



SOCIETY OF ACTUARIES

Article from:

# The Financial Reporter

June 2006 – Issue No. 65

# Setting the Level of Margins in a Principles-Based Valuation Using a Cost-of-Capital Approach with Exponential Utility

by Stephen J. Strommen



One of the central problems in accounting for insurance is the valuation of the liability for future benefits under an insurance policy. Since the liability is uncertain in amount, an estimate is required. And under most accounting frameworks, the estimate of an uncertain amount should be conservative, that is, it should contain some sort of margin for uncertainty. However, it is not clear how to set the size of that margin. This paper provides a mathematical connection between that margin and the market price of risk as measured by the cost of capital. The connection makes use of the theory of utility and an exponential utility curve.

There are three sections to this paper. First, the main concepts to be applied in the paper are reviewed. Then the mathematics of applying an exponential utility curve to the valuation of an insurance liability is developed. Following this is a discussion of the results and their implications.

## Conceptual Background

Several concepts form the backdrop for the mathematical relationships in this paper. The concepts include the need for capital in connection with risky (uncertain) liabilities, the cost of capital as the market price of risk, declining marginal utility of increases in wealth and the release-from-risk framework of accounting for risky liabilities.

### The need for capital

When an insurer issues a policy, there is usually a chance that the benefits eventually paid will exceed the initial premium. In order to avoid insolvency, the insurer must have some extra money to pay the claims in such a situation. That extra money must be available before the policy can be issued, and that extra money is at risk as long as the policy is in force. That extra money is the capital that the insurer must maintain.

### The cost of capital as the market price of risk

The owners of an insurer expect a return on their capital. Since the capital has been put at risk, the return that they expect is greater than most market interest rates. And the greater the risk, the larger the expected return. The excess of the return they expect over the return they would get based on risk-free interest rates is the cost of capital. This cost must be provided for in the pricing of an insurance policy so that the insurer can provide the expected return to the owners of its capital.

The cost of capital can be considered the market price for risk, because it is the compensation that owners of capital demand for putting their capital at risk.

### Declining marginal utility

The idea that a dollar has more value to a poor person than to a rich one is very helpful in modeling economic decision-making. The same concept can be extended to the valuation of uncertainty or risk in an insurance setting. Insurers want to be profitable, but they must remain solvent. So a dollar of profits in excess of those expected has less economic impact than a dollar of benefit costs in excess of those expected. One can simulate this mathematically by weighting incremental or marginal profits with a weight that declines exponentially as profits increase. The weighting applied to each dollar is referred to as its utility. When this is done, the utility-weighted value of an uncertain profit will be less than the best estimate expected value, and the utility-weighted value of an uncertain benefit payment will be greater than its expected value.

### The release-from-risk framework for accounting

The release from risk framework for insurance accounting recognizes any profit on an insurance policy over time as the risk in any insurance policy gradually disappears. The full profit is not recognized until the policy expires.

Under a release-from-risk framework there is no profit or loss when an insurance policy is first put in force—all profit or loss occurs later as time passes and risk expires. For this to happen, the valuation of the liability for the policy must be done on assumptions that make the present value of premiums equal to the present value of benefits and expenses at the time of issue. That equality will only be achieved under valuation assumptions that produce zero future profits. In other words, the margin included in valuation assumptions must be equal to the expected (best estimate) profit margins included in the price of the policy.

This framework only makes sense for policies that are priced to return a reasonable profit under best estimate assumptions. The evaluation of what is reasonable requires some professional judgment. However, the mathematical framework presented here allows some quantification of reasonableness in terms of a degree of risk aversion.

### Mathematical Development

Consider a simple insurance policy such that a premium is paid to the insurer at the beginning of the year, and an uncertain amount of claims or benefits is paid by the insurer during the year. We will characterize the uncertain amount of benefits as having a value to be taken from a Gaussian or “normal” distribution. The parameters of the distribution are:

- $\mu$  = The mean or expected present value of benefits (best estimate)
- $\sigma$  = The standard error of the present value of benefits

We will refer to the ratio  $v = \sigma/\mu$  as the volatility ratio.

The actual present value of claims will be  $X$ . We can define  $x = (X - \mu) / \sigma$  is a unit normal random variable with mean zero and variance 1.

The probability density function of  $x$  is

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

We will assume that a company holding this liability holds an amount of capital equal to  $r * \sigma$ . That is, the amount of capital held is sufficient to cover an unfavorable variance from expected benefits in the amount of  $r$  times the standard error. Typically one might assume that  $r$  would be greater than two, probably greater than three, so that the company could survive all but the most extreme adverse events.

