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Actuarial Guideline ZZZ and Option Pricing

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Summary: This session provides an understanding of the content and status of the draft NAIC Actuarial Guideline ZZZ and the work of the AAA Task Force on Variable Products with Guaranteed Living Benefits. The focus of discussions is on reserving and investment considerations, including:

- *Option replication strategies and the "hedged-as-required" criteria of Guideline ZZZ;*
- *The Black-Scholes projection method and its acceptability to regulators;*
- *Reserving method for variable products with guaranteed living benefits;*
- *Valuing options and option pricing techniques.*

Mr. Joseph H. Tan: As the work progresses, two of the methods, namely the amortized-type option cost-based method and the option cost-based method, were discarded, and the Black-Scholes projection method was accepted as an adaptation of the market value reserve method (MVRM).

To help us understand the intricacies of these various methods and the option valuation associated with them we have assembled a group of three experts. Larry Gorski will start off the discussion by giving the conceptual framework of Guideline ZZZ and the regulatory concerns associated with equity-indexed product reserving. Then Tom Ho will discuss various option valuation methodologies and their applications for equity-indexed products. Lastly, Brian Kavanaugh will provide a hands-on application of reserving under Guideline ZZZ.

†Dr. Ho, not a member of the sponsoring organizations, is President of BARRA, Inc. in New York, NY.

Note: The handout is reproduced at the end of the manuscript.

Larry Gorski is an FSA, a member of the Academy, and has been a life actuary for the Illinois Department of Insurance since 1976. As you know, he's a frequent speaker at Society meetings and he's active in several NAIC groups, including the Life and Health Actuarial Task Force (LHATF) and the Invested Asset Working Group. Dr. Tom Ho is executive vice president of BARRA, Inc. where he heads up the research group in New York City. He joined BARRA when the firm merged with Global Advanced Technology. Tom received his Ph.D. in mathematics from the University of Pennsylvania and taught at New York University's Stern School of Business. Tom has extensive publications in various prestigious financial journals and is a frequent speaker at SOA Enriched Management Conferences. Brian Kavanaugh is president of Integrated Actuarial Services. He is a frequent speaker at actuarial meetings and a contributor to actuarial and trade publications. He is a member of the Academy and is on the Academy committee for ZZZ and the working group finalizing the wording for ZZZ. He is the author of the MVRM, which is one of the methods specified in ZZZ.

Mr. Larry M. Gorski: My task today is really twofold. One, it's to bring everyone up to speed at a big picture level as to the framework of ZZZ and to discuss, in some detail, some of the significant changes to the 1997 version of ZZZ. First, maybe a brief history. The project to bring ZZZ to where it is now was about a two-and-a-half year project. There was a lot of interface between both the AAA Equity-Indexed Products Task Force and the LHATF. Of course, you're not going to get all the details on the reserving methods and other examples in this session, so if you're really interested in this subject take a look at the 1997 Academy Report. It's a terrific document. One word of caution though: when you read that document, don't blindly apply the formulas to real-life examples. All the work in there is premised on certain product designs. As product designs change, those formulas may not be appropriate anymore. I think Brian is going to talk about the formula underlying the computation of the strike price for option pricing. That's one of those areas where, depending on your product design, that formula may be somewhat different from the one that's in the report, so be careful on those little details.

There are a couple of unique features of ZZZ. In the first place, it does not define a single, specific valuation method. It actually defines two sets of methods—what we call a Type 1 method and a Type 2 method. There's only one computational method that's classified as a Type 1 method. There are two methods classified as a Type 2 method. We were trying to define a method that we all felt comfortable in calling consistent with the Commissioner's Annuity Reserve Valuation Method (CARVM), so we had to come up with a way of describing a methodology that we, meaning the regulators and the Academy Task Force, felt was consistent with CARVM; hence we have these two sets of methods.

The Type 1 method is basically a book value method. One Type 1 method that's recognized is something called the enhanced discount intrinsic method (EDIM). The Type 2 methods are essentially market value methods. There's the CARVM with updated market values (CARVM-UMV) and the MVRM. Since there were two different sets of methods and clearly the values that they developed were different, there needed to be some way of ensuring that the methods all could be called consistent with CARVM. We hit upon the idea of making the use of that method contingent upon meeting the "hedged-as-required" criteria. I'll be talking more in detail on all these issues. I just want to give you a big picture framework of how this hangs together.

In order to use a Type 1 method, one has to meet the hedged-as-required criteria. For the Type 2 methods, since they're market-value methods and regulators felt that these methods were more responsive to changes in the equity markets, one does not have to meet the hedged-as-required criteria. However, to use one of the Type 2 methods, the MVRM, since it is to some degree a simplification or approximation to the CARVM-UMV method, one has to comply with some additional, general requirements. The capstone to the whole guideline are the requirements for several different types of actuarial certifications. There are really two classes of actuarial certifications. There's one class dealing with certifying as to the reasonableness and appropriateness of the assumptions underlying the options that go into the reserving methodologies. The other class of certifications deals with certifying as to compliance with the hedged-as-required criteria. I'm going to talk about some of the details concerning some of the 1998 changes to ZZZ. Brian will talk about some of the computational issues that are related to different methods.

Who is completely unfamiliar with Guideline ZZZ? Maybe a quarter of the people here. Who is moderately familiar? Maybe about half the people. Everyone here is moderately familiar, so glossing over some of the computational issues at this level shouldn't cause any problems. The things that I want to talk about are the changes to ZZZ that took place over 1998. The first one is the hedged-as-required criteria. Again, the hedged-as-required criteria are applicable to those companies that want to use the Type 1 or the book-value method. One of my responsibilities with the Illinois Department is to review policy form filings. Over the last six months or so, I've seen very few filings in which the company is electing the Type 1 method. Most companies are electing the method I will be talking about in a few minutes, the Black-Scholes projection method, so this may not be a real issue for companies. However, there may be some companies who issued products in the past who want to use the Type 1 method, so for those companies it is a real issue. One of the features of Guideline ZZZ as it's going to be implemented by states is that it is retroactive. It's going to apply to all business—not only business that was issued after the effective date or adoption of the guideline. It applies across the board.

That decision came about very early in the process. We, as regulators, like to have a level playing field; companies like to have a level playing field. We didn't want companies rushing into the market with products with strange reserving methods that could give them an advantage for a time, so we said up front, "When this is adopted, it's going to apply to everyone whether you like it or not." But that's the regulator in me speaking.

Hedged-as-required criteria. The basic goal is for the criteria to ensure that the hedging portfolio develops payoffs that match the equity-enhanced liabilities associated with the single dominant benefit in the product. This idea of a single dominant benefit came about when we were grappling with the fact that even though most contracts today focus on the end-of-term benefit as the reason why everyone is buying the contract, there's no reason why you can't have a significant equity enhancement to interim death benefit or some type of other benefit. We tried to develop a guideline that was robust enough to deal with future product developments. The guideline talks about the single dominant benefit, which everyone would agree is the reason for that product. The company is selling it for that benefit; people are buying it for that benefit. It's not a concept you can define by using a mathematical formula. It's something you can only understand and appreciate if you pick up the policy form and all the marketing material. Common sense will tell you that we're selling this as a seven-year term product; we're selling this as a one-year term product; we're selling this because we have this really great equity-enhanced death benefit in there, or what have you. It's something that is not amenable to strict mathematical analysis; nevertheless, it's an important concept in the guideline. So that's the goal of the hedged-as-required criteria.

What are the hedged-as-required criteria? Again, they're criteria that need to be met by a company that wants to use a Type 1 or book-value method. There are five points that break down into a couple of different classes. Again, these are all laid out very clearly in Guideline ZZZ. The first two points deal with this single dominant benefit. Point 1 says you need to have a hedging portfolio that has the same characteristics as the options embedded in the liabilities. Point 2 again refers to a single dominant benefit and gives a rule for how much of the hedging portfolio you need—you may need to be 100% hedged, 80% hedged, 20% hedged, etc. Points 1 and 2 deal with the nature of the hedging portfolio and the amount of the hedging portfolio. Point 3 recognizes the fact that there may be other benefits in the contract that are equity enhanced, which means you may need a plan to hedge those additional interim equity-enhanced benefits. Point 4 says you must have a system in place to monitor the effectiveness of your hedging strategy. Point 5 says you should have a tolerance between the actual and expected results and be able to report on whether you're meeting your expected results.

If you look at these criteria, you walk away with the belief that these criteria were designed specifically for a buy-and-hold strategy. Let's say you're selling a seven-year point-to-point, equity-enhanced product. If you read the criteria, they would lead you to believe that you need to buy seven-year options because of Point 1. In fact, the first version of ZZZ required a buy-and-hold type of hedging strategy in order to meet the hedged-as-required criteria. Obviously, many companies felt that they wanted the flexibility of an option replication strategy. It took some time to get that idea in place, but that's what happened over 1998. When you take a look at the guideline and the hedged-as-required criteria, there are two criteria: a basic set (buy-and-hold) and a second set, which is Point 1, the option replication strategy. There are two forms of the hedged-as-required criteria: (1) basic, which is based on a buy-and-hold strategy, and (2) option replication. The penalty for failing to meet the hedged-as-required criteria, if you are using a Type 1 method, is that you have to move over to a Type 2 method. The Type 2 method, since the reserve or liability valuation brings into play fair values of the embedded equity options, is felt to be more responsive and reflective of current expected costs to hedge the portfolio, so if you fail to meet the hedged-as-required criteria you have to move over to the more responsive-type MVRM.

What is this option replication hedged-as-required criteria? There was a session that dealt with ZZZ and managing ZZZ products. That session was devoted to a discussion of managing the equity exposure in equity-indexed products through option replication strategies. The perspective may be a little bit different than I'll be taking here, but I would advise you to get the recording of that session because it was a good one.

From the regulatory perspective and option replication strategies, an investment strategy that uses short-dated options, futures, and cash-like instruments to replicate the performance of a long-dated option is an alternative to a buy-and-hold strategy. It's an idea that certain companies wanted us to recognize within ZZZ. What are the regulatory concerns? One, it's a very complex strategy to implement. Different people will interpret and define the terms that are needed to implement the strategy differently. The chances of not pulling the strategy off are in some cases pretty strong—maybe not so much from the investment standpoint but from operational standpoints, so there are a lot of regulatory concerns. Two, some of us were concerned that this strategy would simply be used as a screen to speculate and we're using derivatives. We were reluctant to move ahead with recognizing the option replication strategies.

