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# Market-Consistent Term Insurance Premiums and Liabilities

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What does it mean to be market-consistent? Can insurance products be priced and valued in a market-consistent fashion?

John Jacob discusses market consistency in his article "Actuaries and Assumptions" in the March 2011 issue of this newsletter.[1] The emphasis in his article is the selection of inputs or assumptions. He makes the observation, for example, that a truly market-consistent approach would use a binomial function for the lapse assumptions.

Anna Rita Bacinello uses a binomial function in her demonstration of calculating a fair premium for an annual premium participating life contract.[2] She assumes that a contract terminates if its cash value exceeds what she refers to as the "continuation value." The contract does not terminate if the value to continue is greater than the value to surrender. Bacinello uses market-consistent techniques for the financial elements of the pricing and refers to the result as a fair premium. "In this connection, we will term *market values* (or *prices*) the outcomes from the valuation of purely financial elements, and *fair values* (or *fair premiums*) the final results obtained by combining financial and actuarial valuation tools."[3]

There appear to be limits to which market-consistent techniques are, and perhaps can be, applied to insurance. This article addresses the limits to market consistency by questioning whether there in fact can be a market-consistent level annual premium for a term life insurance contract. For simplicity, the example considers only mortality. Interest, expenses and margins are disregarded. Expected mortality is from the 1990–95 Society of Actuaries basic male age nearest birthday table.

## PREMIUM CALCULATION

Consider a two-year term insurance contract. The policyholder pays the first premium for a death cover for one year and for the option to renew for a second year. If the market for insurance had the characteristics of markets contemplated by the term "market-consistent," there would be a robust market for term insurance and the policyholder would, at the end of the first year, consider if he should pay the second premium or cancel his contract and purchase a new one-year contract. If the policyholder is healthy, he will purchase a new contract if the premium is less than the second-year premium on his original contract. This means that the insurer that issues the two-year contract must charge a one-year select term rate in the second year of the contract to avoid having only unhealthy lives in its portfolio after the first year.

The premium for the first year then must be the amount needed for the death benefits in the first year and for the option to renew. The option to renew is easily priced. It is the amount of extra mortality associated with the second year after underwriting as compared to the expected mortality for one year of newly underwritten mortality.

Because of the option to renew, there is value in the contract. The presence of the value means that the insurer does not anticipate surrenders. If, at the end of the first year, the policyholder were to decide that he no longer needs insurance, he would not terminate his contract; rather, he would sell it for its value. It would remain in-force and continue to be an obligation of the insurer.

The following table and calculations illustrate this concept.

**Table 1: Expected benefits for a one-year and two-year term insurance contract, both terminating at age 55**

| Death benefits for 10,000,000 exposure | | |
|---|---|---|
| | Attained age | |
| | 53 | 54 |
| Two-year term at age 53 | 1,406 | 1,990 |
| One-year term at age 54 | | 1,470 |

The price of the option, the excess benefits in the second year of a two-year contract as compared to a one-year contract, is 520 = 1990 - 1470. The first-year premium must be 1926 (1406+520), which is the price of the insurance for the first year plus the price for the guarantee that the policyholder can purchase insurance for the second year at the same rate as a person who has just been through underwriting and has qualified for new insurance. The policyholder pays for the guaranteed ability to purchase insurance in the second year. With these market constraints in effect, the first-year premium is 1926 and the second-year premium is 1470. The premium pattern is not level. It also doesn't follow the select and ultimate mortality pattern associated with annually renewable term insurance.

The pricing approach can be extended to longer terms. If a policyholder purchases an n-year term policy, the premium in each of the renewal years must be the same as for a newly underwritten contract for the remaining term. The premium for the first year then is the first-year mortality plus the extra mortality over the next n-1 years as compared to a new contract for n-1 years.

The following tables illustrate premiums for term contracts ranging from one to 10 years in duration, all terminating at age 55. Table 3 shows the premiums based on the expected payments in Table 2.