Finally, we will assume that the company's utility for profits at a level other than best estimate is characterized by  $U(x) = e^{-kx}$ . That is, the utility of profits equal to best estimates is  $U(0) = 1$ , and this utility decreases by a factor of  $e^{-k}$  for each standard error by which profits increase. Conversely, utility increases proportionally for each standard error by which profits decrease.

The variable  $k$  represents the degree of risk aversion, with higher values implying greater aversion to risk. The value  $k = 0$  implies no aversion to risk, that is, risk neutrality.<sup>1</sup>

A corresponding utility curve can be used to value the liability. The utility curve for benefit costs is  $V(x) = e^{kx}$ . Here the sign of the exponent is reversed because higher benefits mean lower profits. Based on this utility curve, the utility of benefits at the expected (best estimate) level is  $V(0) = 1$ .

Noting that the random variable for the present value of benefits is  $X = \mu + x\sigma$ , we have:

The release-from-risk framework for insurance accounting recognizes any profit on an insurance policy over time as the risk in any insurance policy gradually disappears.

continued on page 10 >>

<sup>1</sup> In other literature on exponential utility, the variable  $\tau=1/k$  is more commonly used, and is referred to as the degree of risk tolerance rather than the degree of risk aversion.

Liability value, ignoring utility =

$$\int_{-\infty}^{\infty} (\mu + x\sigma) \cdot f(x) dx = \int_{-\infty}^{\infty} (\mu + x\sigma) \cdot \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} dx = \mu$$

Liability value, reflecting utility =

$$\begin{aligned} \int_{-\infty}^{\infty} (\mu + (V(x) \cdot x\sigma)) \cdot f(x) dx &= \int_{-\infty}^{\infty} (\mu + e^{kx} x\sigma) \cdot \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} dx \\ &= \mu + k\sigma e^{k^2/2} \end{aligned}$$

One can observe that larger values of the risk aversion parameter  $k$  lead to larger liability values reflecting utility.

We will denote the excess of the value reflecting utility over the value ignoring utility as  $M$  for margin. We will denote the margin as a proportion of expected benefits by  $m = M/\mu$ . We can use our definition of  $m$  as a proportional margin to derive another expression for  $m$  using the formula above for liability value. We have  $\mu \cdot (1+m) = \mu + k\sigma e^{k^2/2}$ , which leads to  $m = kve^{k^2/2}$ .

The liability value reflecting utility is the price an insurer would charge for taking on the liability. Since this will be greater than the expected benefits by an amount  $M$ , the price would include a present value of profit equal to  $M$  if the best estimate of benefit costs is realized. We can express this profit in terms of a pre-tax return on equity if we make two additional assumptions. Let:

- $r =$  The level of capital as a multiple of  $\sigma$ . That is, capital is equal to  $r \cdot \sigma$ .
- $i =$  The pre-tax investment return on invested assets.

Then the pre-tax return on equity created by a pricing margin of  $M$  is:

$$ROE = i + \frac{M}{r\sigma} = i + \frac{m\mu}{r\sigma} = i + \frac{ke^{k^2/2}}{r}$$

These expressions for the return on equity behave in a very intuitive way as the parameters are changed. Based on this expression, if one holds the level of risk aversion constant, one can observe all of the following implications.

- If pricing margins are increased, the ROE increases.
- If the level of capital ( $r\sigma$ ) is increased while pricing margins ( $m\mu$ ) remain unchanged, the ROE declines.
- Smaller percentage margins ( $m$ ) are needed to achieve the same ROE when the volatility ratio  $\sigma/\mu$  is smaller.
- As the level of risk aversion ( $k$ ) is increased, the ROE arising from this formula is increased.

A surprising result is that the ROE is independent of the volatility ratio, as indicated by

$$ROE = i + \frac{ke^{k^2/2}}{r}$$

This result occurs because we assumed that the company holds capital proportional to the standard deviation of the liability value. As the volatility ratio increases, the amount of capital increases and the cost of maintaining the capital increases in dollar terms, but the rate of return under this mathematical framework does not change.

### Discussion

The mathematical framework presented here provides a way to connect the market price of risk as measured by ROE with the margins that would be consistent with valuation assumptions to be used in a release from risk accounting framework. Regulators, in particular, could use this approach to select a level of risk aversion and translate it into guidance on the level of risk margins that would be appropriate for statutory valuation.

### Regulatory Minimums

If regulators wish to set some sort of minimum on aggregate margins for a product, they could use the formula for ROE in a particular way. The formula

$$ROE = i + \frac{M}{r\sigma} = i + \frac{m\mu}{r\sigma}$$

includes an add-on term that implies a price for risk.

