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INTRODUCTION TO ASSET/LIABILITY MATCHING

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This session will briefly discuss the asset/liability matching process, use of process, and the strength and weaknesses. A case study will also be presented and discussed.

MR. MICHAEL M.C. SZE: I assume that we have all performed an actuarial valuation, and most of us have also performed an actuarial projection. Are valuations and projections the same kind of actuarial exercise? Not really. A projection is performed for a different purpose from an actuarial valuation. Companies do pension actuarial valuations because they are required to do them. However, they do projections because they want to.

There's a definite purpose and need in the projection exercise. It is very important for the actuary to know the client's needs. Typically, they do asset/liability modeling to learn something: such as how to set investment policies; to test out their investment policies under different liability and economic constraints; or to make sure that their asset/liability strategies are in tune with the company objectives. We, in doing projections, need to reflect those objectives.

What are the typical objectives of a company? Companies don't always just want to minimize contribution. Typical companies would like to do financial planning ahead of time. They want a stable contribution rate and a stable expense trend. They want to minimize risk. What kind of risk? Investment risk and interest rate fluctuation risk. How can we help in minimizing these risks? We help by looking at assets and liabilities together, and by matching assets and liabilities; it helps to minimize the unexpected risk of intolerable fluctuations in contributions and expenses.

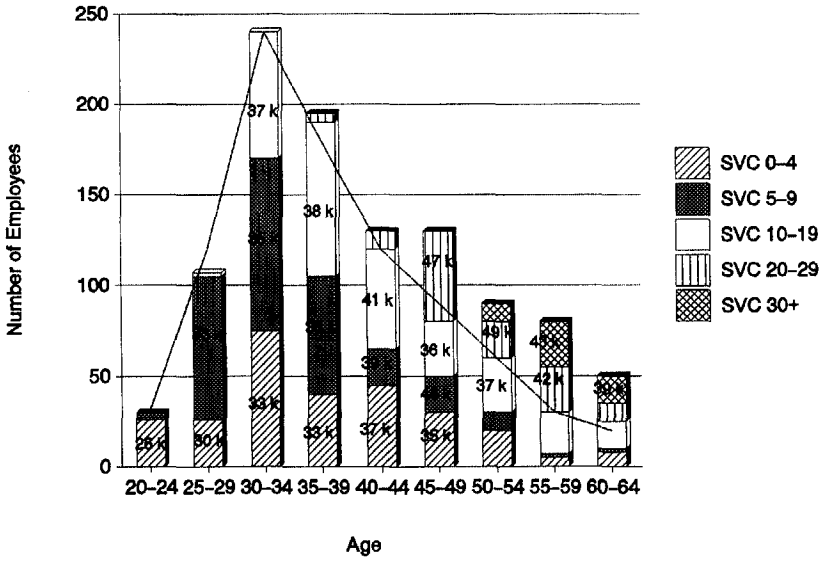
We shall now discuss the process of an asset/liability modeling study. Typically, such a study starts with a liability projection study and an independent and concurrent asset study is done. After a review of these results, an integrated asset/liability study is performed, in order to piece the information together.

Our own actuaries might ask why we need to perform the first two independent studies. Why don't we do the asset/liability studies together? The answer is quite simple. An asset/liability study is a very complicated project. You want to start with two simple procedures to get your bearings. Both the actuary and the client company are more familiar with a liability study, and through this type of study, they will better understand what kind of constraints the company has and what the taboo situations are to guard against in terms of assets and liabilities. Why do we do a concurrent asset study? We do it so that we can analyze what kind of portfolio would be useful for further analysis. After we get our bearings on assets and liabilities, we can piece them together. The process of piecing them together is done through simulation. Many results will be generated. Unless we do the preparatory groundwork, our clients wouldn't know what we are talking about. We may not

even know where we are heading ourselves. So it is very important to go through assets and liabilities separately first before we piece them together.

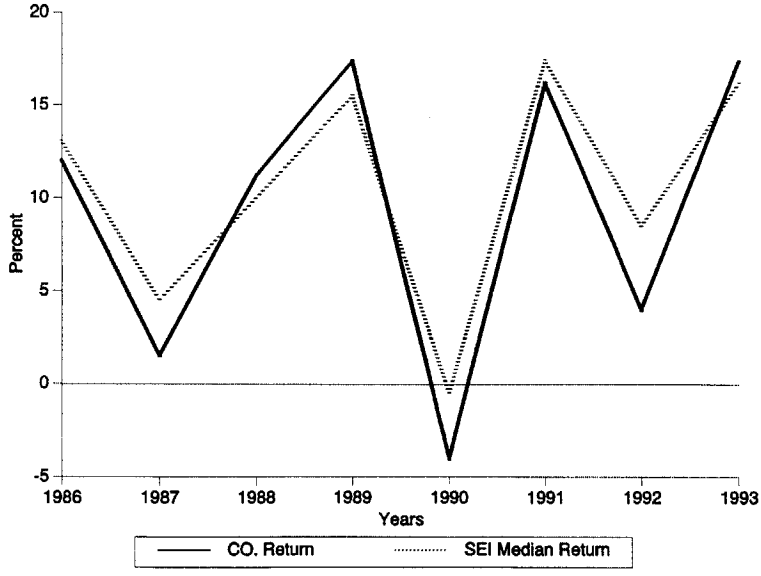
I'll use a modeling job that I did in May 1990 as an example for discussion. Of course, I changed the name. Before we start doing any of these projection studies, it is very important to know the background information, such as the underlying population, the pension plan, the funded status, and the contribution trend that the company had in the past. Look at this four dimensional graph on two dimensional paper (Chart 1). The horizontal axis shows the age groups. The height of each column represents the number of people in each age group. Each age column is subdivided into four segments representing different service groups for each age. For instance, the lowest segment of each column represents employees with service from zero to four years. The next segment represents service from five to nine, and so on. The number inside each box is the average pay for the age/service group. Now looking at the trend of the population statistics, we can tell it is quite a normal kind of population. The only thing that stands out on the graph is at the tail end. They have some high-age service groups. With these kinds of groups, we can anticipate that, in the future, there may be cash-flow strains as the older employees retire. We can also expect that an early retirement window will be costly. These are concerns that we should have before embarking on a projection. We need to test out our concerns when we start on the projection. As a matter of fact, the sponsor of this plan actually called for an early retirement window. We tested out the projected cost. After seeing the cost, the client decided not to do it.

CHART 1
POPULATION STUDY AS OF JANUARY 1, 1993



Let us now look at their investment returns on the graph (Chart 2). The solid line is the asset returns in the past for the company and the dotted line represents the median returns of pension plans in Canada. Their returns are tracing the average return of other pension funds. There's one thing that stands out. They actually have more fluctuation than the average pension assets, and that wasn't supposed to happen according to the stated investment policy. This is another piece of background information that we investigated upfront before we started on the asset/liability study. This again is a piece of groundwork done in order to ascertain the base situation first.

CHART 2
HISTORY OF INVESTMENT RETURN



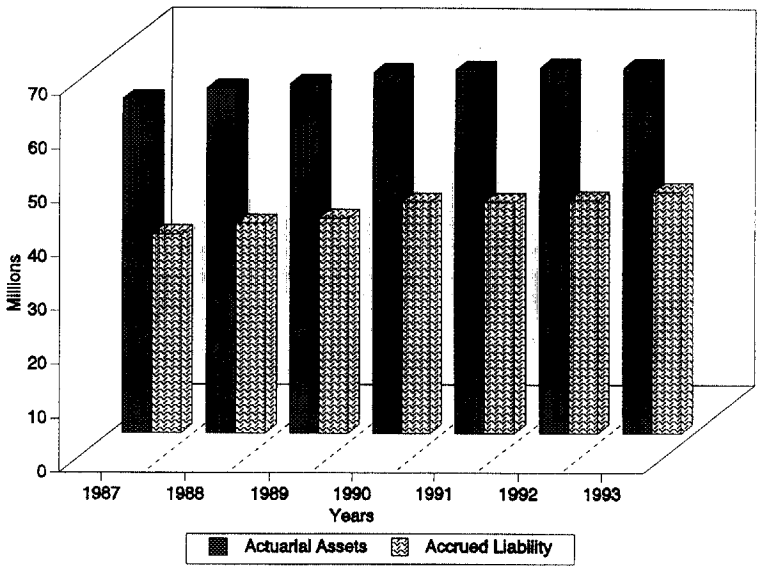
Assets versus liabilities. In Chart 3, the lighter columns are the actual accrued liabilities and the darker columns are the assets.

