RECORD OF SOCIETY OF ACTUARIES
1995 VOL. 21 NO. 4A

DESIGNING AND ADJUSTING AN INVESTMENT STRATEGY: REAL LIFE EXAMPLE

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Walk through and participate in the development of an investment strategy for a single premium deferred annuity (SPDA) product.

MR. BRYAN EDWARD BOUDREAU: The topic of this session is designing and adjusting an investment strategy for an SPDA product. I work for MetLife in New York in the retirement and savings center. We also have with us Mark Abbott from Global Advanced Technology (GAT) in New York. GAT specializes in fixed-income consulting, analytic and research. Mark has been with GAT for about five years and consults extensively with insurance companies and other financial institutions on asset/liability management and risk management issues.

I'm going to talk about some basic concepts and then present an SPDA liability example. Then Mark will talk about some more advanced asset/liability management techniques. The views expressed are the views of the presenters only, not necessarily those of our employers.

For asset/liability management purposes, liabilities can generally be classified as one of two basic types. The first type comprises fixed liabilities, characterized by cash flows that are fixed at issue. Examples would be a GIC and a structured settlement or other payout-type annuity. Because liability cash flow are essentially fixed at issue, all you can manage is the asset portfolio. So, any asset/liability management decisions are, in effect, asset-only decisions.

The other type of liability is interest-sensitive. SPDAs are one example. Interest-sensitive liabilities can be managed after issue, which implies that you generally have the ability to pass investment experience through to the contractholder. Of course, you have to stay within the bounds of good marketing, equity, and the contract specifications. Interest-sensitive liabilities require asset/liability strategies that are linked or at least should be linked if the asset/liability management process is coordinated properly.

We are going to be looking at the effect of different asset/liability strategies on economic value. Economic value is a pretty simple concept for most of the assets that insurance companies buy. There is a fairly active trading market for many fixed-income securities, and even some of the more illiquid assets are appraised from time to time. So, economic value generally means something that approximates market value.

For liabilities the notion of economic value is much less clear. There is not an active market for trading liabilities, other than perhaps the reinsurance market. There are

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several approaches that could be used to look at economic value for liabilities, and this a fairly hot topic right now.

One basic approach to the liabilities is to realize that much of the liabilities that insurers write have fixed-income-type characteristics. So to measure economic value, we start by applying fixed-income techniques to those liabilities. For fixed cash-flow liabilities, fixed-income valuation would generally be based on discounting cash flows along a single path. That might be the path of forward rates implied by the current spot curve. You look at a single path of cash flows, and discount them back at Treasuries plus a spread, and that would be a reasonable proxy for economic value.

Interest-sensitive liabilities are slightly more tricky. Here what we have to realize is that the cash flows are not going to be fixed for any given interest rate scenario, since they may be a function of credited rates or policyholder behavior. Any concept of economic value has to consider that there are multiple possible paths. We will use the same technique of discounting along interest rate paths, but we have to consider a set of alternative paths that spans a reasonable array of possibilities for what might happen to interest rates.

To ensure consistency in economic valuation between an interest-sensitive liability and a fixed liability, we also need to impose some sort of condition on the interest rate paths that we look at. Generally that's called arbitrage free. If you have a set of scenarios that are arbitrage free, one condition satisfied by those paths is that, if you valued a fixed cash flow back along the different paths and average the results together, you should get the same answer that you would get if you just looked at the current Treasury rates.

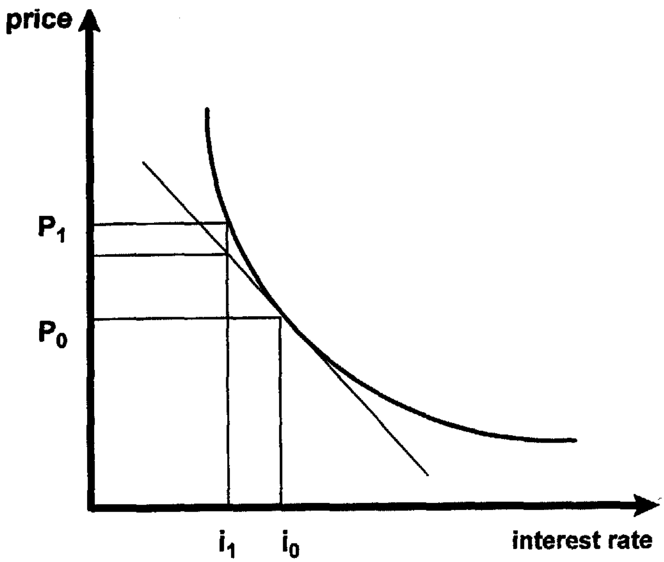
I'd like to discuss one more point about economic value in contrast to book value. Basically, differences between economic value and book value are really just timing differences. If you're looking at longer-term strategies, models should produce fairly similar results whether you look at economic or market value results or book value results. However, in the shorter term you could have some fairly significant differences depending on how things like capital gains are treated.

Related to economic value are the concepts of duration and convexity. The simplest definition of duration is the Macaulay duration which is, in effect, the mean term of the liabilities, or the weighted average time until a liability cash flow occurs. It is really only valid for liabilities that have fixed cash flows. A more robust measure of duration and convexity is the effective duration and convexity. Effective duration and convexity measure the change in the economic value of an asset or liability for a given change in interest rates.

Effective duration is defined using the formula minus the first derivative of present value with respect to interest rates, divided by the present value. Convexity is the curvature term, the second term in the Taylor expansion of the price function. An extension of effective duration and convexity would be option-adjusted duration and convexity. That would be the same type of definition, where you are looking at the change in present value with respect to a change in interest rates, but you also take into account differences in cash flows resulting from the exercise of options.

To see pictorially how duration and convexity look, Chart 1 is fairly standard. What you are looking at in the curved line is a price performance profile, for either an asset or a liability. What it shows is how the price or economic value of that asset or liability changes as interest rates change. The point on the bottom labeled i_0 is where interest rates are right now. The duration component of the change in price is basically the area from P_0 to the line between P_0 and P_1. It is how much the value would change if there were a linear relationship between price and interest rate. Convexity is the curvature, the rest of the difference up to P_1.

CHART 1
DURATION AND CONVEXITY



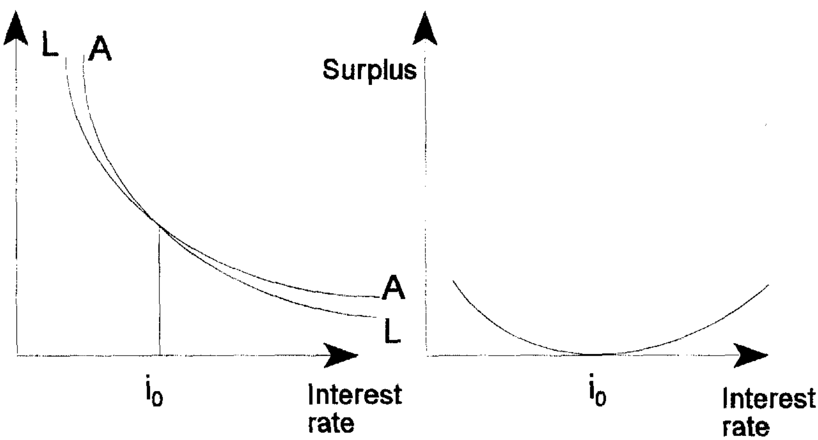
Positive convexity is curvature in the up direction. It means that economic value increases more, when rates go down, or decreases less if rates go up. Convexity is basically a good thing if you have it, that is, if you are long convexity. It's a bad thing if you have sold it. We are going to see in the SPDA example that some of the product features would imply that you have actually sold convexity, and you have a potential problem on your hands if you don't manage that.

