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NONLINEAR INTERPOLATION WITH EXCEL TO CONSTRUCT U.S. TREASURY BOND YIELD CURVE

by Andrew Chan

Excel offers many powerful mathematical and statistical functions that allow us to solve numerous business problems. One of the typical challenges is that we don't have all the information we need! Using U.S. Treasury bonds as an example; it only has yield rate for certain maturities, e.g., one year, two years, three years, five years. If we want yield rate for a four-years maturity bond, then we may have to find a mathematician/statistician to interpolate the value for us. Fortunately, we can be the mathematician/statistician if we have Microsoft Excel. Excel has a few tools that we can use to interpolate values; and we are going to demonstrate a few of them in this article.

Trendline

I went to the U.S. Treasury website and copied the following yield rates to my Excel Worksheet.

US Treasury bonds

Year	Yield %
1	0.28%
2	0.56%
3	0.85%
5	1.64%
7	2.35%
10	2.99%
20	3.82%
30	4.06%

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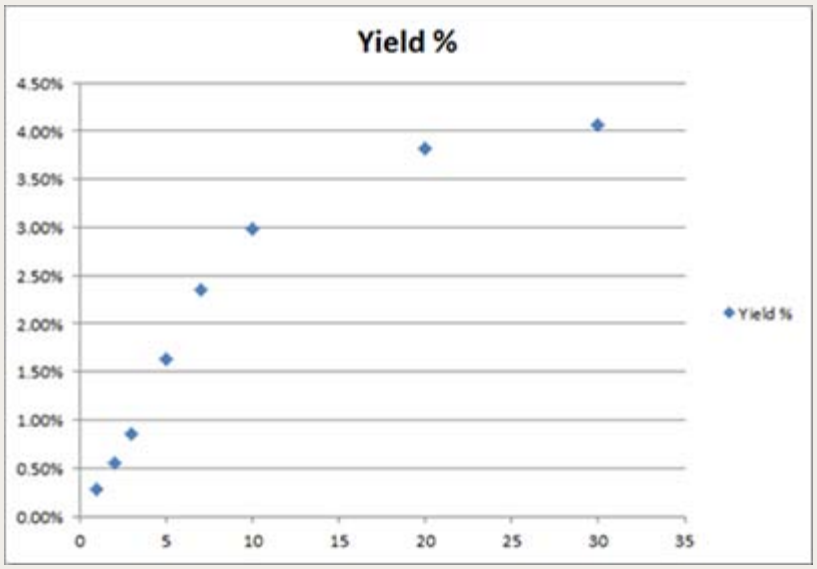
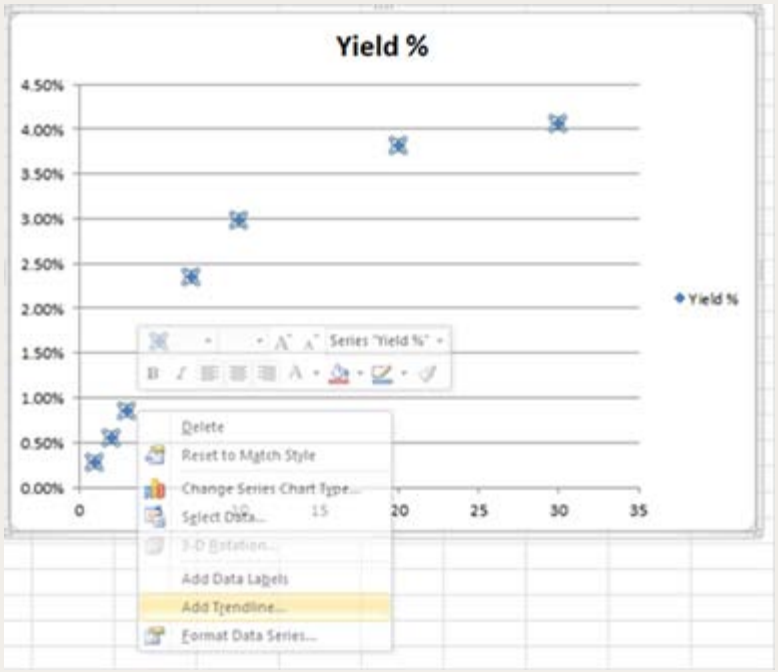
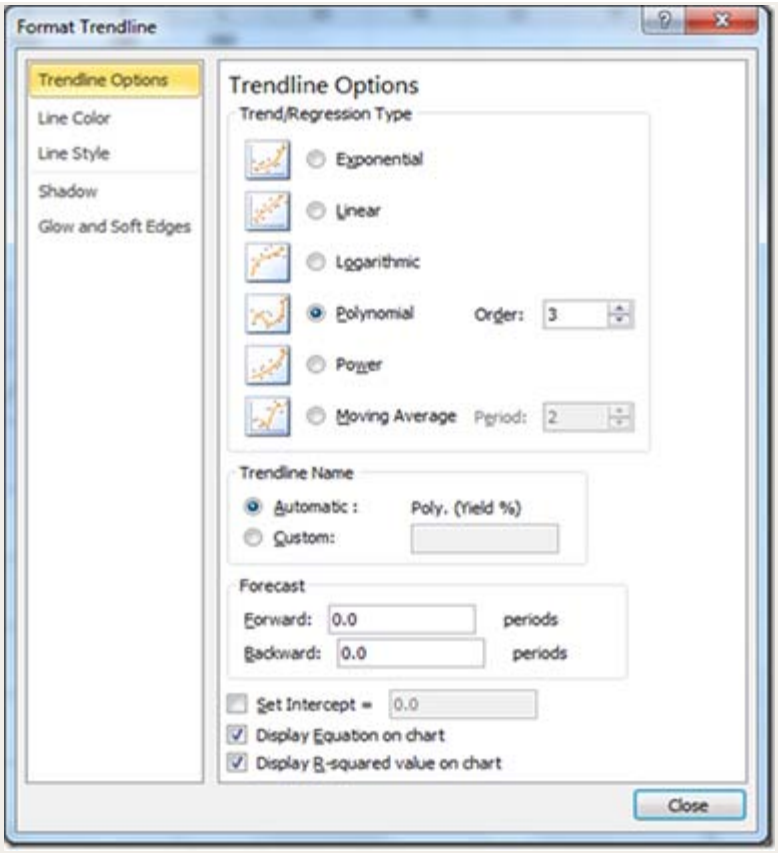
Once we have the data, we can create a trendline. The first step is to create a chart (graph) based on the yield rate.



