Equity-Based Insurance Guarantees Conference

Nov. 14–15, 2016 | Chicago, IL

Presented by the Society of Actuaries and Annuity Systems, Inc.

A Study of Exotic Equity-Linked Guarantees: Pricing, Projections, Hedging and Performance

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A study of exotic equity linked guarantees
pricing, projections, hedging, and performance

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EBIG 2016, November 14th, 2016 (1045 – 1215 hours)

Agenda

- Introduction
- Market model choice for exotic derivatives
- Historical pricing
- Historical performance
- Hedging difficulties
- Summary

FIA Crediting Mechanics

- For FIA, index credits are awarded depending on performance

  – FIA Crediting Mechanics

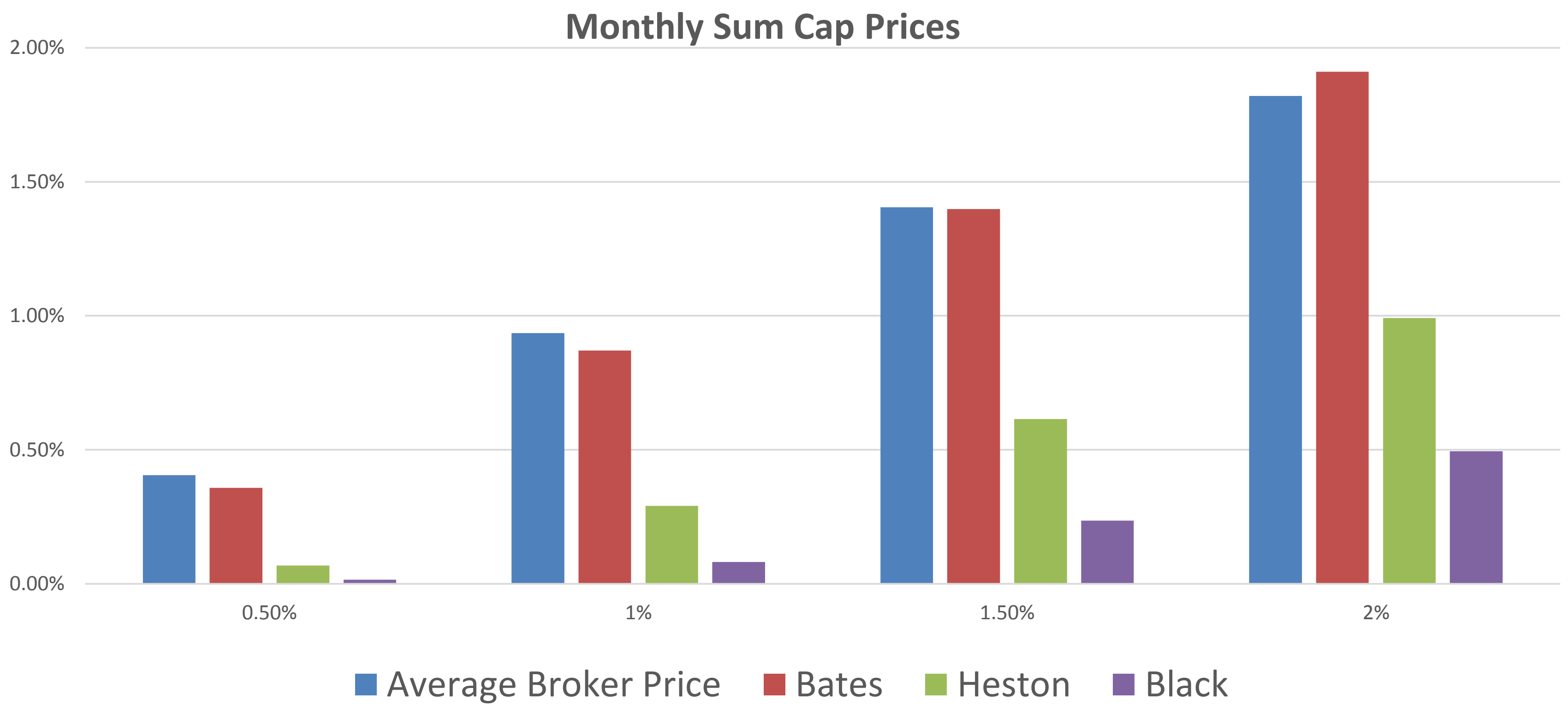
$$PTP = min\left(cap, \max\left(0, \frac{S_T}{S_0} - 1\right)\right)$$

$$Cliquet = max\left(0, \sum min\left(cap, \frac{S_i}{S_{i-1}} - 1\right)\right)$$

- Market risk is transferred from policyholder to insurer. Necessitates need to hedge FIA underlying

Cliquet pricing issues

- Two counterparty quotes received for 2/24/15
- Models calibrated to market data