The company's assets have always been higher than liabilities and, as a matter of fact, in terms of the absolute dollar amount, the surplus is increasing. However, by proportion, the liability is getting closer to the assets because the company has not been contributing in recent years. The client wants to keep what Canadians call a contribution holiday around for as many years as possible. We will utilize an asset/liability study to test out the viability of this objective. We will simulate the behavior of the pension plan, the pension fund, and the contributions and expenses under various economic scenarios. What if there's a severe market downturn? What if, on top of an asset collapse, the liability actually increases because of an interest rate decrease? What would the impact be of all these on pension funding and

expending requirements? This kind of analyses typically included in a liability projection tells us what taboo situations we need to guard against.

Let's look at some of the liability projection results that came out. We performed the projection study under expected, optimistic and pessimistic scenarios. I presume that you have all seen similar scenario assumptions.

CHART 3
ACTUARIAL ASSETS VERSUS ACCRUED LIABILITY



Let us look at the output in Chart 4. On the expected basis there's no contribution requirement for any projection year because of the rich funded status we have just seen.

Even on the pessimistic scenario, they would not have contribution requirements until the very last two years. This says immediately that the funding contribution is not a problem. We should analyze something else.

What about pension expense as a percentage of pay? (See Chart 5.) Now the company is enjoying a 5% of pay pension income every year. If expected things were to happen, that trend will continue. Under the optimistic scenario, the pension income increases radically. If that would happen, should they just operate the pension plan and close out all other operations? Of course not, now let's look at what a pessimistic scenario would give them. They do have some possible problem with expense if pessimistic things were to happen. Under the pessimistic scenario we assume that they will be getting only a 5% return. It is not even zero investment return yet. Thus it is not an impossible scenario.

CHART 4
LIABILITY PROJECTION—1994
PENSION COST UNDER DIFFERENT SCENARIOS

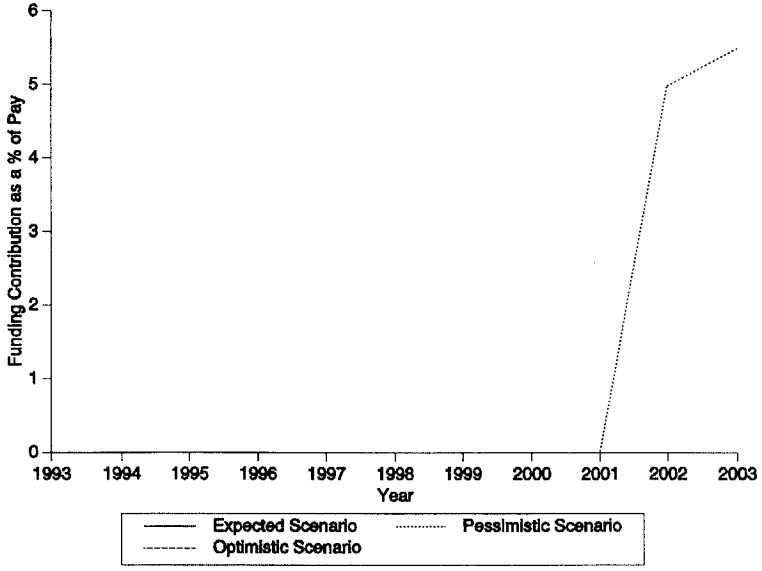
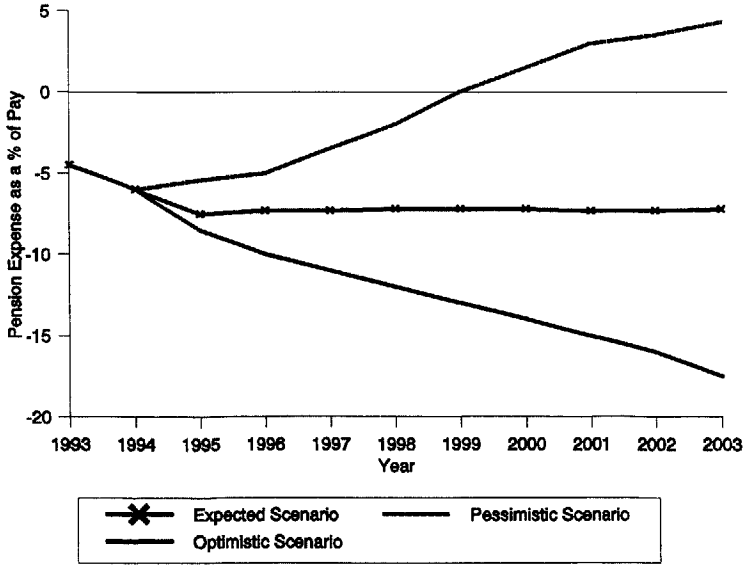
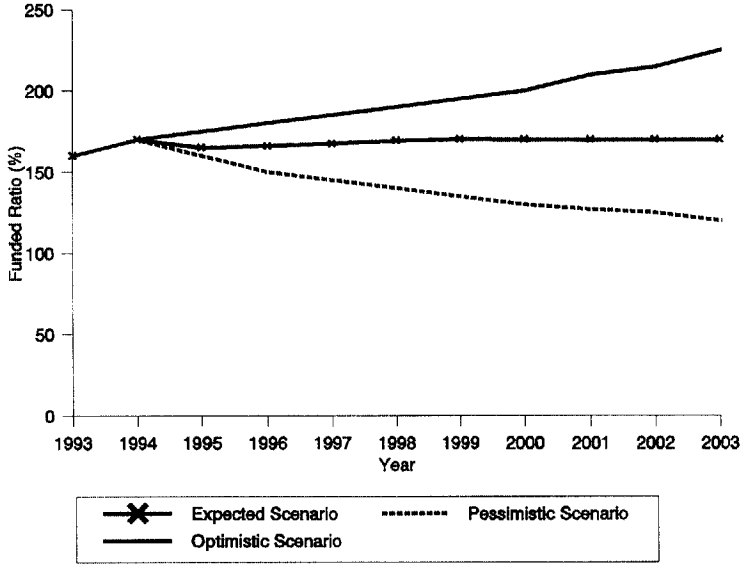


CHART 5
LIABILITY PROJECTION—1994
PENSION COST UNDER DIFFERENT SCENARIOS



What about the projected funded ratio? Chart 6 shows that the funded ratio is no problem whatsoever. As a matter of fact, the funded ratio is ridiculously high under the optimistic scenario. Under such circumstances, you want to find ways to bring down the funded ratio. Even the pessimistic scenario is not causing any problem.

CHART 6
LIABILITY PROJECTION—1994
PENSION COST UNDER DIFFERENT SCENARIOS



Now, what happens if we have a severe market downturn in the form of a 15% negative return for one year? As indicated by the "one bad year" line in Chart 7, funding is required after five years. What if there are two years of bad returns back to back? Then funding is required immediately. To put things in perspective, we have not had two years of back to back 15% negative returns in the past 50 years. It is only under a very pessimistic scenario that they would have a funding problem.

