



The Actuary

The Newsletter of the Society of Actuaries

VOLUME 2, No. 8

OCTOBER, 1968

ON THE SUBJECT OF SPECIALIZATION

by Arthur Pedoe

It was at a meeting of the Society of Actuaries in June 1955, during a discussion on *Selection, Education and Training of Actuarial Students*, that I held forth against specialization in the qualifying Fellowship actuarial examinations. I was against qualifying a man as "a pension actuary, a fraternal actuary, or other specialty."

Following the meeting, a member buttonholed me with the following story. At a diplomatic gathering watched by an old lady and her niece, a man in blue uniform with an abundance of gold braid was much in evidence. The old lady asked her niece to find out who he was. On her return she told her aunt that he was a Naval Surgeon. "A navel surgeon," exclaimed the old lady, "my, how they specialize these days!"

I was reminded of this incident at a meeting this year of the Younger Actuaries, in Toronto, which appeared to be a continuation of the discussion at the Society meeting in October 1966 on *The Future of the Actuarial Profession as It Appears to the Younger Actuaries*.

The speakers were all out for actuarial students being examined in Operations Research, of which a notable example was the running and designing of buses in London, also sampling techniques, consumer psychology, and particularly management science. One of the openers at the Toronto meeting suggested that a worthy subject for the Education Committee in their choice of reading might be *How to Become President of a Life Insurance Company*.

This was all most intriguing to one of the older members of the Society (I attended my first meeting in New

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GOVERNMENT HONORS CANADIAN ACTUARIES

Five prominent members of the Canadian Institute of Actuaries—W. M. Anderson, J. G. Beatty, B. T. Holmes, Arthur Pedoe and N. E. Sheppard—were recently honored by the Government of Canada in being awarded Centennial Medals. These Medals were instituted as part of the celebration in 1967 of the Centennial of Confederation and were awarded to outstanding individuals in various walks of life for their contributions to the community.

All five recipients are members of the Society. Messrs. Anderson and Holmes are Past Presidents and Mr. Beatty a Past President of the American Institute of Actuaries. Mr. Pedoe is an actuarial author of renown on both sides of the Atlantic. Professor Sheppard has been for many years in the Department of Mathematics at the University of Toronto and many members of the Society are his former students.

The Society takes a proper pride in this recognition and tenders congratulations to the individuals so honored. □

HEARINGS ON PROPOSED INTEGRATION RULES

by E. F. Boynton

Substantial opposition to the proposed new rules for integration of private pension plan benefits with Social Security was expressed by most witnesses at the hearings on proposed integration rules held on Sept. 16 and 17.

Under discussion were proposed amendments to the Income Tax Regulations, published on July 6, 1968, and a proposed Revenue Ruling which would implement this regulation, released as Announcement 68-49, on July 15, 1968. In addition to the oral testimony, of which no record was made except notes

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THE HEALTH OF THE MATH OF FINANCE

by James C. Hickman

The study of compound interest and annuities certain is a declining activity in North American colleges and universities. The expulsion of the subject from departments of mathematics, except from those that feature work in actuarial science, has proceeded at a rapid pace. Even within collegiate schools of business, the subject is seldom taught as an independent course but instead appears in various disguises within courses in accounting and finance.

The impact of the influential Gordon-Howell report (*Higher Education for Business*, Columbia University Press, 1959), which stamped the subject as being "sub-collegiate", accounts in part for its reduced acceptability to colleges of business. A natural economic consequence of the declining market is that few substantial college text books on compound interest have been published in recent years. As a result of these trends, many actuarial students have found the portion of part 3 of the Society's examinations devoted to compound interest to be one of the most perplexing topics in the early part of the examination series.

Impact of Technology

In a computer-oriented age, it is appropriate to inquire about the impact of the new technology on the study of compound interest. Traditional courses on the subject discuss two basic problems: (1) the determination of the value of a stream of payments given an interest rate, and (2) the determination of the rate of return defined by a stream of payments purchased by an initial investment.

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The Actuary

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Published monthly (except July and August) by the SOCIETY OF ACTUARIES,
208 S. LaSalle St., Chicago, Illinois 60604: Morton D. Miller, President; Fredrick
E. Rathgeber, Secretary, and Herbert L. DePrenger, Treasurer.