Models: Black-Scholes (1973)

$$dS_t = (r(t) - q(t))S_t dt + \sigma(t) S_t dW_t$$

- Classic stochastic equity model, but…
  - Assumes constant volatility across strikes
    - Empirically false!
  - Not enough flexibility to capture volatility smile/skew
  - Cannot capture correlation between volatility and the asset
  - Cannot model mean reversion of volatility through time
  - Cannot capture skew and kurtosis in EQ return distribution
- Need a more advanced model

Models: Heston (1993)

- Heston makes volatility stochastic by adding a second correlated stochastic factor (dz) to the Black-Scholes framework

$$
\begin{aligned}
dS_t &= (r(t) - u(t))S_t dt + S_t \sigma(t)\sqrt{z_t}\, dW, \\
dz_t &= \overline{\kappa}(t)(1 - z_t)dt + \overline{\xi}(t)\sqrt{z_t}\, dV, \\
\langle dW\, dV \rangle &= \rho(t)dt, \\
z_t\big|_{t=0} &= z_0, \qquad S_t\big|_{t=0} = S_0.
\end{aligned}
$$

- Heston produces more realistic equity return dynamics than Black
  - More accurate PV
  - More accurate Greeks (sensitivities)

Models: Bates (1996)

- Bates extends the framework even further by incorporating jumps to capture skew and kurtosis

$$\frac{dS_t}{S_t} = (r_t - u_t - \lambda_t m_J)\, dt + \sigma_t \sqrt{z_t} dW + J dN_t,$$

$$dz_t = \overline{\kappa}_t (1 - z_t) dt + \overline{\xi}_t \sqrt{z_t} dV,$$

$$\langle dW\, dV \rangle = \rho_t dt,$$

$$z_t\big|_{t=0} = z_0, \qquad S_t\big|_{t=0} = S_0.$$

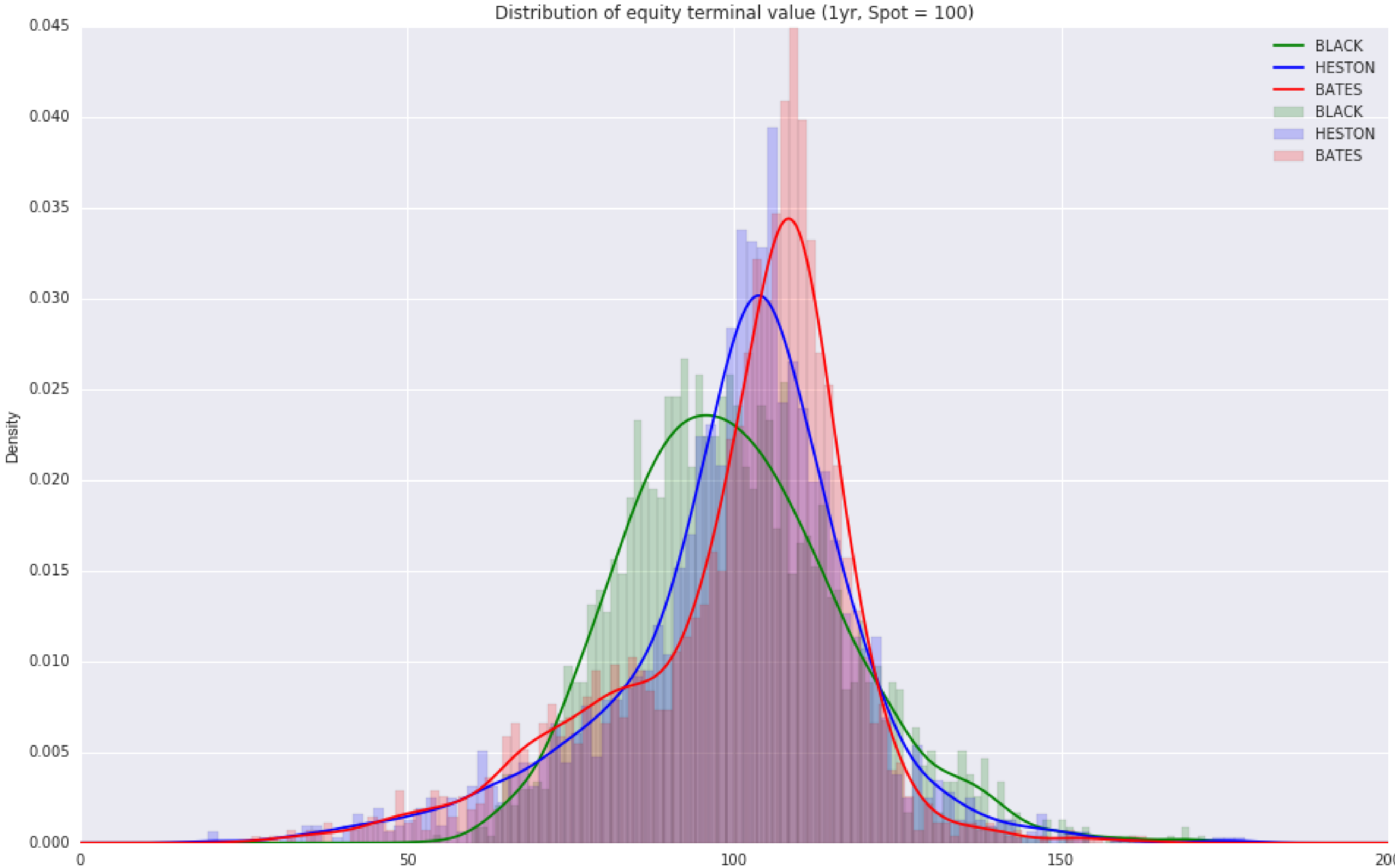
- Bates produces more realistic equity return dynamics than Black or Heston
  - More accurate PV (especially for short dated options where Heston tends to struggle)

