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Calculation of Tax Expense in a Principles-Based Reserves Environment

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It appears that one of the major questions of the Principles Based Reserves (PBR) era is, “If it is accepted that reserves should be pre-tax, why should required capital be post-tax?” This article attempts to resolve that issue.

First, let’s examine why statutory reserves should be pre-tax, and why that issue is related to the need to fully admit deferred tax assets (DTAs) and liabilities (DTLs) on the statutory balance sheet to accomplish “system integrity.”¹ Assume a 35 percent tax rate.

Definitions

- $R(s)_z$ Statutory reserve, end of year z, pre-tax.
- $R(t)_z$ Tax basis reserve, end of year z.
- ACF_z Actual pre-tax cash flows during year z, *i.e.*, premiums, benefits, expenses, excluding interest earned (i) on prior year reserve
- ECF_z Expected pre-tax cash flows, year z, excluding interest earned on prior year reserve.

The DTA with respect to this item is thus equal to $(.35)*[R(s)_z - R(t)_z]$ at end of year z, assuming full admissibility.

Statutory after-tax book profit (SP_z) can then be seen, given a fully taxable organization as:

Formula (2) can be intuitively described as 65 percent of the pre-tax emerging margin. Thus, for example if $ACF = (-) 40$ and $ECF = (-) 60$, then the post-tax emerging margin equals 13 (*i.e.*, 65 percent of 20).

This is a logical result and demonstrates that a pre-tax reserve provides an appropriate reserve for both pre-tax cash flows and taxes when there exists a fully admissible deferred tax asset. Please see the October, 2006 edition of the *Actuarial Practice Forum*² (<http://www.soa.org/library/journals/actuarial-practice-forum/2006/october/october-2006-detail.aspx>) for a more expansive paper on this issue, and the illogical result if the statutory reserve were to be calculated in a post-tax manner.

The reserve is exactly adequate if the present value of statutory book profits post-tax [PV(SP)] is zero.

From Formula (1) it can be seen that PV(SP) equals 65 percent of the algebraic sum of (i) and (ii), as follows:

- (i) Present value of actual cash flows (PV(ACF))
- (ii) $\sum (v^t)*(R(s)_{t+1}*(1+.65*i) - R(s)_t) = R(s)_0$, where $v = 1/(1+.65*i)$. Since $R(s)_0$ is pre-tax, then this reflects the pre-tax nature of the proper statutory reserve.

$$\text{Thus PV(SP)} = .65*[PV(ACF) + R(s)_0]. \quad (3)$$

Now let’s move to the Total Asset Requirement (TAR) issue, from which Required Capital (RC) is derived. Under “Principles-Based” concepts, $RC_z = TAR_z - R(s)_z$. Current authoritative guidance is such that RC is nondeductible under tax law but does not contribute to the company’s DTA.

$$\text{Define distributable earnings (DE}_z\text{):} = SP_z + RC_{z-1}*(1+.65*i) - RC_z. \quad (4)$$

As shown in the above calculation of SP_z , Formula (4) can be intuitively described as post-tax statutory book profit (SP_z) plus the releases of required capital, plus post-tax

Formula

$$SP_z = .65*ACF_z - [R(s)_z - R(s)_{z-1}] + .35 * [R(t)_z - R(t)_{z-1}] + .35*[R(s)_z - R(t)_z] - .35 * [R(s)_{z-1} - R(t)_{z-1}] + .65*(i)*R(s)_{z-1}$$

Explanation

(Actual cash flows net of taxes thereon) (Minus increase in reserves net of taxes thereon) (Plus increase in DTAs) (Plus post-tax interest earned on prior reserve)

$$\text{Thus } SP_z = (.65)*ACF_z - (.65)*[R(s)_z - R(s)_{z-1}] + .65*i*R(s)_{z-1} \quad (1)$$

But $R(s)_z - R(s)_{z-1} + i*R(s)_{z-1} =$ The dollar-for-dollar offset to ECF_z . (Thus, for example, this number is positive if ECF is negative.) Thus $[R(s)_{z-1} - R(s)_z] = -ECF_z - i*R(s)_{z-1}$.