The guideline again sets out two sets of criteria—one specifically for those companies who want to use the option replication variety of the hedged-as-required criteria. There are three key concepts: (1) target of the option replication strategy;

(2) the compliance and the evaluation criteria; and (3) understanding the penalties if you screw up.

There actually is a problem with Guideline ZZZ in this particular section. It's something we have to clear up between now and December. It has to do with that first point—the target of the option replication strategy. If you can remember those five points I mentioned earlier, the first two dealt with the characteristics and amount of the hedging portfolio and hedging instruments. When you deal with the option replication variety of this, the target takes the place of the embedded options and the liabilities, so you're asked to have a hedging portfolio that matches up with the target of the option replication strategy in terms of both the characteristics and the amounts. There's a direct linkage between the two sets of criteria.

The next two points are basically the same. It's not until we get to Point 5 where there's any significant difference. That's where the "b" and "c" components of this come into play. First is the compliance evaluation criteria. In the guideline there is a rule for measuring the effectiveness of your option replication strategy. Basically, you're told to measure, on a weekly basis, the change in the fair value of the hedging portfolio and the change in the fair value of the options embedded in your liabilities. Calculate those two amounts, take the difference of those two and compare that to the fair value of the options embedded in your liabilities at the beginning of the quarter. It's a weekly test that calculates a difference of two numbers and relates it back to the fair value of the equity options embedded in your liabilities at the beginning of the quarter. That's the test. It's very close to the idea of a hedge variance report that was discussed in the earlier ZZZ session. There was quite a bit of discussion as to how one monitors the effectiveness of your hedging strategy. This is analogous to that. We did not want to get into a description of using the beta, kappa, gamma, and all the other Greek letters because not everyone may be using that approach. There may be other ways of managing a hedging portfolio, so we stay generic and simply define the test that we wanted actuaries to use to evaluate the effectiveness of the strategy. Now that we know what the compliance evaluation criteria is, how is that used? What are the penalties for not having a strategy that meets the criteria? The cutoff point is 10%. If during any quarter the deviation that I define is less than 10%, there's no need to worry about it. However, if it exceeds 10% more than once in a quarter, then there's a notification requirement to the Commissioner. If it's more than 25% in any quarter, then there are some additional notification requirements to the Commissioner. If it exceeds 35%, then your strategy is deemed to be ineffective and then you have to start worrying about changing to the Type 2 method. It's all spelled out in the guideline, but this gives you at least a big picture understanding of it.

The big question from a regulator and a company is, will compliance with the hedged-as-required criteria guarantee meeting the goal that I spoke about before—the goal of having a hedging portfolio to produce the right kind of payoffs relative to the liabilities? I think we're all pretty comfortable that the buy-and-hold strategies that are subject to the basic requirements will meet their goal. When it gets to the option replication strategies, I have to say I don't know. The expectation is that it's going to work, but in some peculiar economic environments it may not work. There may be some things that we didn't consider. When I say we, I'm speaking of myself, the regulators, and the people who are on the Academy Task Force because they had a major hand in working on this also. I think we all think it's going to work but we're not sure, which brings us to a reminder in Guideline ZZZ that asset adequacy analysis is still required. If it's required for the company, it still has to be done for this product. Some actuaries took the position that since we're defining market value methods in the guideline, the actuaries could forget about asset adequacy analysis. Within the guideline there is a reminder that you're still subject to any requirements concerning asset adequacy analysis.

The other major change to the guideline is the recognition of another computational method that is defined as a variant of the MVRM. This method was felt to be necessary because of the problems associated with applying the existing methods to annual ratchet designs. One of the small, but important, comments in Guideline ZZZ is that methods described in the guideline are those that can be used. Other variations are not permitted, so in order to get the Black-Scholes projection method as a permitted variation, we had to very specifically bring it into the guideline itself. What is the Black-Scholes projection method? Basically, if it's designed for annual ratchet-type products, you calculate the cost of a full hedging call option as a percentage of the account value. You accumulate the percentage to the end of the period (usually a year) and then you use that to accumulate the percentage cost as the projected growth rate for the account value during the period. If you have a seven-year product, let's say, but it's an annual ratchet-type design where you may have a first-year guaranteed participation rate of 100% and then renewal guarantees of 50%, you look at each year separately. You calculate an option cost for the account value at the end of each year, taking into account the guaranteed parameters of the product for that year and come up with the cost of the call option. Express that as a percentage of the beginning account value and accumulate forward at the risk-free rate of return and that becomes your factor for growing the account value to the next year. You do that in this case seven times. In practice, most products we see have nominal renewal guarantees, which, in effect, make any calculations beyond the first year of little significance.

There are the other rules associated with the MVRM. Again, it's all spelled out in the guideline, but the basic idea is to look at the product one year at a time, calculate an option cost for each year, and use that option cost expressed as a percentage of the beginning account value. Use that percentage and accumulate to the end of the year. That becomes a factor to grow the account value. There's one question that comes up quite a bit though. In setting the assumptions for the option cost for each of the seven years, some actuaries are forgetting the fact that this methodology assumes that you're calculating an in-the-money option cost for each of those seven years because each year you're bringing your account value up and then determining an option cost for an at-the-money option. Some actuaries are forgetting that fact and calculating option cost for future years based on implied volatilities that would be appropriate for out-of-the-money options, so you have to remember that each year you're dealing with an in-the-money option.

A couple of reminders. Since the Black-Scholes projection method is a variant of the MVRM, a company using this method must comply with the general requirements associated with the MVRM. That has to do with being able to identify that single dominant benefit and reserving for that single dominant benefit. The other reminder is that Guideline ZZZ does not recognize other variants, so you're limited to three basic methods—one method with the variant.

The last change to Guideline ZZZ is in the scope section. The scope of Guideline ZZZ obviously says it applies to equity-indexed annuities (EIAs), but it also has an embedded statement that applies to variable annuities (VAs) with guaranteed living benefits. If you think about a VA with a guaranteed living benefit, it's almost like an EIA. An EIA is a floor with upside appreciation; a VA with a guaranteed living benefit is a base product with upside appreciation. You're protecting yourself with a floor, so it's sort of like the same thing. Initially, I wanted Guideline ZZZ to apply to those products, even though I knew there was going to be a lot of practical problems in doing that. We put that in the regulation as a way of finding out who is selling these products to encourage the Academy to begin work on a project to define appropriate reserving methodologies for that kind of product and concerns of that sort. After about a year and a half, we finally decided that the Academy was far enough along; everyone understands the regulatory concerns with these VAs with guaranteed living benefits, so it was time to bring that out of the scope and let nature take its course. That's one change you're not going to see in your version of the guidelines, but in fact it did occur. With that I'll turn it over to Tom.

Dr. Thomas S.Y. Ho: My task here is to describe some of the methodologies people have for evaluating equity options. People associate valuing equity options with Black and Scholes' 1973 paper. In fact, option pricing has a very long history. A lot of economists, even ten years before Black-Scholes, have been trying to price equity options. You should not think that an equity option is like a Standard & Poors (S&P) stock option where, say three months from now, you have the right to buy the S&P

index at the strike price. In fact, a stock option is any payout that is formulistically dependent or implied by the underlying index. That index has to be tradable; in our case the S&P index, that you tie anything to, is, to us, an option. Since 1973, a lot of research has been done on discussing variations of the methodology and pricing of stock options.

It is very important for us with respect to ZZZ and the regulation because we have to extract the underlying option within the product. What is the value and how would you hedge it? In my presentation I would like to give an overview of some of the methodologies and the basic principles behind using some of the models. I think it's very important, as Brian pointed out to me this morning, not to assume that the Black-Scholes model can do all the pricing models (and price all kinds of equities and annuities) but to think of its shortfalls. We'll discuss those and what we can do about those shortfall situations.

I think it's appropriate to begin with why we have the Black-Scholes model and what people did before that. If we were buying an option before 1973—say three months from now we had the right to buy an S&P index at strike—how would we price such a thing? We would say, "That's good. Let's get an analyst and project what the S&P is going to be like and give the possible payoff at distribution of the S&P distribution and then discount back." That's the present value of the option. I think we applied that to many of our products too when we priced them.

The difficulty is that economists often disagree. One aspect about which they disagree a lot is the discount rate. What is the right discount rate? I'm talking about going back 30–40 years now. This is particularly important when we are talking about long-dated options, a year or two years from now, and the discount rate, particularly now that options or derivatives are considered risky. If we require a 10% discount rate for S&P index, we must be using a 30% discount rate for options. This becomes a very different number, whatever number you get. That is what is driving the whole problem. The insight behind the Black-Scholes model is that we are doing something wrong here. That is because if we, by definition, say that option is tied mathematically to an index, the S&P for instance, and if we already agree that we are requiring 10% for the return of the stock market, then there must be a mathematical way of relating that information directly to the pricing of options. Really, it should not matter whether you dislike the risk or you like the risk because someone must have a way of tying the stock index return directly to the return of an option because they're mathematically related. What is this relationship? That is the insight of the whole Black-Scholes model. I think that is crucial for us as we move ahead to deal with the whole ZZZ regulation and so on because that is the crucial point.

That's very important, so don't come up with a model and say that we don't like this risk. We put another risk premium behind it and so on. We're bringing ourselves back 30 years again, by Black-Scholes insight into the following. If you buy a stock option—call stock option—what you are really doing is borrowing money to buy stock; you are buying stock at a margin. The only difference is that you are changing the ratio all the time so that if you adjust the ratio continuously, you should also get a payoff of the call stock option by just simply buying stock at a margin. Let me repeat that again. You buy a call option, it should be identical to, given a particular dynamic strategy of buying stock at the margin. It's just changing the margin ratio. Maybe at some point, when you're out of the money option, you should borrow $90 and only put $3 in stock; altogether it's $90 plus $3 or $93 in stock. Borrow $90, so the leverage ratio can be very huge in the stock option when you're out of the money. Over time you can change that ratio.