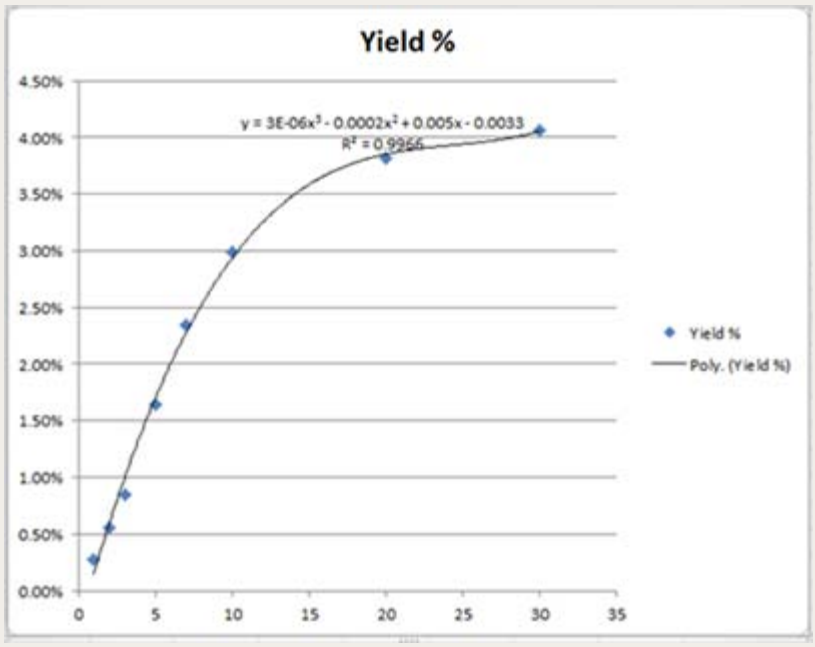
It is very simple to add a Trendline: just right-click the data series in the chart. From the pop-up menu, select **Add Trendline**.