18. Internationaler Kongreß
der Versicherungsmathematiker



MÜNCHEN · 4. - 11. Juni 1968



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LETTERS

Osculatory Interpolation

Sir:

I thought you would be interested in knowing that Jane Goodsell, in her "Soup of Nonsense" column in the *Michigan AFL-CIO News* dated June 26, 1968, quotes a West German medical magazine. It reports that, on the basis of life insurance actuarial studies, the husband who kisses his wife every morning before leaving for work can expect to live a median of five years longer, hold a job paying 20-35% more, lose up to 50% less time from illness, and be involved in significantly fewer car accidents than the husband who doesn't.

Her poem reacting to this news item follows:

If you'd succeed in business, lad
Your wisest course is this:
Betake yourself a wife, lad,
And plant on her a kiss
Before you go to work, lad,
On each and ev'ry morn,
And you will up your income, lad,
As sure as you were born.

It takes but just a moment, lad;
To give your wife a peck,
And it's mighty good insurance,
To avoid an auto wreck.
For statistics plainly show, lad,
That each time you leave the house,
You risk a tangled fender
If you fail to kiss your spouse.

Actuarial statistics
Show as plain as plain can be
That a man improves his chances of
Long life expectancy
If he bestows a prudent kiss
Each morn upon his wife.
'Tis a paltry price to pay, lad,
To ensure a good long life.

So go to work unshaven, lad.
Skip breakfast if you're late.
But whatever else you do, lad,
Don't neglect to kiss your mate.
'Tis worth your time and trouble, lad,
For science recommends
Morning kisses as investments
That pay solid dividends.

Martin J. Frank

(Editor's note: We are indebted to Press Associates, Inc., of Washington, D.C., for permission to reprint this poem.)

(Continued on page 3)

SINCE a picture is worth a thousand words we might dispense with an editorial. The picture of the Deutsches Museum in Munich supplements Mr. Moorhead's article on the International Congress. It does not show the actuaries in full blast nor does it show the "Isar rolling rapidly" alongside the building. The rushing river could scarce compete with the many-tongued Versicherungs Mathematiker assembled on its banks.

To the left is the insignium of the Congress—our old friend qx streamlined and followed by 18 IKVM standing for Internationaler Kongress der Versicherungs Mathematiker.

In Germany Versicherungsmathematiker identifies the actuary and perhaps the German people are thereby more aware of the nature (but not the scope) of the profession. Not for them the shudders and horror of the ignorant audience (usually at a cocktail party) when one member of the group announces that he or she is an actuary. This simple Germanic definition has apparently been adopted by our mentors in public relations. A recent press release describes the Society as the principal professional organization of life and health insurance mathematicians in the United States and Canada. This must be the effect of the Munich Congress negating the efforts of many actuaries, past and present, to define their profession.

Whether or not you like the definition (and we no not) at least the press release in which it appeared brought good news. And so we close on a note of welcome to Charles "Barry" H. Watson as Executive Director of the Society, a fortunate addition to the sometimes harassed but always willing staff at 208 South La Salle Street.

—A.C.W.

A PARTICIPANT VIEWS THE CONGRESS IN MUNICH

by E. J. Moorhead

Many younger actuaries will someday be wondering, "Shall I make the effort to attend an International Congress?"

Perhaps it may be the 19th Congress, in Norway in 1972—or it may be the 20th to be held possibly in Japan or Australia in 1976. Maybe the "effort" will consist mainly of persuading your employer to underwrite your trip. Whatever and whenever it may be, perhaps this little account of the 18th Congress in Munich may be useful. The three questions I shall attempt to answer are: What is a Congress like? How is it organized? What benefits can one expect to get from it?

What Is a Congress Like?

A Congress is an immense and rather slow-moving assembly of usually middle-aged actuaries from many parts of the world. Immense indeed—in Munich there were 1,200 actuaries from 46 countries and almost 1,000 of what may be reasonably described as camp-followers. Slow-moving—largely because of language difficulties.

The solution offered for the language problem was twofold. Each of us was given a badge marked with name, country and one or more solidly colored circles. The color of each circle was a code indicating the language spoken by the wearer. The size of the circle indicated how fluently he or she could speak that particular language. This was very helpful.

At most of the major sessions ear-phones were provided for simultaneous translations into five official languages; English, French, German, Italian and Spanish. This was a brave attempt but somewhat ineffective, because it was impossible to provide translators who were both linguists and masters of actuarial jargon.

How Is a Congress Organized?

The host country—in this case, Germany—takes almost the entire responsibility for the program, for the physical arrangements and for the social functions. Naturally the success of any Congress depends upon the wisdom of the organizers in the choice of topics, their efficiency in planning the meeting and the opportunities that they provide so

that actuaries from different countries may meet one another at social and business events.