What about pension expense? This is where a problem may arise. The dotted line in Chart 8 indicates what happens after one year of bad investment returns. All the pension income is gone. Most managements would not be able to tolerate that. If there are two bad years back to back, either the management gets fired or the actuary does.

What if they have a bad investment return and a bad interest discount rate? (See Chart 9.) This is not something that the company can tolerate either. Even a 1% drop in the discount rate will result in more than a 50% decrease in pension income.

CHART 7
LIABILITY PROJECTION—1994
SEVERE MARKET DOWNTURN

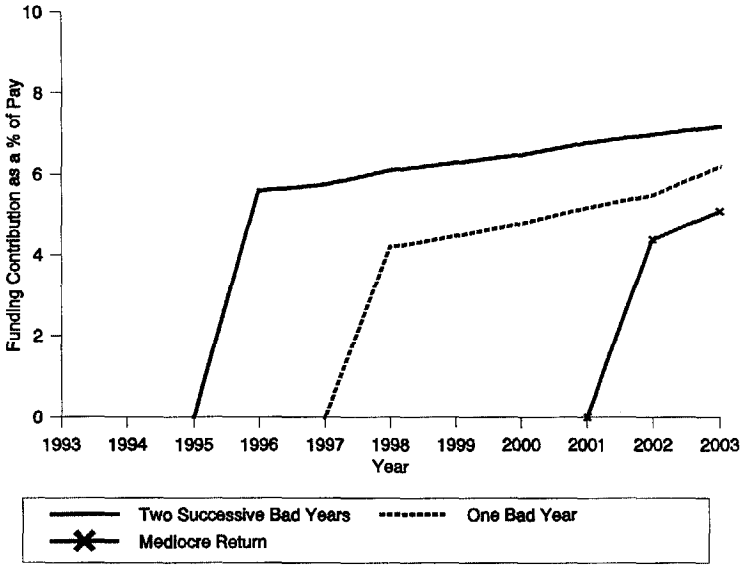


CHART 8
LIABILITY PROJECTION—1994
SEVERE MARKET DOWNTURN

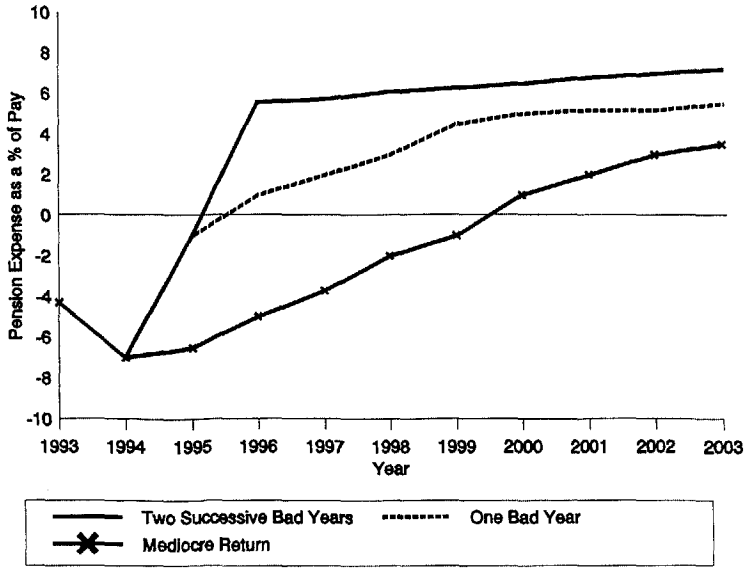
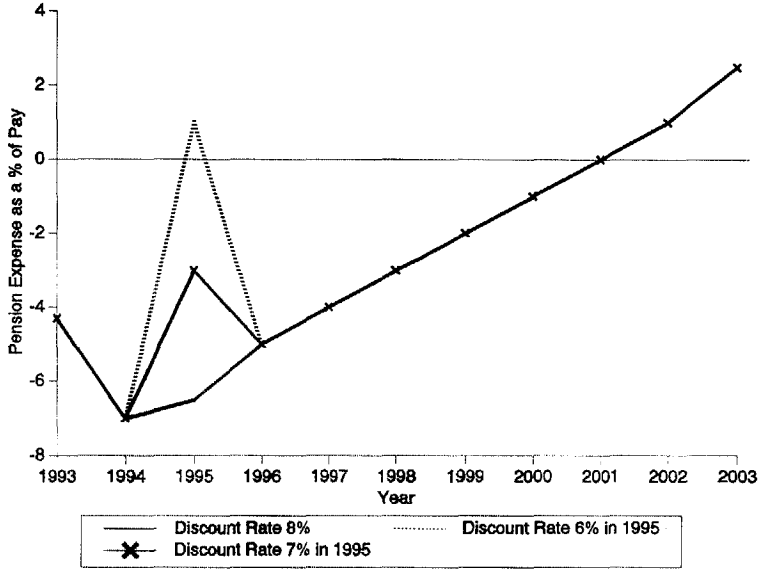


CHART 9
LIABILITY PROJECTION—1994
IMPACT OF VARYING DISCOUNT RATES



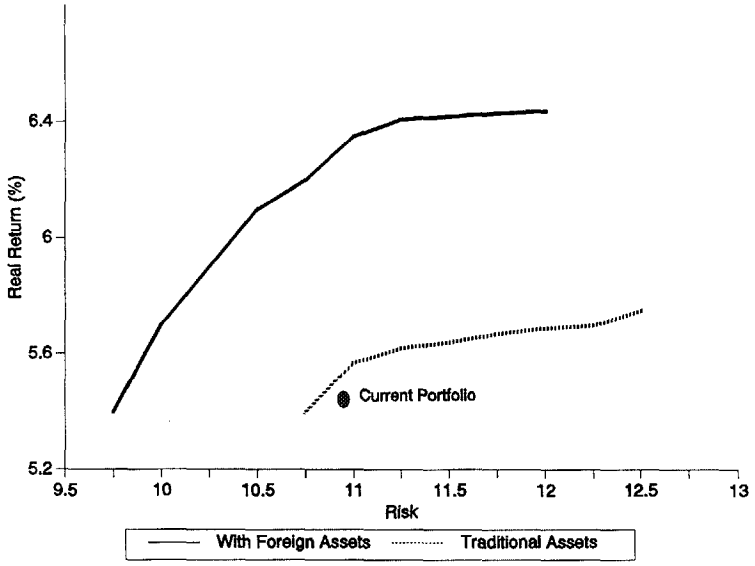
In summary, the liability projection study shows that:

1. Funding is no problem.
2. The funded ratio is no problem.
3. It is pension expending we need to be concerned about.
4. We need to test out expending under bad economic scenarios. Even though we are not anticipating two bad scenarios back to back, just one bad year, or one bad year on top of the discount rate decrease may pose serious problems for the company.

What is the purpose of doing an asset study? The purpose is to maximize returns and minimize risk. How do we do it? Through an efficient frontier study, we simulate various investment portfolios and analyze the real return versus the standard deviation of the return for each portfolio. The standard deviation is chosen as a proxy for the investment risk.

In Chart 10, the bullet represents the real return and the standard deviation of the current portfolio. The dashed line represents the efficient frontier obtained by using the traditional asset classes only. Using the traditional assets of stocks, bonds, and cash without going foreign would not improve the company's situation that much. But if they just introduce some foreign investment, foreign stocks and foreign bonds, the efficient frontier goes up substantially.

