



SOCIETY OF ACTUARIES

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### RISK THEORY: AN INTERESTING APPLICATION

*Techniques learned through study for the actuarial examinations frequently find unexpected applications. Mr. Burton D. Jay found the risk theory part of the actuarial syllabus rather esoteric when he first encountered it. In fact Mr. Jay admits having only a hazy recollection of the term linear algebra from an undergraduate course and "for all I know a Markov chain is a device to keep Russian convicts from escaping."*

*In the following article Mr. Jay describes how United Benefit Life was able to benefit from his study of risk theory.*

As an example of the power of a relatively simple tool explained in the "risk theory" study note of the Society's syllabus, I will explain how we applied the Monte Carlo method to solve a problem proposed by our Agency Department. The solution of this problem by classical mathematical procedures, if at all possible, would have been exceedingly complex.

A fascinating sales contest had been conceived called GIBINGO. United's Guaranteed Insurability supplement is referred to as the "GIB" or Guaranteed Issue Benefit. And, the contest was a form of the game bingo; hence the name.

The rules were to be as follows: for every GIB sold, a letter B, I, N, G or O was to be randomly awarded according to the last digit of the policy number. Sales were to be divided into policies

below \$10,000 face amount and those above.

When the salesman had collected one set of all five letters in either size group, he had a BINGO and was to be awarded \$50 for the smaller size group or \$100 for the \$10,000 and over size group. Whenever a new policy was issued as the result of an election of a Guaranteed Insurability option, the salesman was to be given a "wild card" or letter which could be used as any letter needed to help complete a BINGO.

The questions asked by the Sales Department were simple. What is the contest likely to cost and what are the chances of a salesman getting a BINGO if he sells five, six, seven etc. GIBs?

We were able to arrive at only a partial solution to the second question by applying a multinomial distribution to the five variables representing the number of each of the five letters awarded for a total of N sales. The calculations grew very burdensome as N increased beyond eight sales or so. The Monte Carlo method was then applied to give complete answers to both questions.

In round numbers, the cost was found to be \$10,800 with a standard deviation of just under \$2,000. Appealing to the central limit theorem, which states that the distribution of means around their mean will follow a normal curve, one could make the statement that if

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our assumed distribution of sales proved to be accurate, the probability is .95 that the cost will run from \$6,800 to \$14,800. This is not a strikingly narrow range, but nevertheless it gives some insight as to the probable magnitude of the cost.

As a postscript, it might be pointed out that the contest was cancelled before our cost solutions were made available because it appeared that with any reasonable volume of sales the prizes would be too small to provide any real incentive. For example, for the salesmen making 10 sales (only .6% of salesmen are expected to hit or exceed this level in a 5 month period if sales are assumed double those in the distribution) the expected number of bingos is .5341 by the Monte Carlo method so that he will earn only  $\$100 \times .5341$  or \$53.41 prizes on the average.

Nevertheless, we learned much, and we had a great deal of fun following the method through. The knowledge acquired, when coupled with several additional refinements that could be made, will be useful in solving future problems which will arise from time to time.

*The Actuary*

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