Applying these three tests to the 18th Congress this observer would give very high ratings for the first two of these, but did feel personally disappointed about the third. It does seem that the Congress authorities must strive hard to introduce actuaries from different countries to each other, particularly those who are attending their first Congress. The social events at this meeting—the opera "Fidelio," the bus trip through the countryside, the reception at Castle Schleissheim, the receptions and dance—were delightful and should, I think, have proved more effective "mixers" than they did.

What Benefits Can One Expect?

First, ideas from the formal papers. There is much excellent material on six major subjects in the printed volumes. Those measure more than 5 inches in thickness, even before the inclusion of discussions to be printed later. It is strongly recommended that you take the trouble to obtain and look through the Transactions of the Munich Congress. You will find much to reward your search.

Second, ideas gained from the oral discussions in the various sessions. These are not easily gleaned but are worth seeking.

Third, the opportunity to present your own ideas to the actuaries of other countries. One feature of the Congress that astonished me was the large amount of informal extemporaneous discussion and argument.

Fourth, already mentioned, the opportunity to meet people from other lands. And fifth, never to be overlooked, the fun and advantage of going places and seeing things before, during, and after the Congress.

I hope this note conveys the message that I, for one, think that attending an International Congress is indeed worthwhile. Through it ideas do flow across international boundaries much more readily than otherwise. We learn that we in the United States and Canada are not the only reservoir of insurance ideas. We have much to contribute, of course, but we do well if we attend a Congress with determination to learn from the experiences of others. □

Letters

(Continued from page 2)

Truth in Lending

Sir:

As one who sponsored truth-in-lending legislation in Vermont, I was interested to read the articles by Robert Myers (*The Actuary*, February 1968) and David Good (*The Actuary*, June 1968) relating to two methods of calculating the effective annual interest rate.

The enactment of the federal legislation now makes it clear, that the rate to be disclosed will not be the effective annual interest rate, but rather the rate calculated according to the "actuarial" method, often referred to as the "United States rule."

The federal legislation defines the applicable annual percentage rate to be "that nominal annual percentage rate which will yield a sum equal to the amount of the finance charge when it is applied to the unpaid balances of the amount financed, calculated according to the actuarial method of allocating payments made on a debt between the amount financed and the amount of the finance charge, pursuant to which a payment is applied first to the accumulated finance charge and the balance is applied to the unpaid amount financed."

Thus, the rate to be disclosed is the actual periodic rate multiplied by the number of periods in a year. This is obviously not the effective annual interest rate, except in the case where interest is paid annually in arrears.

I think the rate to be disclosed on a loan or sales contract should be generally comparable to that rate disclosed as available for savings accounts, usually a nominal rate—4%, compounded quarterly. Or, it can be argued that the rate to be disclosed on a consumer loan should be determined in the same manner as that rate advertised as available on home mortgage loans.

The main purpose of this letter, however, is to inquire whether any of your readers know how the term "actuarial method" came to be used to describe the method of computing the nominal rate. One does not need to be an FSA to multiply the monthly rate by 12.

James H. Hunt

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A DISCUSSION OF NEGATIVE RESERVES

by Donald B. Warren

Several types of insurance coverages give rise to negative reserves. Such reserves occur most frequently in connection with decreasing term risks (family income and mortgage protection plans, payor insurance, and disability benefits). They can also occur under other situations as, for example, in the first policy year at age 0 on a whole life plan and for many ages and durations in connection with accidental death benefits.

The usual procedure in computing mean reserves is to assume that any negative terminal reserve entering into the computation shall be adjusted to a value of zero, or alternatively to require that no mean reserve be less than one-half the net premium. It is interesting to note that the Society's Committee, which computed the 1958 CSO Monetary Tables for life plans, allowed negative reserves to reduce the mean reserves below one-half the net premium although the "one-half net premium minimum" was observed in the disability and accidental death benefits tables. The Committee stated that: "The required minimum mean reserves vary by state, some states requiring zero, some states one-half the net level premium, and some one-half the term cost for the current policy year."

Conditions for Negative Reserves

It is the purpose of this note: (1) to suggest that none of the three methods mentioned by the Society's Committee provides adequate mean reserves in certain cases where terminal reserves are negative, and (2) to propose a minimum reserve formula which does produce adequate mean reserves in all cases.