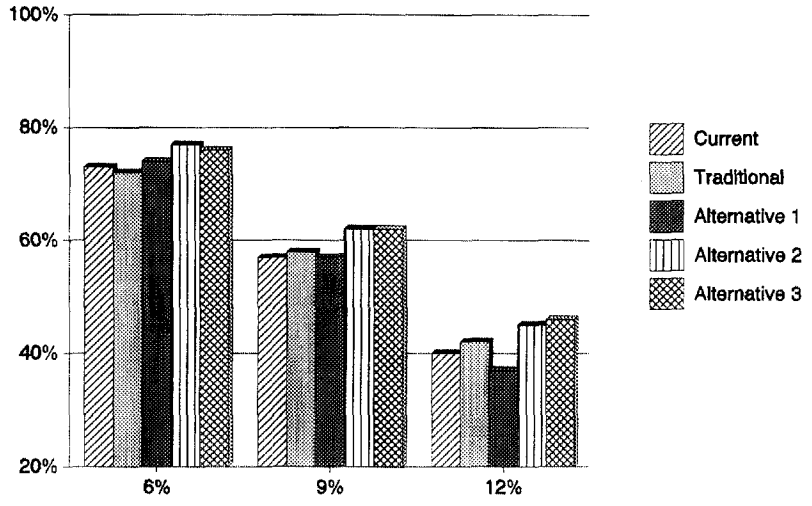
CHART 10
ASSET STUDY—1994
EFFICIENT FRONTIER



As a result of the asset study, the company chose five portfolios to be tested in the integrated asset/liability study. (See Chart 11.) First, their current portfolio has 42% Canadian stocks, 49% Canadian bonds and 9% cash equivalent. This is a very conservative portfolio, almost 60% bonds. The second portfolio is more aggressive but still Canadian; there's nothing foreign. This is the traditional second portfolio. A third portfolio has foreign bonds instead of foreign stocks, a very conservative portfolio. The next two portfolios go into foreign stocks, and we see how they behave.

And now a brief description of the asset/liability study. Typically we would do 500 simulations here. We would do the simulation for each projection year, for each asset class, for liability and for inflation. These simulations are performed under some input assumptions which I'm not going to detail. Based on these simulated results, we will compute the liabilities, the assets, the contribution and the expense for each projection year, and go into the next year and repeat the procedure. In total, how many actuarial valuations do we perform? We do 500 simulations for ten projection years for each of the chosen portfolios. This results in information overload. For this kind of process, what matters is how you condense the results, how you make sense out of them, and how you analyze the results and discuss your results with a client. It doesn't matter how big a computer program you have to generate the results. Always, keep focused on the objective; keep focused on the taboo situation, keep focused on what kind of things we should be analyzing. What do we focus on in our current study? We focus on the pension expense! We focus on what if we have a bad investment scenario. And what if we had a bad discount rate happening at the same time!

CHART 11
ASSET/LIABILITY STUDY
TARGET 3-YEAR RETURN



Let's look at some projection results. The projection results are analyzed for each of five investment policies chosen. The typical projection output includes the asset trend, the funding, the contribution, the expense and so on under each of the 500 simulated scenarios. However, we will stay focused in our discussion.

All other trends will be discussed very quickly. The meat of the matter is the pension expense and the taboo situations. Attention is directed to five key probabilities of occurrences: the 15th, 25th, 50th, 75th, and 95th percentiles. These are not the only results that come out: these represent a brief summary of the output. Because of the voluminous output, we must condense the results so that they are in readable and comprehensible form.

Look at the asset trend graphs. (See Charts 12–15.) There is much growth under all economic climates. Under a favorable climate, assets grow more than two times. Even under unfavorable climates, they increase over 30%. Even the fifth percentile event still does not present much problem for assets. This confirms the fact that assets are not what we need to worry about.