Market Model choice for exotic derivatives

- Market Model has significant impact on the pricing results
- What drives such a big difference?
  - Distribution study
  - Compounding effects
- What are the implications?
  - Problems with projections
  - Problems with hedging

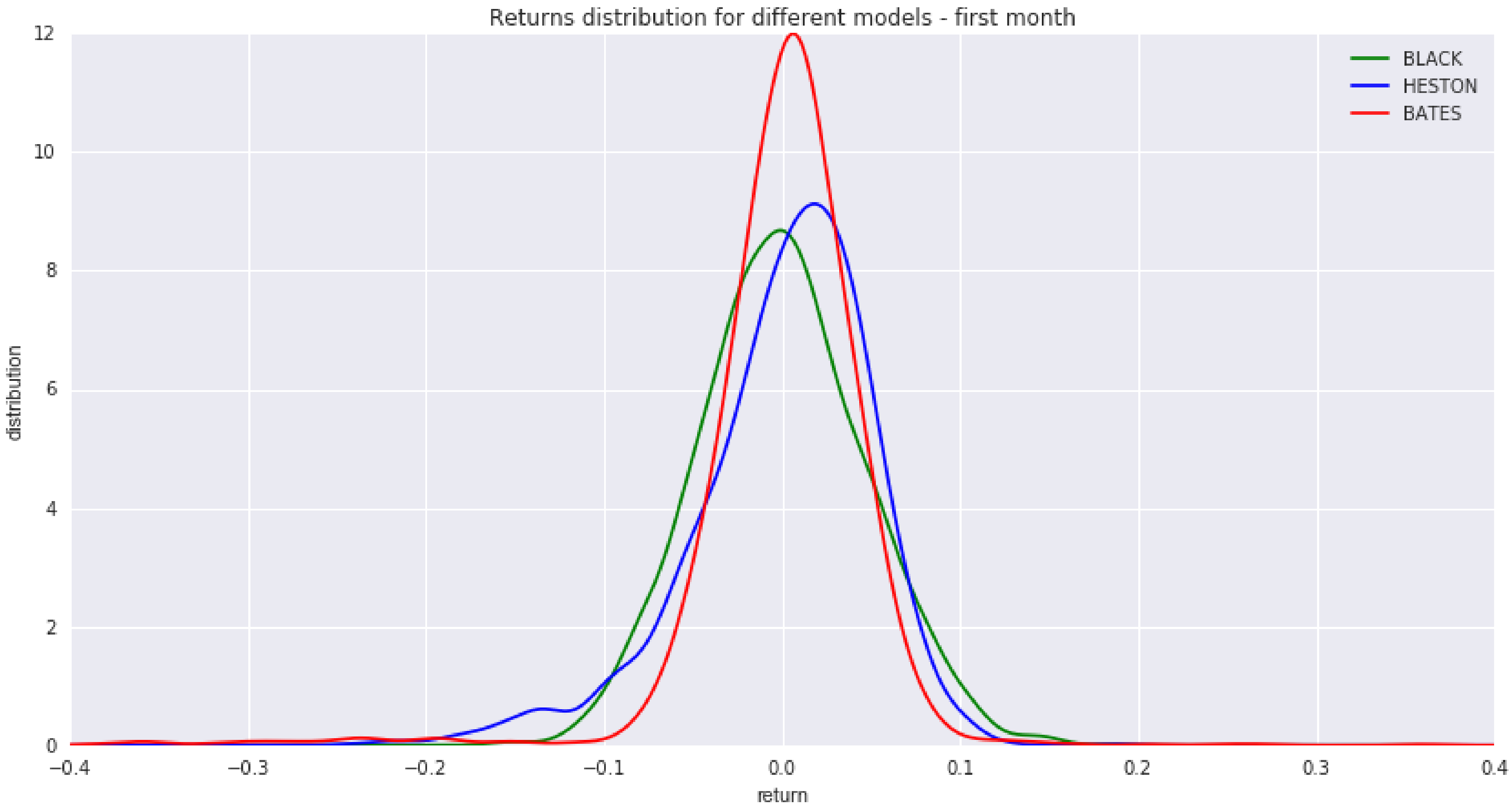
Market Model choice for exotic derivatives