A negative reserve arises whenever the accumulated cost of insurance to date exceeds the accumulated net premiums to date. Since net premiums are by definition computed to provide exactly the contractual insurance benefits over the total policy period, it follows that a negative reserve means future net premiums have been borrowed against to meet already accrued claim liabilities. This assumes that all future contract premiums will be paid. If, however, a policy lapses before the

reserve becomes positive, the insurer has overstated his surplus if he has assumed in his reserve calculations that all premiums for the negative reserve period will be collected.

Normally the effect of lapses is neglected in reserve calculations because, where reserves are positive, the insurer's reserve position will be strengthened by future lapses and it is therefore conservative to assume that there will be no lapses. In the case of negative reserves, the opposite situation holds since the insurer's reserve position will be weakened by future lapses. If the aggregate negative reserves were only a miniscule percentage of any insurer's total positive reserve liabilities, then the various mean reserve approximations with respect to negative reserves might be acceptable for practical reasons.

Need for Rigorous Standards

With the rapidly increasing popularity of family income and reducing term insurance, which can give rise to substantial negative reserves, it appears that more rigorous reserve standards might well be developed and applied. For example, the maximum negative reserve per \$1,000 initial amount on a typical 20-year mortgage protection plan issued at age 45 is \$7.53 compared with the level net premium of \$6.67, as shown in the table which follows. Plans of this type are often issued to insureds in their forties, and even in their fifties. The high ages and long durations for which mortgage insurance is currently being issued are, actuarially, somewhat surprising.

A Suggested Basis

It is suggested, therefore, that consideration be given to the following minimum reserve basis for plans which would otherwise develop negative reserves:

No plan of insurance nor any disability or accidental death benefit shall be permitted to have other than positive reserves at any duration. The minimum reserves for any plan or benefit, which would otherwise develop negative reserves, shall be derived from a series of net premiums, according to the mortality table and rate of interest

used therein, so computed that they produce reserves which are not less than zero in all cases and where no net premium in the series is greater than its predecessor; provided that if a modified reserve valuation method is used, the first year net premium only may be less than the second year net premium so long as the first year reserve is not less than zero; provided further that the first year net premium (on a net level basis) or the second year net premium (on a modified reserve basis) shall be the smallest premium which will satisfy these conditions. For any such plan or benefit the deficiency reserve (if any) shall be the present value according to such mortality and interest standard of an annuity of the excesses of the net premiums over the corresponding gross premiums for the period during which such excesses occur, with no offsetting credit allowed for any excesses of gross premiums over the corresponding net premiums.

Rationale for Basis

The reason for the requirement that no net premium be greater than its predecessor (except in the case of first-year modified premiums) is to avoid having an entire series of reserves equal to the one-year term costs of insurance. This would have the effect simply of rearranging surplus inadequacies. An interesting possibility arises with respect to varying gross premiums; the most logical such arrangement might provide a level gross premium for about half the premium paying period with a lesser level gross premium for the balance of the period. Normally this would eliminate the possibility of negative reserves. However, it may nevertheless be considered desirable to extend the minimum reserve rule to provide for consistent treatment in event of varying gross premiums.

Based on the 20-year mortgage protection plan issued at age 45, described above, the three recognized reserve bases and the basis recommended in this note would produce the mean reserves as shown in the table on page 5.

**20 Pay 20-Year Mortgage Protection Policy
1958 CSO 3% Curtate**

Policy Year	Death Benefit During Year	Yearly Term Cost of Ins.	Conventional Terminal Reserves (P=6.66953)	Alternative Mean Reserves			
				(1)	(2)	(3)	(4)
				$\left(\frac{P_{\frac{t}{2}}V_{\frac{t}{2}} + V}{2}\right)$ ← 0	$(i) \frac{P}{2}$	$(i) \frac{C}{2}$	Recommended In This Note*
1	\$1000	\$5.19	\$1.53	\$4.10	\$4.10	\$4.10	\$4.42
2	973	5.51	2.79	5.50	5.50	5.50	6.14
3	945	5.84	3.75	6.60	6.60	6.60	7.58
4	915	6.17	4.41	7.42	7.42	7.42	8.74
5	883	6.52	4.73	7.90	7.90	7.90	9.60
6	849	6.86	4.72	8.06	8.06	8.06	10.14
7	812	7.18	4.38	7.88	7.88	7.88	10.36
8	774	7.48	3.70	7.38	7.38	7.38	10.26
9	733	7.75	2.73	6.55	6.55	6.55	9.88
10	690	7.97	1.49	5.44	5.44	5.44	9.22
11	644	8.13	.03	4.10	4.10	4.10	8.34
12	596	8.22	-1.59	2.56	4.34	4.11	7.31
13	544	8.21	-3.27	.90	3.34	4.10	6.20
14	489	8.07	-4.89	0	3.34	4.04	5.10
15	431	7.78	-6.30	0	3.34	3.89	4.19
16	369	7.29	-7.27	0	3.34	3.64	3.64
17	303	6.54	-7.53	0	3.34	3.27	3.27
18	233	5.50	-6.71	0	3.34	2.75	2.75
19	159	4.10	-4.39	0	3.34	2.05	2.05
20	81	2.28	.00	1.14	3.34	1.14	1.14

