuation will show that caps will become very expensive and floors will become relatively cheap.

One example of this is for a swap. A company wants to shorten duration by about five years, and the company enters into a seven-year fixed-for-floating swap. It can be shown that the price changes for the swap will look much like the price changes for the seven-year treasury instrument or high-quality corporate issue. There are two possible solutions to actually shortening. The first possible solution is to do the plain vanilla fixed-for-floating swap. The swap curve implies paying seven-year treasury plus 35 basis points, which is approximately 6.50% right now, and receiving LIBOR as the floating rate, which is 3.25%. There is no fee for entering into a swap. The cost in the unchanged environment is 290 basis points. By going forward seven years in time and discovering that the forward curve was wrong and rates remained at initial levels, the company would be paying 290 basis points a year for this swap. Consider the same parallel duration idea priced off of a different part of the curve in which the company enters into a seven-year swap with a five-year duration. Rather than bearing the cost of the expectation of the front end of the curve rising dramatically and fairly quickly, which is built into the forward expectation and therefore built into the pricing, the floating leg is based off of something further out on the curve. So, the company could get paid on a floating basis, for example, the reset of the ten-year treasury, which presently is about 6.15%. The parallel duration is now hedged five years, the same five years as in the fixed-for-LIBOR swap, but the cost in the unchanged environment is now only 100 basis points. This is true because the expectation of the ten-year CMT rising is much less than for LIBOR because of the shape of the forward curve.

An issue in swap land right now is counterparty credit risk when entering into a swap agreement. There is an obligation to either pay or receive a fixed leg with another counterparty for some period of time, perhaps as long as five, seven, or ten years.

This allows exposure to the risk of that counterparty's credit, like owning the bond of the company that is the counterparty. The difference is the notional amount. There is no principal flow at the beginning and there is no principal flow at the end. The real credit exposure in any swap is not the notional amount. With a \$100 million notional amount entered into today, which is an on-market fixed-for-LIBOR swap, there is no up-front cost. If the swap is unwound, its value is still zero. There is no credit risk. What counts is the exposure to the counterparty when unwinding and trying to replicate the swap. The at-risk amount is not the notional, but rather the at-risk amount can be thought of as the market value of that swap. One of the things brokers actually do is set up "AAA" subsidiaries. Firms like Morgan Stanley, Merrill Lynch, and others, which are "A" credits or thereabouts, are now setting up "AAA" subsidiaries, which basically protect against the "A" credit of the company. A better mechanism, but sometimes problematic for insurers, is the idea of a two-way mark to market. If the swap goes off market in one direction or the other, the party who enters the swap must post collateral to protect the party who has the credit risk. If Morgan Stanley writes a swap that goes in its favor, there is a market value to that swap. The counterparty would be asked to post collateral equal to the value of that swap. This is a good insulator against counterparty credit risk and, therefore, allows longer dated swaps to be done.

MR. FRANCIS P. SABATINI: There is growing interest in the application of instruments such as swaps, caps, and floors. Presented in this section are three examples of approaches that have been used by insurance companies. These examples are made up and are not based on current pricing, but I'll try to give you an idea of what current pricing might look like. The examples do not represent perfectly executed actions, because it takes some work to figure out the best maturity, strike price, and such.

Generally, people believe interest rates are going to rise or stay the same. Very few people think they will go down, and certainly the forward curve would suggest that interest rates are going to rise. But recent experience has shown several years of declining interest rates, where every time it seems they've hit bottom, they keep going lower. At the same time, most insurance industry portfolios rapidly decline in terms of their yield performance. Portfolio rates are declining and the rates credited on interest-sensitive products may not be declining as fast, which creates a fair amount of spread compression. If rates stay the way they are or even go lower in another year or two, contractual minimums will become a concern, especially on contracts such as universal life, which, in some cases, have 5% or 5.5% minimum guarantees. The first problem that will be discussed is that of ABC Life. It has a \$100 million book of universal life contracts with 5% minimum guarantees, which present a material risk in the view of the management of that company. The solution is to purchase an interest rate floor contract as insurance against supportable rates falling below contract minimums.