The next concept I want to look at is C-3 risk, arising from asset/liability mismatches. What does that mean? Well, in the most general form C-3 risk arises from cash-flow mismatches. If you use that definition, pretty much any portfolio has C-3 risk because you're rarely going to have a portfolio that has asset/liability cash flows perfectly synchronized under all future scenarios. A better working definition of C-3 risk would be economic surplus risk, arising from duration or convexity mismatch. This would exist when economic surplus changes as interest rates change.

One way to control C-3 risk is cash-flow matching. The objective here would be to try to get an asset/liability portfolio where the asset cash flows match the liability cash flows in all scenarios. You're probably never going to do it. So, generally we rely on immunization conditions. An immunized portfolio would satisfy the condition that the duration of the assets equals that of the liabilities, and the convexity of the assets is greater than or equal to the convexity of the liabilities.

Chart 2 shows how that would look. The graph on the left shows the economic value profiles for an asset and for a liability. Again i_0 is where rates are currently. If this portfolio were immunized, then you'd have a tangency between the performance profiles for the asset/liability. For small rate changes the lines would approximate each other. However, for large rate changes where convexity becomes significant, you would like to have the convexity of the assets greater than the convexity of your liabilities. If you had that, then for an instantaneous change in rates the surplus picture looks like Chart 2. Any movement in either direction helps you, and you'd tend to increase surplus if rates moved.

CHART 2
AN IMMUNIZED ASSET/LIABILITY PORTFOLIO



I would like to review a couple of SPDA preliminaries. First, the SPDA is fundamentally an investment product, even though it is called an annuity. I think people buy these for the most part because of their credited rates and because of the tax deferral on the accumulation. So, it is really a product sold on its investment-type attributes.

We're going to assume for our example that the product design is fixed. Realize that part of the product development process should take into account that some of the features that you build into the product have economic value implications. Because the SPDA is an interest-sensitive liability, what we really need is a coordinated asset/liability strategy. What we're going to model is a fixed-liability strategy, and then try to determine an appropriate asset strategy for that liability.

I'm going to assume here that we have a 7% up-front acquisition cost on this product. Let's further assume that we reset the credited rate annually and can't set it any lower than

3%. We'll also assume a declining surrender charge that starts at 7% and drops by 1% per year, with 10% annual free withdrawals with no surrender charge. Finally, I'll assume no surrender charge on deaths and annuitization in this particular product. Overall, it's a fairly standard set of features for an SPDA.

Let's develop an investment strategy for this product. What are some of the requirements of the investment strategy? If we are going to have any sales or sustain any business here, we need to make sure that we maintain an adequate credited rate. The investment strategy must be able to support a reasonable strategy for setting rates. In addition, we need to have adequate profit margins, or we probably shouldn't be in the business to begin with.

Given a target liability strategy and required margins, we need to consider what variables we can change in the investment strategy. These might be asset mix type decisions, like the percentages of callable and noncallable bonds, mortgage-backed securities (MBS), or common stocks. We also want to control C-3 risk, so we need to tell the portfolio manager the duration to run this portfolio at. We may also want to tell him or her something about the convexity of the liability. In addition, we also need to consider things like portfolio quality and liquidity.

How do we select a strategy? One approach is to model the liabilities without regard to the assets. We want to get a feel for how the SPDA looks without considering how we invest for it, so we are going to calculate liability statistics such as economic value, duration, and convexity. Then we will model alternative investment strategies, using an asset/liability model over different scenarios to see what the range of results is. The range of results for different strategies will allow us to evaluate risk/return tradeoffs.

I'll talk about just a few of the considerations that you have to take into account when modeling the liabilities. First, you need a cash-flow generator, a subroutine or a function or a procedure that projects the accumulation value of the SPDA. The function will have interest rates as an input, will assume some sort of credited rate strategy and policyholder behavior, and produce cash flows.

We need to model policyholder options. The most significant policyholder option is lapse, which will tend to happen if your credited rate lags behind the competition as rates move up. That behavior is called disintermediation and it has a cost that should be reflected in the model, so we need a lapse function in our model. We also have to take into account the insurer options. The insurer has the right to change credited rate. What we are going to do is assume a credited-rate strategy that is based off of the five-year Treasury rates and assume a certain amount of movement toward new money rates. As a base case, I'll assume we move about 25% in the direction of new money. We also need to take into account the 3% minimum rate, which also has a cost.

Using these assumptions, we run the SPDA liability model through arbitrage-free interest rate paths. The interest rate paths that I used in this particular example are from the GAT interest rate path generator. They span a range of possible interest rate paths that could occur, given today's starting yield curve. I'd like to thank my colleagues from GAT, Basil Rabinowitz and Pavel Bortnovskiy, and Bryan and Glen Salisbury from MetLife for their assistance in developing the SPDA examples that follow.

Table 1 depicts the liability market value results. The market value here is calculated using discounting at Treasury rates flat. The first line here is the base model, where we move 25% toward new money rates in resetting credited rates. The market value for a $10,000 SPDA at issue is $9,567. However, we have a 7% acquisition cost. So, on a $10,000 SPDA we are starting with only $9,300 cash, even though the present value of the liability at Treasury flat discounting is a little under $9,600. In effect, you are starting out with a small deficit, which means that you have to earn on average about 28 basis points over the Treasury curve to get back to paying off your liabilities.

TABLE 1
SPDA MARKET VALUE RESULTS

	Market Value	Duration	Convexity	Required Option-Adjusted Spread (in basis points)
Base model	9,567	3.47	15	28
Less responsive interest rates	9,592	3.95	21	31
More responsive interest rates	9,546	2.88	8	25
Reduce lapse sensitivity	9,520	3.58	13	22
Increase lapse sensitivity	9,611	3.41	15	34
Increase minimum rate to 4.5%	9,583	4.02	103	30

By looking at economic value for the set of interest rate paths plus and minus 50 basis points, we can calculate duration and convexity for the SPDA. For our base case the duration is about 3.5, and the convexity is about 15.

Now let's look at some modifications to the SPDA model. On the second line we use less responsive interest rates. Instead of moving 25% towards new money, we are moving only 20% towards new money. Duration goes up by about a half a year and convexity also goes up. Option-adjusted spread increases slightly.

The third line assumes more responsive interest rates. Instead of moving 25% in the direction of new money, we assume movement of 33%. Market value still doesn't change all that much, but duration shortens by about a half a year to 2.9. Convexity decreases but there is not too much effect on required spread.

On the fourth and fifth lines, I alter the assumed policyholder behavior. First, we assume that there are no interest sensitive lapses. The base case assumed an increasing propensity for lapse as rates went up. By removing the interest rate sensitivity in the fourth line, the value of the SPDA went down slightly. This is telling you that the liability is not as valuable if the policyholders don't exercise their options as efficiently. Duration and convexity are essentially the same. You will also notice that the cost of funding that SPDA drops by about six basis points, which represents the cost of the policyholder behavior that we assumed in the base case. If we increase the lapse sensitivity, we get the opposite results: the SPDA is more valuable. Again, you don't get a big change for duration and convexity, but the cost of funding now increases by about six basis points.