The add-on term is  $\frac{m\mu}{r\sigma}$ , or  $\frac{m}{rv}$ . Regulators could

set a minimum for this add-on, call it  $Z$ . If we use

the expression  $Z = \frac{m}{rv}$  then the minimum amount

of margins as a percentage of the present value of benefits is  $m = Zrv$ . If one selects values<sup>2</sup> for  $r$  and  $Z$  of, say, three and 0.08, then the minimum percentage margin becomes  $0.24v$ . That is, the minimum margin as a fraction of the expected present value of benefits is 0.24 times the volatility ratio. More volatile (or uncertain) business would require higher margins.

The application of this approach in a regulatory context requires the valuation actuary to quantify and document two items:

1. The volatility ratio  $v$  applicable to the present value of benefits. Note that this is not the volatility ratio of the reserve (which would be net of future premiums), so the present value of benefits must be computed and some stochastic analysis applied to determine the volatility ratio of that present value.
2. The aggregate amount added to the reserve due to the introduction of margins in valuation assumptions. This amount must be at least  $Zrv$  times the present value of benefits under best-estimate assumptions.

This sort of framework for minimum margins moves the focus of a valuation to the underlying best-estimate assumptions. Controls on overly aggressive “best-estimate” assumptions could be applied at the best-estimate level before any required margins are added. In addition, it is not necessary to determine a margin separately for each assumption used in the reserve calculation. The focus is directly on the aggregate effect of all margins included in the reported reserve.

### Quantifying prudence

In the United States, the term “prudent best estimate” has been proposed to characterize the level of assumptions that should be used in valuation for statutory purposes. Prudent best estimates can be characterized verbally in many ways. However, using the concepts in this paper, we can quantitatively characterize the “prudence” we have introduced into our estimate of the benefit costs. We can do this by determining where our risk-averse price falls in the distribution of benefit costs. That is, we can determine the percentile point on the distribution corresponding to the risk-averse price.

Under the mathematical framework presented here, if the actual distribution of benefit costs is Gaussian with mean  $\mu$  and standard error  $\sigma$ , and we have a risk aversion parameter of  $k$ , one can show that the

distribution underlying our risk-averse valuation has mean  $\mu + \sigma k e^{k^2/2}$ . Since we know that  $\mu$  is the 50 percent point on the distribution, we know that  $\mu + \sigma k e^{k^2/2}$  is at a percentile greater than 50 percent. Given a value for  $k$ , we can easily look up the associated percentile in standard tables. For example, if  $k = 0.75$ , the associated percentile is 84 percent.

But we don’t know the value of  $k$ . Fortunately, the mathematics we’ve developed allows us to determine the value of  $k$  that is consistent with pricing for a given pre-tax return on equity, when equity is equal to a given multiple of the standard deviation of benefit value, and we also know the return on invested assets.

Recall that  $ROE = i + \frac{ke^{k^2/2}}{r}$ . It would be ideal

if we could solve this to obtain an expression for  $k$  as a function of  $ROE$ ,  $i$  and  $r$ . That has proven difficult. In the absence of an expression, one can use Newton’s method to solve for values of  $k$  that satisfy the equation, given values of  $ROE$ ,  $i$  and  $r$ . The table below shows values of  $k$  as they depend on  $ROE$  and  $r$  when the investment return is 6 percent.

Values of  $k$  as a function of  $ROE$  and  $r$  assuming investment yield of 6%

		r (capital measured in standard deviations)			
		2	3	4	5
pre-tax ROE	8%	0.040	0.060	0.080	0.100
	10%	0.080	0.119	0.158	0.196
	12%	0.119	0.177	0.234	0.288
	14%	0.158	0.234	0.305	0.373
	16%	0.196	0.288	0.373	0.452

The author feels that realistic values of  $k$  are near the diagonal from bottom left to top right in the table above. That puts  $k$  in the general neighborhood of 0.2. The percentile points associated with  $\mu + \sigma k e^{k^2/2}$  when  $k = 0.2$  is 58 percent. This is substantially less than the 80 percent or so that has often been discussed as the expected level for reserves. Pricing for a percentile level of 80 percent implies  $k = 0.67$  which, based on the table above, implies pricing for

continued on page 12 >>

<sup>2</sup> The values selected here are purely arbitrary and not meant as suggested regulatory criteria.

Controls on overly aggressive “best-estimate” assumptions could be applied at the best-estimate level, before any required margins are added.



a pre-tax return on equity well in excess of normal market returns for insurance companies.

The surprising conclusion is that the principles-based reserve for our simple one-year contract in a release-from-risk accounting framework should be held at something closer to the 60 percent level rather than the 80 percent level. If reserves are more conservative, then a loss will normally be reported upon issue of products that are priced to achieve reasonable levels of profit.

One can only speculate as to why this has not been widely understood previously. Part of the answer may be revealed by a discussion of contracts with long-term guarantees. The margins needed in the first year of a contract with long-term guarantees are greater than those needed for a one-year contract because of the typically greater risk and greater capital requirement. The next section discusses the implications of this and extends the mathematical development presented above.