CHART 12
ASSET/LIABILITY STUDY
ASSET TRENDS—AVERAGE ECONOMIC CLIMATE

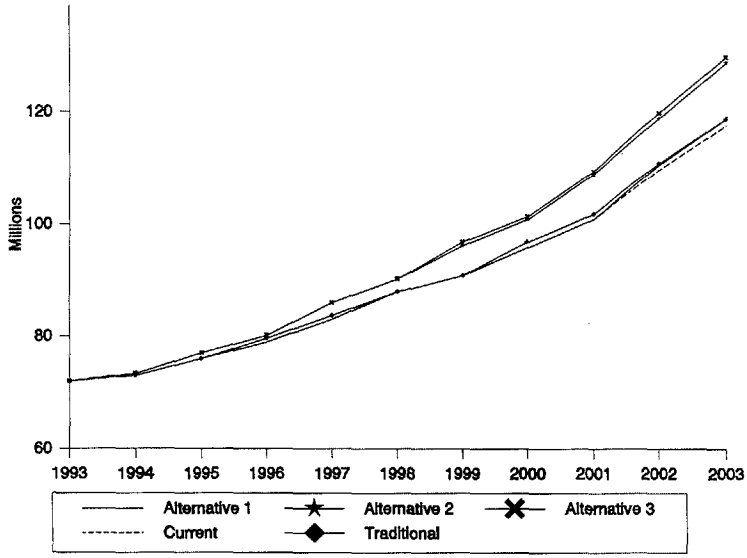


CHART 13
ASSET/LIABILITY STUDY
ASSET TRENDS—UNFAVORABLE ECONOMIC CLIMATE

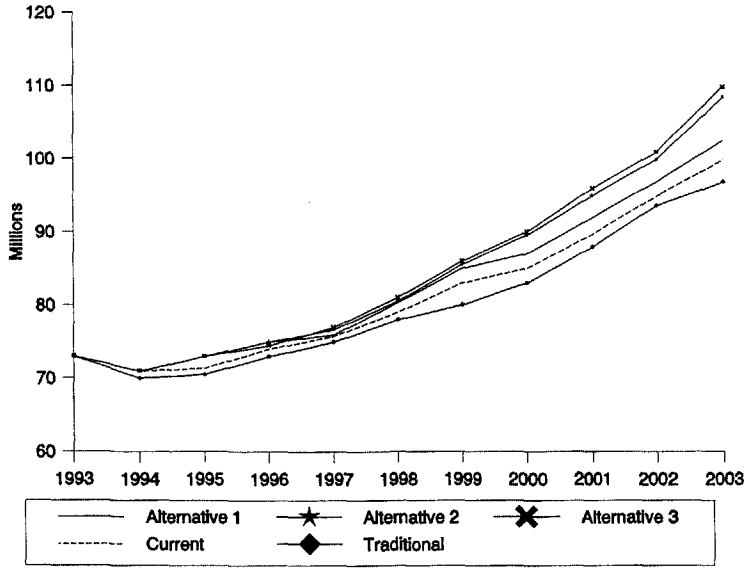


CHART 14
ASSET/LIABILITY STUDY
ASSET TRENDS—EXTREMELY UNFAVORABLE ECONOMIC CLIMATE

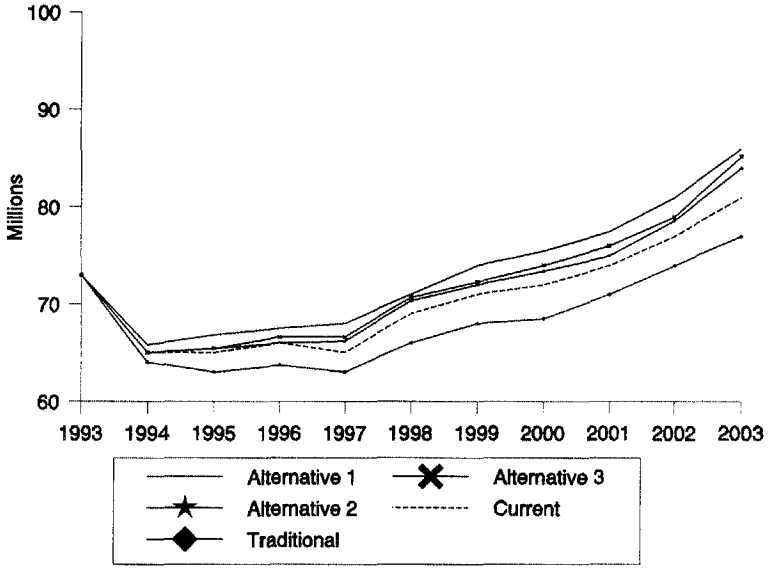
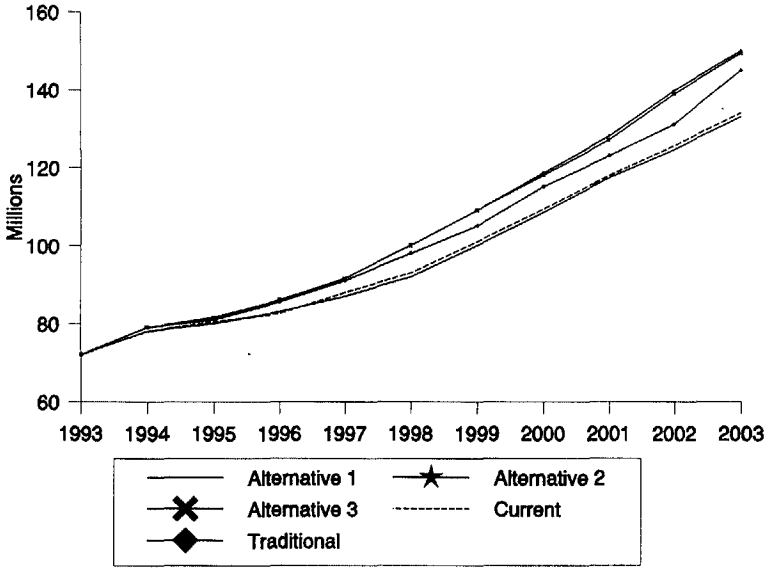


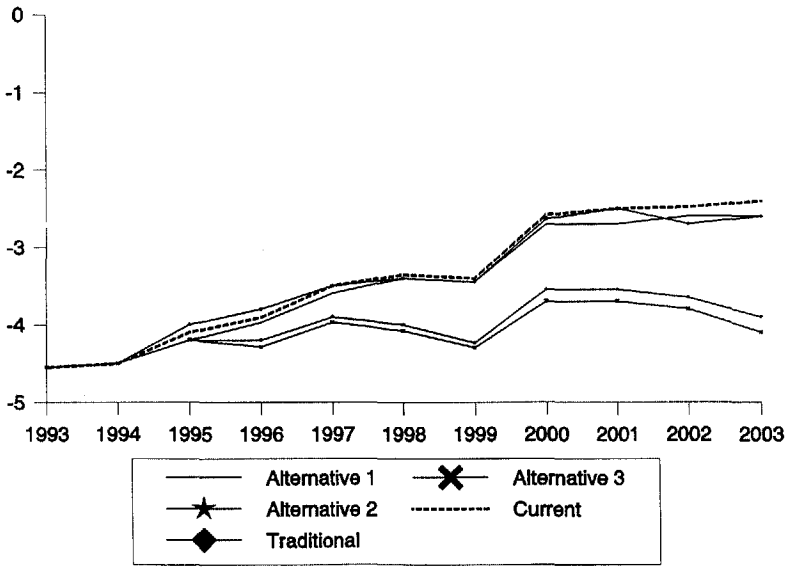
CHART 15
ASSET/LIABILITY STUDY
ASSET TRENDS—FAVORABLE ECONOMIC CLIMATE



Let us now consider the pension expense. We may want to spent a little bit more time here. Consider an average economic climate represented by the 50th percentile events.

Chart 16 shows that if the company adopts an investment policy represented by either alternative two or alternative three, the expense income is stabilized. On the other hand, under the other investment policies there may be substantial decrease in pension income.

CHART 16
ASSET/LIABILITY STUDY
PENSION EXPENSE%—AVERAGE ECONOMIC CLIMATE



Let us now analyze the taboo situation which is represented by the 25th to 50th percentile events. Under the 25th percentile event, irrespective of investment policy, the funded ratio comes down a great deal. (See Chart 17.) The funded ratio will decrease to about 120%. Consider the fifth percentile event. (See Chart 18.) When everything goes to pieces, investments are not hauling in the money, and yet the interest discount rate is low so liabilities go up. Under such an extremely unfavorable scenario, the funded ratio would dip substantially below 100% no matter what the investment policy is.

CHART 17
ASSET/LIABILITY STUDY
FUNDED RATIO (FUNDING BASIS)—UNFAVORABLE ECONOMIC CLIMATE

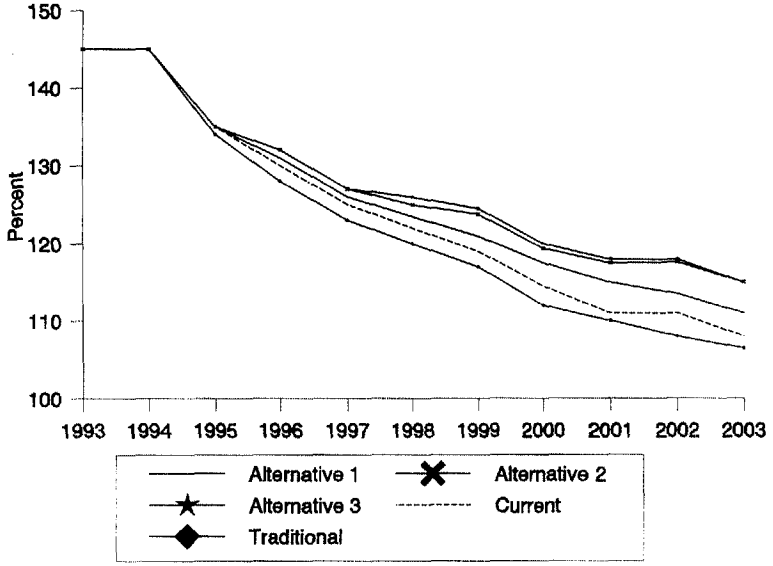
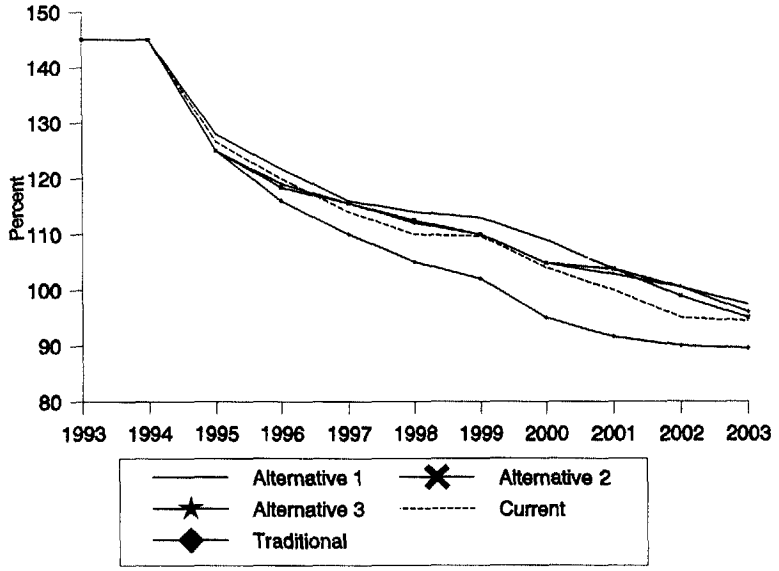
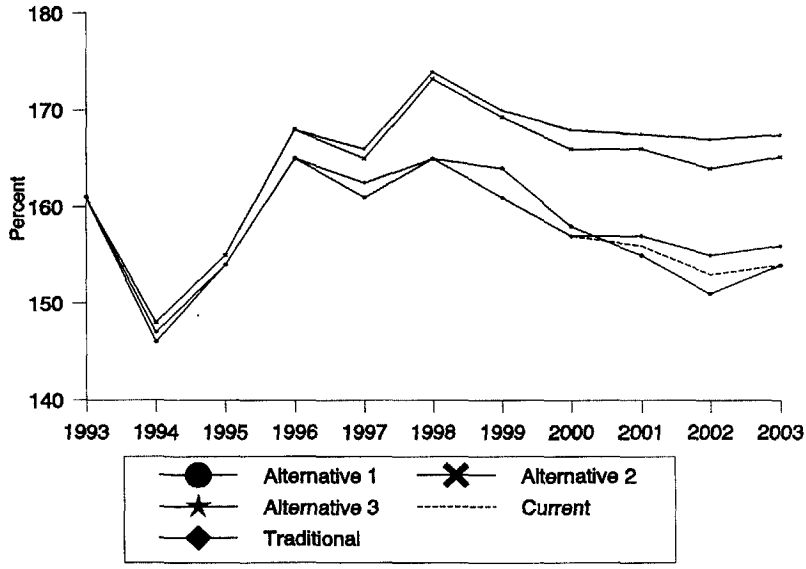


