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R CORNER: EXTREME GREY SWAN SCENARIOS–DMFBM

by Steve Craighead

In "The Black Swan," Taleb[1] makes a distinction between extreme scenarios that can be modeled (Mandelbrotian Grey Swans) and those that cannot (Black Swans). A Grey Swan model would consider the fractal nature of the markets that Mandelbrot first described in the 1960s.[2] Mandelbrot and Van Ness in Fractional Brownian Motions, Fractional Noises and Applications outlined how to model Fractional Brownian Motion (FBM).[3]

I recently located the paper Basic properties of the Multivariate Fractional Brownian Motion by Amblard, et al. I was pleasantly surprised that this article outlined how to create dependent multivariate fractional Brownian motion (DMFBM) scenarios. I have created the DMFBM R function that implements their procedure.The rest of this article will outline how to use the DMFBM function and the next article will discuss how the function works and the assumptions and limitations of it.

The DMFBM function and the two associated functions buildportfolio and HurstExp require the use of three packages and their supporting packages. These packages are tseries, dvfBm, and fBasics. When you use the package installer to install these three packages, make sure that the install dependencies switch is set so that all of the supporting packages are included.

The DMFBM function is available here. The buildportfolio function, described in the last newsletter, is here and it is used to build a historical portfolio of assets indicated by their ticker symbols. The

HurstExp function is here and is used to determine the Hurst Exponent for each asset index within a portfolio. Open each of these in a browser and copy and paste the contents into your R workspace.

The DMFBM function requires the input as described in Table 1.

Table 1 : DMFBM Calling Parameters

Parameter Name	Description
HurstList	A list of Hurst exponents, which are determined from each asset in the price return portfolio.
covar	This is the covariance matrix of the price return portfolio.
correl	This is the correlation matrix of the price return portfolio.
projperiod	This is the projection period associated with the scenarios. This defaults to 120 months.
numscen	This is the number of scenarios generated.
delta	This variable allows the change of modality. For example, if the price return portfolio has daily frequency, using delta = 21, the scenarios will be approximately monthly (there are approximately 21 working days a month).
GenKernel	Generation kernel. The default value is NULL. If supplied DMFBM will not regenerate the kernel and will proceed directly to scenario generation.

The output of DMFBM is described in Table 2:

Parameter Name	Description
cusumX	The cumulative sum of the innovations, which corresponds to hydrological applications. The dimension is (numscen)(asset count) (projperiod+1).
cuprodX	The cumulative product of 1+innovations, which corresponds to return indices. Its dimension is (numscen)(asset count)(projperiod+1).

DelX	The individual innovations from the projection. Its dimension is (numscen)(asset count)(projperiod+1).
GenKernel	Generation Kernel used in the projection process.

As outlined in the prior newsletter, the sample historical equity portfolio is built by these commands:

```
(library(tseries))
(ticker<-c("^DJI","^GSPC","^FTSE","^GDAXI","^N225","^HSI"))
(startdate <- "1990-12-03")
(enddate <- "2010-08-02")
(freq<-1)
test<-buildportfolio(ticker,startdate,enddate,freq)
```

If you pasted the entire contents of the buildportfolio html, the commands above should have already been processed in the R workspace.

The test portfolio contains two elements, which are test$PricePortfolio and test$PRPortfolio. Our interest is in the price return component test$PRPortfolio. The next required step is to determine the covariance and correlation matrices of the price return portfolio and estimate the Hurst Exponents for each equity index. This is done with these commands:

```
(covar <-var(test$PRPortfolio))
(correl<-cor(test$PRPortfolio))
(HurstList<-HurstExp(test$PRPortfolio)$HurstList)
```

Now, since all of the components for DMFBM are now available, a small set of scenarios are generated using this command:

```
a<-DMFBM(HurstList,covar,correl,projperiod=12,numscen=10)
```

The DelX or the cuprodX components will be key in using the scenarios for models. If a simulation model requires price indices (or RBC-C3- phase II wealth factors) use a$cuprodX, but if price returns or total returns at a point in time are required, use a$DelX.