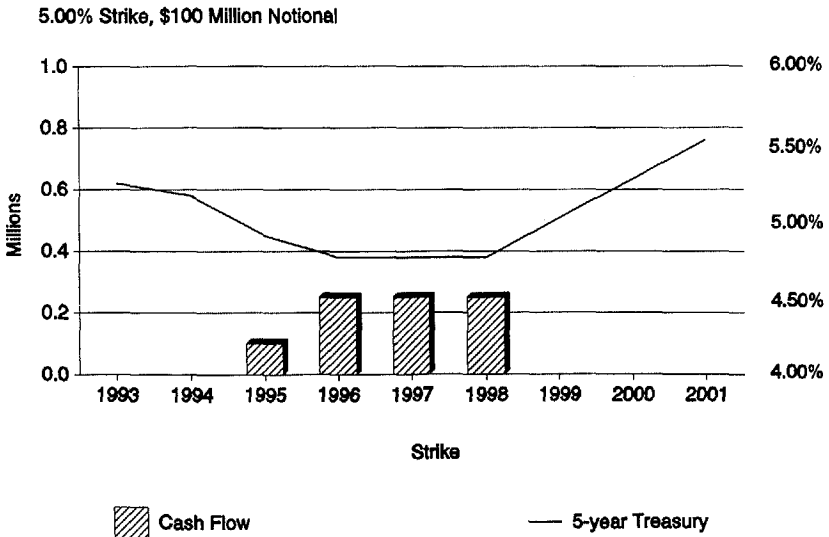
The first things to consider are the contract terms of the floor. Assume a \$100 million notional amount, which is the same as the size of the portfolio to be hedged against. The strike level is 5%, which is the same as the minimum rate guaranteed by the contracts, and the contract term is ten years. The index will reset quarterly. The underlying index will be the five-year treasury, which for purposes of this example, is about 5.25%. The payment to the purchaser of the floor contract would

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be the difference between 5%, the strike price, and the value of the index. That value can't be less than zero, so at the point in time the contract is entered into, it is really 25 basis points out of the money. The premium for the floor in this example is 1% of this \$100 million notional amount, but actual pricing would probably be much less than that. The premium is paid up front to the issuer of the contract. Floors are very cheap now, simply because the forward curve would suggest a rise in interest rates, not a fall.

In this example, let's say interest rates fall to about 4.50%, as shown in Chart 1. Starting above the strike yield, interest rates fall. In 1995, the index falls below the strike yield, so the contract starts to generate income. As long as the index stays below the strike yield, the purchaser of the contract continues to receive income. In 1999, in this example, interest rates move back up above the strike level, and the contract doesn't produce any income after that point in time. It is important to note that if the interest rate path were above the 5% strike level, an agreement would have been entered in which a premium is paid but no income is received. That is not much different than many other types of insurance. It is like homeowners' insurance. Premiums are paid for years and years, and if the house never burns down, all those premiums were paid without receiving any value. But, insurance really pays off the day the house burns down.

CHART 1
Interest Rate Floor Contract – Illustration



Look at Table 5 in the context of this insurance company for the first three years of a possible transaction, with five-year treasuries as the index. The index starts at 5.25%, moves down to 4.75% in year 2, and then down to 4.50% in year 3. A portfolio rate for this company starts at 7.7%, declines to 7.15% in year 2, and then down to 6.93% in year 3. The credited rate on the contract is priced at 200 basis points off the portfolio rate, subject to contractual minimums, so it is 5.70% in 1993,

5.15% in 1994, and 5% in 1995. The net income before introduction of the floor contract on a year-by-year basis is just the 2% spread times the amount of in force, which is \$100 million. It is producing \$200 million a year for the first two years, but in year 3, when it should have credited 4.93% to get its 200-basis-point spread, it instead had to credit 5% because of the minimum, it had reduced income. With the floor contract, a 1% up-front fee is paid in cash, but accounting rules allow amortization of the cost of that contract over its life. So, for a ten-year contract, \$1 million would be amortized over ten years at \$100,000 per year. In year 1, income is reduced by \$100,000 by having this insurance. In year 2, however, the index has fallen below the strike price by 25 basis points, generating \$250,000 of income, which produces more income than if the agreement had not been entered into. In the third year, which actually dips below targeted profitability to normal operations, and the floor contract provides more than enough income. Of course, this is an efficient execution, but it certainly makes the point that floor contracts are great insurance for protection against contractual minimums. They are also useful in other parts of portfolio management, but this is one example that makes the point very well.