CHART 18
ASSET/LIABILITY STUDY
FUNDED RATIO (FUNDING BASIS)—
EXTREMELY UNFAVORABLE ECONOMIC CLIMATE



What about the funded ratio on an expensing basis? (See Charts 19 and 20.) As you know, the funded ratio on an expensing basis is quite different from that on a funding basis. For the funded ratio on a funding basis, the underlying interest rate is not changing. For that on a expensing basis, the discount rate would actually change. What happens to the expensing basis funded ratio as the interest rate changes? The ratio fluctuates in the opposite direction. This fluctuation is not something that any plan can easily tolerate. This problem can be remedied with a suitable investment policy. With investments structured in sync with the liabilities, the assets and liabilities will fluctuate in the same direction and thus produce a stable funded ratio even when the economic environment changes radically.

CHART 19
ASSET/LIABILITY STUDY
FUNDED RATIO (EXPENSING BASIS)—AVERAGE ECONOMIC CLIMATE



What about pension expense expressed as a percentage of pay? (See Chart 21.) As the graph indicates, under unfavorable situations, portfolio alternatives two and three actually produce more stable income streams. However, under an extremely unfavorable situation, no conventional investment policy is going to help much. (See Chart 22.) Alternatives two and three help out some; however, the ultimate fluctuation in expense is still not what a company can tolerate. This all points out the fact that just changing the portfolio mix may not make assets and liabilities behave in tandem. This diverts the extreme risk of assets going to pieces and yet the liability goes up because of the interest rate downturn. We must go through an asset/liability matching process. This is a very important and technical process. We are happy to have an expert to lead us through this process. With that I give you Dave LeSueur. He is going to tell you how to do it.

CHART 20
ASSET/LIABILITY STUDY
FUNDED RATIO (EXPENSE BASIS)—UNFAVORABLE ECONOMIC CLIMATE

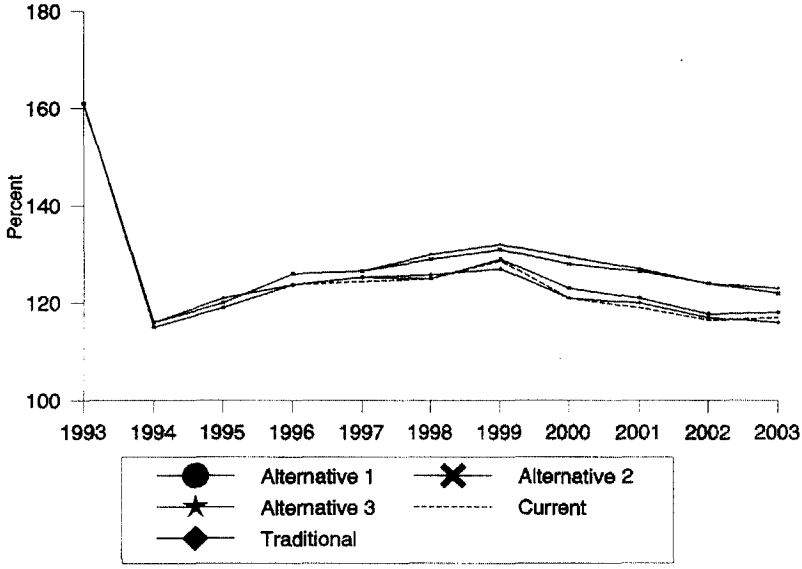


CHART 21
ASSET/LIABILITY STUDY
PENSION EXPENSE %—UNFAVORABLE ECONOMIC CLIMATE

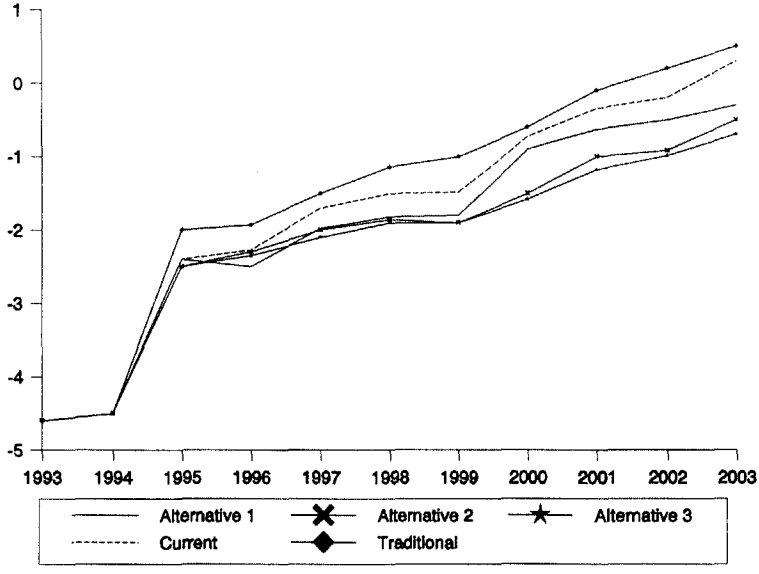
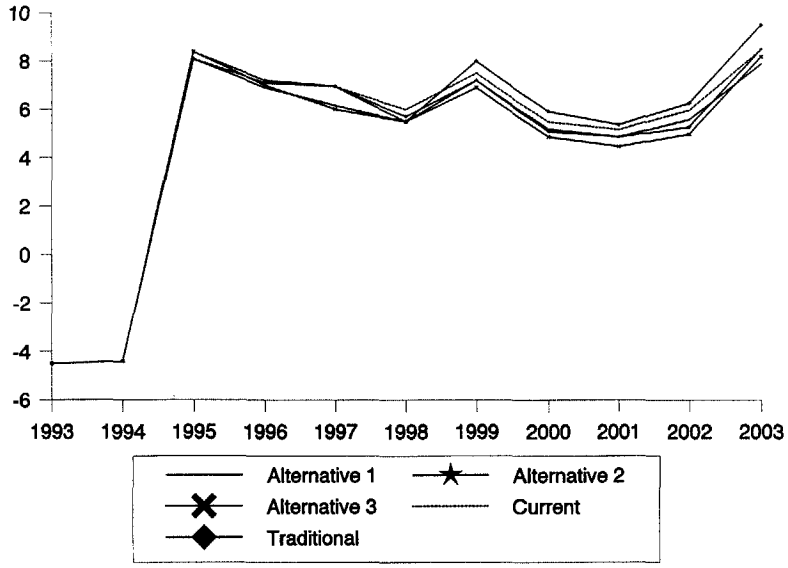


CHART 22
ASSET/LIABILITY STUDY
PENSION EXPENSE %—EXTREMELY UNFAVORABLE ECONOMIC CLIMATE



MR. DAVID C. LESUEUR: My portion of the program is entitled "Introduction to Duration." Duration is a concept I want to discuss very briefly. It can be described or defined as the rate of change of an asset or a liability as the discount rate changes. The formula for duration is: the duration of the present value of a liability or asset is the negative partial derivative of that liability or asset with respect to the interest rate divided by that present value. Now technically this is called "modified duration," but I don't think it's important for our purposes to know the difference between modified duration and other kinds of duration.