**Table 2: Expected payments for 10,000,000 exposure for contracts with terms ranging from one to 10 years**

| Age at issue | Term | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 45 | 10 | 685 | 968 | 1,232 | 1,493 | 1,795 | 2,173 | 2,648 | 3,148 | 3,631 | 4,178 | 21,951 |
| 46 | 9 | | 759 | 1,039 | 1,287 | 1,580 | 1,952 | 2,390 | 2,907 | 3,483 | 4,049 | 19,447 |
| 47 | 8 | | | 836 | 1,100 | 1,346 | 1,701 | 2,147 | 2,647 | 3,205 | 3,872 | 16,853 |
| 48 | 7 | | | | 903 | 1,190 | 1,489 | 1,878 | 2,336 | 2,859 | 3,480 | 14,135 |
| 49 | 6 | | | | | 978 | 1,309 | 1,665 | 2,087 | 2,551 | 3,099 | 11,689 |
| 50 | 5 | | | | | | 1,077 | 1,457 | 1,875 | 2,329 | 2,797 | 9,534 |
| 51 | 4 | | | | | | | 1,199 | 1,632 | 2,119 | 2,612 | 7,562 |
| 52 | 3 | | | | | | | | 1,344 | 1,835 | 2,406 | 5,586 |
| 53 | 2 | | | | | | | | | 1,406 | 1,990 | 3,395 |
| 54 | 1 | | | | | | | | | | 1,470 | 1,470 |

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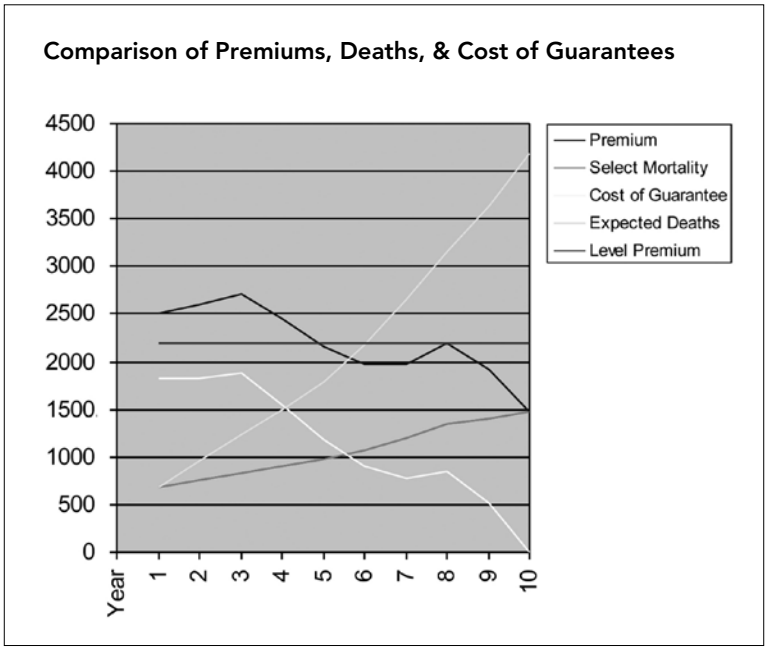
**Table 3: Premiums for term insurance of one to 10 years ending at age 55**

| Age at issue | Term | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 45 | 10 | 2,504 | 2,594 | 2,718 | 2,446 | 2,155 | 1,972 | 1,976 | 2,191 | 1,925 | 1,470 | 21,951 |
| 46 | 9 | | 2,594 | 2,718 | 2,446 | 2,155 | 1,972 | 1,976 | 2,191 | 1,925 | 1,470 | 19,447 |
| 47 | 8 | | | 2,718 | 2,446 | 2,155 | 1,972 | 1,976 | 2,191 | 1,925 | 1,470 | 16,853 |
| 48 | 7 | | | | 2,446 | 2,155 | 1,972 | 1,976 | 2,191 | 1,925 | 1,470 | 14,135 |
| 49 | 6 | | | | | 2,155 | 1,972 | 1,976 | 2,191 | 1,925 | 1,470 | 11,689 |
| 50 | 5 | | | | | | 1,972 | 1,976 | 2,191 | 1,925 | 1,470 | 9,534 |
| 51 | 4 | | | | | | | 1,976 | 2,191 | 1,925 | 1,470 | 7,562 |
| 52 | 3 | | | | | | | | 2,191 | 1,925 | 1,470 | 5,586 |
| 53 | 2 | | | | | | | | | 1,925 | 1,470 | 3,395 |
| 54 | 1 | | | | | | | | | | 1,470 | 1,470 |




The following graphic shows a comparison of the premiums for the 10-year contract to a level premium and to the expected deaths. It also shows the annual charge for the renewal option (or cost of the guarantee), along with the one-year select mortality. The peculiar shape of the premium and option curves are a result of the pattern of first differences in the mortality rates. They may reflect genuine characteristics of mortality curves or they may reflect that the smoothing underlying the construction of the table did not anticipate its use for this purpose.

As already noted, the premium each year is the same regardless of the issue age. It is a function of the attained age and the remaining term of the contract. Meanwhile, the insurer will have a liability that is a function of the age at issue and the original term.