As will be discussed in the next newsletter, the generation kernel a$GenKernel uses the greatest amount of process time. The process time increases as the generation period increases. Because of this, it is wise to first generate a small set of scenarios and reuse the generation kernel to generate a larger scenario set. For instance, the following command will generate 10,000 scenarios, by reusing

a$GenKernel:

big<-DMFBM(HurstList, covar, correl, projperiod=12, numscen=10000, delta=1, GenKernel=a$GenKernel)

From the six price indices over 10,000 scenarios, a collection of box plots through time, also known as a funnel of double, for each asset in the portfolio is displayed using these two commands:

```
par(mfrow=c(3,2))
for(i in 1:6) boxplot(big$cuprodX[,i,],main=ticker[i])
```


Figure 1: Funnel of Doubt
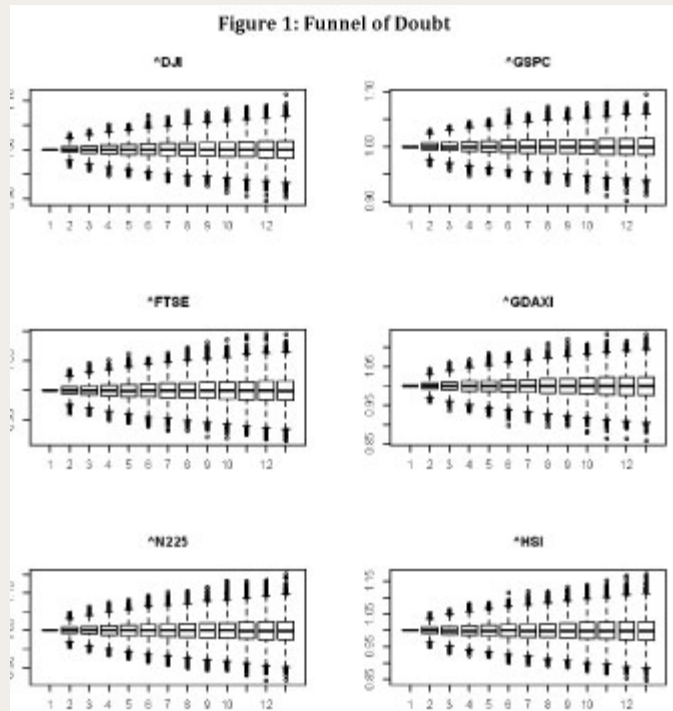
Don't forget to use the summary command to display actual statistics on the entire set of scenarios for a specific index through time. For example,

```
summary(big$cuprodX[,1,])
```

displays various sample statistics on the Dow Jones scenarios (results not shown).

Using the same par(mfrow=c(3,2)) configuration as before, to display scenario 5 as in Figure 2, use the command:

```
for(i in 1:6) plot(big$cuprodX[5,i,],type="l",main=ticker[i])
```