I'll go through a couple of simple examples, the first for an asset. If you have a bond worth $100 at 8% interest and it is worth $90 if interest rates go up to 9%, then the duration of the bond is simply calculated as 100 minus 90 divided by 100 times 1%. That's the change in the present value of the bond divided by the change in interest rate divided by the present value. The duration of this bond is ten. Another way to think of it is that a 1% change in interest changes the value of the bond in the opposite direction by 10% and so we say the duration of that bond is ten.

Now let's look at a simple example for pension liabilities. Suppose that the present value of the liability for a group of retirees is $100 million when the discount rate is 8%, and that the liability changes to $95 million when the discount rate increases to 9%. We say that the duration of the liability of that group is five and, again, it's because the liability changes by 5% when the discount rate changes by 1%. Now this very simple example is very powerful and can be very useful when we try to estimate changes for groups of either active employees or retirees or vested

terminated employees. It is fairly common that the actuary will be sitting in a meeting with financial representatives of the pension plan and be asked, "What would happen to the liabilities if discount rates go up or down by 0.5%?" You don't have time to go back to the office and do a calculation on the computer. If you know the duration of the liabilities for that particular plan, then you can estimate the new liability fairly easily. If you don't know the duration for that plan, you can use some rules of thumb which are based on an average plan and population. For example, the duration of the projected benefit obligation for an active group is usually somewhere around 15. Obviously, it's going to be lower or higher for different groups. The duration of the service cost or normal cost is usually somewhat longer than that. But you can make some estimates off the top of your head fairly quickly that way. Vested terminated employees durations usually are 15–20 and for retirees usually around seven or eight. Now, obviously, they're going to vary depending on whether there are cost-of-living adjustments included and so forth. But the more familiar you become with duration, the easier it will be to make estimates.

So very simply, knowing the duration can be useful because if you want an asset or a liability to change in the same way when interest rates change, then one way to achieve that is to try to match the durations of the assets and the liabilities. I should point out that the implication of trying to find the duration of assets only readily applies to fixed-income-type assets. Typically the change in the value of equities does not correlate as directly as fixed-income assets do to interest rate changes. One logical question is why would you want to match the duration of assets and liabilities, since generally, in order to do so, you're going to have to give up some potential return. (That happens because equities are expected to have a higher return than fixed-income investments and in order to do duration matching you're going to have to go to fixed income.) So you are giving up something normally in expected returns so why would you want to do that? Here are several possibilities.

One is to preserve a surplus. For example, let's suppose that one company is going to be sold to another company in three months. A pension surplus exists in the retirement plan and let's suppose it is a fairly important part of the deal. At least it's being counted on in the purchase price. So it's important that the surplus still be there when the deal closes. One way to try to ensure that the surplus is still there is to match up the duration of the liabilities and the assets at least for that three-month period so that the surplus is preserved. Then, after that, the new company can go ahead and change the asset mix if it wants to.

To stabilize contributions is another reason to match the duration of the assets and liabilities. This particular reason is not usually the motivation for duration matching. Companies seem to be able to live with unstable contribution patterns. Mike's study showed you that if a company wanted to achieve more stabilization of contributions, then it would have to do some duration matching.

Another possible reason that the plan sponsor might want to match the assets and liabilities is to stabilize pension expense. Finally, another reason might be to conserve a FAS 88 settlement gain. In fact, there was a real situation which I worked on, which was the reason I first became interested in this subject. In this case my client needed to show an increase in its bottom-line results during the fourth quarter because of the particular industry they were in and bond covenants and so forth. So

one of the ways that they tried to achieve this was to buy annuities for the retiree group, and through the workings of the *FAS 88* settlement accounting they were able to accelerate recognition of many of the gains that otherwise were being deferred. We can argue about whether this is good practice to have accounting rules drive your investment decisions, but nevertheless that's reality. They were making these decisions in October but knew that because of the time required to make an annuity purchase, it wouldn't be until December that it actually would occur. So it became important to them that the surplus that they were calculating for the gain from this annuity purchase was actually going to still be there by the time they did the annuity purchase. So what they did for that two-month period was to set aside assets from within the pension trust that were equal to the liabilities for the retiree group based on current interest rates. They invested those assets in securities with the same duration as the liability. And as it turned out, interest rates went down so that the liabilities of the retired group went up. Because the assets they had invested in had the same duration, the value of the assets also went up and they were able to conserve their settlement gain.

On that particular project, I was working with another actuary who was representing the investment banker putting together this proposal for my client. The two of us worked together on that and we ended up writing a paper on the whole subject of duration which actually contains the details of the formulas that I'm going to be talking about here. But in case you're interested, the paper is contained in the June 1993 *Pension Forum*. We suggested that the general approach for determining the duration of assets that would be needed to stabilize surplus or to stabilize contributions or pension expense, or to accomplish any other objective you had is to first write a formula for the element that you're trying to stabilize. Second, take the partial derivative with respect to the interest rate. Third, set the partial derivative equal to zero because you want that change to be as small as possible and then solve for the duration. That's the general approach.

Here is an example. Suppose you wanted to preserve the funding surplus in a plan. If that were your objective, then first write the formula for the funding surplus: assets minus the accumulated benefit obligation (ABO) for retirees, minus the ABO for active employees. That's the surplus. Then take the derivative of the right-hand side of the equation with respect to interest, set it equal to zero and solve for duration. Now the details of that equation would go on for several pages and are in the paper that I referred to. The answer is that the duration of the assets that you would need to preserve the funding surplus is equal to the ABO for retirees times the duration for retirees, plus the ABO for actives times the duration of the actives ABO, divided by assets. If you think about it, the solution is fairly logical. All that means is that the duration needed for the whole plan is the weighted average of the durations for the individual groups. And maybe you could have figured that out without going through the whole formula, but some of the other solutions are a little more complicated. But I was happy to see that this result was logical.

We took a single plan and said, OK, what if we wanted to calculate the duration necessary to do each of those things: preserve surplus, stabilize contributions and so forth. What we found was that the duration needed to accomplish each of those objectives was different in each case, so you can't just find a single duration and have it stabilize contributions and stabilize pension expense, preserve the surplus and

accomplish all those things at the same time. So what that points out is that you have to determine what your objective is and then calculate the duration that you need to accomplish that objective. And I just want to remind you again that usually that means that you're going to have shift some assets from equities to fixed-income type of investments.

Mike mentioned immunization in his remarks. Immunization is a special case of preserving a surplus when you're just looking at retirees. What you do is set-up a portion of your assets that can be used to match the expected cash flow for your retirees. The most radical way to do this is to set-up what's a dedicated bond portfolio. If you know from your projections what the cash-flow is expected to be each year (benefit payments to retirees), then you can establish a bond portfolio such that the cash flow coming out of the bonds (the coupon payments) will exactly match the cash payments to retirees. Now, clearly, this isn't going to work out exactly because of the estimates involved in the cash-flow projection. But in theory, you wouldn't even care what happened to interest rates because you would hold the dedicated bond portfolio to maturity and all you'd be worrying about is the cash flow. A less radical way of accomplishing the same thing is to match the duration of your assets to the liabilities. So rather than matching the cash flow, you match the duration. Whether interest rates went up or down or stayed the same, the value of the assets would stay equal to the value of the liabilities.

