



SOCIETY OF ACTUARIES

Article from:

The Actuary

October 1968 – volume 2 - Issue 8

A DISCUSSION OF NEGATIVE RESERVES

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Several types of insurance coverages give rise to negative reserves. Such reserves occur most frequently in connection with decreasing term risks (family income and mortgage protection plans, payor insurance, and disability benefits). They can also occur under other situations as, for example, in the first policy year at age 0 on a whole life plan and for many ages and durations in connection with accidental death benefits.

The usual procedure in computing mean reserves is to assume that any negative terminal reserve entering into the computation shall be adjusted to a value of zero, or alternatively to require that no mean reserve be less than one-half the net premium. It is interesting to note that the Society's Committee, which computed the 1958 CSO Monetary Tables for life plans, allowed negative reserves to reduce the mean reserves below one-half the net premium although the "one-half net premium minimum" was observed in the disability and accidental death benefits tables. The Committee stated that: "The required minimum mean reserves vary by state, some states requiring zero, some states one-half the net level premium, and some one-half the term cost for the current policy year."

Conditions for Negative Reserves

It is the purpose of this note: (1) to suggest that none of the three methods mentioned by the Society's Committee provides adequate mean reserves in certain cases where terminal reserves are negative, and (2) to propose a minimum reserve formula which does produce adequate mean reserves in all cases.

A negative reserve arises whenever the accumulated cost of insurance to date exceeds the accumulated net premiums to date. Since net premiums are by definition computed to provide exactly the contractual insurance benefits over the total policy period, it follows that a negative reserve means future net premiums have been borrowed against to meet already accrued claim liabilities. This assumes that all future contract premiums will be paid. If, however, a policy lapses before the

reserve becomes positive, the insurer has overstated his surplus if he has assumed in his reserve calculations that all premiums for the negative reserve period will be collected.

Normally the effect of lapses is neglected in reserve calculations because, where reserves are positive, the insurer's reserve position will be strengthened by future lapses and it is therefore conservative to assume that there will be no lapses. In the case of negative reserves, the opposite situation holds since the insurer's reserve position will be weakened by future lapses. If the aggregate negative reserves were only a miniscule percentage of any insurer's total positive reserve liabilities, then the various mean reserve approximations with respect to negative reserves might be acceptable for practical reasons.

Need for Rigorous Standards

With the rapidly increasing popularity of family income and reducing term insurance, which can give rise to substantial negative reserves, it appears that more rigorous reserve standards might well be developed and applied. For example, the maximum negative reserve per \$1,000 initial amount on a typical 20-year mortgage protection plan issued at age 45 is \$7.53 compared with the level net premium of \$6.67, as shown in the table which follows. Plans of this type are often issued to insureds in their forties, and even in their fifties. The high ages and long durations for which mortgage insurance is currently being issued are, actuarially, somewhat surprising.

A Suggested Basis

It is suggested, therefore, that consideration be given to the following minimum reserve basis for plans which would otherwise develop negative reserves:

No plan of insurance nor any disability or accidental death benefit shall be permitted to have other than positive reserves at any duration. The minimum reserves for any plan or benefit, which would otherwise develop negative reserves, shall be derived from a series of net premiums, according to the mortality table and rate of interest

used therein, so computed that they produce reserves which are not less than zero in all cases and where no net premium in the series is greater than its predecessor; provided that if a modified reserve valuation method is used, the first year net premium only may be less than the second year net premium so long as the first year reserve is not less than zero; provided further that the first year net premium (on a net level basis) or the second year net premium (on a modified reserve basis) shall be the smallest premium which will satisfy these conditions. For any such plan or benefit the deficiency reserve (if any) shall be the present value according to such mortality and interest standard of an annuity of the excesses of the net premiums over the corresponding gross premiums for the period during which such excesses occur, with no offsetting credit allowed for any excesses of gross premiums over the corresponding net premiums.

Rationale for Basis

The reason for the requirement that no net premium be greater than its predecessor (except in the case of first-year modified premiums) is to avoid having an entire series of reserves equal to the one-year term costs of insurance. This would have the effect simply of rearranging surplus inadequacies. An interesting possibility arises with respect to varying gross premiums; the most logical such arrangement might provide a level gross premium for about half the premium paying period with a lesser level gross premium for the balance of the period. Normally this would eliminate the possibility of negative reserves. However, it may nevertheless be considered desirable to extend the minimum reserve rule to provide for consistent treatment in event of varying gross premiums.

Based on the 20-year mortgage protection plan issued at age 45, described above, the three recognized reserve bases and the basis recommended in this note would produce the mean reserves as shown in the table on page 5.