#### **Long-term contracts**

The mathematical presentation has been in the context of a one-year contract under which all profit emerges in a single period. However, the same concepts apply in the multi-year case that is more common with life insurance.

In the multi-year case we still wish to express the level of capital as  $r\sigma$ . This requires the mean  $\mu$  and standard error  $\sigma$  of the value of benefits to be based on the present value of all future benefits, not just those in the first period. That way capital of  $r\sigma$  still provides the same probability level of ultimate security, but this time over the life of the policy rather than over just one year.

Now recall that the return on equity for a single year is  $ROE = i + m\mu/r\sigma$ .

This formula uses the proportional margin  $m$ , which is now a proportion of the present value of all future benefits  $\mu$ . Let's determine what the margin is as a proportion of just the benefits in the current year. To do this we start using the subscript ( $t$ ) to refer to policy year, and define the following terms:

$b(t)$  = Present value of benefits in policy year  $t$ , measured at beginning of policy year  $t$

$P(t)$  = (Present value of all future benefits)/ $b(t)$ , measured at beginning of policy year  $t$

Note that in the last year of a contract  $P(t)$  is 1.0, and in earlier years  $P(t)$  is generally greater than 1.0.

We then have  $\mu = P(t)b(t)$ , and therefore the total margin for a single year is  $m\mu = mP(t)b(t) = [mP(t)]b(t)$ . Since  $b(t)$  is the claim cost for the current year, we see that the proportional margin is  $mP(t)$ . Since  $P(t)$  is typically greater than one, the margin as a proportion of current year claims is greater than  $m$ , potentially several times greater.

If the margin to be released in each year of a long-term contract (before the last year) is greater than for a one-year contract, then the reserve for a long-term contract is at a higher probability level than the reserve for a short-term contract. This explains why something like an 80 percent probability level has been discussed in connection with long-term life insurance reserves even though we've shown that reserves for short-term contracts might logically be near the 60 percent level.

Let's consider how one might use this approach to determine the aggregate reserve margin needed for a long-term contract. Recall that the margin needed for a one-year contract is  $M = Zr\sigma$ . For a long-term contract, we need to release margins in an amount based on that formula each year. The present value of those annual margins is the aggregate margin needed on the valuation date. We know that  $Z$  and  $r$  do not vary by duration, but  $\sigma$  does vary by duration. We can therefore express the aggregate margin as follows:

$$M = \sum_t Zr\sigma_t(1+i)^{-(t-1)}$$

If we observe that  $r\sigma_t$  is the economic capital held at the beginning of year  $t$ , we see that the aggregate margin is  $Z$  times the present value of future economic capital associated with this liability.

### Possible regulatory application

The above discussion of long-term contracts suggests a simple approach to determine whether aggregate reserving margins are reasonable. One could require disclosure of the value of  $Z$  associated with the reserves being held. Expressed verbally,

$$Z = (\text{Reserve actually held} - \text{Best-estimate liability value}) / (\text{present value of economic capital})$$

Expressed mathematically,

$$Z = \frac{M}{\sum_t r\sigma_t(1+i)^{-(t-1)}}.$$

Higher values of  $Z$  correspond to more conservative reserves. With time, regulators would gain experience in evaluating the value of  $Z$  and determining when a company seems outside the normal range.

Considerations that arise from this approach include the following:

- Knowing the value of true economic capital is difficult. However, one could use current regulatory risk-based capital requirements as a proxy. One can expect that, over time, regulatory capital requirements will change to better reflect economic capital.
- This approach may allow the focus of regulatory oversight of principles-based reserves to shift from the level of margins to the level of best-estimate assumptions. This could lead to improved monitoring of whether actual experience is in line with reserving estimates.
- To the extent that minimum regulatory capital is generally smaller than economic capital, higher values of  $Z$  should be expected from the formula above than might correspond with market returns on equity.

### Summary

This article has shown how the concept of risk aversion can be applied mathematically to the pricing of a single-period insurance contract, and how the same concepts can be applied to determine reasonable



aggregate reserving margins in long-term contracts within a release from risk accounting framework.

One surprising result is that the probability level at which the liability is held under a release-from-risk accounting framework is not always the same. It is higher for long-term contracts than for short-term contracts. This has important implications for “principles-based” reserving, where it has often been assumed that reserves for most all contracts should be set at some common probability level.

If one cannot rely on the probability level to help determine an aggregate reserve margin, another approach is needed. The  $Z$ -factor approach of this paper was developed with that end in mind. §



Stephen J. Strommen, FSA, MAAA, is senior actuary with Northwestern Mutual in Milwaukee, Wis. He may be reached at [stevestrommen@northwesternmutual.com](mailto:stevestrommen@northwesternmutual.com).