Finally, I wanted to look at the cost of the interest rate floor. So, I changed that from 3% to 4%. What you see is a small change in the market value of the annuity. However, both the duration and convexity go up significantly. That means that the curvature of the liability line increases since you have shorted a more valuable interest rate floor which, from an asset/liability perspective, is not a good thing to do.

Using our base case assumptions and instantaneous changes in interest rates, I wanted to see what the liability risk profile looked like (Table 2). Market value, duration, and convexity are the same as you see in the base case. As interest rates move up and down by 100 bp and 200 bp, what we get is an asymmetrical picture. As rates go down, there is a significant lengthening in the duration of the SPDA and the convexity goes up quite markedly because that interest rate floor you have written is starting to become closer to being in the money. If rates go up, the market value decreases, which is what you would expect, but the duration and convexity don't change all that much. You only shorten by about a tenth of a year, in contrast to lengthening by close to a year if rates go down.

TABLE 2
SPDA LIABILITY RISK PROFILE

	Market Value	Duration	Convexity
Rates down 200 basis points	10,311	4.36	67
Rates down 100 basis points	9,905	3.64	73
Current rates	9,567	3.47	15
Rates up 100 basis points	9,242	3.43	16
Rates up 200 basis points	8,932	3.37	21

Note: Effect of interest rate floor is to increase liability duration and convexity in falling rate scenarios.

Chart 3 shows how this SPDA with positive convexity might look in an asset/liability portfolio if you had callable assets. Here, we have the opposite picture from what we saw before. Liabilities now have the more positive convexity. If you have callable assets in your portfolio, your assets are going to have less or perhaps negative convexity. While your assets and liabilities may have approximately equal values at current rates, as interest rates go down, callable assets increase in value less than your liability because of the call and prepayment features.

You end up with the right side of Chart 3. You get deterioration in your economic surplus for instantaneous changes in rates. You will have an accumulated loss over time if interest rates are volatile. That's not all bad news because you are getting paid for taking this risk. Presumably, the more convexity risk that you take, the more spread you are going to get. What you have is a tradeoff between how much spread you earn and how much you lose due to negative convexity as interest rates fluctuate.

Finally, to get a reality check on the liability-only modeling, we also ran the same SPDA through an integrated asset/liability model. The model projected asset/liability cash flows, with reinvestment and borrowing as appropriate. We ran the same 51 scenarios, looked at the results, using the present value of GAAP profit over a 20-year horizon. We looked at the average present value of GAAP profit, and the standard deviation of this value.

Chart 4 shows what the output of this model looks like. Across the bottom are different portfolio strategies. Each one of those strategies represents rebalancing the portfolio to a constant target duration, from two to five years. What you see for each strategy is the average present value of profit plus and minus one standard deviation.

CHART 3
EFFECT OF CALLABLE ASSETS
NET NEGATIVE PORTFOLIO CONVEXITY

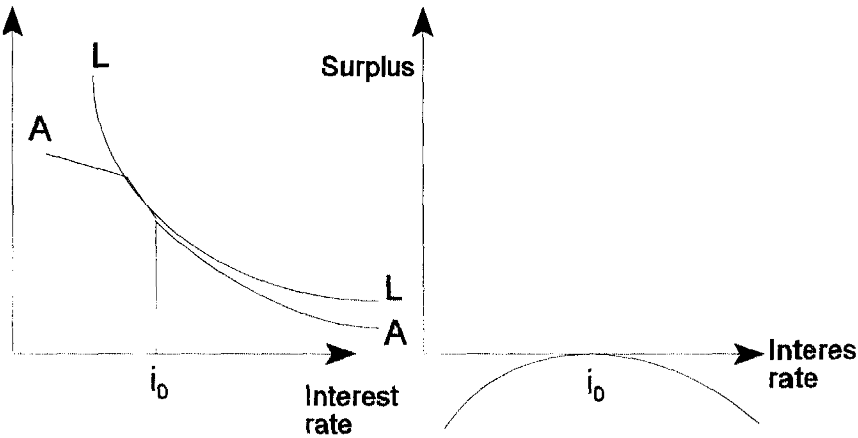
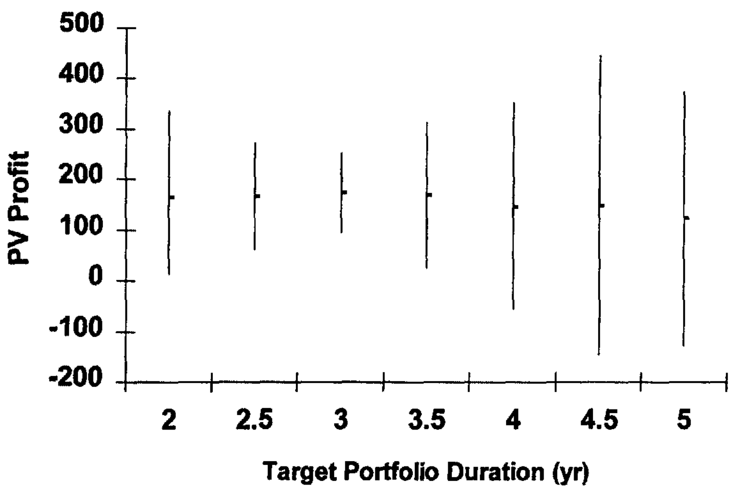


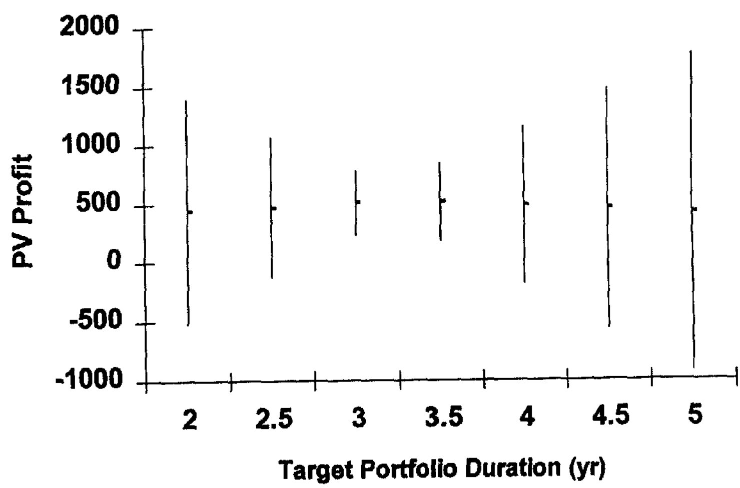
CHART 4
SPDA INVESTMENT STRATEGY ALTERNATIVES
EFFECT OF TARGET DURATION (SINGLE PREMIUM)



The results are fairly consistent between the asset/liability model and the liability only model. What I mean by that is that it looks like the three-year target portfolio duration has the best average return and minimum risk. The liability duration is shortening as you move through time, with 3.5 the duration at issue, so the average portfolio duration probably ends up being slightly shorter than 3.5. In the asset/liability model, a three-year duration looks like a reasonable investment strategy.