$$\text{Substituting into formula (1), } SP_z = (.65)*ACF_z + (.65)*[-ECF_z - i*R(s)_{z-1}] + .65*i*R(s)_{z-1} = (.65)*[ACF_z - ECF_z] \quad (2)$$

¹ The one exception to full admissibility would be to make allowance for a significant possibility that the entity will not realize the tax benefit of the deferred tax asset. GAAP refers to the consequent reduction of a DTA as a “valuation allowance.”

² Society of Actuaries.

interest on the prior year end required capital. The tax cash flows are fully considered in SP_z , so that Formula (4) is a true post-tax economic reflection of distributable earnings, since tax cash flows should be considered in distributable earnings and in the present value of distributable earnings [PV(DE)].

$$PV(DE) = PV(SP) \text{ plus } \sum (v^t) * (RC_{z-1} * (1 + .65 * i) - RC_z) \quad (5)$$

$$= PV(SP) + RC_0.$$

Put differently, given Formulas (3) and (5),

$$PV(DE) = .65 * [PV(ACF) + R(s)_0] + RC_0. \quad (6)$$

If the ACF_z values were to include adverse scenarios, such that a positive value of RC_z is necessary to generate a zero value of PV(DE) for a particular adverse post-tax result, then Formula (6) is indicative of the fact that Required Capital needs to be calculated on a post-tax basis.

Interestingly, if RC did fully contribute to the DTA (which it does not), then DE_z would equal:

$$DE_z = SP_z + RC_{z-1} * (1 + .65 * i) - RC_z + .35 * (RC_z - RC_{z-1})$$

$$= SP_z + .65 * (RC_{z-1} - RC_z) + RC_{z-1} * .65 * i \quad (7)$$

Putting formula (1) into formula (7) this becomes:

$$= (.65) * ACF_z - (.65) * [R(s)_z - R(s)_{z-1}] + .65 * i * R(s)_{z-1} - (.65) * [RC_z - RC_{z-1}] + .65 * i * RC_{z-1}$$

$$= .65 * ACF_z - .65 * [(R(s)_z + RC_z) - (R(s)_{z-1} + RC_{z-1})] + .65 * i * (R(s)_{z-1} + RC_{z-1}) \quad (8)$$

It is obvious in this hypothetical case that formula (8) is analogous to formula (1), simply by substituting TAR (*i.e.*, $R(s) + RC$) for the statutory reserve ($R(s)$) alone. Thus, following the logic of formula (2), DE_z would equal 65 percent of the margin involved comparing the TAR to the actual pre-tax cash flows.

It can be seen from the above that, since ACF is a pre-tax number, if the change in $[R(s) + RC]$ in a given financial

period is exactly sufficient to provide for pre-tax excess claims in that period (*i.e.*, for distributable earnings to equal zero), then both $R(s)$ and RC would be pre-tax numbers in this hypothetical scenario.

Thus, we now have the linkage between statutory reserving and required capital, and the reason that the former is pre-tax while the latter is post-tax, that is:

- Reserves are partly deductible, and any excess of statutory reserves over tax-basis reserves is to be taken up by the DTA.
- Required Capital is non-deductible, but does not contribute to the DTA.

Unfortunately, the current authoritative regulatory guidance does not admit the entire gross DTA; rather, as an element of statutory conservatism, it generally only admits that small portion related to temporary differences that reverse over the next 12 months from the statement date. Thus the above analysis does not reflect current regulatory constraints; what constitutes a prescribed “valuation allowance” exists equal to the non-admitted portion of the DTA. This prescribed “valuation allowance” can be far larger than true economics would dictate, since temporary differences on reserves typically take many years to reverse. PBR will be imperfect if this issue is not addressed by the NAIC in a more effective manner.