Specialization

(Continued from page 1)

York in 1923). As one who was hurt years ago and whose brother, Professor Dan Pedoe, a well-known mathematician, was seriously hurt recently in trying to board a London bus while it was in motion, I would not wish to blame any actuary for the design of the London buses (see *The Actuary*, March 1968).

However I stick by my guns and praise the Faculty of Actuaries in Scotland for concentrating on a uniform standard for its Fellowship qualification and leaving specialization for that long period of life *after* a man qualifies. I regret that the Society and the Institute (of London) have departed from this. One can express surprise regarding the Society, for the profession of actuary in the U.S.A. owes much to the Scots.

Some History

As for *How to Become President of a Life Insurance Company*, the following tid-bit of our history should be of interest. In 1930 Ben Holmes and I started the Fellowship Study Circles (which are still in operation). In 1935 we passed the supervision on to six of the younger Fellows. All six have become leaders of the actuarial profession and three of them presidents of their companies: Alistair Campbell of the Sun Life, Harold Lawson of the Glens Falls and George Ryrice of the North American Life.

Again, when I became Chairman of the Educational Committee of the Society in 1934 and introduced major changes in our syllabus, etc., two of the Committee who took the most active part in our work were Henry Beers of the Aetna and the late Horace Bassford of the Metropolitan. So, engaging in the educational work of the Society increases the chances of becoming President of your company.

Regarding *Management Science*, I can appreciate the problem of the clever young actuary who, on qualification, expects the salary of a junior executive but dodges the responsibilities of the position in relation to the other members of the staff. As companies get larger and the business more complex,

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Based on the following net premiums and terminal reserves:

Policy Year	Net Premium	Terminal Reserve	Policy Year	Net Premium	Terminal Reserve
1	\$6.98	\$1.85	11	\$6.98	\$4.37
2	6.98	3.44	12	6.98	3.27
3	6.98	4.75	13	6.98	2.14
4	6.98	5.76	14	6.98	1.09
5	6.98	6.46	15	6.98	.31
6	6.98	6.84	16	6.98	0
7	6.98	6.89	17	6.54	0
8	6.98	6.65	18	5.50	0
9	6.98	6.12	19	4.10	0
10	6.98	5.34	20	2.28	0

Under certain conditions, where term insurance costs fluctuate substantially from year to year, the computation of the minimum series of net premiums which would meet the proposed requirements could be a difficult matter of trial and error. Under most conditions which will be met in actual practice, however, the comparison of the averages of progressive sums of the yearly term costs of insurance with the next succeeding term cost will indicate the approximate areas over which level premiums should be computed.

Negative reserves for level accidental death benefits normally occur only on

continuous premium plans and then only at the younger issue ages (up to a possible age 32 under the 1959 Table) but may run in a few cases for over 40 years. Negative reserves for disability benefits occur quite generally on the shorter term plans and in a few cases on waiver or income to ages 55 or 60 with co-terminous premiums. While reserves, in general, for accidental death benefits and waiver of premium benefits are small dollar-wise per \$1,000 of basic insurance, the methods recommended in this note can result in doubling at some durations the mean reserves obtained by present methods. □

Specialization

(Continued from page 5)

team work and leadership become of major importance. Should not our aim be to qualify the young man while his mind is still fresh and receptive and *then* see that he becomes conditioned to the responsibilities of his position, if this is needed?

The senior staff of a Canadian life insurance company was recently "business oriented" by an after-dinner address by a professor of management science on *Decision Making*, based on some special research recently completed by the school of business of a university. To one of the listeners the "seventy-minute address" could be summed up as: get your facts before you make a decision.

What an overwhelming discovery! Yet I have attended business meetings where a committee of ten men is expected to come to a decision on a matter and no effort was made, prior to the meeting, to distribute the facts on which a decision could be based; in some cases even the subject was not announced previously.