I also ran the same type of model, assuming a 20% annual growth in a book of business, so the duration should be slightly longer on average. What you get in Chart 5 is a result fairly consistent with that, with a duration between three and three-and-a-half years minimizing the variation in your present value of book profit.

CHART 5
SPDA INVESTMENT STRATEGY ALTERNATIVES
EFFECT OF TARGET DURATION
(20% ANNUAL GROWTH IN BOOK)

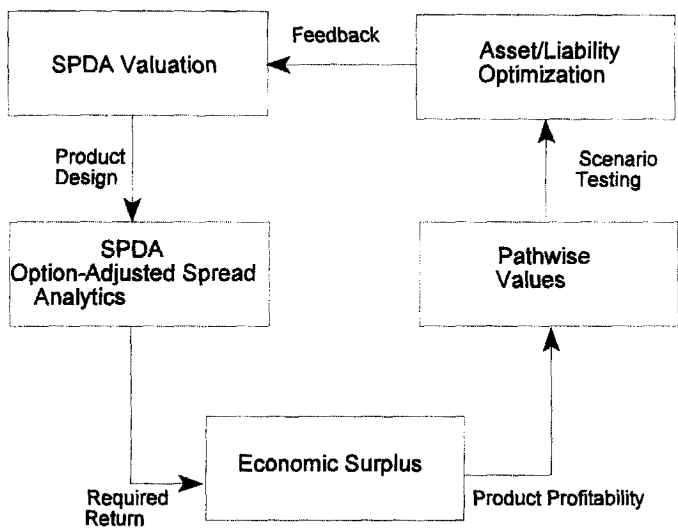


I hope to have established the first steps in developing investment strategy with this example. We look at the liability characteristics and requirements, and use some simple modeling to get a feel for what kind of duration and convexity we might want to run the portfolio at. Now I'm going to turn it over to Mark at this point who's going to talk about some more advanced techniques.

MR. MARK C. ABBOTT: Let me summarize the points that I hope to make. I will be starting with the total return approach as we view it, various tools for risk measurement and risk management, portfolio management techniques, liability modeling techniques, and some of the product features. I'll build on the concept of effective duration and move to nonparallel key rate sensitivities and key rate durations (KRDs). Then I'll again analyze the SPDA and discuss benchmarking and methodologies to develop a custom liability index. Finally, I'll discuss required option-adjusted spread and then proceed to the more advanced techniques, pathwise valuation, optimization, and arbitrage free bond canonical decomposition (ABCD).

Let me start with an overview of the total return approach (Chart 6). This is a framework that needs to encompass immediate, prospective, and retrospective analysis. There are feedback loops that help you to frequently calibrate your models so you get the best predictors going forward. You need to make sure that your asset/liability modeling is consistent and that, as you learn more about the policyholder behavior, you reevaluate your models. I think there's a great deal of work yet to be done in this area. We must develop a method of marking to market both the investment portfolio and the liabilities. That is necessary for total return management and performance attribution where we attribute value to both sides of the balance sheet. Finally, we need to be consistent with the long-term view of book value accounting.

CHART 6
TOTAL RETURN APPROACH



Since we're dealing with an SPDA, I'll briefly discuss a feedback loop for looking at the model itself, the product design and generating analytic that can be used in evaluating or even determining an optimal investment strategy. Those analytics are the price (or fair value or market value), option-adjusted spread, required option-adjusted spread, duration, and convexity. You can then look at profitability in an economic surplus methodology as you walk forward in time. What happens if you stress the scenarios? What happens to your market value under those extreme interest rate path situations? Ultimately you can optimize an investment portfolio to match the characteristics of the liability.

Bryan covered effective duration, which is sensitivity of price with respect to the parallel movement of spot rates. Positive duration in this case represents the discounting effect on fixed cash flows; the price movement is the opposite of yield movement. I'll talk about a negative duration and what that means in just a moment. KRD is a concept that Tom Ho of GAT introduced about the same time that Bob Reitano at John Hancock was introducing partial durations. They both came up with the idea about the same time of a method to measure sensitivity to nonparallel movements in the yield curve.

Linear path space (LPS) is an arbitrage-free interest rate space; it's a more efficient method than Monte Carlo simulation. In addition, this is a framework that allows you to systematically compare cash flows. The next step is to look at market values as you discount along these paths. This can be used consistently for assets and liabilities. We have developed the pathwise valuation profile that matches and contrasts the pathwise values of the liability versus the assets.

Our SPDA model cash flows would basically be from lapses, the payment of accumulation at some distant horizon, deaths, and any expenses like commissions. We would discount those cash flows at Treasury spot rates plus some spread depending on the option characteristics within the particular product.

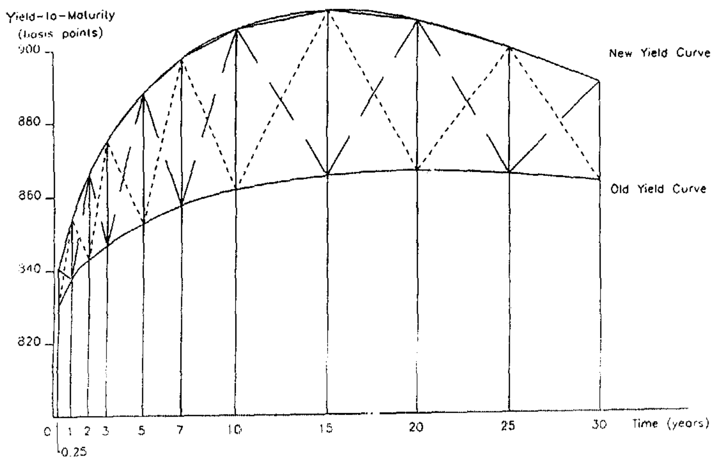
I'll also talk about ABCD, which is a superior way of explaining the embedded options in a product rather than just giving it a single option-adjusted spread. ABCD is a building block approach to replicating the embedded options by matching pathwise value by using option-free building blocks, option-embedded building blocks, and various types of path-dependent building blocks. Once you can explain all of the options in a particular liability, it becomes clearer to an investment manager what must be done to eliminate some of that risk by techniques such as diversification to capture some gains by taking appropriate strategic action and assuming other risks.

Portfolio management techniques that are commonly used for asset/liability modeling (ALM) are duration matching at the onset and rebalancing at regular future intervals or after major market moves. I think that one of the points that Bryan was going to make is that, if you look at what the duration of a portfolio is, you also should look at what happens in a few years from now because the duration is going to change. It's going to change due to any change in the yield curve. It's also going to change due to the fact that time has elapsed. You need to keep track of what is going on with the liability and its sensitivities, as well as that of the investment portfolio; it's a linked problem.

Let me now define KRD. KRD uses the same formula as duration, except now we're just looking at price change for a sensitivity to a key interest rate term. One of the good relationships is that for infinitesimal changes the effective duration is equal to the sum of the KRD. We've actually gone further recently and applied the same concept to key rate convexities, which has applications in the value-at-risk (VAR) framework. For VAR there are additional dynamics in terms of the interest rate risk and the correlations between the various terms that you need to consider.