- Distribution of returns – terminal (1yr)



Market Model choice for exotic derivatives

- Distribution of returns – first month



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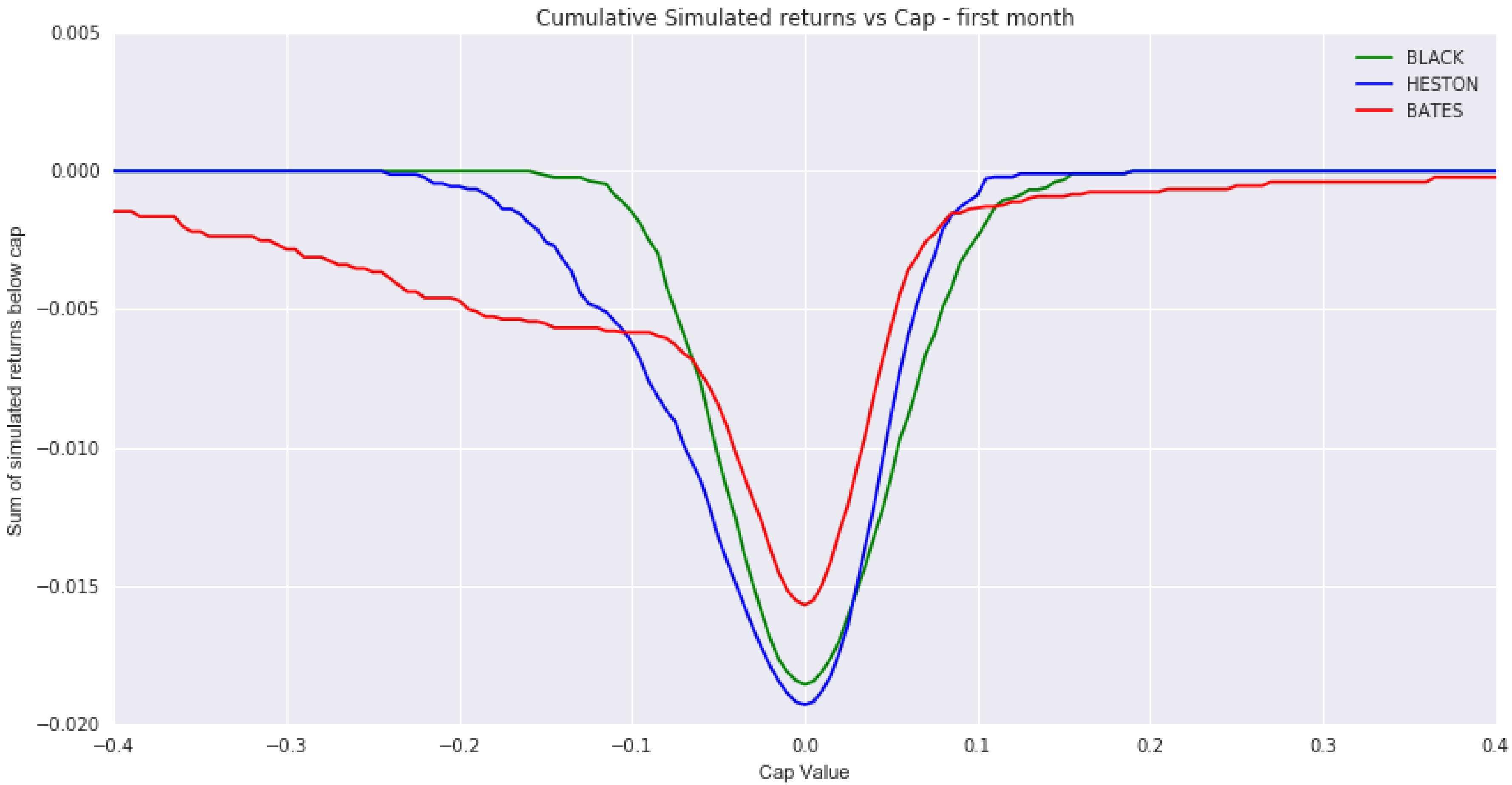
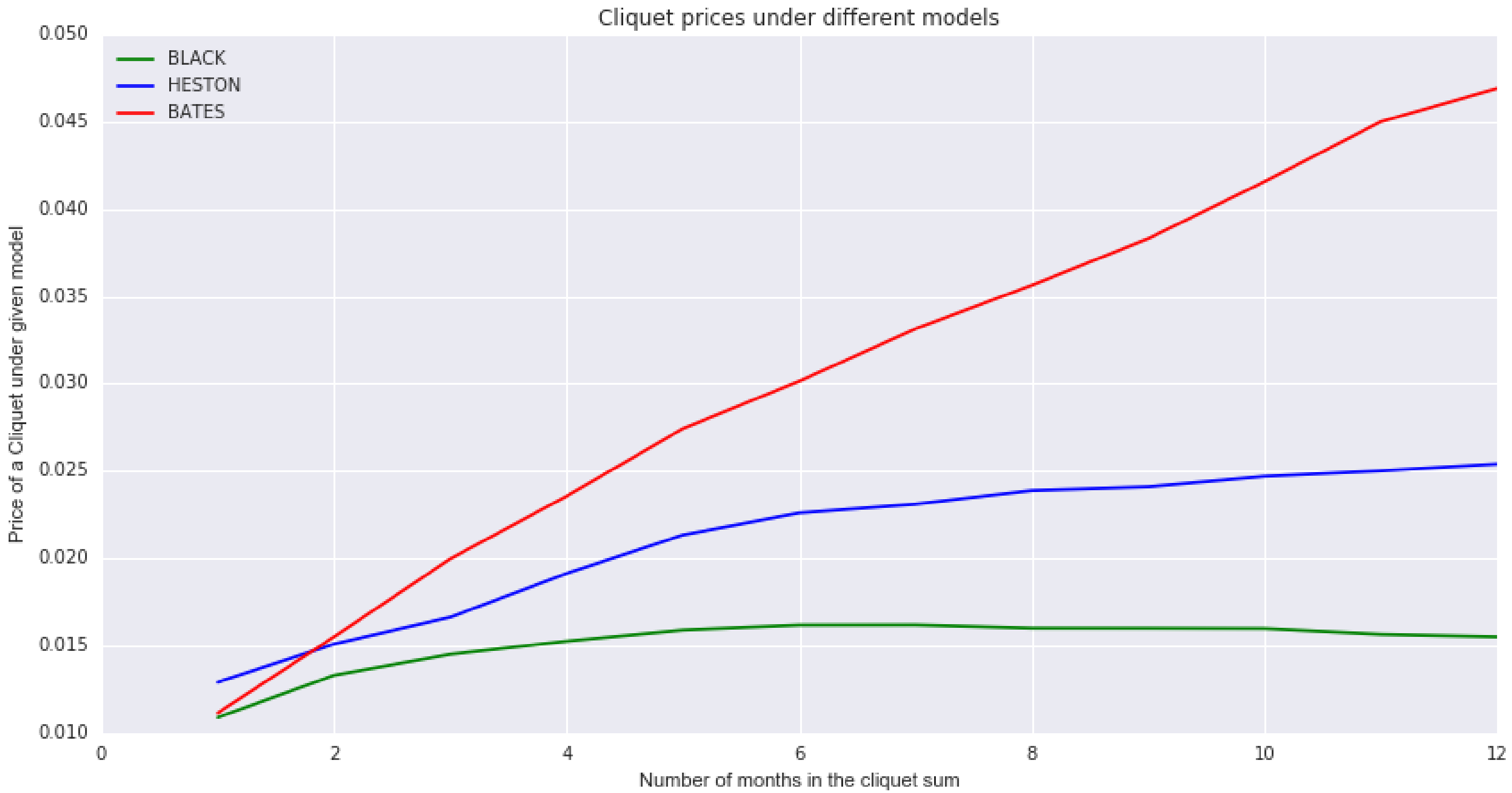
Market Model choice for exotic derivatives