If you follow this argument, the question we are asking is, what is this ratio? The Black-Scholes model argues if we know the following crucial list of assumptions. For us in the practical world, we're always very nervous when we look at the list of assumptions because they look so unrealistic. That's the art of research.) We go through the list of assumptions and see when C risk is violating these assumptions and making this methodology fail. First, let's assume interest rate is constant. That's heroic by itself. Let's assume the interest rate is fixed. In fact, in the Black-Scholes model even assume one interest rate—5% flat. The second assumption is what we call the S&P index and it has to be something tradable. The S&P itself can't be tradable so we have to proxy it with the futures on the S&P or a basket of stock. If it's not tradable, then the theory would be wrong. For example, you can buy options on S&P so S&P itself is not a tradable index. That's a fair problem. For us, S&P is fine; you have proxies for that. It's a tradable security, and this tradable security has to be continuously tradable all the time.

The third assumption turns out to be more important. We must also know the volatility of this index movement. If you know this, then I can formulate a strategy of finding this index, shorting borrowing money—so that three months from now, whatever the payoff is, I can replicate it for you.

We need today to put a hedge on. Should I be borrowing $90 and put in $3 of my own principal amount so it is $93? That is the option pricing. We work backwards a little bit. You need to know the exact mathematical relationship of S&P index to your payout to work out a hedging strategy. You then determine the position you should hold now. That is the option pricing. Now, why is this so popular? Because 1973 also coincided with the recent introduction of the S&P index stock option. This was a reality check, and like a lot of models presented by economists where you can't check things, traders in those days wrote down the Black-Scholes model, calculated the hedge ratio, and saw how much it cost to do the trade. As time goes on they revise the trade and actually book profit and loss on almost a daily basis, so they kind of check the models whether they're correct or not in a very short time and give you a very exact way of how to replicate it. Theoretically the price is $93, but there's no way to verify that. It's not economic value; you can't trade it. Here you quickly trade and know exactly the arbitrage you're taking. In three months you can book profit and loss, so in that sense, it is an extremely successful formula because you have a way of implementing it and checking the results.

Let me go back to the assumptions and see why we have problems and how we can adjust them. The first one I talked about is the interest rate. When the Black-Scholes model first came out, it was very much for the Chicago Board of Trading Stock Options so a three-month or six-month expiration was not a problem, assuming the interest rate was 5%. It really doesn't affect the value too much, but when we are talking about long-dated options, then there are problems.

First, if this is one year, the U curve may not be flat, so we can have a downward sloping curve. Which rate should be picked? It is important to adjust our model to fit the shape of the U curve. That's one point.

Larry referred to the second problem, which is when we have a series of options and we are using the Black-Scholes projection methodology to find out what the in-the-money options look like and then discount back. The in-the-money option depends on the interest-rate level. Knowing the in-the-money option is not enough. You also need to know the discount at that future time and the interest-rate level with respect to the at-the-money or the in-the-money option. That is how the volatility of interest rates become part of the valuation. It would affect it. In a similar product, in fact, if the lapsing or surrender behavior is related to the interest-rate level, then you're confounding what you think the option is they have offered plus the surrender behavior of the lapse, which is also tied to interest. To deal with this problem, we have to deal with both stock and interest-rate uncertainty together. Researchers call these two-factor models—they have an interest-rate risk as well as a stock risk. This really depends on the product design. You can take the option out and call it Black-Scholes or you can't. An example of where you can't is when you bring in the lapsing behavior, which is tied to the interest rate as well as the value of the future option, which is also tied to the interest-rate level. It becomes unclear how to actually factor everything out. This is the interest-rate uncertainty problem. The solution to that is either through the model you identify or by removing the Black-Scholes simple equity option. When you can't separate it, you need to go through a two-factor model approach to value it.

Another major problem with option pricing models is what I call volatility. It is assumed that traders know how to hedge this option. In my mind they think they know the volatility, the uncertainty of the index, when in fact they don't. It may never work. We use historical volatility and that clearly is a mistake because very often our capital market gets caught because of the market changes and everyone is caught by surprise because we are looking at the past in the rear-view mirror. Option pricing is very much forward-looking because we view that with a lot of uncertainty in the future, the option price go up. So you keep looking at the past, which is not reflecting what people are looking forward to reflect uncertainty. Option pricing is extremely sensitive to this volatility number and that's a problem, so how do we do that? In fact, it's not that simple so I need to describe a little bit about some of the research in this area.

First of all, the volatility is not one number. If you simply look at S&P three-year expiration options versus the long-dated options, one year or two years, you will find the volatility that traders use is different, particularly nowadays. For example, we are going through a very uncertain period now, so you will find that the volatility people use for short-dated options is very high. Long-dated options are lower and we call that the term structure of volatility. We have a seven-year ratcheting equity-linked annuity. Which volatility should we be using? Clearly, I don't think it should be one number. We have some short-dated ones and some long-dated ones, so there has to be some way of putting these numbers together to find out exactly how you add all the options together.

The conclusion of this is that implied volatility is a better way. That means using publicly traded options so that we have some forward-looking concept to price our option and hedgings so when we report the replicating portfolio, the hedge portfolio, it will depend on the volatility number. It's not just valuation, so the whole hedge ratio depends on the volatility number. It would be more useful if, in the report on your research, you say that the market is trading these options at these prices; therefore, we can refer back from the market-prices what volatility they are using. For that reason they are using those numbers to price our embedded options. But then you have another problem. Historically, people find that with options traded out-of-the-money, there's a much higher volatility than if the stock option traded is at-the-money. So, in other words, given an option with the same expiration date, people disagree whether the option should be in-the-money or out-of-the-money. They make separate points about why that is the case and so on, but for our purpose we just want to be aware of that to talk about the volatility change as the option becomes in-the-money or out-of-the-money.

I would like to go through the variations on options and how we would solve this problem with the ZZZ requirements. I want to discuss the Black-Scholes model because it's an essential concept of what an option is and what issues are; in fact, most options are not the Black-Scholes type. The way I'd like to describe it is by using the geographic approach. The Black-Scholes model really deals with a type of option called the European option. You can only buy and sell that option; you can't do anything until the expiration date, when you have the right to buy the option. That's the European option.

The American option allows you to exercise the option at a series of strike prices—anytime from now to the expiration date. Why is that so different? If we allow our policyholders to change or surrender, that would be American. If you allow them to have that option of anytime before the expiration date, the decision-making process is open. If I don't exercise the option, I can always exercise tomorrow. Given that I can exercise tomorrow, that depends on the following date. In other words, there exists a kind of interrelationship between future expectations and today's position and that makes a lot more complicated. If it's European, you just trade it and the only decision you will make will be at the end. But if it's American, because we allow policyholders to change all the time, their decisions will be interrelated to all the decisions subsequent to that. This "Americanness" is very important. Researchers call it an early exercise premium. We need to calculate that.

The third type is called the Bermuda option. The Bermuda option is somewhere in-between. This option is quite often in our equity-linked annuities. You allow the policyholder to make these early decisions at regular intervals, so that's the ratcheting part. Very often it depends on the design of the contract; for example, they can ratchet up on a quarterly basis. Their decision of whether to take money out or not depends on what they think of the following quarter. It's sometimes not separable. It's not a series of options. It is, in fact, interrelated options over the lifetime; therefore, researchers show that you just can't unbundle them. There's not a simple way to say that. It's just a series of Black-Scholes models. It depends on the contracts and how you tie the options together through the optimal decision of the policyholder.

The final type, which is the most common for equity-linked annuities, is the Asian option. The Asian option depends on historical behavior. It comes out to the payoff expression date, so that is called Asian. You can think of this as partly dependent. The payoff is dependent on how the S&P index has behaved over that period. It's not just the expiration date.

Bear in mind that there are various types of these options. Using basic research, you can actually show that all the options basically can be built from the Black-

Scholes model. There are all these different types of valuation models to handle that.

I want to quickly discuss dynamic hedging. In dynamic hedging, if you have a Black-Scholes model, then the Black-Scholes model would quickly give you the hedge ratio of how much stock you need to buy and how much you need to borrow. But in all of the other cases, it will be difficult, so you need a Black-Scholes model, you use an option pricing model to price whatever better option you have. Then you find a small change of S&P index relative to small change in the value of the option. That's how you compute the option price and the hedge ratio for stock. For bonds, it's not necessarily the risk-free rate of cash anymore. In fact, you need to borrow maybe a two- or a three-year bond depending on the strike date. This is a common mistake. You really just borrow cash and buy stock. In fact, it is very sensitive to the strike date you choose, so you're really borrowing bonds to buy stocks.

On the standard hedging side—what is called the portfolio replication—the question we pose is the following: If we have an equity-linked annuity and you develop a valuation model and take that out as a stock option model, what are the publicly traded capital market options that we can put together so we can replicate it efficiently? One methodology is to simulate all the cash flow you can out of all these scenarios and find the methodology of putting all the options together so that they line up all this cash flow in these scenarios. There are a number of procedures in doing that. In fact, there's extensive research literature in that area called portfolio replication or static hedging methodology.

In summary, the purpose of my presentation is really twofold. First, I wanted to talk about the basic insights into this relative valuation or contingent claim valuation or arbitrage-free valuation approach to pricing equity options. And second, I wanted to talk about various types of options that are available that might fit into the design of equity-linked annuities and the general methodology for putting a static hedging or a dynamic hedging together.

Mr. Brian Kavanaugh: In general, what I'm trying to do is indicate which reserve method you need to use in certain situations to determine market values because I find that there's a fair amount of confusion in talking with various actuaries who are in this area with regard to the establishment of straightforward things such as exercise prices, how to determine the amount of the option that needs to be purchased, how to treat certain basic concepts and policies such as caps, annual ratchet designs, and even annuities. Some companies are actually selling EIAs, so I hope when I finish talking to you today you'll be very knowledgeable in all these areas and be able to determine exactly what the reserve should be.

The complexities of GGG added to the complexities of ZZZ may not be exactly the same as XXX, but some of the comments of the actuaries whom I've talked to certainly could be rated XXX.

Regarding hedged-as-required, we tend to look at things as valuation actuaries and we tend to feel, for example, that we'd like a nice simple reserve methodology. Bear in mind the people who are handling the investments are on the other side of that street, so to speak. Some companies are running computers in a parallel sense, turning out results 24 hours per day. I think that if, as the valuation actuary, you meet with these actuaries who work in this area and say, "You could really make my life a lot easier if you could meet the hedged-as-required criteria because then my valuation instead of taking three hours will take two-and-a-half hours and please, if you would do that, I'd really appreciate it," you better have tenure because these people are not very interested in your valuation problems. They're interested in the very high technical area where they believe, given a free rein, they can increase profits on annuities by up to 3% of the premium consideration. I also think that Illinois is a very good test of how ZZZ will be applied in practice because it has been in effect for over a year. From what Larry has seen in submissions to the Illinois Insurance Department, I believe only one company has indicated that it would use EDIM. In practice, that may be a mistake when people realize their limitations that they may put on their investment strategies. Even though it's in the regulation, it may not get a lot of use.