Chart 7 gives an example. If I were to look at the short rate shift, I would have from zero months a parallel line 25 bp above the original curve connecting to the three-month point, and then a line moving down to the original curve at the one-year point. Everything else would be right along the original yield curve. If you moved out to the ten-year point, you can see a dotted linear shift going up from seven to ten years and down again from ten to fifteen years. Basically, by this construction, the sum of all these partial changes would equal the parallel spot curve shift for an effective duration.

CHART 7
YIELD CURVE SHIFT AS THE SUM OF KEY RATE SHIFTS



Charts 8A–H are recent of examples of KRD profiles. In Chart 8A, the 20-year Treasury zero coupon bond has no sensitivity to changes in short rates. Because all of your money is out at the 20-year point in time, price sensitivity is just to that duration. In Chart 8B, a 30-year coupon bond, you can see the distribution of coupon and principal. The examples in 8C and 8D look at the various callable corporate. You can see that the calls to the higher coupon premium bond basically shorten duration and decrease the sensitivity in the longer key rates. Chart 8E is an example of the KRD on a European call and put. For the put you're buying an option to sell a 30-year bond in ten years. You're buying an option and shorting a bond at this point, so you have the positive sensitivity at the ten-year point to purchasing the option, with the shorting of the bond indicated by the negative key rates beyond that. Another good example of negative duration is in Chart 8H. The Government National Mortgage Association (Ginnie Mae) interest only represents the interest only payments in a MBS where you have a large amount of negative sensitivity to the ten-year index rate sensitivity of the prepayment model.

There are three areas of interest rate sensitivity in the SPDA model: the crediting rate function, the discounting rate function, and the lapse function. Chart 9 is an example of cash flows and the sensitivities of the cash flows. This graph shows the variability of future cash flows along the implied forward curve for three different surrender charge schedules. The first is the seven-year surrender charge. The next one is a five-year surrender charge. The third one in each group has no surrender charge. You can see by the cash flows that, if there is no penalty, lapses occur immediately as forward rates rise and the competitor rate exceeds the crediting rate. If you look out about five years, you see the five-year surrender charge release. Out seven years, you see the seven-year surrender charge release. At the end we're looking at the accumulated cash balance being paid out in full at the 30-year horizon in this example.

CHART 8 a–d
KEY RATE DURATIONS

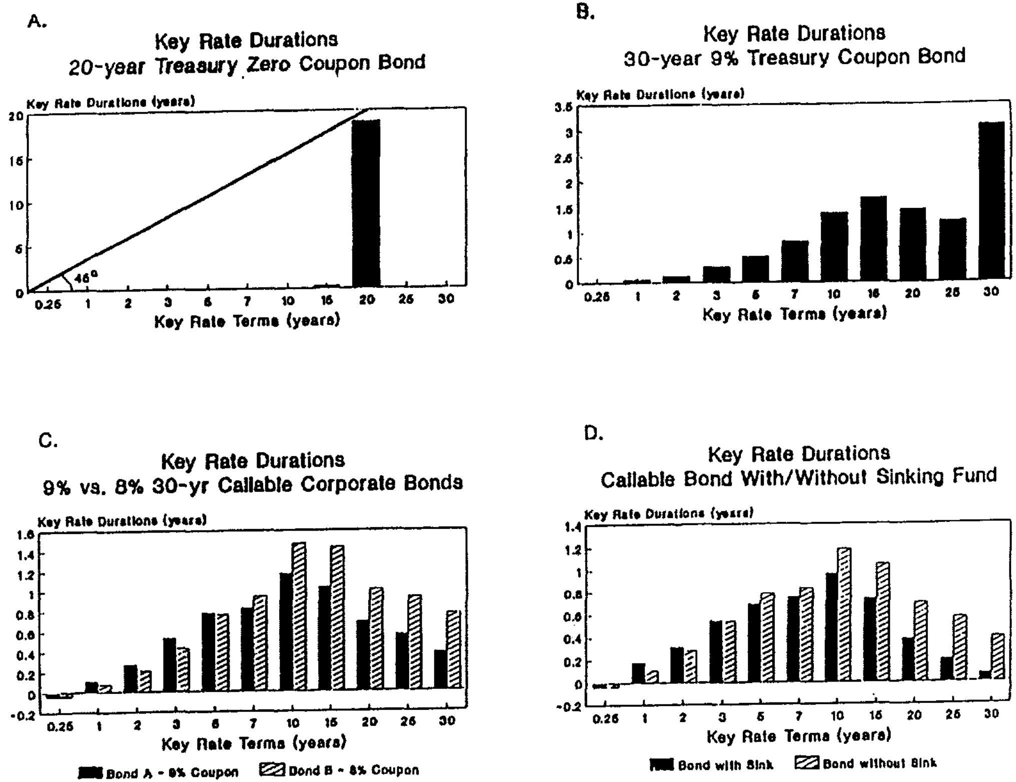
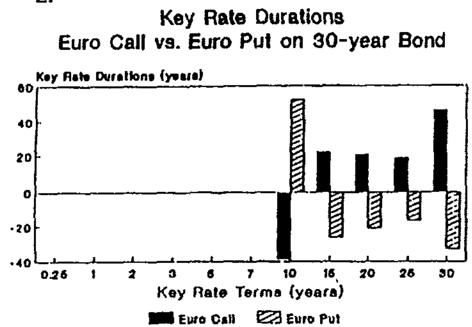


CHART 8—CONTINUED
8e-h: KEY RATE DURATIONS

E.



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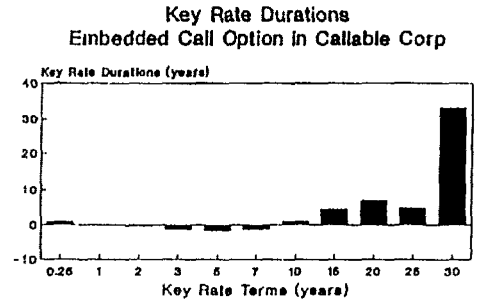

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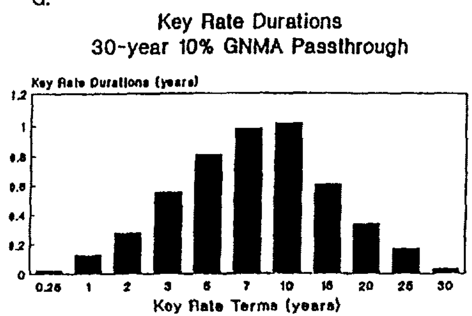

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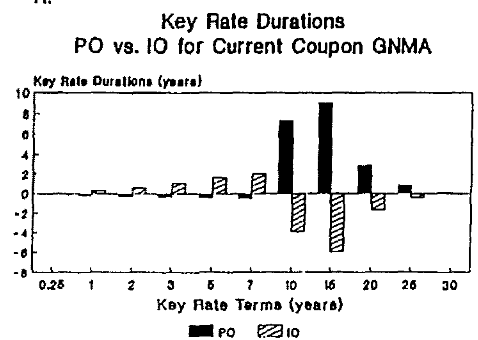
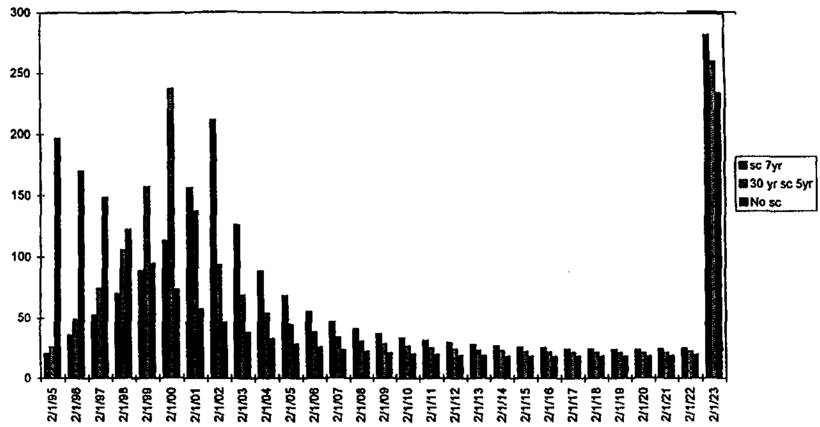