Study and examinations can only accomplish so much. There is the story of the comment made by a wise Presbyterian minister to a young man in Scotland leaving his village to go to the university. "Noo laddie, remember, the university can gie ye philosophy, biology, mathematics and the like, but if ye havena got common sense, it canna gie ye that." □

Math of Finance

(Continued from page 1)

In solving problems of the first type, ingenious manipulations are often taught so that the answer could be determined using tabled functions. Problems of the second type are often solved in traditional courses by clever approximation and iterative methods.

Today, standard computer programs for solving polynomial equations can be used to solve yield rate equations and the value of a stream of payments at a fixed interest rate is usually determined by direct calculation rather than through table look-up. (See, for example, "Computer Algorithms for Finding the Exact

Rate of Return," by S. Kaplan, *Journal of Business*, October 1967).

Random Components

The recognition that business transactions do not take place in a world characterized by certainty has led to the development of economic and financial models that explicitly incorporate random components. In the past it was common to ignore uncertainty about the occurrence and the value of future payments. This new stochastic view of business processes has also tended to reduce the stress placed on learning the fundamentals of compound interest and shifted the emphasis to probability.

However, it would be a mistake to conclude that the study of compound interest is dead. The subject is currently flourishing in the field of capital budgeting. In the *Journal of Business*, the *Journal of Finance*, and in *Management Science*, a series of interesting papers has used compound interest in developing a theory to guide managers in selecting alternative investment projects.

One of the most influential of these papers was by Lorie and Savage ("Three Problems in Capital Rationing", *Journal of Business*, October 1955). The paper indicates that if capital projects are to be evaluated by their internal rates of return, the comparison may be confused by the existence of multiple rates of return. That is, if a_j is the amount of money that flows into (-) or out of (+) a project at integer time t_j , $t_0 = 0$, the internal rate of return equation

$$\sum_{j=0}^n a_j v^{t_j} = 0$$

may have more than one solution on the unit interval.

In other words, there may be more than one value of v on the unit interval that solves this equation and this implies that there is more than one positive interest rate ($i = 1/v - 1$) that is an internal rate of return.

Illustrative Problem

In 1960 John Boermeester showed me such a project hidden in the form of an elementary problem:

A states to B: "I want a loan of \$208 one year from today. The loan period will be one

Canadian Note

The Toronto Press is not behind Dallas (see *The Actuary* Editorial June, 1968) in recognizing and explaining the actuarial profession.

The July 12 edition of the *Toronto Globe and Mail* carried an interesting article on the profession written around the new President of the Canadian Institute, Colin E. Jack.

Mr. Jack reported among other items that his civilian occupation—actuary—was classified 999 in the Army. This highly mathematical number when decoded meant "Others including piano tuners, taxidermists, and actuaries." Apparently a harmonious but stuffy profession!

year. In return, I will pay \$100 today and \$108.15 two years from today at the end of the loan period."

What is the interest yield to B if he accepts?

It is easy to show that $i = .03$ and $i = .05$ will both solve the internal rate of return equation. A letter to the Editor from Daniel Harris published in *The Actuary*, September 1968, contained a similar example. The apparent paradox may be resolved by making additional assumptions about reinvestment rates and/or loan rates. However, an interesting mathematical problem remains: What pattern of cash flows in and out of an investment project will lead to multiple positive internal rates of return?

A recent paper by Jean, entitled "On Multiple Rates of Return" which appeared in the March 1968 issue of the *Journal of Finance*, answers this question. By using Budan's theorem from the theory of equations, Jean shows that when, as in the Boermeester example, a reverse flow of funds occurs in the middle of the project's time range, multiple positive internal rates of return will result.

Actuaries may amuse themselves! constructing other examples, or they may be interested in tracing some of the references in Jean's extensive bibliography. □

CLAIM EXPERIENCE ON LONG TERM DISABILITY

by R. A. Hall

(Editor's note: This is a summary of comments presented at an informal discussion at the Regional Meeting, Philadelphia, April 8-9, 1968).

The claim experience on LTD coverage depends on both the rate of incidence of disability and the disabled life annuity claim value. This value depends on the disability termination rate. Together, these two determine the level of claim experience.