- Cumulative sum of returns (conditional)



Market Model choice for exotic derivatives

- Is a single month distribution of returns on its own able to explain price differences? No – cumulative (monthly) effect is very important too!



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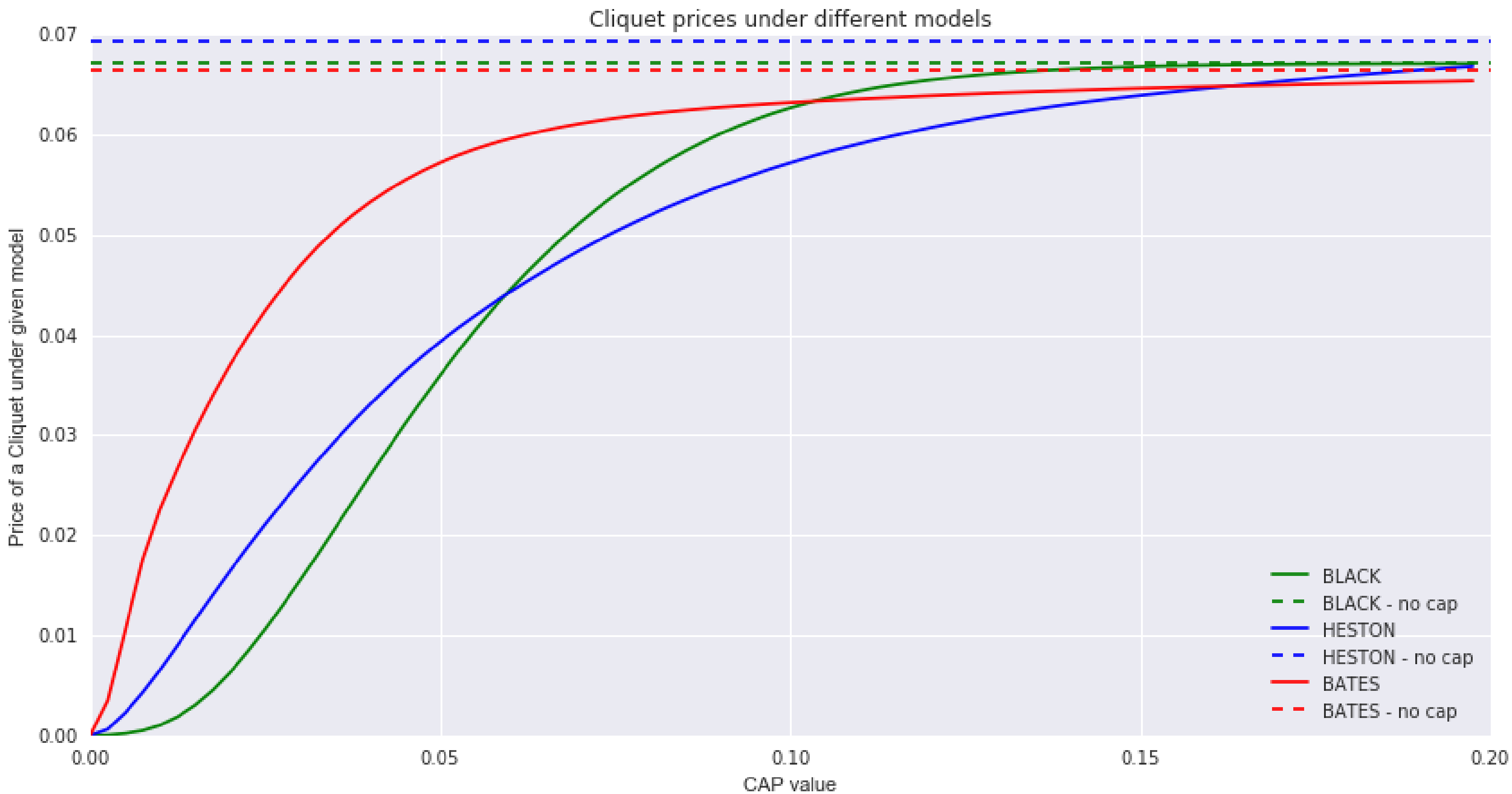
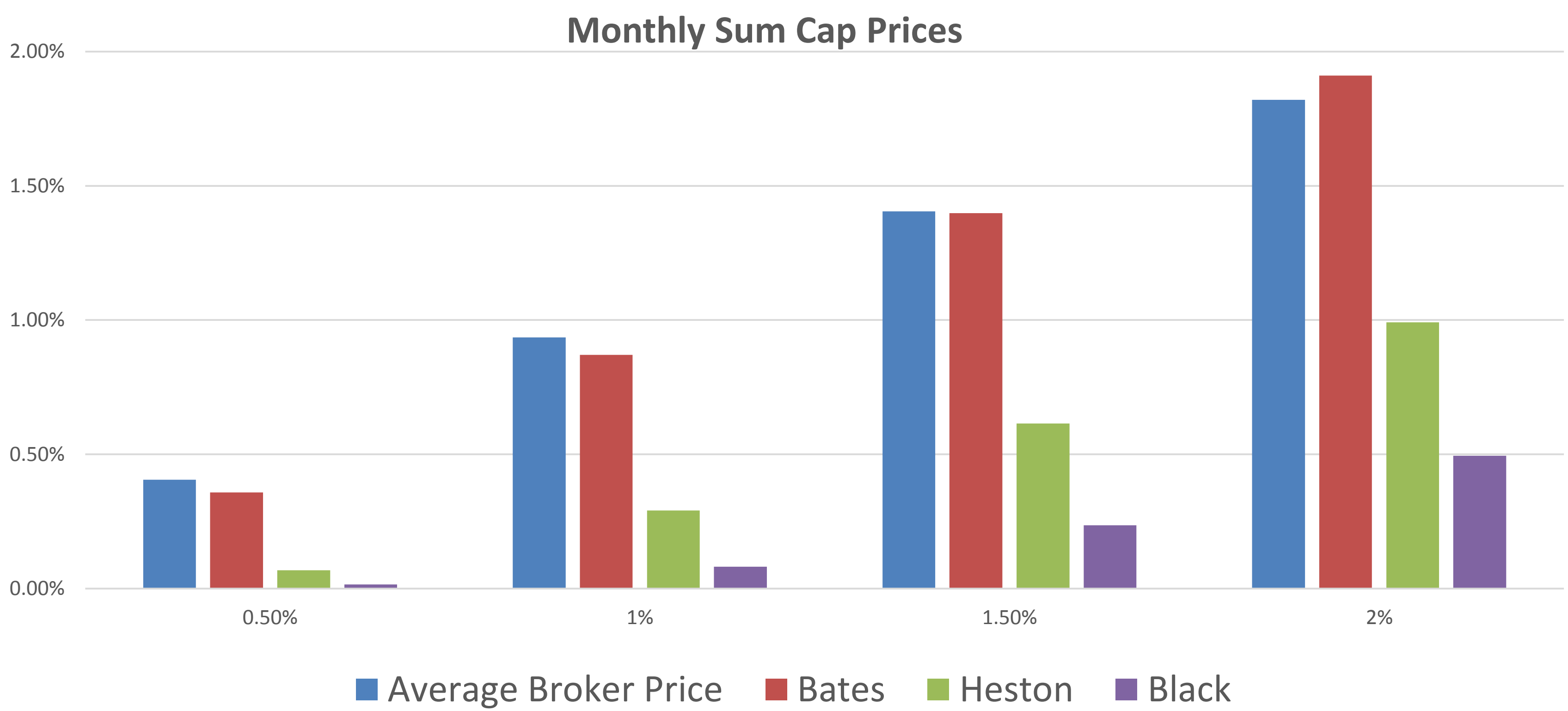
Market Model choice for exotic derivatives