TABLE 5
Interest Rate Floor Contract – Example

Uses Floor Contract – 5.00% Strike Price – to Protect Against Universal Life 5.0% Contractual Minimum (\$100 Million in Force)			
	1993	1994	1995
5-year Treasury	5.25%	4.75%	4.50%
Portfolio rate	7.70	7.15	6.93
Credited rate	5.70	5.15	5.00
Net income	\$2,000,000	\$2,000,000	\$1,930,000
Floor contract			
Cost @ 1%	(\$100,000)	(\$100,000)	(\$100,000)
Income	0	\$250,000	\$500,000
Income (after floor)	\$1,900,000	\$2,150,000	\$2,330,000

The second example is an interest rate swap, in which an insurance company in the GIC business has figured out that its traditional approach to investing behind these contracts is unacceptable from a risk perspective. It is faced with the possibilities of either getting out of the business or finding another way to remain in the business. Deciding that it didn't want to get out of the business, it looked to the interest rate swap market. Something that doesn't come intuitively is the idea of swapping liabilities. Instead of working with a fixed rate that is guaranteed for five years by using the swap market, it converts the liability into a floating-rate liability which allows a totally different and somewhat unique investment strategy.

This can be illustrated by doing some stochastic pricing, which will first be done for the traditional approach: The liability for a five-year contract is the five-year treasury plus 45 basis points. Benefit withdrawals occurred at the rate of about 5% annually. The investment strategy used traditionally is buying five-, six-, and seven-year fixed-rate bonds that are A and BBB rated, as shown in Table 6. It is getting 100-115 basis points over treasuries so, given the shape of the yield curve, it is getting about 130 basis points over the five-year treasury in terms of its investment yield. It is

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paying 45 basis points over the five-year treasury for its liability cost, which gives a spread of about 85 basis points. The six-year average life on the portfolio is probably a duration that is a bit longer than the liability, which has a five-year maturity and some benefit withdrawals. The liability is between four and five years durationally, and the assets are greater than five years. There is mismatch inherent in this portfolio, and this particular company thought it needed to have that mismatch to maintain a competitive position in the marketplace. Included in the pricing exercise is required surplus of 4.4%.

TABLE 6
Stochastic Pricing and Risk Analysis – Traditional

- Liability: 5-year treasury + 45 basis points
- Liability benefit withdrawals: 5% annually
- Investment strategy:

Asset	Asset Spread Assumptions
5-year "BBB" fixed-rate bonds	5-year treasury + 115 basis points
6-year "BBB" fixed-rate bonds	6-year treasury + 115 basis points
7-year "BBB" fixed-rate bonds	7-year treasury + 115 basis points
5-year "A" fixed-rate bonds	5-year treasury + 100 basis points
6-year "A" fixed-rate bonds	6-year treasury + 100 basis points
7-year "A" fixed-rate bonds	7-year treasury + 100 basis points