Because of the confusion in the calculation of the exercise price, I've developed a formula that is kind of general.* I'm not going to discuss it in too much detail. If you look at the exercise price from two points of view, the first equation looks at it from the point of view of the policy provisions or as the insured may look at it; that is, he or she has a basic benefit that has had some appreciation out of which may be subtracted some expenses such as fees. That's what the first form expressed.

From an insurance point of view and a company point of view, they look at it a little bit like they have a certain amount of money they know they need, such as the guaranteed cash value, in a payout that under the terms of ZZZ has to exactly hedge. Putting those two equations together to eliminate the forward index gives a more or less generalized form for determining what the exercise price should be, which is the third equation. It looks a little long because I made it a little bit general in nature. Many times most of these values will be one or relatively straightforward; nevertheless, as Larry pointed out, if you have some peculiar designs, it is necessary to be able to put these into practice.

***Note:** The handout is reproduced at the end of the manuscript.

Further, you need to know the amount of the option that you need to purchase based on the single premium and the percentage of it being affected not only by the participation rate, but also the surrender charge.

Now in theory it's nice to be able to work these things out. Also, with regard to the MVRM, it's based on indexes and in general the dominant benefit such as a point-to-point, seven-year maturity value. You determine what the index would be at that point in time, and you assume compound growth between valuation and the maturity, which gives you the index at any intervening part. Here are the two formulas which will enable you to do that calculation.

Let's apply these particular formulas to a situation where there was a guaranteed cash value which was 90% accumulated 3%. In the fifth year there was a surrender charge of 3%. There was a participation rate of 45% for this particular cash value, which would give you an exercise price equal to 116.8% off the single premium. In addition, because of the same parameters, the amount of option you should purchase equals 43.65%. I think it's very wise when you do these very particular designs to actually go through them from a policy provisions point of view. I do it under three scenarios: one where it's out-of-the-money; one where it's exactly on-the-money, which means out-of-the-money; and one where the index is above the strike price. In each of these three scenarios, rationally, there should be no payout needed from the equity portion if you're out-of-the-money. If you're exactly on-the-money, that is if the index equals the strike price at some future time, both the amounts available from the insurance company should exactly equal the equity value. If the index is above the strike price, there is a certain payout.

Again, from the insurance point of view the effects of the situation should prevail, and if you've done your calculation of your exercise price correctly and the amount of purchase correctly, both these two values should be exactly equal—23.2%. With regard to the Updated Market Value (UMV) method, for each policy year and benefit, determine what a market value is. Using the exercise price and the Black-Scholes in its point-to-point, seven-year design gets a good answer and determines the fair value. From that fair value, you can determine the amount of the purchase adjusted for participation rates and any surrender charges. You then project that to the end of the year at the valuation rate to get the additional benefit, which must be added to the guaranteed benefit. If I remember correctly, $900 accumulated 3% is $927 and if you add $16.96 to it, you get the benefit projected to the end of the year. You do a similar type of calculation for the other two types of benefits—basic benefits which would be annuitization. To use the UMV method initially, you need to have the market values of 21 different call options in order to determine what the reserve should be.

With regard to the MVRM, it just takes the seventh year value, accumulates it at the valuation rate, and determines what an index growth would be. Based on that index growth, it can specify what the index would be for each of these years. Using that index, we can determine, based on the policy provisions, what the cash value, annuity value, and death benefits should be. Obviously, once you have these values you can apply CARVM.

The simplest of all is EDIM, which only requires that you have available the reserves under either one of the other two methods at issue and assumes the compound growth from issue to maturity, to which is added an intrinsic value that is the discounted appreciation. I've shown what the reserve would be based on a situation where the index has gone down and where the index has gone up.

Again, you need to go through what needs to happen with caps, for example. If there's a cap on your contract, you want to use Black-Scholes. You can easily do that as the difference between two market values: the market value without the cap and the market value created by not having to pay anything above the cap. You use the cap as the exercise price, determine the two values, and subtract them. That will enable you to use Black-Scholes to value options with caps.

With regard to annual ratchet, the method specified in ZZZ is Black-Scholes. The example I have is an Asian design where Black-Scholes doesn't work, but the methodology that can be employed is still the same. The method I use to determine the fair values of the options uses a net worth and expansion, which is not Black-Scholes. Black-Scholes generally cannot be used except in point-to-point designs.

Again, you go through the same justification to show that you're exactly meeting the perfect protection. You have to do the annual ratchet design on a year-by-year basis. You determine at the end of the first year after valuation what the index would be at the end of the year and use the index. If it's an average index, you then have to deduce from that what the actual ending index would be. Use that as a starting index. Go into the second year and project various things such as volatility and utilize those in order to come up with what the market value appreciation would be in the second year. You can then do a CARVM-type calculation, just like you saw before, because if you know the index you can therefore apply the policy provisions to determine what each of these values should be.

The last thing I just want to touch briefly on is the effect of volatility. In general, all three methods give very good answers, but you have to be a little careful with short-term annuitization rates in the application of UMV. You may not have to be that careful if you're using one of the other two methods.

Based on these simplified assumptions determine what will happen if short-term volatility in particular suddenly goes through the roof. Table 1 (see handout) shows what happens if the short-term volatility rate moves from 15% to 25%. This is what happens to the asset increases. Again, I'm trying to complete this in a relatively short period of time. However, you can see the dramatic difference even when the volatility remains, say, at 15%. There is a fluctuation in the earnings on the UMV method that is not present in either the MVRM or the EDIM. They're relatively the same. The actual losses, because of the increase in volatility in this example, could be $33 per 1,000. The principal reason for this is that the actual reserve is driven by the second-year annuitization rates. Because of the short-term nature of that second-year annuitization rate, and because they are more greatly effected by volatilities, this can result in a substantial increase. Be very, very careful in your designs if you're using UMV. Check all your annuitization rates in the various scenarios both in up markets and down markets because they can have a dramatic effect on the reserve levels. There's a major problem. Since this regulation is retroactive to all your equity-indexed products and if, in the past, for example, you've not been careful of your annuitization rates, in today's market you may find your reserve substantially higher than it otherwise would normally be. For example, today the volatility has gone to 35%, and not to 25%. The actual fluctuations could be $70 per 1,000 and not $30 per 1,000, so it is extremely important that you pay attention to every different type of benefit available under the contract because under GGG. Remember, you had to have integrated benefits and you have to check every possible utilization. If you have an annuitization rate at the end of the first policy year after valuation. In the second policy year, the CARVM mechanism assumes that that annuitization is used by 100% of the people even if it historically may well be less than 1%, and can create a mismatch between your assets and liabilities which would give you this type of result that will fluctuate up and down depending on the volatility assumptions.

To finish up, which method should you use? EDIM may be preferable because of its simplicity, but I think you will have a tremendously adverse reaction from your investment people unless you're a small company with a very limited investment capability. If you can't use the EDIM and if under the guideline it's permissible, I'd use the MVRM because it's relatively simple. In addition, it isn't as prone to fluctuation because of short-term volatility. It tends to concentrate on benefits seven years down the line and assumes a reasonable growth between those two periods. If, because the way the regulation is written, you can't use the MVRM, then you're left with UMV. If you don't like the consequences of that, then obviously you can't play in this particular ballpark, so I think as far as a decision as to which method to use, it really is very straightforward. Can you live with the hedged-as-required? By all means, use EDIM. If you can't, try to use the MVRM. If you can't do that, your only other alternate would be the UMV.

From the Floor: My question is, I guess, for Larry. In projecting the option value by Black-Scholes or other designs, what implied volatility and what implied dividend yield should you use to do that? Are they prescribed and what are they based on?

Mr. Gorski: If you're using the Black-Scholes projection method, which I think is the method you were talking about, with an annual ratchet product, the guideline says use the implied volatilities, the risk-free rate of returns, and the dividend yields off the forward curve. Let's say you have a seven-year annual ratchet product; for each one of those seven option costs you would use the appropriate assumption from the forward curve, so you wouldn't use one year values at issue but off the forward curve. That's built into the guideline.

Mr. Mazavee: For example, for some exotic option like discreet look-back would you be using the same implied volatility as you would for Black-Scholes?

Mr. Gorski: You would be using an implied volatility appropriate in that the market and you would be charging for that type of option. Built into the guideline is a requirement that the assumptions for determining fair value of options on the liability side would be both reasonable and consistent with the option that you're holding on the asset side. I'm assuming that if you have an annual discreet look-back product, you're holding options roughly comparable to that. When you fair value those options, you'd be using market-based implied volatility, so there is a need for consistency there. The implied volatility using the liability side should be consistent with what you would be using on the asset side.

Mr. Paul Curly: It was mentioned that the Black-Scholes valuation method doesn't apply directly to Asian options. I've been attempting to adapt it using the notion that an average is an integral and that an integral can be approximated by a weighted sum of point values. If you then adapt the technique a bit to keep the discount period constant, but to create a notional option that defers payment until the end of the average period, you can then apply the Black-Scholes type of thinking and sum things up, which becomes very easy to do because of distributivity with the discount factors. I wonder if anyone has attempted anything like that or if you see pitfalls in it?

Mr. Gorski: It seems to me I have seen people try that approach over the last two years and it seems like the approach doesn't grapple with the path dependence of an Asian option versus the nonpath-dependence of a European option. I'm not sure if you bring that into your thinking or not.

Mr. Curly: This would be a weighted average of European options, basically, with a deferred payment.

Mr. Gorski: Does it really address the path of dependence?

Mr. Curly: I'm not certain.

Mr. Gorski: That has been the stumbling block.

Dr. Ho: I think path dependence has two meanings. In your case, it's averaging. That's sort of Asian. That's average price over period or you can have a look-back where the last two months were at the lowest price. The second type would be much harder because you really have to keep track of the minimum/maximum over that period. As far as I know, that approximation is not so bad for that type. In fact, there are several closed-form methodologies of changing this Asian back to making it look like a Black-Scholes consistent with what you're saying. Did you refer to some of the research papers written in that area?