Some simplified numerical illustrations follow, assuming a 35 percent marginal tax rate and a statutory reserve that represents the present value of pre-tax negative cash flows. Those simplifications include:

- The existence of the DTA doesn’t change the level of invested assets. That is, the company is not invested any less heavily because of the existence of this asset. This is mathematically consistent with reality, in that under current regulatory guidance DTAs and DTLs are not discounted at interest.
- All the negative cash flows in the table occur at the end of the year.
- Tax DAC (pursuant to Code Section 848) is ignored.

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Table 1 illustrates the fact that, if actual negative cash flows are at 90 percent of the reserve expectations, the post-tax statutory book profit will be 65 percent of the resulting emerging margin (*i.e.*, 65 percent of the 10 percent difference between actual and expected negative cash flows). Thus the table is a demonstration that the pre-tax approach to statutory reserve calculation is appropriate.

Table 1

Pattern of Emergence of Post-Tax Statutory Book Profit

Inv.Inc.Rate	5.00%
Discount Rate	0.952381
Tax/Stat Ratio	85.0%
Act/Expected CIs	90.0%

Can vary. Doesn't change final column below.

Year	Expected	Res, Beg.of Year		DTA Beg.of Yr.	Actual C.Flows	Inv.Inc. on Stat. Res	Stat Book Profit	Ratio
	Pre-Tax Cash Flows*	Statutory Reserve	Tax Reserve					
1	(100.00)	523.96	445.37	27.51	(90.00)	26.20	6.50	65.0%
2	(90.00)	450.16	382.64	23.63	(81.00)	22.51	5.85	65.0%
3	(81.00)	382.67	325.27	20.09	(72.90)	19.13	5.26	65.0%
4	(72.90)	320.80	272.68	16.84	(65.61)	16.04	4.74	65.0%
5	(65.61)	263.94	224.35	13.86	(59.05)	13.20	4.26	65.0%
6	(59.05)	211.53	179.80	11.11	(53.14)	10.58	3.84	65.0%
7	(53.14)	163.05	138.60	8.56	(47.83)	8.15	3.45	65.0%
8	(47.83)	118.06	100.35	6.20	(43.05)	5.90	3.11	65.0%
9	(43.05)	76.14	64.72	4.00	(38.74)	3.81	2.80	65.0%
10	(38.74)	36.90	31.36	1.94	(34.87)	1.84	2.52	65.0%

* Excluding investment income

Legend

Expected Pre-Tax Cash Flows (CF _t):	Given
Statutory Reserve, beg. of Yr (SR _t):	(SR _{t+1} - ECF _t)/(1.05)
Tax Reserve beg. of yr (TR _t):	(.85)*(SR _t)
DTA	(.35)*(SR _t - TR _t)
Actual Cash Flows (ACF _t)	(.90)*(ECF _t)
Investment Income on (SR _t):	=.05*(SR _t)
Statutory Book Profit post-tax (SP _t):	65*(ACF _t) +(SR _t - SR _{t+1}) +(.35)*(TR _{t+1} - TR _t) + DTA _{t+1} - DTA _t + (.65)*(Inv.Inc.on SR _t)
Ratio	(Stat.Bk Profit)/(10% of expected Pre-tax Cash Flow)

Table 2 takes the same assumptions, except that it assumes an extremely adverse set of values for actual negative cash flows and calculates Required Capital on an after-tax (AFIT) basis. It shows that the Required Capital calculation should be AFIT in order to arrive at a zero value of distributable earnings.

Table 2

Required Capital is Exactly Sufficient to Cover Costs Not Provided for in Reserves

Inv.Inc.Rate	5.00%
Discount Rate	0.952381
Tax/Stat Ratio	85.0%
Act/Expected Cls	150.0%

Can vary. Doesn't change final column below.