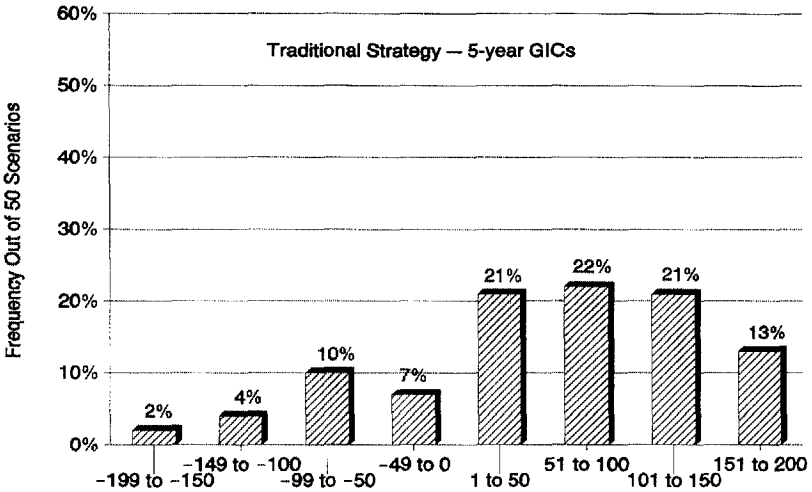
- Required surplus: 4.4%

Let's look at the pricing, in which the results are measured for each of 50-yield curve scenarios in the form of realized pretax average annual gross spread. As shown in Chart 2, the distribution is somewhat skewed, because a number of scenarios end up with negative spread results. The mean of this distribution is 53 basis points, which is much less than initially, in which there was an 85-basis-point-spread just from a straight, mechanical, pricing exercise. The range goes from a negative 199 to a positive 188 spread, with a fairly large standard deviation of 89 basis points and 11 negative scenarios. This is clearly an indication of the risk with which this company was uncomfortable. Looking at the distribution of these results in the context of achieving an ROE with roughly a 4% capital requirement to achieve a 15% ROE, a mean expectation of at least 90 basis points is required. Even with a mean expectation of 90 basis points, the standard deviation is 89 basis points, showing the uncertainty of results. This is a win-big or lose-big situation, and that is not an approach to life that this particular company wanted to take, so it considered an alternative.

I will explain how swaps are used to convert the liability from fixed to floating. The liability is the five-year treasury plus 45 basis points. For the swap, receive fixed and pay floating. What is paid in the way of a floating rate is the three-month treasury plus 55 basis points. The net liability cost should be the floating rate of three-month treasury plus 55 basis points. What is important here is that the swap contract changes the nature of the liability from fixed to floating. Without the swap, a fixed rate is paid every year. By entering into a swap, the swap counterparty is going to pay cash every reset period equal to the initial five-year treasury plus 45 basis points. The receipt of that cash allows for payments to be made to the contract holder, but

then the person on the other side of the swap transaction has to be paid three-month treasury plus 55 basis points, which ends up being the liability.

CHART 2
Stochastic Pricing and Risk Analysis Results
Distribution of Annual Pretax Gross Spread



Gross spread (incl. interest on required surplus) in annualized basis points

Most of the swap transactions are really LIBOR based. There is a fairly close relationship between the three-month treasury and LIBOR. The relationship between the two does fluctuate, so when investing in treasuries and entering into a swap it creates something called basis risk, which is the relative movement of the three-month treasury to LIBOR. That is something that can be measured and assessed, but that risk is not nearly as big as some of the other risks under the traditional strategy.

With a liability of three-month treasury plus 55 basis points for a five-year contract, reflecting a five-year swap, and assuming the same liability withdrawals for benefit payments of 5% annually, a much different approach is taken to invest behind this liability, as shown in Table 7. The portfolio is a mix of fixed-rate bonds, five- and six-year "BBB" and "A" bonds, at the same spreads over treasuries as in the traditional strategy, but also now includes a variety of floating-rate instruments. About 75% of the portfolio is now being invested in adjustable rate mortgages, floating-rate mortgage-backed securities, and straight floating-rate securities, with the other 25% invested in the five-, and six-year bonds. Also, LIBOR caps are purchased 200 basis points out of the money. For a floating-rate liability, the company ideally would invest in a floating-rate asset that moves parallel with the liability. The problem is that the markets do not allow floating-rate assets to be bought at a large-enough spread to reach profit targets, so the company invests 25% of the portfolio in longer-term securities; i.e., the five-, and six-year bonds. The LIBOR caps bring in some insurance protection against the possibility that interest rates go way up, that the

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liability cost increases, and that there is less response in terms of performance from the assets purchased in this portfolio.