The **Format Trendline** dialogue will be displayed (see below). For this example, we will choose Polynomial with Order 3 as the trend type and we also select to **Display Equation** on chart and **Display R-squared value on chart**.



After we click the Close button, we would see a trendline is added to the chart with the equation and R^2.



Worksheet Functions

Excel provides many functions to project values:

- **FORECAST**
- **TREND**

- **GROWTH**
- **LINEST**
- **LOGEST**

We are going to use **LINEST** in this demonstration, which generates statistics for a "least squares" linear regression (for complete documentation on the function parameters, see Microsoft's website http://office.microsoft.com/en-us/excel-help/linest-HP005209155.aspx?CTT=1). Since the yield curve is a third order polynomial function, we have four variables.

*Yield = a1 * Years3 + a2 * Years2 + a3 * Years + a4*

Enter the following formula in our Excel worksheet to identify the coefficients a1 through a4 in the formula above:

=INDEX(LINEST(Yield, Years^{1,2,3}),1,1)
=INDEX(LINEST(Yield, Years^{1,2,3}),1,2)
=INDEX(LINEST(Yield, Years^{1,2,3}),1,3)
=INDEX(LINEST(Yield, Years^{1,2,3}),1,4)

Yield and **Years** are the defined range names containing the Y and X points. We can see the coefficients a1, a2, a3 a4 and even R2 are very close to the results from **Trendline**.

a1	2.70683E-06
a2	-0.000198231
a3	0.004970323
a4	-0.003253473
R2	0.996593232

The formula for R2 is: **=INDEX(LINEST(Yield, Years^{1,2,3},TRUE,TRUE),3,1)** Note the additional parameters in the **LINEST** function—the last one (TRUE) tells the function to return additional regression statistics.

Now we have all coefficients, we can interpolate the yield rates that we need.

Years	US Treasury Bond Yield Rate	Projected Yield Rae
1	0.28%	0.15%
2	0.56%	0.59%
3	0.85%	0.99%
4		1.36%
5	1.64%	1.70%
6		2.00%
7	2.35%	2.28%
8		2.52%
9		2.74%
10	2.99%	2.93%
11		3.10%
12		3.25%
13		3.38%
14		3.49%
15		3.58%
16		3.66%
17		3.73%
18		3.78%
19		3.82%
20	3.82%	3.85%
21		3.88%
21		3.88%
23		3.91%
24		3.93%
25		3.94%
26		3.95%
27		3.97%
28		3.99%
29		4.02%
30	4.06%	4.05%

Solver

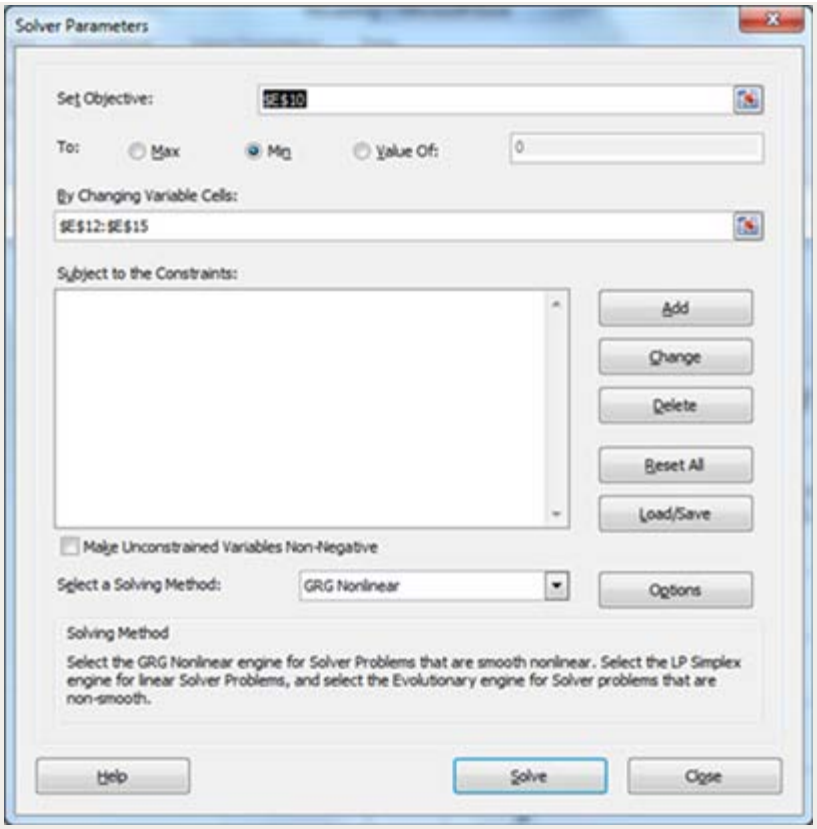
If we want to use some custom functions, e.g., the Nelson-Siegel function to interpolate the yield value, "Solver" is a good tool for the job. The Nelson—Siegel function is a modified Exponential function:

*Yield = A1 + (A2+A3) * (Beta / Years) * (1-e-Years/Beta) - A3 * e-Years/Beta*

	A	B	C	D	E
1		Years	Yield %	Nelson-Siegel Projected Values	Squared Residual
2		1	0.28%	9.48%	0.008467328
3		2	0.56%	8.65%	0.006539386
4		3	0.85%	7.92%	0.004996286
5		5	1.64%	6.95%	0.002822621
6		7	2.35%	6.42%	0.001658696
7		10	2.99%	6.00%	0.000905846
8		20	3.82%	5.50%	0.00028224
9		30	4.06%	5.33%	0.000162138
10					0.025834541
11					
12				Alpha1	0.05
13				Alpah2	0.05
14				Alpha3	0.05
15				Beta	1

Column D shows the projected values that are based on Nelson-Siegel function (using arbitrary initial parameters) and Column E is the Squared Residual value between the projected and actual values. We can use "Solver" to minimize the sum of all Squared Residual values (cell E10) by adjusting the function parameters (cells E12 through E15).

"Solver" is accessed from the "Data" ribbon bar in the "analysis" section at the far right. Solver setup is quite straight forward: we specify the **Objective** (and whether to solve for Minimum, Maximum, or a specific value), and **Variable Cells** (the parameters that will be changed). Also choose the option shown below for **Select a Solving Method**. [*Editor's Note: Excel 2007 "out of the box" does not have this option in the dialog. See the "Addendum: Advanced Solve Functionality Setup in Excel 2007*]. When everything is ready, we click the **Solve** button.



The variable cells have been changed to their optimal values.

	A	B	C	D	E
1		Years	Yield %	Nelson-Siegel Projected Values	Squared Residual
2		1	0.28%	0.30%	3.3678E-08
3		2	0.56%	0.50%	3.34644E-07
4		3	0.85%	0.88%	1.0457E-07
5		5	1.64%	1.69%	2.34899E-07
6		7	2.35%	2.33%	5.48605E-08
7		10	2.99%	2.96%	1.017E-07
8		20	3.82%	3.80%	2.88858E-08
9		30	4.06%	4.09%	9.58936E-08
10					9.8913E-07
11					
12				Alpha1	0.046669726
13				Alpah2	-0.040938811
14				Alpha3	-0.063510791
15				Beta	1.654403566

Again, we can now use the coefficients to interpolate the yield rate.

The R2 is 0.9993 vs. 0.9966 from **LINEST**.