Figure 2: Scenario 5
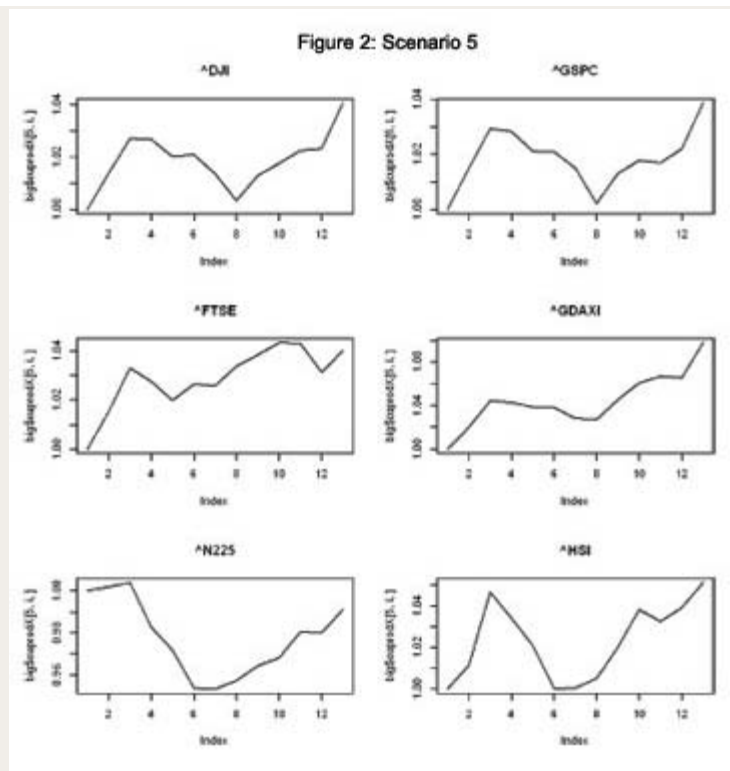
On a personal note, in the early 1990s, I created a collection of independent Grey Swan scenarios using FBM and determined what the reserve requirements were for GMDB riders on a block of variable annuities. I found that the reserves calculated with these scenarios were three to five times larger than when using lognormal scenarios. At that time, I was frustrated with having to create single variable independent scenarios, because there was no easy technique to model multivariate and dependence into the modeling as was for regular Brownian motion. Even though these results were considered too extreme and were disregarded, it caused us to realize that our reserves were woefully inadequate if certain major institutions failed, such as the U. S. government and Wall Street.

This article has introduced a method to create dependent multivariate fractional Brownian motion scenarios. However, to fully implement Grey Swan scenario generation an additional feature is required. This feature is called MMAR, which is short for Multifractal Model of Asset Returns. MMAR adds the capability to DMFBM to have a stochastic trading time component. However, in the market, asset returns reflect various trading ranges. There are times when the market is slow and the volatility is insignificant. At the other extreme, when the market is fast, the volatility changes frequently and it becomes difficult for market observers to keep up with what is happening. Also, during an extremely fast market, the market resets, correlations tighten and prices are discontinuous.

An MMAR model is a composition of an FBM BH(t) or a DMFBM

BH1,...Hn(t)) with a cumulative distribution function ?(t) of a multiracial measure. The MMAR resulting process is X(t) = BH(?(t)). The function ?(t) supplies a random trading time to the MMAR that models both slow and fast markets and the corresponding volatility.

In the next article, I will outline what MMAR is and how it is constructed and what it can do. I will describe what ?(t) actually is and how to compose it with Brownian motion and DMFBM models.

[1]Nassim Taleb (2010), "The Black Swan, the Impact of the Highly Improbable, 2nd Edition," Random House Trade Paperbacks, New York, NY.

[2]Benoit Mandelbrot (1997), "Fractals and Scaling in Finance, Discontinuity, Concentration, Risk," Springer-Verlag, New York, NY.

[3]The variance (var{B(t) = t} of a Brownian motion becomes infinity as time t goes to infinity. The variance (var(FBM(t) = t^(2*H)) of an FBM also goes to infinity as time t increases. The constant H is called the Hurst Exponent and it takes on values between 0 and 1. When H = 1/2 FBM corresponds to Brownian motion. If 0 < H < 1/2, FBM is called anti-persistent and if 1/2 < H < 1, FBM is called persistent. Persistent means that if an FBM moves upward, the next move will tend to be upward, or if it moves downward, the next move is downward. If H < 1/2, the FBM tended to be noisier and more jagged as H gets closer to 0. If H > 1/2 the FBM becomes smoother as H approaches 1. All FBM paths are continuous but not differential. See my paper Chaotic Analysis for an in-depth discussion around the properties of the Hurst Exponent and other chaos topics.