CHART 9
GLOBAL ADVANCED TECHNOLOGY CORPORATION
REDUCED SURRENDER CHARGE



If we change the policyholder behavior, we see variations in the cash-flow pattern (Chart 10). A cool money policy would represent individuals less likely to take their money out. The hot policy would represent the people who are more investment savvy. They would see a better competitor rate out there and would actually do something about it.

Table 3 shows the detailed results. The first line is a base case SPDA. As the surrender charge is reduced, the market value increases. As you reduce the minimum guarantee to 0%, the market value decreases. Notice also that, as you reduce the minimum guarantee, you shorten duration. Hot money, where there is a greater propensity to lapse, costs the insurer more, reflected in the market value and reduced duration and lower convexity. Cool money has the greatest convexity because the 3.5% minimum guarantee affects the largest balance of retained business.

The next thing to look at are the KRDs. The no-surrender-charge policy has the largest first-year KRD sensitivity. At the fifth year you see that the maximum duration sensitivity results for the five-year surrender. To understand the negative durations, you have to think of the positive discounting duration offsetting the negative index duration.

In Chart 11 we see the similarity between the KRD from the SPDA and a five-year constant maturity Treasury (CMT) floating rate note. Notice the same sort of leverage that occurs between the discounting and index KRD. This suggests some possible use of swaps or floating rate instruments to replicate the price sensitivities of the liability product.

I'd like to review some of the benefits suggested by the above results. We've calculated the option value in a framework that's consistent with the assets. We've looked at the parameter sensitivity, and I think the education factor comes very high here, relative to measures that impact design, profitability, and risk. These can facilitate better communication with the portfolio managers to help them to develop a better portfolio strategy to match some of the embedded options in a particular product.

CHART 10
GLOBAL ADVANCED TECHNOLOGY CORPORATION
HOT MONEY VERSUS COOL MONEY

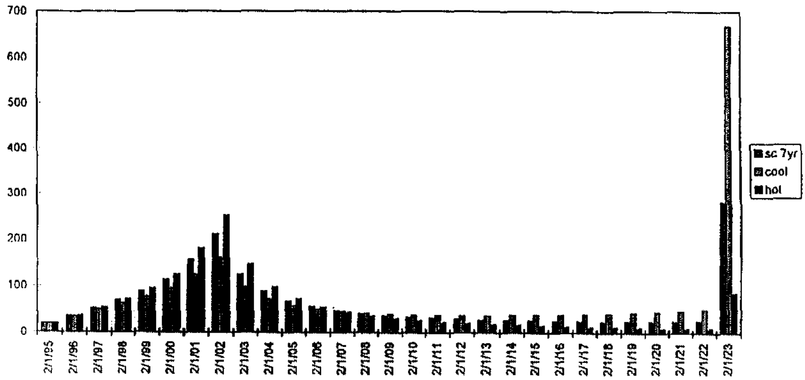
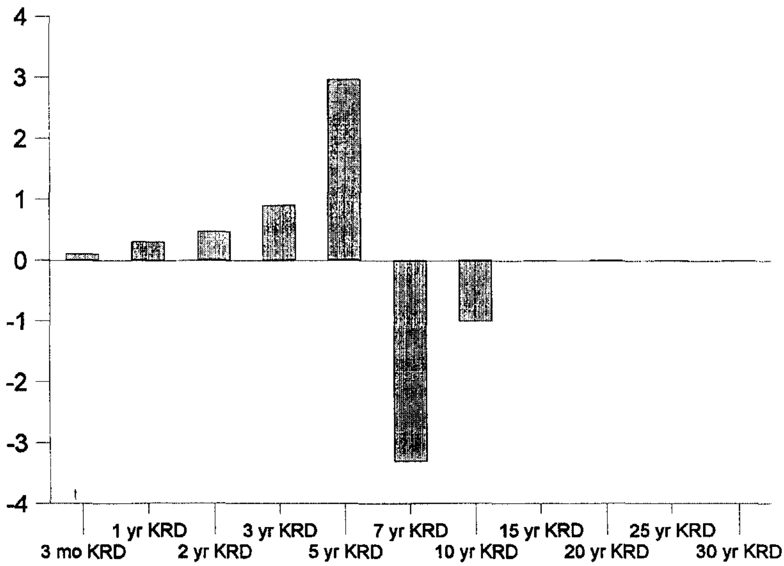


TABLE 3
GLOBAL ADVANCED TECHNOLOGY
SIMULATION RESULTS

	SPDA						
	30-year Surrender Charge = 7 years	30-year Surrender Charge = 5 years	30-year No Surrender Charge	30-year Surrender Charge = 7 years[a]	30-year Surrender Charge = 7 years[b]	cool 30-year Surrender Charge = 7 years[c]	hot 30-year Surrender Charge = 7 years[d]
Market Value	911.991	930.632	960.946	908.085	902.576	886.167	928.285
Duration	4.098	3.755	2.502	3.839	3.133	4.600	3.794
Convexity	0.688	0.634	0.709	0.664	0.108	0.846	0.573
OAS	0.000	0.000	0.000	0.000	0.000	0.000	0.000
KRD 3 months	−0.082	−0.082	−0.082	−0.082	−0.082	−0.082	−0.082
KRD 1 year	0.093	0.092	0.300	0.093	0.094	0.094	0.092
KRD 2 years	0.240	0.244	0.504	0.240	0.242	0.242	0.238
KRD 3 years	0.725	0.814	0.867	0.728	0.740	0.726	0.724
KRD 5 years	1.661	2.151	0.939	1.680	1.738	1.626	1.684
KRD 7 years	1.942	1.268	0.000	1.991	2.188	1.752	2.086
KRD 10 years	−0.735	−1.162	−0.450	−0.714	−0.697	−0.691	−0.798
KRD 15 years	−0.244	−0.053	0.026	−0.251	−0.682	−0.150	−0.338

[a]2.5% minimum guarantee
[b]No minimum guarantee
[c]temp. = 0.5
[d]temp. = 2

CHART 11
GLOBAL ADVANCED TECHNOLOGY CORPORATION
KEY RATE DURATIONS—FLOATING RATE NOTE



Duration, convexity, and KRD provide benchmarks for investment strategies to improve ALM. *Instruments that have some floating rate sensitivity should probably be used if they* match the KRD profile of a particular liability. The asset/liability management department or the risk management function in your corporation can monitor the sensitivities of your assets and liabilities with their interest rate bets. They can suggest to the portfolio management group the type of structures to hedge out some of the risk that they don't want and to maximize return.