- Pricing of an Monthly SumCap for different caps



Market Model choice for exotic derivatives

- Given results from counterparties one should choose Bates to price cliquets



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Historical Pricing

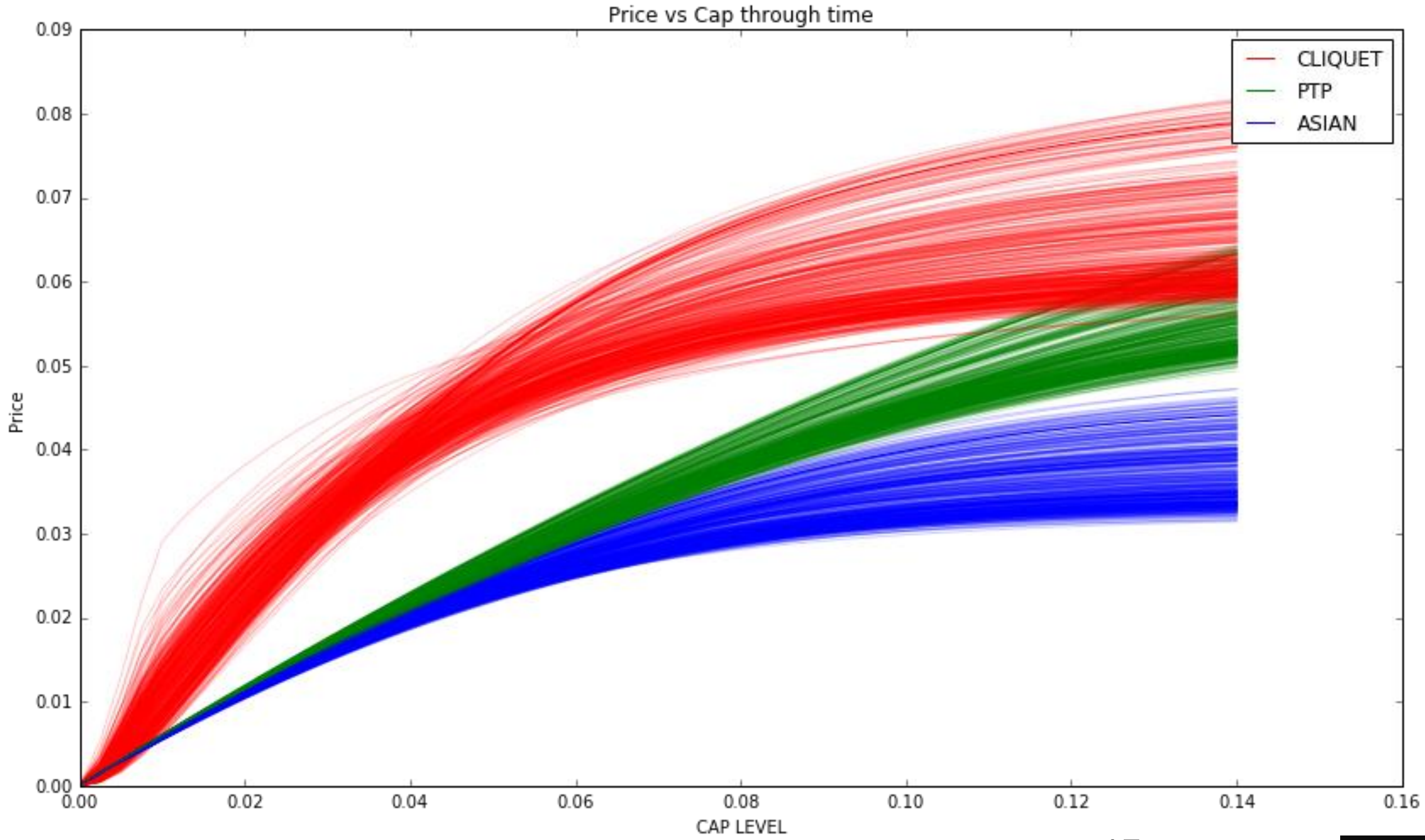
- Bates model was calibrated to two years of historical data (mid 2012 – mid 2014)