Years	US Treasury Bond Yield Rate	Nelson-Siegel Projected Values
1	0.28%	0.30%
2	0.56%	0.50%
3	0.85%	0.88%
4		1.30%
5	1.64%	1.69%
6		2.03%
7	2.35%	2.33%
8		2.57%
9		2.78%
10	2.99%	2.96%
11		3.11%
12		3.23%
13		3.34%
14		3.43%
15		3.52%
16		3.59%
17		3.65%
18		3.71%
19		3.76%
20	3.82%	3.80%
21		3.84%
21		3.84%
23		3.92%
24		3.95%
25		3.98%
26		4.00%
27		4.03%
28		4.05%
29		4.07%
30	4.06%	4.09%

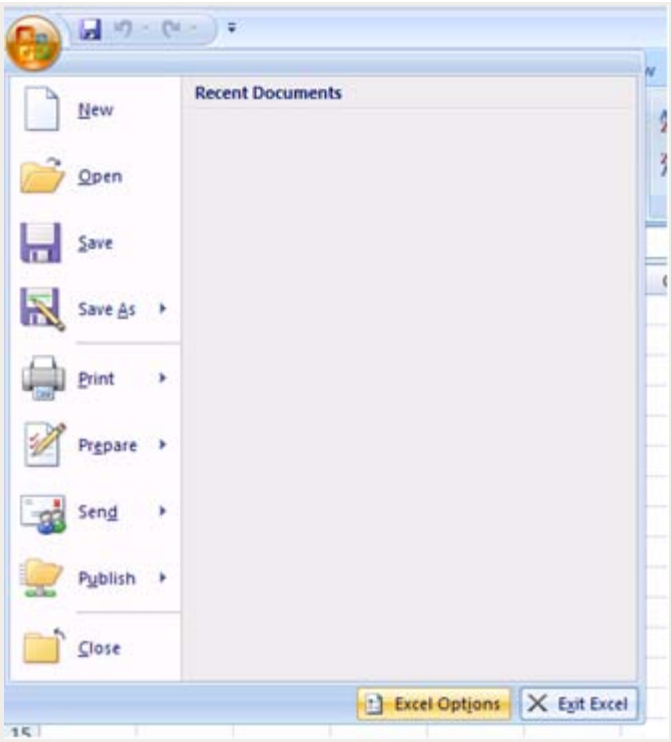
Others

There are other Excel tools that we can use to project/interpolate values, such as Analysis ToolPak or Microsoft Solver Foundation. I will discuss these in future articles.

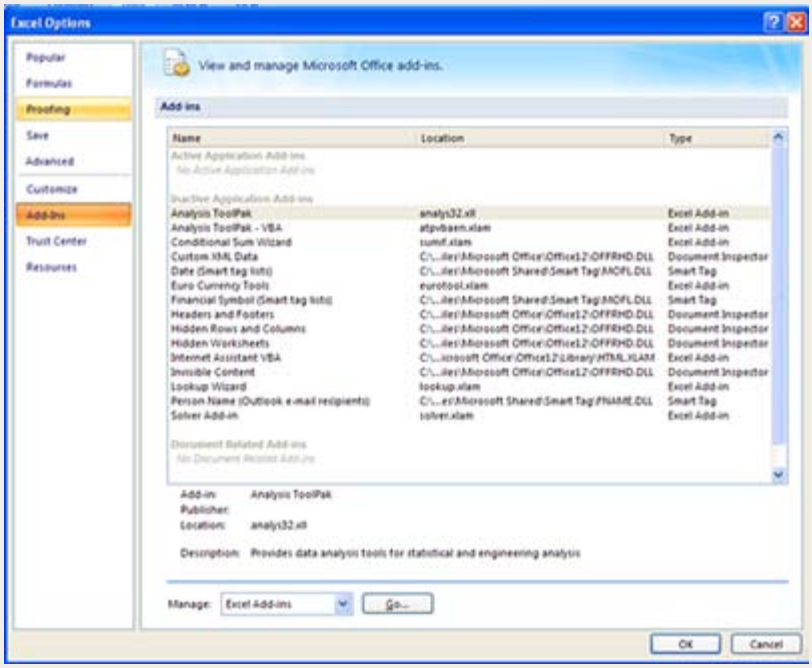
Addendum: Advanced Solve Functionality Setup in Excel 2007

Advanced solve functionality is available in Excel, it just needs to be enabled through an Add-In. It is very easy to enable it in Excel 2007:

Click Excel Options



Select Add-ins and click Go button



Select Solver Add-in and click OK button.

You can start using Solver!

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Actuaries
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