**20 Pay 20-Year Mortgage Protection Policy
1958 CSO 3% Curtate**

Policy Year	Death Benefit During Year	Yearly Term Cost of Ins.	Conventional Terminal Reserves (P=6.66953)	Alternative Mean Reserves			
				(1)	(2)	(3)	(4)
				$\left(\frac{P_{\overline{20} } + V}{2}\right)$ 40	$(i) \frac{P}{2}$	$(i) \frac{C}{2}$	Recommended In This Note*
1	\$1000	\$5.19	\$1.53	\$4.10	\$4.10	\$4.10	\$4.42
2	973	5.51	2.79	5.50	5.50	5.50	6.14
3	945	5.84	3.75	6.60	6.60	6.60	7.58
4	915	6.17	4.41	7.42	7.42	7.42	8.74
5	883	6.52	4.73	7.90	7.90	7.90	9.60
6	849	6.86	4.72	8.06	8.06	8.06	10.14
7	812	7.18	4.38	7.88	7.88	7.88	10.36
8	774	7.48	3.70	7.38	7.38	7.38	10.26
9	733	7.75	2.73	6.55	6.55	6.55	9.88
10	690	7.97	1.49	5.44	5.44	5.44	9.22
11	644	8.13	.03	4.10	4.10	4.10	8.34
12	596	8.22	-1.59	2.56	4.34	4.11	7.31
13	544	8.21	-3.27	.90	3.34	4.10	6.20
14	489	8.07	-4.89	0	3.34	4.04	5.10
15	431	7.78	-6.30	0	3.34	3.89	4.19
16	369	7.29	-7.27	0	3.34	3.64	3.64
17	303	6.54	-7.53	0	3.34	3.27	3.27
18	233	5.50	-6.71	0	3.34	2.75	2.75
19	159	4.10	-4.39	0	3.34	2.05	2.05
20	81	2.28	.00	1.14	3.34	1.14	1.14

Specialization

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York in 1923). As one who was hurt years ago and whose brother, Professor Dan Pedoe, a well-known mathematician, was seriously hurt recently in trying to board a London bus while it was in motion, I would not wish to blame any actuary for the design of the London buses (see *The Actuary*, March 1968).

However I stick by my guns and praise the Faculty of Actuaries in Scotland for concentrating on a uniform standard for its Fellowship qualification and leaving specialization for that long period of life *after* a man qualifies. I regret that the Society and the Institute (of London) have departed from this. One can express surprise regarding the Society, for the profession of actuary in the U.S.A. owes much to the Scots.

Some History

As for *How to Become President of a Life Insurance Company*, the following tid-bit of our history should be of interest. In 1930 Ben Holmes and I started the Fellowship Study Circles (which are still in operation). In 1935 we passed the supervision on to six of the younger Fellows. All six have become leaders of the actuarial profession and three of them presidents of their companies: Alistair Campbell of the Sun Life, Harold Lawson of the Glens Falls and George Ryrice of the North American Life.

Again, when I became Chairman of the Educational Committee of the Society in 1934 and introduced major changes in our syllabus, etc., two of the Committee who took the most active part in our work were Henry Beers of the Aetna and the late Horace Bassford of the Metropolitan. So, engaging in the educational work of the Society increases the chances of becoming President of your company.

Regarding *Management Science*, I can appreciate the problem of the clever young actuary who, on qualification, expects the salary of a junior executive but dodges the responsibilities of the position in relation to the other members of the staff. As companies get larger and the business more complex,

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Based on the following net premiums and terminal reserves:

Policy Year	Net Premium	Terminal Reserve	Policy Year	Net Premium	Terminal Reserve
1	\$6.98	\$1.85	11	\$6.98	\$4.37
2	6.98	3.44	12	6.98	3.27
3	6.98	4.75	13	6.98	2.14
4	6.98	5.76	14	6.98	1.09
5	6.98	6.46	15	6.98	.31
6	6.98	6.84	16	6.98	0
7	6.98	6.89	17	6.54	0
8	6.98	6.65	18	5.50	0
9	6.98	6.12	19	4.10	0
10	6.98	5.34	20	2.28	0

Under certain conditions, where term insurance costs fluctuate substantially from year to year, the computation of the minimum series of net premiums which would meet the proposed requirements could be a difficult matter of trial and error. Under most conditions which will be met in actual practice, however, the comparison of the averages of progressive sums of the yearly term costs of insurance with the next succeeding term cost will indicate the approximate areas over which level premiums should be computed.

Negative reserves for level accidental death benefits normally occur only on

continuous premium plans and then only at the younger issue ages (up to a possible age 32 under the 1959 Table) but may run in a few cases for over 40 years. Negative reserves for disability benefits occur quite generally on the shorter term plans and in a few cases on waiver or income to ages 55 or 60 with co-terminous premiums. While reserves, in general, for accidental death benefits and waiver of premium benefits are small dollar-wise per \$1,000 of basic insurance, the methods recommended in this note can result in doubling at some durations the mean reserves obtained by present methods. □