Mr. Curly: No, I didn't.

Dr. Ho: You can. Hold's book has a list of them. Asian options can be written in terms of closed-form solutions.

HANDOUT

Equity Indexed Annuities
Reserve Methodology & Call Options

Abstract

The NAIC is about to adopt an Actuarial Guideline on reserves for Equity Index Annuities. The current version specifies the use of two market value type methods, CARVM-UMV and MVRM which are not subject to investment restraints and one method, EDIM which is subject to "hedge as required" investment constraints. Each method to some degree requires the determination of the values of call options normally acquired by an insurer for protection against index increases. This article examines how to determine investment parameters based on policy provisions required to establish the value of a call option and how such values are used to calculate statutory reserves.

1. Methods

The methods recommended to the NAIC by the Academy of Actuaries are a departure from existing methodology and requires some knowledge of investment terminology. The Glossary given in Appendix A gives definitions which may be helpful in understanding how to apply the methods. The reader should also be knowledgeable in provisions of Actuarial Guideline ZZZ.

1.1 CARVM-UMV

The Updated Market Value (UMV) method is based on the values at valuation of a set of call options which match policy provisions and which cover every possible equity based addition to each elective and non-elective benefit for each future policy year end. In this review, the values of options used for all methods are Fair Values (FV). Market values or premium values could also be used and there should be consistency between assets and liabilities.

Each FV value is accumulated to the appropriate policy year end at the benefit's statutory valuation interest rate and added to the floor for the benefit to determine the Equity Benefit Value. Guideline 33 is applied to these benefits subject to any minimum guarantees.

On the asset side, investments to protect the insurer against index increases are held at FV if FV's were used to determine liabilities.

1.2 MVRM

The Market Value Reserve Method (MVRM) projects market values as in UMV. However, only one FV is used for certain selected policy year ends. Such a policy year end occurs when there is:

a. **Fully Vested Equity Additions**. An unrestricted fully vested equity additions to an elective cash value, as in point-to-point designs, or
b. **Interim Indexes**. An index determination is needed to establish an Exercise Price, as in annual ratchets, or
c. **Scheduled Payouts**. A scheduled payout determined or redetermined, as in equity based annuities, or
d. **End of "Term."** A "term" ends when there is dominant benefit available which will most likely be taken. In this situation, the market value used to establish the index would be the one for this dominant benefit.

MVRM does not use a projected FV's directly but calculates an equity index which would produce this value. Indices for policy year ends not determined in this manner are obtained by assuming compound growth between those so determined and known indices, such as the index at valuation. Benefits for each policy year end are determined from these indices based on policy provisions and Guideline 33 is applied.

This definition is a generalization of the MVRM definition given in the current draft of the AG 33. It is consistent with uses of MVRM noted in the AAA report on EIA reserves and the recent adoption by the NAIC of a submission by Noel Abkemeier on the Black-Scholes Method for MVRM designed for annual ratchet designs. It also reflects the intent of the author of MVRM but its use in any given situation depends on an actuary's judgement as to compliance with the finalized guideline.

On the asset side, investments to protect the insurer against index increases are held at FV.

2.3 EDIM

The reserve for the Enhanced Discounted Intrinsic Value Method (EDIM) at any time is the sum of:

- **a.** **Fixed Component**. The Fixed Component at issue is the reserves produced by either UMV or MVRM and, at maturity time n, it is the guaranteed benefit. At any other time, it is obtained by assuming compound growth between these values and
- **b.** **Equity Component**. The Equity Component is the expected payout, assuming the index at valuation stays the same, discounted back to the valuation date at the valuation rate.

The guideline states that components should take into consideration the elections expected at maturity, e.g. 90% surrender and 10% annuitization.

On the asset side, call options are valued at amortized cost plus the Equity Component. Note that, instead of matching call options, option replication also qualifies as "hedge as required".

3. Purchased Amounts and Exercise Prices for Call Option

UMV requires the calculations of Purchase Amounts and Exercise Prices of call options based on policy provisions for every equity based benefit at each future policy year. Purchase Amounts are not needed by MVRM to determination indices which are based on 100% of FV's of call options with appropriate adjustment to the benefits determined from these indices.

3.1 Formulas

For single premium products, let a Single Premium Unit (SPU) equal the starting index and let the policy provisions and indices be defined:

I_s

Both index at issue and SPU.

I_t

Index at end of any period t, expressed as % of SPU.

P_t^b

Participation rate for benefit b in year t.

GB_t^b

Minimum guaranteed benefit assuming no equity payments, expressed as % of SPU.

IB_t^b

Policy index base from which the index appreciation is measured, expressed as % of SPU.

VB_t^b

Policy value base to which equity appreciation is added to determine the equity benefit value, expressed as % of SPU.

FB_t^b

Policy fund value base on which asset fees are accessed, expressed as % of SPU.

F_t^b

% of FB_t^b to arrive at asset fee.

BC_t^b

Benefit charge as a percent of the Equity Benefit Value (EBV) . This usually occurs only for the cash value benefit.

EP_t^b

Exercise Price, expressed as % of SPU.

FV_t^b

Fair value of call option.

i_t^b

Valuation interest rate.

Policy based provisions given above are not meant to be all inclusive, e.g. some companies may have devised charges which are not given above.

3.1.1 Purchase Amounts

Call options are normally for 100% of the appreciation. The generalized formula for the required Purchase Amount is:

$$PA_t^b = I_s * P_t^b * (1 - BC_t^b)$$

That is, instead of buying options which pay a fraction of the appreciation, the same payout can be achieved by buying the same fraction of the call option paying 100% of the appreciation.

3.1.2 Exercise Prices

Based on policy provisions, the benefit value per Unit of Single Premium for benefit b in year t is:

$$I_s * (VB_t^b + P_t^b * (I_t - IB_t^b) - F_t^b * FB_t^b) * (1 - BC_t^b)$$

From the company's point of view, the source of the payout will come from the guaranteed amount required under policy provisions assuming no equity payout plus the equity payment, if any, based on the appreciation of the index over the Exercise Price for the matching call option:

$$I_s * (GB_t^b + PA_t^b * (I_t - EP_t^b))$$

Eliminating I_t, I_s and replacing the $PA^b{}_t$ with the formula from 3.1.1 gives a generalized Exercise Price formula as a percent of I_s based on the policy provisions given in 3.1:

$$EP_t^b = ((\frac{GB_t^b}{1 - BC_t^b} - VB_t^b) / P_t^b + IB_t^b + F_t^b * FB_t^b / P_t^b)$$

which can be determined at issue but will require redetermination if there are any post issue changes in guaranteed parameters.

3.1.3 Indexes Required under MVRM

The projected index is the FV for a 100% Purchase Amount accumulated at the valuation rate to the projected payout date plus the Exercise Price. Therefore, assuming the period from issue to payout is n:

$$I_t = EP_t^b + FV_t^b * (1 + i_t^b)^{n-t}$$

The growth between any two indices at l and m:

$$g = (I_m / I_l)^{1/(m-l)} - 1$$

The index as a percent of I_s at any time t between l and m:

$$I_t = I_l * (1 + g)^{t-l}$$

3.2 Examples

Table A gives the assumptions used in Tables B through J . For example, the Exercise Price as a percent of I_S for the 5th policy year surrender value under UMV is:

$$((\frac{.9*1.03^5}{1-.03}-1)/.45+1)=116.80\%$$

That is, the index at the end of 5 years would have to be greater than 116.80% of the issue index before the equity surrender value would be greater than the guaranteed cash value.

The percent of the appreciation as a payout is:

$$(1-.03)*.45=43.65\%$$

That is, it is only necessary to purchase 43.65% of a call option paying 100%.

If there were surrender charge free withdrawal, it would be also necessary to calculate EP and PA for this benefit separately by the setting the surrender charge to 0 in the above formulas giving an EP of 109.63% and a PA of 45% before adjustment for the portion that is charge free. If the free charge amount were 10%, then the PA would be 10% of 45% and the PA for the surrender balance would be 90% of 43.65%.

3.3 Demonstration of Compliance

The following tables demonstrate that an insurer on a surrender is not financially affected by index movement as required by the definition of Exercise Price.

	Scenario 1	Scenario 2	Scenario 3
Index at End of Five Years	110.00%	116.80%	170.00%
Based on Policy Provisions			
100% of Appreciation	10.00%	16.80%	70.00%
Vested at 45%	4.50%	7.56%	31.50%
Equity Account Value	104.50%	107.56%	131.50%
Surrender Charge at 3%	3.13%	3.23%	3.95%
Equity Surrender Value	101.37%	104.33%	127.56%
Not less than			
Guar. CV= .9*1.03^5	104.33%	104.33%	104.33%
Required Equity Addition	**0.00%**	**0.00%**	**23.22%**
Option Payout			
Exercise Price	116.80%	116.80%	116.80%
Index Appreciation over EP	0.00%	0.00%	53.20%
Option Payout at 43.65%	**0.00%**	**0.00%**	**23.22%**

Similar demonstrations can be shown for annuitization and death benefits.

3.4 Caps

3.4.1 CARVM-UMV

Under UMV, when there is cap on an equity addition, it is necessary to determine the reduction in Fair Value for the amount not paid due to the cap. This Fair Value would be subtracted from the unrestricted Fair Value before projection.

In the above surrender example - Scenario 3, if the index at the end of any year is capped at 10% annual growth, the cap in the fifth year would be 1.15 = 161.05%. This also would be the Exercise Price used in determining the Fair Value of the reduction. The Purchase Amount would be the same.

Scenario 3

	No Cap	**With Cap**	**Deduction**
Index at End of Five Years	170.00%	161.05%	170.00%
Based on Policy Provisions			
100% of Appreciation	70.00%	61.05%	8.95%
Vested at 45%	31.50%	27.47%	4.03%
Equity Account Value	131.50%	127.47%	4.03%
Surrender Charge at 3%	3.94%	3.82%	0.12%
Equity Surrender Value	127.56%	123.65%	3.91%
Not less than			
Guar. CV= .9*1.03^5	104.33%	104.33%	0%
Required Equity Addition	**23.22%**	**19.31%**	**3.91%**
Option Payout			
Exercise Price	116.80%	116.80%	161.05%
Index Appreciation over EP	53.20%	44.25%	8.95%
Option Payout at 43.65%	**23.22%**	**19.31%**	**3.91%**

Note that the Fair Value of the deduction is theoretical as if would only exist if the insurer actually issued such an option to a third party.