Year	Expected	Res. Beg. of Year			DTA	Actual	Inv.Inc. on	Stat Book	Ratio	Required	Release	Post-Tax	DE
	Pre-Tax	Cash	Statutory	Tax									
1	(100.00)	523.96	445.37	27.51	(150.00)	26.20	(32.50)	65.0%	183.17	26.55	5.95	0	
2	(90.00)	450.16	382.64	23.63	(135.00)	22.51	(29.25)	65.0%	156.62	24.16	5.09	0	
3	(81.00)	382.67	325.27	20.09	(121.50)	19.13	(26.33)	65.0%	132.46	22.02	4.31	0	
4	(72.90)	320.80	272.68	16.84	(109.35)	16.04	(23.69)	65.0%	110.44	20.10	3.59	0	
5	(65.61)	263.94	224.35	13.86	(98.42)	13.20	(21.32)	65.0%	90.34	18.39	2.94	0	
6	(59.05)	211.53	179.80	11.11	(88.57)	10.58	(19.19)	65.0%	71.95	16.85	2.34	0	
7	(53.14)	163.05	138.60	8.56	(79.72)	8.15	(17.27)	65.0%	55.10	15.48	1.79	0	
8	(47.83)	118.06	100.35	6.20	(71.74)	5.90	(15.54)	65.0%	39.62	14.26	1.29	0	
9	(43.05)	76.14	64.72	4.00	(64.57)	3.81	(13.99)	65.0%	25.36	13.17	0.82	0	
10	(38.74)	36.90	31.36	1.94	(58.11)	1.84	(12.59)	65.0%	12.19	12.19	0.40	0	

Legend (continuing from Table 1 legend)

Required Capital Beg. of Yr (RC_t): $(RC_{t+1} - SP_t) / (1 + .65 * .05)$
 Release of RC_t: $RC_t - RC_{t+1}$
 Post-Tax Inv.Inc. on RC_t: $(.65) * (.05) * RC_t$
 Distributable Earnings (DE): $SP_t + (\text{Release of } RC_t) + (\text{Post-Tax Inv.Inc. on } RC_t)$

For those interested, there remains the following question: "What if the Required Capital value were to generate a deferred tax asset?" The answer would be that Required Capital would simply be in the nature of a non-deductible reserve. Table 3 illustrates this hypothetical situation. Note that Required Capital would then be calculated pre-tax (BFIT) once a deferred tax asset is added to the calculations, in order for distributable earnings to be zero. [We register no opinion as to whether a deferred tax asset on Required Capital would be appropriate accounting.]

Table 3

Hypothetical Result, if Required Capital Generated a DTA

Year	Additional Pre-Tax CF's	Additional Required Capital	Additional Tax Reserve	Additional DTA	Pre-Tax Inv.Inc. on Req.Cap	Additional Distributable Earnings
1	(50.00)	261.98	0	91.69	13.10	0
2	(45.00)	225.08	0	78.78	11.25	0
3	(40.50)	191.33	0	66.97	9.57	0
4	(36.45)	160.40	0	56.14	8.02	0
5	(32.81)	131.97	0	46.19	6.60	0
6	(29.52)	105.76	0	37.02	5.29	0
7	(26.57)	81.53	0	28.53	4.08	0
8	(23.91)	59.03	0	20.66	2.95	0
9	(21.52)	38.07	0	13.32	1.90	0
10	(19.37)	18.45	0	6.46	0.92	0

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Legend

Additional Pre-Tax Cash Flows (AdCF) _t :	Actual minus Expected Cash Flows (Pre-Tax)
Required Capital (RC) _t :	$(RC_{t+1} + (AdCF)_t)/1.05$
Additional Tax Reserve:	None, as RC is nondeductible.
Additional DTA (AdDTA) _t :	35% of (RC) _t
Investment income on ((ADTA) _t):	$.05*(RC)_t$
Distributable Earnings:	$(.65)*(AdCF)_t + (RC_t - RC_{t+1}) + AdDTA_{t+1} - AdDTA_t + (.65)*(Inv. Inc. on RC)_t$

Note that the RC_t value would then be a pre-tax calculation. Yet the resulting zero Distributable Earnings value would fully allow for all cash flows including taxes.