We talked about rebalancing a portfolio as you go forward in time and how important that is. Given that duration rebalancing is only helping with respect to parallel changes in the yield curve, doesn't it make sense to start using KRD for rebalancing? One of the ways that we work with our clients is by helping them create a custom liability index benchmark portfolio. We select the initial universe of assets and calculate all of the individual KRDs. Then, we come up with the mix of assets with the same KRD profile as your liability for some additional diversification. We call that process KRD optimization.

We use a dynamic process of reevaluating the KRD match as the yield curve changes and as time elapses. Most investment departments like dynamic processes and understand portfolios of real assets. They like to continually analyze a portfolio, trying to maximize the return by buying cheap securities, making sector bets, and testing other investment strategies. A custom index gives them a nice target that allows them to better meet the constraints of the liability and perform their job in terms of investment management. Certainly this framework helps in terms of establishing and evaluating asset/liability benchmarks.

Sometimes we need an investment proxy to model the liability crediting rate. The more you know about the liability product behavior, the better you can make that investment proxy. The investment department needs some sort of liability proxy so that its people can manage their money against that particular benchmark, and attribute any differences that are made between the actual and the proxy.

Option-adjusted spread (OAS) and required spread are unified for this consistent framework. OAS is the amount of additional discounting above Treasuries that is needed simultaneously for every path for a security being valued in a path dependent manner. For liabilities we go one step further and say that the required option-adjusted spread (ROAS) is what's needed for a break-even market value once we've included expenses. We can look at what happens to ROAS as the yield curve changes and time elapses.

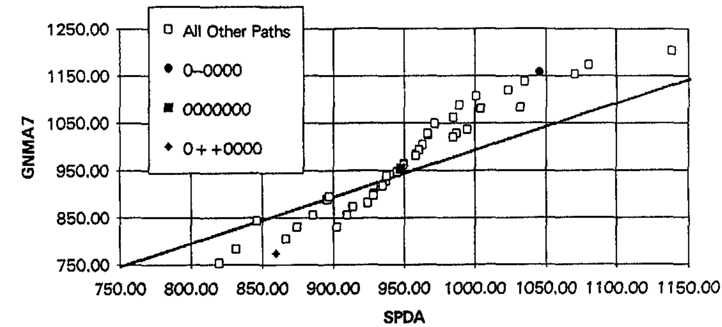
We implement ROAS at GAT using LPS, which is built on top of our arbitrage-free lattice framework. LPS starts with the binomial lattice, where successive nodes represent moving forward one month in time. Every point of the yield curve is determined by the term structure of volatilities and the initial interest rates as we go forward. The term structure of volatilities results in mean reversion. The problem with the binomial lattice is that there are just too many paths. The solution is systematically slicing up the interest rate space into different levels and the movements from level to level using a trinomial process that we call LPS.

The original construction that we use at GAT has trinomial gates at one-year, three-year, five-year, seven-year, ten-year, 20-year, and 30-year. For most long insurance portfolios this seems to be an effective structure for the interest rate space, but we're looking at variations shifting gate locations closer. By counting all the underlying binomial paths through for each representative trinomial path movement, we can assign a probability weight to that particular path. The probability weights are actually very important in our calibration process. We are currently doing more research and have just developed a two-factor LPS interest process that models short rate movements with long rate movements in a recombining lattice.

If you value path-dependent liability products and assets like MBSs or CMOs, you need to slice up interest rate space. We order possible paths through the lattice by probability, generate the cash flows and discount the cash flows to derive the pathwise market values along all scenario curves. A pathwise profile is basically a plot of asset market values along every single scenario versus the liability market values along the same scenario. A static hedge can be constructed by the method of pathwise dedication, an optimization simultaneously along many scenarios constraining cumulative cash-flow shortfall or surplus to some range. Pathwise immunization is good for dynamic hedging strategy. It constrains the market value difference between assets and liabilities to be within some threshold or tolerance. Tom Ho outlined these concepts in his 1991 working paper, "The Total Return Approach to Performance Measurement."

Chart 12 shows an example. The axes are the liability on the bottom and the mortgage backed assets along the vertical. We see profits when interest rates fall in the upper right-hand quadrant. We see losses when rates rise reflected by the diamonds in the lower left-hand corner. One strategy might be to hedge to get a duration match in your portfolio. The effect of this is a rotation to align more with the 45 degree line.

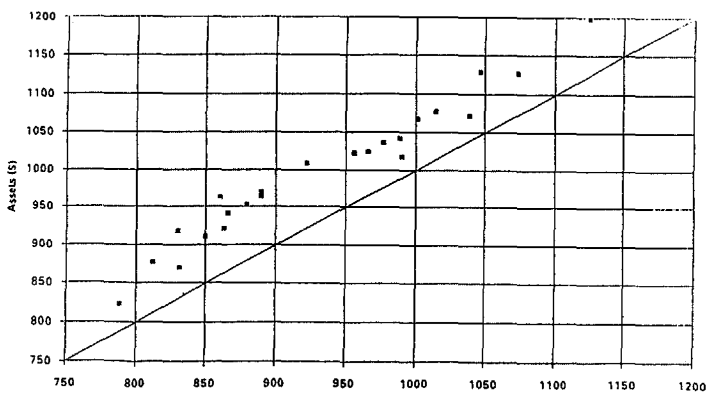
CHART 12
ASSET/LIABILITY PATHWISE VALUES: HIGH SURPLUS DURATION IMPLIES PROFIT WHEN RATES FALL, LOSS WHEN RATES RISE



probabilities 1.316 3.247 14.434 40.800 20.485 15.533 2.870 1.316

An example of pathwise dedication is shown in Chart 13. In this example, I'm using a constraint that the surplus has to be positive. In this case I had very few options, no corporate bonds, and no derivatives. The linear programming solution, in terms of the assets that it chose to meet the cash-flow requirement of no shortfall under every single scenario, was an asset mix at zero spread.

CHART 13
PATHWISE DEDICATION: WITHOUT DERIVATIVES, 0% REINVESTMENT



A pathwise immunization is shown in Chart 14. The constraint on pathwise values is that the market value of the assets/liabilities must be within a range. We were able to achieve a nice solution from the pathwise profile. What this method doesn't guarantee is no shortfall. In fact, for this example, we have excess cash flows right at the beginning and shortfalls later on. This risk can be eliminated by dynamically rebalancing in the future.

The last area I'd like to touch on is the impact of several theorems by Tom Ho and Michael Chen for what they call ABCD. They can be summarized by the following:

- Primary Decomposition: compare with option free bonds to understand the embedded options.
- Secondary Decomposition: determine the state dependent options for the best replicating portfolio.
- Tertiary Decomposition: analyze the residual to assess the impact of remaining path dependency.

The primary decomposition determines what embedded options are in your liability. To do that we limit the asset universe to a set of zero coupon bonds. We then match the liability cash flows exactly along the implied forward curve.