3.4.2 MVRM

The same approach can be used as in UMV. However, since MVRM projects the index, the cap can be applied to the full index in determining policy values.

4. Application of MVRM to Other Policy Designs

There are policy designs which require additional clarifications before MVRM can be applied.

4.1 Annual Ratchet Designs

Determine the index at the end of the immediate policy year after valuation. Use the year end index as the starting index for the following policy year. Repeat this process until all needed policy year end indices are obtained. Assume compound growth to determine other needed indices during a policy year.

Sample calculations are given in Exhibit K for a three year Asian design with valuation taking place at the middle of the second policy year.

The function used to calculate Fair Value's uses an analytical approximation based on an Edgeworth expansion of the probability distribution of the undetermined component of the average. This is reliable for volatilities of up to 30% which makes it historically suitable for indices such as S & P.

In determining an Exercise Price, the available cash value at the end of any policy year is the higher of the guaranteed cash value and the vested equity surrender value assuming no new equity additions in that policy year. Exercise Prices for the three years are:

$$EP_1^{sv} = 1000 * ((\frac{.9 * 1.03}{1 - .1} - 1.00) / .95 + 1.00 + 1.00 * .005 / .95) = 1036.84$$

$$EP_2^{sv} = 1000 * ((\frac{.9 * 1.03^2}{1 - .07} - 1.0235) / .9 + 1.09 + 1.0235 * .005 / 9) = 1099.22$$

$$EP_3^{sv} = 1000 * ((\frac{1.03447}{1 - .0} - 1.03447) / .85 + 1.2247 + 1.03447 * .005 / .85) = 1230.77$$

In the third year, it is necessary to use the vested equity surrender value as it is greater than the guaranteed surrender value.

It would not be consistent with the definition of Exercise Price to deduct the asset fee from the equity payout instead of adjusting the Exercise Price since there may be no or a limited payout. The financial effect on the company would not be neutral.

The percent of the appreciation as a payout is:

Year	Purchase Amount
1	(1 - .10) * .95 = 85.50%
2	(1 - .07) * .90 = 83.70%
3	(1 - .00) * .85 = 85.00%

The following table demonstrates that an insurer is not affected by index movement.

Second Policy Year

	Scenario 1	Scenario 2	Scenario 3
Index at Start	109.00%	109.00%	109.00%
Aver. Index at End	105.00%	109.92%	120.00%
Based on Policy Provisions			
100% of Appreciation	00.00%	0.92%	11.00%
Vested at 90.00%	00.00%	0.83%	9.90%
Asset Fee	00.00%	0.51%	0.52%
Prior EAV	102.35%	102.35%	102.35%
Equity Account Value	102.35%	102.67%	111.73%
Surrender Charge at 7%	7.16%	7.19%	7.82%
Equity Surrender Value	95.19%	95.48%	103.91%
Not less than			
Guar. CV= .9*1.03^2	95.48%	95.48%	95.48%
Required Equity Addition	**0.00%**	**0.00%**	**8.43%**
Option Payout			
Exercise Price	109.92%	109.92%	109.92%
Index Appreciation over EP	0.00%	0.00%	10.08%
Option Payout at 83.70%	**0.00%**	**0.00%**	**8.43%**

That is, the amount of money the company has on hand always equals the amount needed to pay an elective surrender.

In an annual ratchet design, it would be necessary to also determine the indices at the start of each year assuming no cap since these indices are needed to determine the Exercise Price for each year.

4.2 Payout Annuities

Indices are determined as in Annual Ratchet for each policy year end that the annuity benefit is redetermined. Annuities payment are based on these indices.

Exercise Price would normally be the index at the start of each redetermination period as features such as asset fees would not normally occur.

5. Effect of Volatility

Tables E and F give the effect of an increase in short term volatility from 15% to 25% 6 months after issue under UMV. The other two methods are not affected. The following table shows book profits for the first 6 month and, for simplicity, assumes:

a. guaranteed maturity value is provided with a 7% no coupon bond investment of $689.31 yielding $23.72 in the first 6 months,
b. insurer is hedged to pay the maturity benefit for all annuitants,
c. minor effect of elective and non-elective benefits during the 6 month period are ignored,
d. 100% are assumed to elect the maturity benefit under EDIM and,
e. to make the results comparable, amortization of the call option is assumed at the risk free rate of 6.40%. Many companies use a straight line amortization.

	15% Short Term Volatility			25% Short Term Volatility		
Index	**UMV**	**MVRM**	**EDIM**	**UMV**	**MVRM**	**EDIM**
		Asset Increase				
950	(30.28)	(30.28)	(8.52)	(30.28)	(30.28)	(8.52)
1200	90.41	90.41	15.59	90.41	90.41	15.59
			Reserve Increase			
950	(11.01)	(10.85)	11.57	19.35	(10.85)	11.57
1200	115.55	110.50	35.68	147.29	110.50	35.68
			Book Profit			
950	4.45	4.29 *	3.63	(25.91)	4.29*	3.63
1200	(1.42)	3.63 *	3.63	(33.16)	3.63 *	3.63

* The minor difference is due to varying effect on reserves of death benefits in different market scenarios while, for simplicity as noted in b. above, assets ignore any effect of deaths. This does not happen with EDIM as the assets are increased by the Equity Component and the Fixed Component is independent of market conditions.

In these examples, in an up market with 15% volatility, higher early annuitization benefits can come into play under UMV reducing book profits. The effect is increased when short term volatility increases. In fact, the resulting reserves is over 3.5% higher than the needed reserve. If the short term volatility were to increased to 35%, as happened to the S & P Index during 1998, the unneeded reserve increase could be higher than 7%.

On the asset side, an insurer cannot protect itself against these swings since its purchases can only be for expected utilization and not 100% as CARVM assumes. When the method elected is UMV, it would be necessary to have less competitive guaranteed annuitization rates to remove these unneeded swings. Reducing annuitization rates may have adverse marketing consequences and, on average, will reduce tax reserves which are frequently driven by early annuitization due to later elective benefits being discounted at higher tax valuation rates.

6. Critique of Methods

The three methods give reasonable results most of the time. The common problem is the availability of third party option values, especially for options which are theoretical in nature. If the only viable approach is for a company to calculate option values, some objectivity is needed to determine:

- a. **Volatility assumptions**. Implied volatility for major indices are published such as those from SBC Warburg Dillon Read Inc. for the S&P 500.
- b. **Option Loadings**. This occurs when PV’s are required.
- c. **Suitable Formulas**. There a few reliable accepted formulas, the principal being Black Scholes. However, these formula are not comprehensive and different formulas give varying results.

The method least affected is EDIM and UMV is most affected.

6.1 UMV

- a. **Complex**. When option values are available, reserves can be readily determined although the method is rigid and calculations are somewhat complex.
- b. **Interim Indices**. Method cannot be readily applied when interim indices are required to determine option values.

c. **Mismatches**. In practice, UMV is more likely to give results which are contrary to the desirable goal of consistency between assets and liabilities.

6.2 MVRM

Compared to UMV, MVRM is:

a. **Less complex**. Guideline 33 has substantially increased the number of calculation needed to determine reserves. The overlay of UMV will compound this further.
b. **Flexible**. Provides flexibility to avoid unnecessary reserves. This could also be considered a weakness from a regulator's point of view since there may be disagreement on what constitutes unnecessary reserves.
c. **Surrogate Option Values**. Since FV's are only used to determine indices, option values which exactly match policy provisions are not essential. As actual policy provisions are used to calculate reserves, good results can be achieved. This would be useful in situations where there are no acceptable formulas for a particular policy design.

6.3 EDIM

The only real negative to this relatively simple approach is the "hedge as required" restrictions and the consequence of having to change midstream to one of the other methods on the existing in force if the "hedge as required" test fails. Further, some companies may feel that such restrictions reduce potential earnings or the use of investments strategies which cut across products or product lines.

Acknowledgement

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Appendix A

Glossary of Key Terms

Call Option. A call option is the right to receive the appreciation of an index over a fixed price (Exercise Price). Additional information required to determine its value is:

a. **Current index, risk-free interest, dividends and contract period**. These are primarily used to project the index mean. The growth is essentially the difference between the risk-free interest and the dividend rate.
b. **Volatility**. By definition, volatility is basically an annual standard deviation of a random distribution of a index change expressed as a percent. It is subjective and will vary between call option issuers or even with the time of day. Until recently when it has reached 35%, the volatility of the S&P Index has varied between 10% and 30%. The lower the percent, the lower the expected index fluctuations, the lower the FV's.

If the appreciation is based on an average over a period of time, the further information needed is:

c. **Start Date**. The date the average starts.
d. **Frequency**. The sampling frequency, e.g. monthly.
e. **Average to Date**. The average to date when the valuation date is after the Start Date.

Equity Benefit Value. This is a policy benefit after the addition of any equity index appreciation.

Exercise Price. The Exercise Price of a matching call option for a policy benefit is determined so that any payout from the issuer to the insurer is exactly equal to the amount needed to be added to a policy benefit to get the Equity Benefit Value. That is, the insurer is not affected by market performance.

Fair Value of a Call Option. This is the value of a call option which is projected to result in no profit to the buyer or seller.

Market Value of a Call Option. When an option is traded on an exchange, the exchange quote is the Market Value. The type of call options needed to cover insurance risks would not be traded on any exchange.

Premium Value of a Call Option. This is the price that a trader would sell an option and would normally contain margins for profit and expenses in addition to the Fair Value. Margins could also be obtained by increasing volatility assumptions.

Purchase Amount of a Call Option (Purchase Amount). This is the amount of the option that needs to be purchased to cover the equity risk.