A recent Aetna Study of 175,000 life-years' exposure through July 1, 1965 indicated the following variation by age in the annual incidence rates of disability continuing for at least 6 months:

Ages	Rate per 1000
Under 40	1.0
40 — 44	2.2
45 — 49	2.3
50 — 54	5.2
55 — 59	7.2
60 — 64	13.1
All ages	2.7

Termination rate experience was studied for the same claims, but benefits had continued for as long as 2 years for only 15% of the claims and beyond 2 years for only a few claims. For the limited exposure available, actual reserves released by termination were 135% of expected releases computed at the Benefit 2 termination rates from the 1952 Disability Study with interest at 3½%. The study showed no meaningful variation by age.

Social Security Offsets

The effects of Social Security offsets on amounts of benefit payable have a greater impact on LTD experience. During the period of the exposures studied, when the taxable wage base varied little by salary level, Social Security formula benefit amounts have been close to the level assumed. As the taxable wage base rises, these assumptions must be re-evaluated.

The percentage of claimants with dependent children, which affects the average amount of Social Security benefit offset, has been lower than assumed and shows the following variation by age:

Ages	Assumed	Experience
Under 40	87%	60%
40 — 44	80	78
45 — 49	57	70
50 — 54	36	45
55 — 59	14	5
60 — 64	3	—

A review of 317 LTD claims, on which disability commencing after Oct. 1, 1966 had lasted at least one year, showed that the number approved for Social Security benefits was 27% lower than expected, with the following variation in the approval percentage by age:

Ages	Assumed	Experience
Under 40	63%	36%
40 — 44	72	74
45 — 49	80	65
50 — 54	87	78
55 — 59	93	63
60 — 64	98	81
All ages	86%	67%

Substituting this experience for the previous assumptions could produce an average reduction of 35% to 40% in amount of benefit offsets and an increase of 20% to 50% in net amount of LTD benefits for the expected disabilities.

The unexpected results on Social Security offsets may be due in part to the concentration of the exposures among white collar personnel, whose disabilities may not qualify for Social Security approval because they can still perform some gainful employment.

The typical lag in approval of Social Security benefits means that initial overpayment of group benefits is followed by heavy offsets to recover the overpayment. This need not cause an employee reaction problem if explained carefully at the origin of the claim. For claims involving lump-sum Workmen's Compensation awards, the amount of LTD benefit offset depends on the allocation of the award between medical expense compensation and loss-of-income compensation.

In recent years benefit formulas have become more complex with only primary Social Security benefits offset at the basic benefit level—50 or 60% of earnings—and additional Social Security and other disability benefits offset at a higher non-duplication level—70, 75, or 80% of earnings. These formulas are more difficult to make effective and more difficult to explain to the employee.

First year claim levels on new LTD plans averaged 8% above second year claim levels. Much of this difference may reflect claims for long-standing borderline disabilities not really incurred after the effective date.

A review of first-year claims on 135 plans each covering over 200 persons without a pre-existing condition exclusion indicated that 45% to 65% would have been affected by a pre-existing condition exclusion. Benefits for *bona fide* disability claims on pre-existing conditions may reasonably be provided at relatively slight extra cost. It may also be reasonable to cover all disabilities of a pre-existing nature, including some employees performing at subnormal work load, provided the employer is aware of and prepared to pay the additional cost. Coverage of all pre-existing condition claims, however, is inadvisable for small groups, where anti-selection can be most effective and the policyholder net cost cannot reflect claims directly.

A Source of Claim Problems

Emphasis on using announcement literature as a sales tool at solicitation often leads to claim problems if the effects on benefits of non-duplication or offset provisions are not described clearly. The claim department should assume that each claimant is uninformed and explain the calculation of his LTD benefit to reduce follow-up questions.

The extension of LTD from white collar employees to hourly employees appears to be gaining momentum, and recent United Auto Workers bargaining agreements will continue this trend.

In case underwriting, analysis of the group's past experience on short-term disability benefits and on permanent and total disability income benefits under group life can be indicative of future LTD experience. □

Letters

(Continued from page 3)

Sir:

Some folks believe that the wizard in L. Frank Baum's "The Wonderful Wizard of Oz" is an actuary. This theory is based on the belief that the intended title of the book is "The Wonderful Wizard of Odds."

Stuart J. Kingston

Hearings

(Continued from page 1)

taken by the panel, written comments were invited up through Sept. 27, 1968.

At the two-day hearings a total of 22 witnesses (including 10 members of the Society of Actuaries) testified.

They represented law firms, consulting actuaries, four life insurance associations (LIAA, ALC, AALU and NALU), employer associations (Chamber of Commerce, Council on Employee Benefits and others), trade unions and insurance brokerage firms.