TABLE 7
Stochastic Pricing and Risk Analysis – Nontraditional

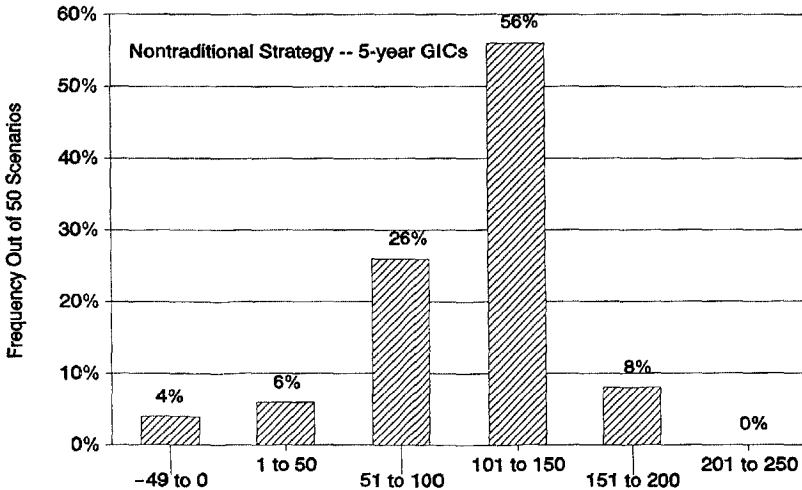
- Liability: 3-month LIBOR + 10 basis points
= 3-month treasury + 55 basis points
- Liability withdrawals: 5% annually
- Investment strategy:

Asset	Asset-Spread Assumptions
5-year "BBB" fixed-rate bonds	5-year treasury + 115 basis points
6-year "A" fixed-rate bonds	5-year treasury + 100 basis points
Adjustable rate mortgages securities	1-year treasury + 130 basis points
Floating-rate mortgage backed	1-year treasury + 175 basis points
Floating-rate notes	LIBOR + 45 basis points
LIBOR caps 200 basis points out of money	

- Required surplus: 4.4%

Chart 3 shows the pricing results, assuming the liability is swapped to floating and the revised asset strategy is used. The pricing results show a much tighter distribution, with a mean of 105 basis points and a standard deviation of 40 basis points.

CHART 3
Stochastic Pricing and Risk Analysis Results
Distribution of Annual Pretax Gross Spread



Gross spread (including interest on required surplus) in annualized basis points

Table 8 shows a comparison of the two pricing results side by side. The mean is doubled, and the standard deviation is cut in half. There is a dramatic improvement in the tail of distribution, with the number of negative results going from 11 to 2. This company has now been able to find a way to really reduce its risk profile and has a better expectation in terms of achieving profit objectives.

TABLE 8
Stochastic Pricing and Risk Analysis – Results Comparison
5-year GIC – Annual Pretax Spread Statistics

	Traditional	Nontraditional
Mean	53 basis points	105 basis points
Standard deviation	89 basis points	40 basis points
90th percentile	-91 basis points	39 basis points
Number of negatives	11	2

Both of these portfolios have about the same degree of mismatch to the liability. With the traditional approach, there might have been about a one-year mismatch, but a one-year mismatch five years out on the yield curve is much different than a one-year mismatch in the three-month to one-year range on the yield curve. Also, the shape of the yield curve is much different at the short end than it is at the long end, which helps with the whole problem. It is obvious from Chart 3 that there really is a much higher expectation and a much lower standard deviation by taking a very nontraditional approach to investing behind the fixed-rate liability.

The last situation is one that many people fear. The product is a one-year reset single premium deferred annuity (SPDA). If interest rates rise, the company is faced with a dilemma. If it does not maintain market crediting rates, the business is going to run, which is not good. Or, if it does maintain market crediting rates, it will end up subsidizing the crediting rate because the portfolio cannot support it. One possible solution, although not the most efficient, is to purchase an interest rate cap as insurance. Here, payments are received if the index goes above the strike yield. Notice that the payment, or the cost of this cap, is 5% (see Chart 4). A ten-year cap with these strike terms, based off of LIBOR, which means it is anywhere between 150 and 200 basis points out of the money, would cost upwards of 10%. This is assuming a somewhat different environment. Caps are very expensive because of the shape of the forward yield curve and the way the street is pricing it. As interest rates rise, they go above the strike yield and the cap generates income. If interest rates stay below the strike, there is no income.