Table A - Assumptions for Table B through I

Policy Year	Volatility Moderate	Volatility High	Annual Rate Dividend	Annual Rate Risk Free
1	15.00%	25.00%	1.85%	5.65%
2	15.00%	25.00%	1.85%	6.06%
3	15.00%	21.00%	1.85%	6.20%
4	15.00%	18.00%	1.85%	6.27%
5	15.00%	15.00%	1.85%	6.37%
6	15.00%	15.00%	1.85%	6.38%
7	15.00%	15.00%	1.85%	6.40%

	Exercise Prices			Option Purchase Amounts		
	Cash value	Annuity	Death Ben.	Cash Value	Annuity	Death Ben.
1	983.87	1,037.50	1,000.00	18.60%	80%	70%
2	1,063.02	1,067.67	1,000.00	23.50%	90%	70%
3	1,117.38	1,103.03	1,000.00	28.50%	90%	70%
4	1,157.61	1,139.45	1,000.00	33.60%	90%	70%
5	1,168.03	1,176.97	1,000.00	43.65%	90%	70%
6	1,175.60	1,215.61	1,000.00	53.90%	90%	70%
7	1,152.69	1,255.42	1,000.00	70.00%	90%	70%

	Participation Rates			Guaranteed Benefits		
	Cash Value	Annuity	Death Ben.	Cash Value	Annuity	Death Ben.
1	20%	90%	70%	927.00	1,030.00	1,000
2	25%	90%	70%	954.81	1,060.90	1,000
3	30%	90%	70%	983.45	1,092.73	1,000
4	35%	90%	70%	1,012.95	1,125.51	1,000
5	45%	90%	70%	1,043.35	1,159.27	1,000
6	55%	90%	70%	1,074.65	1,194.05	1,000
7	70%	90%	70%	1,106.89	1,229.87	1,000

	Valuation Rates			Pv Annuity/	Surrender	Mort. Rate
	Cash Value	Annuity	Death Ben.	Purc. Price	Charge	Per 1000
1	5.25%	6.75%	6.75%	91%	7%	7.290
2	5.25%	6.75%	6.75%	91%	6%	7.782
3	5.25%	6.75%	6.75%	91%	5%	8.338
4	5.25%	6.75%	6.75%	91%	4%	8.983
5	5.25%	6.75%	6.75%	91%	3%	9.740
6	5.00%	6.50%	6.75%	91%	2%	10.630
7	5.00%	6.50%	6.75%	91%	0%	11.664

Table A - Assumptions for Table B through I (cont)

1. **Deaths are assumed to occur before elective benefits in any policy year**
2. **Present value of DB's is an accumulation**
3. **Benefit is the greater of the Equity Benefit and the Guaranteed Benefit**
4. **Reserves are the maximum value of the pv's of either the cash value or the annuity purchased plus the accumulated death benefits**

5. PV of annuity at annuitization is 91% of the amount available for purchase

Table B - CARVM-UMV

Reserves	**At Issue**
Index	**1,000.00**
Volatility	**Moderate**

Policy Year	Fair Value	Fv Of Pur. Amt.	Proj. Equity At Year End	Total Proj. Ben.	Pv Disc.'D For I & Q
			Cash Value Benefits		
1	86.62	16.11	16.96	943.96	890.33
2	91.84	21.58	23.91	978.72	870.25
3	106.15	30.25	35.27	1,018.73	853.46
4	123.55	41.51	50.94	1,063.90	839.24
5	150.95	65.89	85.10	1,128.45	837.52
6	175.71	94.71	126.91	1,201.56	850.34
7	210.19	147.14	207.03	1,313.92	875.25

Policy Year	Fair Value	FV Of Pur. Amt.	Proj. Equity At Year End	Total Proj. Ben.	Pv Ann. At Annuitiz.	Pv Disc. For I & Q
			Annuitization Benefits			
1	59.11	47.29	50.48	1,080.48	983.24	914.35
2	89.79	80.81	92.09	1,152.99	1,049.22	906.90
3	111.97	100.77	122.58	1,215.31	1,105.93	888.01
4	130.49	117.44	152.51	1,278.01	1,162.99	866.92
5	147.62	132.85	184.17	1,343.44	1,222.53	845.37
6	161.30	145.17	211.82	1,405.88	1,279.35	831.52
7	173.81	156.43	243.09	1,472.96	1,340.39	808.49

Policy Year	Fair Value	FV Of Pur. Amt.	Proj. Equity At Year End	Total Proj. Ben.	Pv Disc. For I & Q	Reserve
			Death Benefits			
1	77.60	54.32	57.99	1,057.99	7.23	921.58
2	122.97	86.08	98.09	1,098.09	14.67	921.57
3	160.83	112.58	136.95	1,136.95	22.35	910.36
4	194.00	135.80	176.35	1,176.35	30.29	897.21
5	224.87	157.41	218.21	1,218.21	38.58	883.95
6	250.94	175.66	259.94	1,259.94	47.25	897.59
7	274.80	192.36	303.87	1,303.87	56.38	931.63

highest statutory occurs at end of policy year 7 931.63

Table C – CARVM-UMV

Reserves At	**6 Months**
Index At	**$950.00**
Volatility	**Moderate**

Cash Value Benefits

Policy Year	Fair Value	FV Of Pur. Amt.	Proj. Equity At Year-End	Total Proj. Ben.	Pv Disc.'D For I And Q
1	33.20	6.18	6.34	933.34	906.44
2	48.62	11.43	12.34	967.15	885.48
3	65.39	18.63	21.18	1,004.63	866.63
4	83.38	28.01	33.51	1,046.47	849.98
5	109.46	47.78	60.15	1,103.50	843.30
6	133.68	72.05	94.23	1,168.88	850.74
7	166.94	116.85	160.46	1,267.35	868.24

Annuitization Benefits

Policy Year	Fair Value	FV Of Pur. Amt.	Proj. Equity At Year-End	Total Proj. Ben.	Pv Ann. At Annuitiz.	Pv Disc. For I and Q
1	16.18	12.94	13.37	1,043.37	949.47	915.60
2	47.12	42.40	46.77	1,107.67	1,007.98	903.48
3	70.00	63.00	74.18	1,166.91	1,061.89	884.18
4	89.14	80.23	100.84	1,226.35	1,115.98	862.64
5	106.59	95.93	128.71	1,287.98	1,172.06	840.44
6	120.94	108.85	153.91	1,347.96	1,226.64	825.78
7	134.04	120.64	181.66	1,411.53	1,284.49	802.48

Death Benefits

Policy Year	Fair Value	FV Of Pur. Amt.	Proj. Equity At Year-End	Total Proj. Ben.	Pv Disc. For I and Q	Reserve
1	27.09	18.96	19.59	1,019.59	3.60	919.20
2	72.98	51.08	56.34	1,056.34	11.03	914.51
3	111.12	77.78	91.58	1,091.58	18.67	902.85
4	144.72	101.31	127.33	1,127.33	26.57	889.21
5	175.95	123.17	165.25	1,165.25	34.79	878.09
6	202.81	141.97	203.34	1,203.34	43.38	894.12
7	227.48	159.24	243.46	1,243.46	52.41	920.65
	highest statutory reserve occurs at the end of year				**7**	**920.65**

Table D - CARVM-UMV

Reserves At	**6 Months**
Index At	**$1,200.00**
Volatility	**Moderate**

Policy Year	Fair Value	Fv Of Purc. Amt.	Proj. Equity At Year End	Total Proj. Ben.	Pv Disc.'D For I & Q
Cash Value Benefits					
1	233.47	43.43	44.55	971.55	943.55
2	212.83	50.01	54.00	1,008.81	923.63
3	220.91	62.96	71.55	1,055.01	910.08
4	237.71	79.87	95.54	1,108.49	900.37
5	270.08	117.89	148.41	1,191.76	910.76
6	298.51	160.90	210.42	1,285.07	935.31
7	339.34	237.54	326.19	1,433.08	981.78

Policy Year	Fair Value	FV Of Pur. Amt.	Proj. Equity At Year-End	Total Proj. Ben.	Pv Ann. At Annuitiz.	Pv Disc. For I and Q
Annuitization Benefits						
1	183.56	146.85	151.73	1,181.73	1,075.37	1,037.02
2	209.37	188.43	207.83	1,268.73	1,154.54	1,034.85
3	230.11	207.09	243.83	1,336.56	1,216.27	1,012.72
4	248.13	223.32	280.68	1,406.19	1,279.63	989.15
5	265.32	238.79	320.38	1,479.65	1,346.48	965.51
6	278.62	250.75	354.55	1,548.60	1,409.22	948.70
7	290.80	261.72	394.10	1,623.97	1,477.81	923.26

Policy Year	Fair Value	FV Of Pur. Amt.	Proj. Equity At Year-End	Total Proj. Ben.	Pv Disc. For I and Q	Reserve
Death Benefits						
1	218.23	152.76	157.83	1,157.83	4.09	1,041.11
2	262.18	183.53	202.42	1,202.42	12.55	1,047.39
3	301.90	211.33	248.81	1,248.81	21.29	1,034.01
4	336.85	235.80	296.36	1,296.36	30.37	1,019.52
5	369.28	258.50	346.82	1,346.82	39.87	1,005.38
6	396.18	277.32	397.20	1,397.20	49.85	998.55
7	420.51	294.36	450.06	1,450.06	60.38	1,042.16
	highest statutory reserve occurs at the end of year				**2**	**1,047.39**

Table E - CARVM-UMV

Reserves At	**6 Months**
Index At	**$950.00**
Volatility	**High**

Policy Year	Fair Value	Fv Of Purc. Amt.	Proj. Equity At Year-End	Total Proj. Ben.	Pv Disc.'D For I And Q
			Cash Value Benefits		
1	59.83	11.13	11.42	938.42	911.37
2	93.63	22.00	23.76	978.57	895.93
3	99.69	28.41	32.29	1,015.74	876.21
4	103.32	34.72	41.53	1,054.48	856.50
5	109.46	47.78	60.15	1,103.50	843.30
6	133.68	72.05	94.23	1,168.88	850.74
7	166.94	116.85	160.46	1,267.35	868.24

Policy Year	Fair Value	FV Of Pur. Amt.	Proj. Equity At Year-End	Total Proj. Ben.	Pv Ann. At Annuitiz.	Pv Disc. For I and Q
			Annuitization Benefits			
1	40.46	32.37	33.45	1,063.45	967.74	933.22
2	92.04	82.83	91.36	1,152.26	1,048.56	939.85
3	104.35	93.91	110.57	1,203.30	1,095.00	911.75
4	109.05	98.15	123.36	1,248.87	1,136.47	878.48
5	106.59	95.93	128.71	1,287.98	1,172.06	840.44
6	120.94	108.85	153.91	1,347.96	1,226.64	825.78
7	134.04	120.64	181.66	1,411.53	1,284.50	802.48