Except for representatives of the AFL-CIO, who were interested in phasing out integration altogether, all the witnesses objected strongly to the proposed changes in the rules for integration of private pensions with Social Security on legal, mathematical, and practical grounds.

The testimony was generally divided into two groups. One approach attacked the underlying structure of the proposed rulings along the same lines as the criticism of Announcement 66-58 two years ago. Another group aimed at more practical and workable modifications of the specific rules now proposed. Since all suggestions for an approach to the problem different from that proposed in Announcement 66-58 have apparently been rejected, the general feeling was that I.R.S. is not likely to make substantial changes now after having had two years to consider the same arguments.

Objections

Throughout the testimony there was a recurrence of several specific objections to the proposed regulations and ruling. Most witnesses stated that the old 37½% rule had not been shown to be discriminatory in practice and, hence, it was difficult to rationalize any change which disrupts existing pension plans.

Many felt that the so-called mathematical approach is not mathematically sound and is being used merely as a means to arrive at a predetermined number, particularly in view of the "rounding" of the arithmetically determined 27% result to 30%. There was unanimity, joined in by even the labor representatives, that the approach is unnecessarily complicated to meet the objectives of the law being implemented.

ACTUARIAL MEETINGS

- Oct. 21, Michigan Actuarial Society, Detroit
- Oct. 21, Chicago Actuarial Club, Continental Mid-Day Club
- Oct. 28-30, Society of Actuaries (Annual), Washington, D.C.
- Nov. 14, Baltimore Actuaries Club
- Nov. 17-19, Casualty Actuarial Society (Annual), Washington, D.C.
- Nov. 18, Chicago Actuarial Club, Continental Mid-Day Club
- Nov. 21, Actuaries Club of Indiana, Kentucky, and Ohio, Louisville, Ky.
- Nov. 21-22, Southeastern Actuaries Club, The Riviera Motor Hotel, Atlanta
- Nov. 25, Columbus, Ohio, Actuaries Club
- Nov. 26, Canadian Institute of Actuaries, Toronto
- Dec. 3, Nebraska Actuaries Club, Lincoln
- Dec. 3, Actuaries Club of Hartford
- Dec. 16, Chicago Actuarial Club, Continental Mid-Day Club

Most speakers felt that the moving wage base is unrealistic for practical application and would lead only to further confusion, and that pension formula bend points should be allowed at any level below the current Social Security wage base.

Other Areas of Agreement

Another area of almost unanimous agreement was that existing plans should be allowed to continue without change under a grandfather clause, either indefinitely or until a significant change is made in the benefit formula. Strong support was also expressed for a provision that would relax the integration rules for early retirement benefits. This would recognize the increasing use of subsidized early retirement benefits which represent a socially desirable goal and are not being used in a discriminatory way.

A number of witnesses felt that the change in the rules for final average pay unit benefit plans had no justification. The life insurance industry made a strong plea for a concession to be made in individual policy plans to avoid cancellation of many thousands of existing

policies to meet the new rules. By way of a possible compromise, a number of speakers agreed with the approach suggested by Jack Dyer, among others, that the integration limit be left at 37½% for a plan integrating at \$4,800, and grade uniformly downward to a 30% level at \$7,800 of earnings.

The hearings were conducted in a rather informal atmosphere with periodic questions by panel members, particularly William Gibb, Associate Tax Legislative Counsel of Treasury, and Mrs. Elizabeth Poston of the Actuarial Branch of I.R.S. Then, at the conclusion of the formal testimony there was some informal discussion and questioning of the panel. Particular interest was shown by the panel in the reasons why the moving wage base concept would be impractical, and a possible alternative of a fixed bend point between \$4,800 and \$7,800 was mentioned. There was an indication also that a straight \$7,800 excess plan would be permitted for all years of service, but the integration limit would be restricted to 20% for such a plan.

At the conclusion many observers felt that the panel showed little interest in the suggestions made by the expert witnesses testifying, except perhaps for some minor flaws in the drafting of the proposed ruling, and possibly an alternative approach to the moving wage base problem. The hearing adjourned without any indication as to when the next action from the Treasury on this integration problem might be expected. □

Accreditation

We quote from a statement by Walter P. Reuther, President of the UAW, to the Joint Economic Committee of the Congress, December 1967, on the subject of Federal Legislation and Private Pension Plans:

"In any event it would appear that the implementation of a funding requirement will involve the judgment of competent actuaries, since they must determine the assumptions concerning future experience appropriate for the particular program. It, therefore, appears necessary for standards to be established in order to determine the qualifications of individuals offering to act as actuaries."