CHART 14
PATHWISE IMMUNIZATION—WITHOUT DERIVATIVES, 0% REINVESTMENT

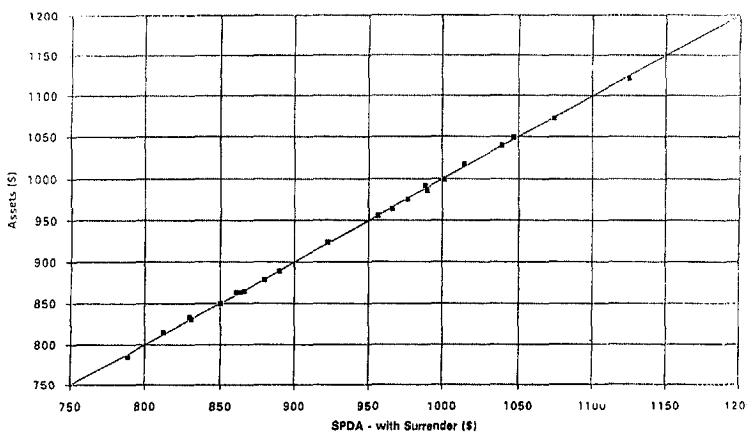


Chart 15 shows an SPDA versus a Treasury cash-flow match. You can see that we're actually quite mismatched from the 45 degree line for most of the scenarios. That shows us the embedded options and the mismatch between the asset and liability. If we had a pure floating rate asset as the liability, we would have a horizontal line relative to the asset portfolio because we would always have the same market value of par regardless of what happened with the interest rate scenarios. Here the SPDA looks very similar to a floating rate instrument.

The secondary decomposition is shown in Chart 16. We add state dependent options to the primary decomposition, run a regression, and match up the pathwise market values. We're now going to buy and sell caps and floors at various strikes and terms. The market values of the Treasury portfolio plus caps and floors is now represented on the x-axis; the vertical axis is the liability market values. You can see everything right along the 45 degree line, but there's a great deal of noise. After the secondary decomposition, all that is left are the path-dependent options relative to the liability. That noise is due to the policyholder behavior that includes the burnout and other things are path dependent. We could follow the same methodology to get an additional decomposition to include a universe of path dependent securities like index amortizing swaps and MBSs to reduce the error.

CHART 15
SPDA PWV VERSUS TREASURY PWV—256 PATHS

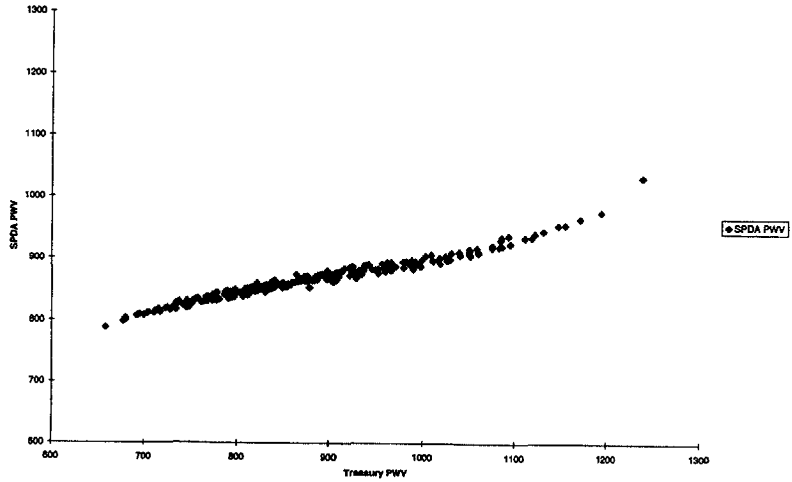
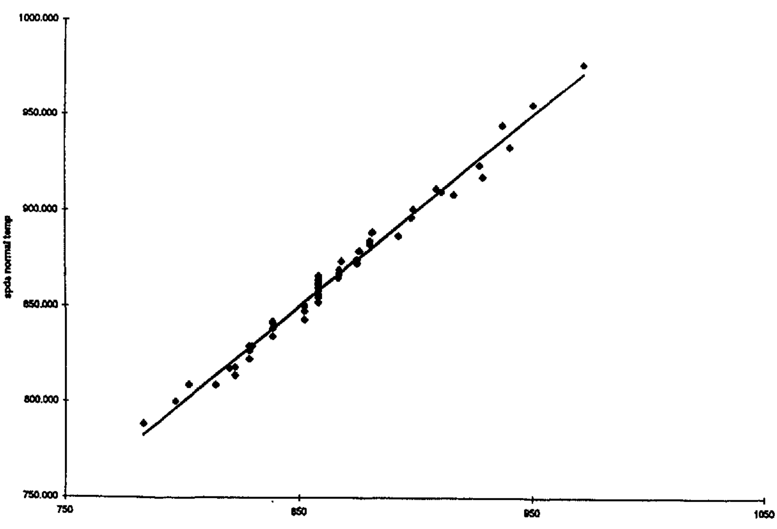


CHART 16
ABCD FOR SPDA WITH NORMAL TEMP



In conclusion, there is an increasing sophistication in techniques that are being used to do investment management. But there's much more work that has to be done to make them all easier to use. I think that the biggest challenge for us over the next couple years is making these methods accessible so people can evaluate the pros and cons of these various investment strategies.

MR. BOUDREAU: The notion of economic value of liabilities is just starting to become a hot topic. Much of what you have presented is predicated on a definition of economic value that everybody agrees to. I think that this is work under development, but we will probably see a great deal more of this in the future.

MR. ABBOTT: How many of you are actually using duration rebalancing techniques in your asset/liability management strategies? Is anybody using KRD?

DR. HANS-JOACHIM ZWEISLER*: I would like to know if you introduce any kind of rebalancing transaction costs in your models, and, if so, what is the effect?

MR. ABBOTT: There might be a number of other considerations. I was talking about just the price behavior in this case, but you could also have diversification constraints. If you are creating a benchmark, you need to think about transaction costs to minimize turnover. That might be something that you want to have as an objective for your investment department. The information from a static hedge put on at policy issue is very important towards minimizing that future turnover. The techniques that I've introduced here, KRD, hedging, portfolio optimization, and some of the other pathwise processes, can help to reduce the need for future transactions.

MR. LARRY J. BRUNING: When we started to evaluate some of the hedges of caps or floors, our conclusion was it got rather costly. There is some level of mismatch or risk that you are willing to accept, because you just can't make it all go away or else you wouldn't even be able to credit the guarantee. That is the struggle, and in some market situations, what might look like a good strategy you just really can't afford at all.

MR. ABBOTT: You might be able to hedge by investing in slightly riskier securities to get lower cost cash flows that will match. There are techniques that you can use. Or you can just assume some risk and accept that outside of certain ranges you are going to have some losses and just live with those. But you can put on partial hedges to accomplish your mission.

MR. MICHAEL J. STRECK: I have a question for Bryan. You showed various investment strategies ranging from a two-year duration to a five- or seven-year duration. My question is, Did you assume that the crediting rate in your product was constant and that there were competitive pressures that forced you to pay a certain return no matter what you invested in?

MR. BOUDREAU: Yes. What I did for that particular model was assume that we stay on the crediting rate strategy. What you do is credit a spread to the five-year Treasury, and always move 25% in the direction of new money. I think there's a strength in that approach in that you need to get a look at what kinds of asset strategies you may want to consider. On the other hand, that approach has a weakness because it doesn't consider the flexibility in rate crediting strategies. It treats the liability too rigidly, as if it were a fixed-income security.

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