Policy Year	Fair Value	FV Of Pur. Amt.	Proj. Equity At Year-End	Total Proj. Ben.	Pv Disc. For I and Q	Reserve
			Death Benefits			
1	53.38	37.37	38.61	1,038.61	3.67	936.89
2	117.51	82.26	90.72	1,090.72	11.34	951.19
3	143.47	100.43	118.24	1,118.24	19.17	930.92
4	162.41	113.69	142.89	1,142.89	27.18	905.66
5	175.95	123.17	165.25	1,165.25	35.39	878.70
6	202.81	141.97	203.34	1,203.34	43.99	894.73
7	227.48	159.24	243.46	1,243.46	53.02	921.26
	highest statutory reserve occurs at the end of year				**2**	**951.19**

Table F - CARVM-UMV

Reserves At	**6 Months**
Index At	**$1,200.00**
Volatility.	**High**

Policy Year	Fair Value	Fv Of Pur. Amt.	Proj. Equity At Year-End	Total Proj. Ben.	Pv Disc.'D For I and Q
			Cash Value Benefits		
1	242.64	45.13	46.30	973.30	945.25
2	251.18	59.03	63.74	1,018.55	932.54
3	252.88	72.07	81.91	1,065.36	919.02
4	256.39	86.15	103.04	1,116.00	906.46
5	270.08	117.89	148.41	1,191.76	910.76
6	298.51	160.90	210.42	1,285.07	935.31
7	339.34	237.54	326.19	1,433.08	981.78

Policy Year	Fair Value	FV Of Pur. Amt.	Proj. Equity At Year-End	Total Proj. Ben.	Pv Ann. At Annuitiz.	Pv Disc. For I and Q
			Annuitization Benefits			
1	199.00	159.20	164.48	1,194.48	1,086.98	1,048.21
2	248.34	223.50	246.51	1,307.41	1,189.74	1,066.40
3	260.92	234.83	276.48	1,369.21	1,245.98	1,037.46
4	266.06	239.45	300.96	1,426.46	1,298.08	1,003.41
5	265.32	238.79	320.38	1,479.65	1,346.48	965.51
6	278.62	250.75	354.55	1,548.60	1,409.22	948.70
7	290.80	261.72	394.10	1,623.97	1,477.81	923.26

Policy Year	Fair Value	FV Of Pur. Amt.	Proj. Equity At Year-End	Total Proj. Ben.	Pv Disc. For I and Q	Reserve
			Death Benefits			
1	229.12	160.38	165.71	1,165.71	4.12	1,052.33
2	291.89	204.32	225.35	1,225.35	12.73	1,079.13
3	323.70	226.59	266.78	1,266.78	21.60	1,059.07
4	348.41	243.89	306.53	1,306.53	30.76	1,034.17
5	369.28	258.50	346.82	1,346.82	40.26	1,005.77
6	396.18	277.32	397.20	1,397.20	50.23	998.93
7	420.51	294.36	450.06	1,450.06	60.76	1,042.54
	highest statutory reserve occurs at the end of year				**2**	**1,079.13**

Table G - MVRM CALCULATIONS

		Reserves Index		At Issue $1,000.00	
Pol. Year	**Fair Value**	**Projected Val. Rate**	**Proj. Value**	**Index At Maturity**	**Index Growth**
7	210.19	5.00%	295.76	1,448.46	5.44%

	Year End Index	Cash Value	Benefit Annuity	Death Ben.
1	1,054.35	939.40	1,043.48	1,038.05
2	1,111.66	964.67	1,100.50	1,078.16
3	1,172.09	996.59	1,154.88	1,120.46
4	1,235.79	1,036.06	1,212.21	1,165.06
5	1,302.97	1,098.28	1,272.67	1,212.08
6	1,373.79	1,177.44	1,336.41	1,261.65
7	1,448.46	1,313.92	1,403.61	1,313.92

	Pv Annuity At		Present Values At Valuation		
	Annuitization	**Cash Value**	**Annuity**	**Acc. Db's**	**Reserve**
1	949.57	886.04	883.04	7.09	893.13
2	1,001.45	857.75	865.61	14.40	880.01
3	1,050.94	834.92	843.85	21.96	865.82
4	1,103.12	817.28	822.29	29.83	852.12
5	1,158.13	815.13	800.83	38.08	853.20
6	1,216.13	833.27	790.43	46.77	880.04
7	1,277.29	875.25	770.42	55.97	931.21

highest stat. res. occur at policy year end 7 931.21

Table H - MVRM CALCULATIONS

Reserves	**6 Months**
Index	**$950.00**

Pol. Year	Fair Value	Projected Val. Rate	Proj. Value	Index At Maturity	Index Growth
7	166.94	5.00%	229.23	1,381.93	5.94%

Year-End	Index	Benefit Cash Value	Annuity	Death Ben.
1	977.79	930.00	1,030.00	1,000.00
2	1,035.82	954.81	1,060.90	1,025.07
3	1,097.30	983.45	1,092.73	1,068.11
4	1,162.42	1,012.96	1,146.18	1,113.70
5	1,231.42	1,067.98	1,208.28	1,161.99
6	1,304.50	1,140.85	1,274.05	1,213.15
7	1,381.93	1,267.35	1,343.74	1,267.35

	Pv Annuity At Annuitization	Present Values At Valuation Cash Value	Annuity	Acc. Db's	Reserve
1	937.30	903.20	903.87	3.53	907.40
2	965.42	874.18	865.33	10.74	884.92
3	994.38	848.36	827.97	18.22	866.58
4	1,043.03	822.77	806.25	26.02	848.79
5	1,099.53	816.16	788.43	34.22	850.38
6	1,159.39	830.34	780.51	42.88	873.22
7	1,222.80	868.24	763.94	52.08	920.33

highest stat. res. occurred at policy year end 7 920.33

Table I - MVRM CALCULATIONS

Reserves	6 Months
Index	$1,200.00

Pol. Year	Fair Value	Projected Val. Rate	Proj. Value	Index At Maturity	Index Growth
7	339.34	5.00%	465.98	1,618.68	4.71%

	Year End Index	Cash Value	Benefit Annuity	Death Ben.
1	1,227.95	969.43	1,182.36	1,159.56
2	1,285.81	1,003.14	1,257.23	1,200.07
3	1,346.40	1,043.79	1,311.76	1,242.48
4	1,409.84	1,092.20	1,368.86	1,286.89
5	1,476.27	1,171.66	1,428.65	1,333.39
6	1,545.84	1,268.32	1,491.25	1,382.09
7	1,618.68	1,433.08	1,556.81	1,433.08

	Pv Annuity At Annuitization	Cash Value	Present Values At Valuation Annuity	Acc. Db's	Reserve
1	1,075.95	941.49	1,037.57	4.10	1,041.67
2	1,144.08	918.43	1,025.47	12.53	1,038.00
3	1,193.70	900.41	993.93	21.23	1,015.17
4	1,245.66	887.13	962.89	30.25	993.14
5	1,300.07	895.39	932.23	39.65	971.88
6	1,357.04	923.12	913.57	49.52	972.65
7	1,416.70	981.78	885.08	59.93	1,041.71

highest stat. res. occurred at policy year end 7 1041.71

Table J - EDIM CALCULATIONS

pol. year	exercise index	FV of PA	MVRM reserve at issue	guar. mat. value	growth	6 months Fix. Comp. reserve
7	1152.69	147.13	931.21	1106.89	2.50%	942.78

continuous annuity from issue at risk free rate of 6.40%	5.68
continuous annuity from valuation	5.35
amortization factor	94.21%
amortized FV of PA after 6 months	138.61
when index at $950.00	
Equity Component	0.00
Reserves	942.78
Increase in reserves from issue	11.57
Call Option Asset	138.61
Increase in asset from issue	(8.52)
when index at 1,200	
Equity Component	24.11
Reserves	966.89
Increase in reserves from issue	35.68
Call Option Asset	162.72
Increase in asset from issue	15.59

Table K - MVRM Annual Ratchet/Asian Design

Average Index Projected	Policy Year 1	2	3
index at valuation/year start	1,000.00	1,075.00	1,224.69
exercise price	1,036.84	1,099.22	1,230.77
average index from year start to settlement		1,055.00	0
sampling frequency		monthly	monthly
expiry date		year end	year end
settlement date		mid year	year start
date when averaging starts		1st month	1st month
fair value		8.44	63.38
valuation rate	5.25%	5.25%	5.25%
actual or projected payout at year end	0.00	8.66	66.71
average index - actual or EP plus proj.	**1,030.00**	**1,107.87**	**1,297.48**

Calculation of Policy Year-end Indices

	Policy Year 1	2	3
policy month end	**actual index**	**actual/ projected Index**	**projected index**
1	1,001.00	1,050.00	1,235.53
2	1,020.00	1,055.00	1,246.46
3	1,005.00	1,020.00	1,257.49
4	1,010.00	1,060.00	1,268.62
5	1,007.00	1,070.00	1,279.85
6	1,050.00	1,075.00	1,291.18
7	1,040.00	1,098.61	1,302.61
8	1,020.00	1,122.74	1,314.14
9	1,022.00	1,147.40	1,325.77
10	1,040.00	1,172.61	1,337.50
11	1,055.00	1,198.36	1,349.34
12	1,090.00	1,224.69	1,361.28
projection solved at		**2.1965%**	**.8851%**
12 month average	**1,030.00**	**1107.87**	**1297.48**

	Policy Year		
	1	**2**	**3**
Equity AV			
par rate	95%	90%	85%
appreciation * par rate	28.50	16.08	61.87
asset fee at 0.5% of initial/prior EAV	5.00	5.12	5.17
year end addition to EAV	23.50	10.96	56.70
year end db/annuitization	**1,023.50**	**1,034.47**	**1,091.17**
Cash Value			
surrender charge %	10%	7%	0%
surrender charge	102.35	72.41	0.00
ECV	921.15	962.05	1,091.17
CV=Max (ECV and GCV)	**927.00**	**962.05**	**1,091.17**
next year surr. charge	71.65	0.00	
min. next year ECV	951.86	1,034.47	
min. next year end CV	**954.81**	**1,034.47**	
reserve calculations - present values at valuation			
accumulated db's disc. at	6.75%	3.90	12.12
annuity discounted at	6.75%	934.49	915.69
cash value discounted	5.25%	934.10	998.22
year end reserve		938.40	1,010.34
statutory reserve			**1,010.34**