In the example, shown in Chart 4 and Table 9, the five-year treasury goes from 5.25% to 6.25%. In 1995, it is 8.25% and in 1996 it is 9.25%. The yield curve does not maintain the same shape, with LIBOR moving from 3.125% to 4.625% to 5.75% to 6.75%. The portfolio rate starts at 8.25% and it inches up gradually to 8.50%, 8.75%, and 9%. At 30 points above the five-year treasury, the annuity rate pricing ends up at 5.55%. This goes to 6.55%, 8.55%, and 9.55% in the following years.

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By targeting a 150-basis-point spread, this starts off in great shape and makes more than the target spread on day 1. In 1994, spread starts to get compressed but it is still healthy. But by 1995, there are only 20 basis points of realized spread through the pricing exercise and by 1996, there is a negative spread of 55 basis points. This is not a totally unrealistic scenario, and this could come about in a few years. The net income stream on the product is a healthy \$2.7 million in year 1, \$1.95 million in year 2, and all of a sudden, it is down to \$200,000 in year 3. Finally in 1996, the company is losing money on the book of business.

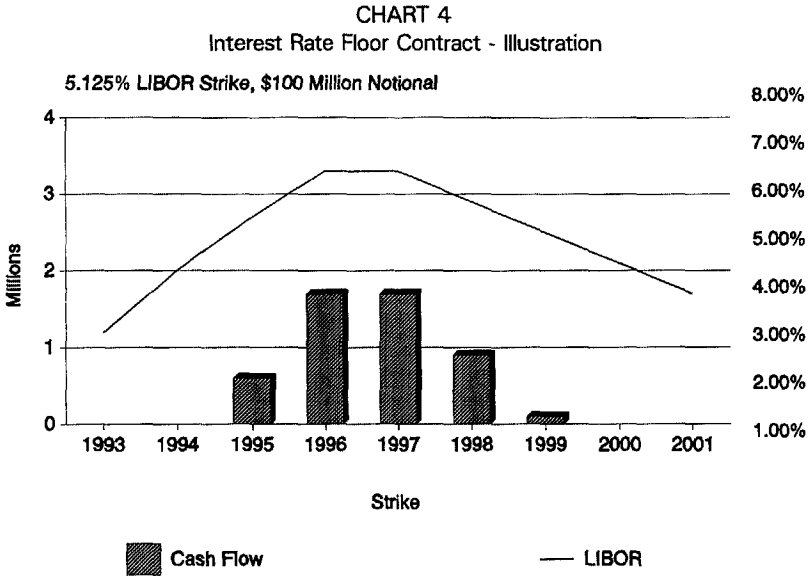


TABLE 9
Interest Rate Cap Contract – Example

- Uses Cap Contract - 5.125% LIBOR Strike Price to Protect Against SPDA Crediting Rate Subsidy (\$100 Million in Force)

	1993	1994	1995	1996
5-year Treasury	5.25%	6.25%	8.25%	9.25%
LIBOR	3.125	4.625	5.75	6.75
Portfolio rate	8.25	8.50	8.75	9.00
Credited rate	5.55	6.55	8.55	9.55
Net income	\$2,700,000	\$1,950,000	\$200,000	(\$550,000)
Cap contract				
Cost @ 5%	(\$500,000)	(\$500,000)	(\$500,000)	(\$500,000)
Income	0	0	\$625,000	\$1,625,000
Income (after cap)	\$2,200,000	\$1,450,000	\$325,000	\$575,000

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In the cap contract, the cost of the cap is amortized over the life of the contract, which is \$500,000 a year. At the trigger point, in 1995, the cap contract generates \$625,000 of income, which improves the result slightly and indicates that this example is not an efficient one. Finally in 1996, when it is needed the most, a cash payment is made through the cap contract. The company bought insurance, which was well worth it in this scenario, and has profitability which it would not have if it had not purchased the cap contract.