## MEASURING THE LIABILITY

The liability is the amount needed for the guarantee of future insurability. It can be thought of as the amount that has been collected for the guarantee for future insurability at the one-year select rate that has not yet been utilized. It can be calculated as the amount of premium collected less the expected death benefits to date. Alternatively it can be calculated as the expected future death benefits in excess of the future premiums. The liability also represents the market value of the contract, given the premise of the paper that there is a robust market in which the contract could be sold. Table 4 shows the liability at the end of each year for the same range of contract terms as in previous tables.

**Table 4: Market-consistent liability for term insurance contracts ranging from one to 10 years, terminating at age 55**

| Age at issue | Term | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 45 | 10 | 1,819 | 3,445 | 4,931 | 5,884 | 6,243 | 6,042 | 5,371 | 4,413 | 2,707 | 0 |
| 46 | 9 | | 1,835 | 3,514 | 4,673 | 5,247 | 5,267 | 4,853 | 4,137 | 2,579 | 0 |
| 47 | 8 | | | 1,882 | 3,228 | 4,037 | 4,308 | 4,137 | 3,681 | 2,401 | 0 |
| 48 | 7 | | | | 1,542 | 2,507 | 2,991 | 3,089 | 2,944 | 2,010 | 0 |
| 49 | 6 | | | | | 1,177 | 1,840 | 2,151 | 2,254 | 1,629 | 0 |
| 50 | 5 | | | | | | 895 | 1,415 | 1,731 | 1,326 | 0 |
| 51 | 4 | | | | | | | 777 | 1,336 | 1,142 | 0 |
| 52 | 3 | | | | | | | | 846 | 936 | 0 |
| 53 | 2 | | | | | | | | | 519 | 0 |
| 54 | 1 | | | | | | | | | | 0 |

## CONCLUSIONS

The premiums calculated on the premise of market consistency are not premiums that would be marketable in the real world. Since so-called market-consistent liabilities are based on realistic cash flows, they are not comparable to the liabilities in Table 4. The illustration demonstrates that real-world term insurance pricing reflects that insurance is not sold in an environment that has the characteristics found in markets that are referenced when searching for market-consistent inputs. If insurance policies were sold in such markets, there would be no need to search elsewhere for inputs. This observation is not new, but the illustrations draw attention to the fact that the term "market-consistent" can be ambiguous and potentially misleading if the actuary does not fully disclose how he has chosen methods and selected inputs.

Most actuarial calculations that are labeled "market-consistent" are in fact hybrid calculations. The fact that not all inputs are market-consistent suggests that actuaries should disclose which inputs and methodologies are market-consistent and which are not. More importantly, actuaries should disclose why use of market-consistent inputs or methodologies is reasonable and appropriate. They should explain the purpose of using market-consistent inputs for some inputs and not for others. These disclosures may be important even if the measurement principle is not fair value or market-consistent. For example, extensive disclosures of the approach to inputs are anticipated in the emerging international accounting standard for insurance.

It is encouraging to see that actuaries are actively addressing the use of concepts from financial economics and the concept of market consistency in particular. In addition to reading Jacob and Bacinello, cited above, interested actuaries should read the transcript of a discussion of real-world versus risk-neutral assumptions by Burden and Ireland at a meeting of the Society of Actuaries in 2005.[4] There is also a good overall discussion in Day, "Financial Economics and Actuarial Practice," by the July 2004 *North American Actuarial Journal*.[5] ■

### END NOTES

1 John Jacob, "Actuaries and Assumptions," *The Financial Reporter*, Issue 84, March 2011, accessed June 29, 2011: *http://www.soa.org/library/newsletters/financial-reporter/2011/march/frn-2011-iss84.pdf.*

2 Anna Rita Bacinello, "Pricing Guaranteed Life Insurance Participating Policies with Annual Premiums," *North American Actuarial Journal*, Vol. 7, No. 3, July 2003, accessed June 29, 2011: *http://www.soa.org/library/journals/north-american-actuarial-journal/2003/july/naaj0307_1.pdf.*

3 Ibid, p. 5.

4 Tamara Burden, Graham D. Ireland, Julia Lynn Wirch, "Back to Basics: Risk Neutral vs. Real World," Record of the Society of Actuaries, Vol. 31, No. 1, accessed June 29, 2011: www.soa.org/files/pdf/003_bk_new-life05.pdf.

5 Tony Day, "Financial Economics and Actuarial Practice." *North American Actuarial Journal*, Volume 8, No. 3, July 2004, accessed on June 29, 2011: *http://www.soa.org/library/journals/north-american-actuarial-journal/2004/july/naaj0403-6.pdf.*