Table 4 is a reconciliation of the initial required capital shown in Table 2. It shows the derivation of the negative present value of distributable earnings if there was no required capital, thus the initial capital need (*i.e.*, the Table 2, year 1 Required Capital amount of \$183.17). One can see that the future “current tax” expense as well as the future “deferred tax” expense are included in the calculation.

Table 4

Reconciliation of Table 2 Required Capital for Year 1			
PV(Total CF's)AFIT	\$549.51	(-)	See Present Value Schedule
Beginning Stat Res (assumes Cash flows BFIT)	523.96	(+)	See Table 1.
PV(Taxes on tax reserve releases)	133.98	(-)	See Present Value Schedule
PV(DTA releases)	\$23.64	(-)	See Present Value Schedule
Net Capital Need	(183.17)		

Present Value Schedule			
Year	Rel TR	Cash Fl*.65	Rel DTA
1	62.73	(97.50)	3.87
2	57.37	(87.75)	3.54
3	52.59	(78.98)	3.25
4	48.33	(71.08)	2.99
5	44.55	(63.97)	2.75
6	41.20	(57.57)	2.54
7	38.24	(51.82)	2.36
8	35.64	(46.63)	2.20
9	33.35	(41.97)	2.06
10	31.36	(37.77)	1.94
NPV	\$382.80	\$549.51	\$23.64
Tax @ 35%	\$133.98		

Legend

Rel TR:	Tax reserve decrease from Table 1.
Cash Fl*.65:	Actual cash flows from Table 2, multiplied by 65 percent.
Rel DTA:	DTA decrease from Table 1.
NPV:	Net present value, at a discount rate of “65 percent of the 5 percent investment income rate.”

Finally, we need to understand the equivalence of Table 2 (no DTA on Required Capital) with Table 3 (as if there existed a DTA on Required Capital). To do this, we need to compare the Required Capital from Table 2 with the “Net Capital Liability” (Required Capital less Additional Deferred Tax Asset”) from Table 3. Logic would indicate that they should be equal, or that there should be a mathematical reconciliation of the difference.

Table 5 shows that there is indeed a difference between the two scenarios, but it is reconcilable. The difference is due to the fact that there are more invested assets under the hypothetical case where an Additional DTA is established (Table 3). The extra invested assets are due to the fact that there is no reduction of investment income for the existence of the Additional DTA; that is, the entire [gross] Required Capital is still being invested. Put differently, the difference is equal to the present value of the investment income due to the invested assets not being reduced for the existence of the Additional DTA. Under the current regulatory guidance for both GAAP and statutory accounting, the Additional DTA is not discounted at interest. Therefore, the existence of the additional DTA does not decrease the supporting invested assets in a given scenario, causing an increase in the gross amount of additional Required Capital and thus an increase in supporting invested assets.

Table 5
Required Capital: Reconciliation of Table 2 with Table 3

Net Capital Liability	Compare to Table 2	Difference
170.29	183.17	12.88
146.30	156.62	10.32
124.37	132.46	8.10
104.26	110.44	6.18
85.78	90.34	4.56
68.75	71.95	3.21
52.99	55.10	2.11
38.37	39.62	1.25
24.74	25.36	0.62
11.99	12.19	0.20

Legend:

Net Capital Liability: Table 3, (Required Capital) minus (Additional DTA)
 Compare to Table 2: Table 2, Required Capital
 Difference: Table 3, $(\text{Difference}_{t+1} + \text{Additional DTA}_t \cdot (.65 \cdot .05)) / (1 + .65 \cdot .05)$

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This article and the accompanying illustrative tables have hopefully achieved two objectives.

First, it provides proof that, where deferred tax assets are calculated without undue constraints, reserves should be calculated pre-tax, while required capital should be calculated post-tax. Note: This is not a perfect world, and current constraints on deferred tax assets might be considered undue.

Second, inasmuch as required capital is merely a reserve by another name, a reconciliation has been provided which equates treatment of required capital to treatment of statutory reserves in the hypothetical case where a deferred tax asset would be calculated on such required capital. ◀