MR. PAUL A. HEKMAN: What is the current statutory reporting status for swaps? Is that an acceptable contract for an insurance company portfolio, and what are the accounting procedures for holding the statement values for those contracts?

MR. SABATINI: Swaps are off balance sheet, so they are a footnote in the statutory statements. With some of the new requirements, they may be a footnote in which the market value is stated, but they do not impact the balance sheet. They are primarily an income statement item, but they have pretty simple amortization of the purchase payment.

MR. KLEIN: The swap, as opposed to a cap or floor, has no initial premium. It is just an exchange of interest rate payments going forward, and the market value initially is zero. This is different than a cap or a floor, in which there is a value initially, and that is why the buyer has paid the premium. The swap is off balance sheet, and there is no value initially to it. It is an income statement item thereafter as interest rates change. An interest rate cap or floor does have value, and that shows on insurance company statutory books at essentially an amortized cost. For example, if you paid 5% up front for a cap with a five-year term, you would amortize 1% a year.

MR. WILLIAM A. ZEHNER: Do you have any statistics as to the success of companies using futures as hedges? Are there any survey results and what are they?

MR. KLEIN: I am not in a position to comment specifically on how they have performed overall for the industry. As hedges, it really is a fairly cut-and-dried type of transaction. Some of the success or failure comes in the decision of how much is to be hedged with futures. Then there is also the aspect of basis risk. Futures are typically available on treasury contracts. Most insurers aren't really invested heavily in treasuries, but rather in corporate bonds or perhaps commercial mortgages. There is a basis risk between what treasury rates are doing and what's going on in the corporate market.

MR. MINTON: In many states, and New York is probably the toughest, you really cannot enter into the futures contract for speculative purposes. There is basically a model investment law that has been proposed that would actually allow futures, options, swaps, and these types of vehicles to be used more, but I do not expect it to go through. If people entered into them understanding the risk that they entered into, they performed pretty well. With futures, there is not only the basis risk, but also a cheapest-to-deliver risk. The duration will fluctuate not exactly like a 30-year bond, because you don't exactly know what the deliverable bond contract is or what the deliverable underlying bond is. With swaps you need to understand that there is a parallel and a nonparallel yield curve shift risk. One bad experience seen with

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swaps was when an agreement was entered and the direction of rates assumption was actually correct. Theoretically, had it been a parallel shift rather than the nonparallel one, life would have been lovely, and it would have been a perfect hedge. But, when the company went to unwind, it had less value because of the change in the steepness of the curve. You must be cognizant of what you are doing and what the second-order or third-order risks in some of these derivatives are. Then, they tend to work out well.

MR. SABATINI: A number of companies have used futures, swaps, caps, and floors and, if properly executed as hedging vehicles, they usually turn out the way they were anticipated. The work must be done up front to understand what you are dealing with and how effective or ineffective the transaction is going to be. They have to be managed as well.

MR. KLEIN: First of all, how many people here work for an insurance company currently or have worked for one in the past? (I see many hands raised.) How many people know that their company has shorted or sold a floor contract? (Only one hand is raised). Now, let me rephrase the question. How many people's insurance companies have sold insurance contracts that have minimum rate guarantees? (Now I see many more hands raised.) Issuing interest-sensitive contracts with minimum rate guarantees is like shorting a floor contract. Frank talked about a universal-life policy that had a minimum rate of 5%. That is really the same thing as shorting or selling a floor to the policyholder. Basically, if market rates go down below 5%, the insurance company still has to pay that difference between the market rates and 5%. I bring this up because sometimes many actuaries are far removed from the investment side of the business. There are clear applications, however, for some of these pricing concepts and so forth in actuaries' product development and pricing with respect to their liabilities. So please bear in mind that many of these floors and caps have application not only on the investment side but also on the liability side in terms of quantifying the cost of certain insurance contract features.