So far, this has been nice in theory but, of course, in reality there are quite a few practical problems. For example, if you're trying to stabilize funding or pension expense, there is no precise relationship between the discount rates that you use for accounting standard calculations and funding calculations and the interest rates that are used for investments. There's a general relationship, but as you know, the *discount rates do not automatically go up and down as interest rates change.*

In addition, if you're analyzing the duration of an entry-age liability or a projected benefit obligation that involves salary projections, you must realize that salaries also change as underlying interest rates change. We've ignored that in all of these calculations. As a result, the duration of salary-related liabilities is a little more complex.

Also, duration is not constant but it changes as the interest rate changes. The duration when interest rates are 9% may not be the same as when interest rates are 8% and this phenomenon is called convexity. It's something that doesn't affect us dramatically but it is a reality.

Assets of fixed-income bonds generally have call provisions that are difficult to adjust for in determining the duration of the bonds.

The last practical problem I will mention is what I call the "Star Trek" problem—the nonparallel yield curve shift. When I saw that term I thought that it sounded like something that Dr. Spock would be saying to Captain Kirk. "We're experiencing nonparallel yield curve shift, Captain." Now what is that exactly? The calculation of durations depends upon the current yield curve, and when that yield curve changes its shape, it could change the duration of your liabilities and assets. It's very likely that the change in the yield curve will have a different impact on your assets than on your liabilities. So even if they were matched before the yield curve change, they

may not have matched afterwards. It's just a real life problem that you have in trying to match liabilities and assets.

Duration matching is, at best, an approximate tool, but I would argue that it's still a very important one. Remember that actuarial liabilities are estimates anyway. Duration matching is a reasonable and effective approach to accomplishing the objectives we have spoken about today.

MR. SZE: Any questions? Obviously, Dave is making things so clear that we all know how to handle the nonparallel yield curve. Well, as always, if you don't ask questions, questions will be asked of you. How many people have done elaborate asset/liability projections? Great. How many of you have done a liability projection? Just about everybody. How many of you have done an efficient frontier study? This is what I actually expect that most of us started out doing as pension actuaries. We are inclined to think of projection as just rolling the population forward and then doing 10 or 15 variations as the client asks us how many years to project. But if you think about it, the projection process is quite different from the actuarial valuation process. It should have been included in the regular actuarial variation process. It's important that you look at both sides of the balance sheet, the assets and the liabilities, in doing funding variations. The concept of comparing assets to liabilities is even more important when you are doing asset/liability projections. Even though the absolute value is important, it is of secondary importance. The most important thing in this kind of study is the trend. The clients want to know when they're going to have a problem and what kind of situation would exist if they have a problem. They want to know, by comparing different strategies, what the best strategy would be. In order to do all this, you have to look at assets and liabilities together and this is the major difference between just doing a liability projection and doing an asset/liability study. I do encourage you to read books on it and try it out yourselves.

The question is, what are the good books that you can use. There are many asset/liability books, but for starters, the Investment Track Syllabus has a lot of very good introductory articles and I do encourage everybody to read it. Actually, the young actuaries have an advantage over us oldies because they have it in their syllabus and we didn't.

MR. DAVID J. DUNCAN: I just wonder if anybody has any familiarity with what some of the investment firms are marketing right now to the public plans where you issue bonds in the amount of your unfunded and the unfunded is basically paid off. Has anybody worked with a group that has done that?

MR. LESUEUR: I think what you're referring to are pension obligation bonds. It's something that public plans are allowed to do. They can issue bonds to the public and currently they might be crediting 6–7%. It has probably gone up by now, but when interest rates were low, the return was in the 6–7% range. Then the public entities could turnaround, invest that money that they got and be earning 8% or 9% in the pension fund. They are allowed, under law, to issue bonds equal to no more than the unfunded actuarial accrued liability. There are a number of plans in the State of California that have done that. I don't know if it has caught on in other places or not, but they did it when the interest rates were low. Right now, it's probably not a

favorable environment to do it, but that's more of an arbitrage kind of maneuver as opposed to an investment or asset/liability matching.

FROM THE FLOOR: Do you have any practical suggestions on how to estimate the asset frontier, the efficient frontier? I mean you show pictures of it on the graph but I assume it's not actually calculable.

MR. SZE: These efficient frontiers were actually simulated from input assumptions based on past statistics, so they are actually calculated. Each point on the efficient frontier actually represents a portfolio.

FROM THE FLOOR: Yes, that's right.

MR. SZE: And the portfolio that we have chosen, the five different portfolios, are all on the respective frontiers.

FROM THE FLOOR: Yes.

MR. SZE: You have the percentage distribution stocks, bonds and so on. All of these were actually represented as points in the efficient frontier.

FROM THE FLOOR: Well, you don't know that they're on the efficient frontier do you? You just know that they're points on the graph. Do you know that there would be, for example, a portfolio that would have a higher return at the same risk rate as one of the ones that you picked? How do you know that you've achieved that efficient frontier?

MR. SZE: That is programming the issue, but let me just address that question anyway. The question is how can we be sure mathematically that the portfolio that's on that curve is an efficient portfolio. This is achieved statistically through a type of programming, called quadratic programming, and in that programming process all possible portfolios with all possible distributions have been tested out.

FROM THE FLOOR: I see.

MR. SZE: And regarding each one of those portfolios on the efficient frontiers, is the portfolio among all portfolios with the same standard deviation that gives the highest possible return. That is why, mathematically, it is the best possible portfolio.

MR. LESUEUR: I'm just going to add that we can prove that the calculations are correct based on the assumptions that are made. Obviously, if future experience doesn't match what our assumptions are, then it's not the efficient frontier. This is all based on the assumption that what we've determined as the expected return and the risk that goes with it are correct and I can tell you right now that they never are correct. We hope that they're close to being correct.

MR. SZE: In an asset/liability study, there will be more wrong than the actuarial variation that you are doing. But that doesn't matter because the emphasis is on the trend comparison. The emphasis is on, when would I reach that taboo situation that I cannot live with.

FROM THE FLOOR: When you discount a liability say for ten years, are you using the yield curve? Are you using a single rate for the entire ten years or are you combining say a one-year rate at the front end of it?

MR. SZE: Yes. In doing the asset/liability study what typically will be done is you simulate the discount rate for each year separately, and at the same time you simulate the return for each asset class. Therefore, the liabilities are calculated using the discount rate and contributions and expenses are calculated for that year. Then the simulation is repeated. In short, we simulate the discount rate for each particular year.

FROM THE FLOOR: When you calculate the discount rate for a given year, everything is discounted at that single rate. Then the next year, when you do the calculation, the discount rate is recalculated based on the interest rate environment at that point in time in the simulation. But it's basically doing a snapshot valuation as of each of those years and each snapshot is using a single discount rate going forward.

MR. SZE: Of course, the underpin of the question is, if that is the case, are we doing ten million actuarial variations. Well, this is a more technical and programming problem. In theory you are. In practice you work out a statistical approximation method that will get you to the same place as doing an actuarial variation without actually doing it. There must be some mystery in everything that we do.

MR. LESUEUR: One thing I forgot to mention in my closing statement is that you'll often see asset consultants do their asset allocation study without looking at liabilities. What they're trying to do is achieve the highest possible return with a given risk that the client or the plan sponsor is able to live with. And then the actuaries will do their liability forecast without looking at assets because we're just trying to show what the cash flow is going to be. There are situations where that's fine, where cash flow isn't a problem and the highest possible return is all you're worried about. But what we hope you've learned today is that there are many situations where that isn't fine, where you should be looking at assets and liabilities together because the liability issues are going to have some influence on the asset allocation and the asset investment policies. I hope you've learned that here.