- Different products were priced using calibrated models
  - Point to Point (P2P or PtP)
  - Cliquet
  - Asian (monthly averaging)

- Are there any relationships between products?

Historical Pricing

- Historical prices produced by Bates model



Cap relationship between P2P and Cliquet

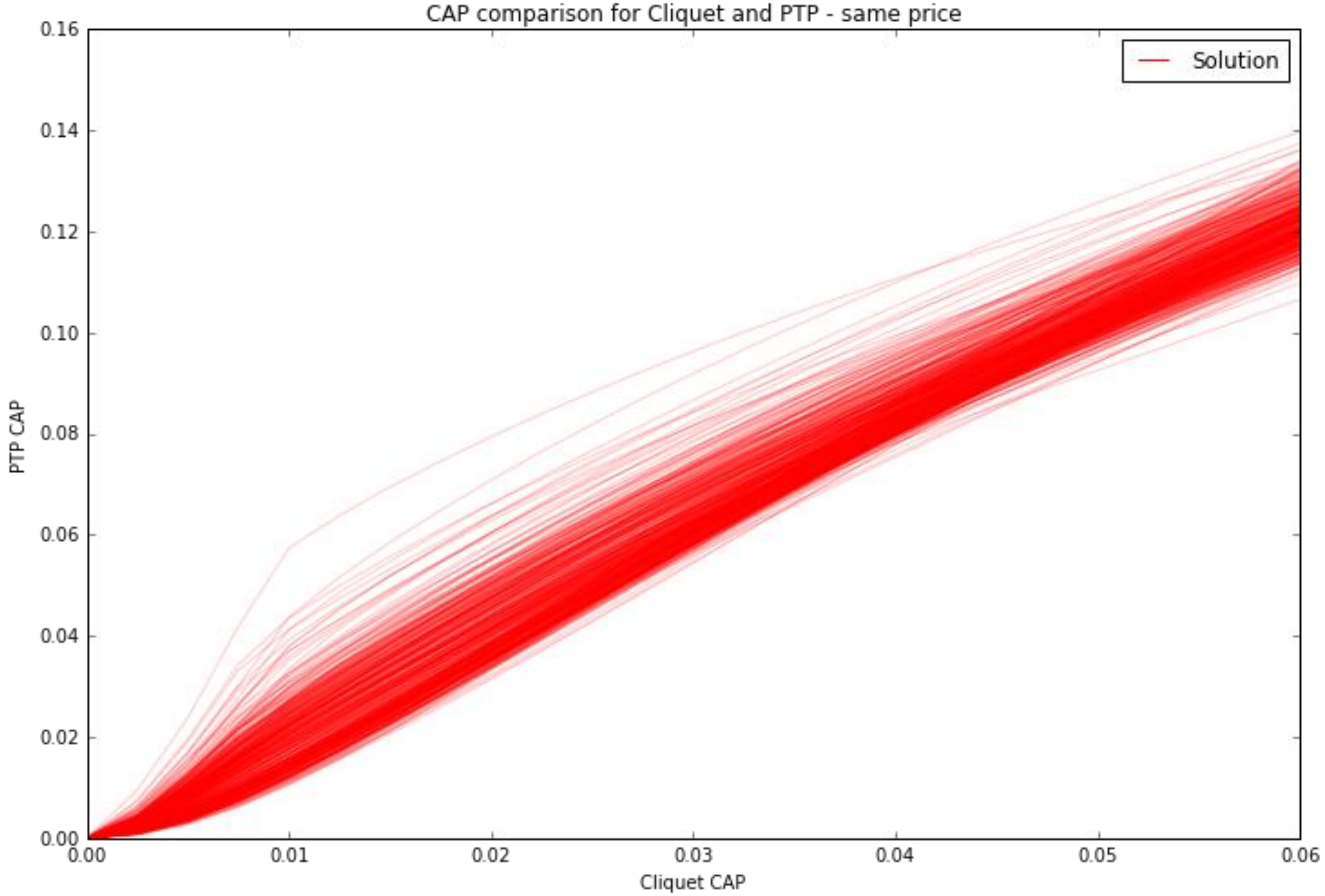
Question: how to compare cap for Cliquet with cap for P2P?

Answer: if budget is given this is equivalent to:

for what cap_{PTP} and $cap_{Cliquet}$ the following relationship holds?

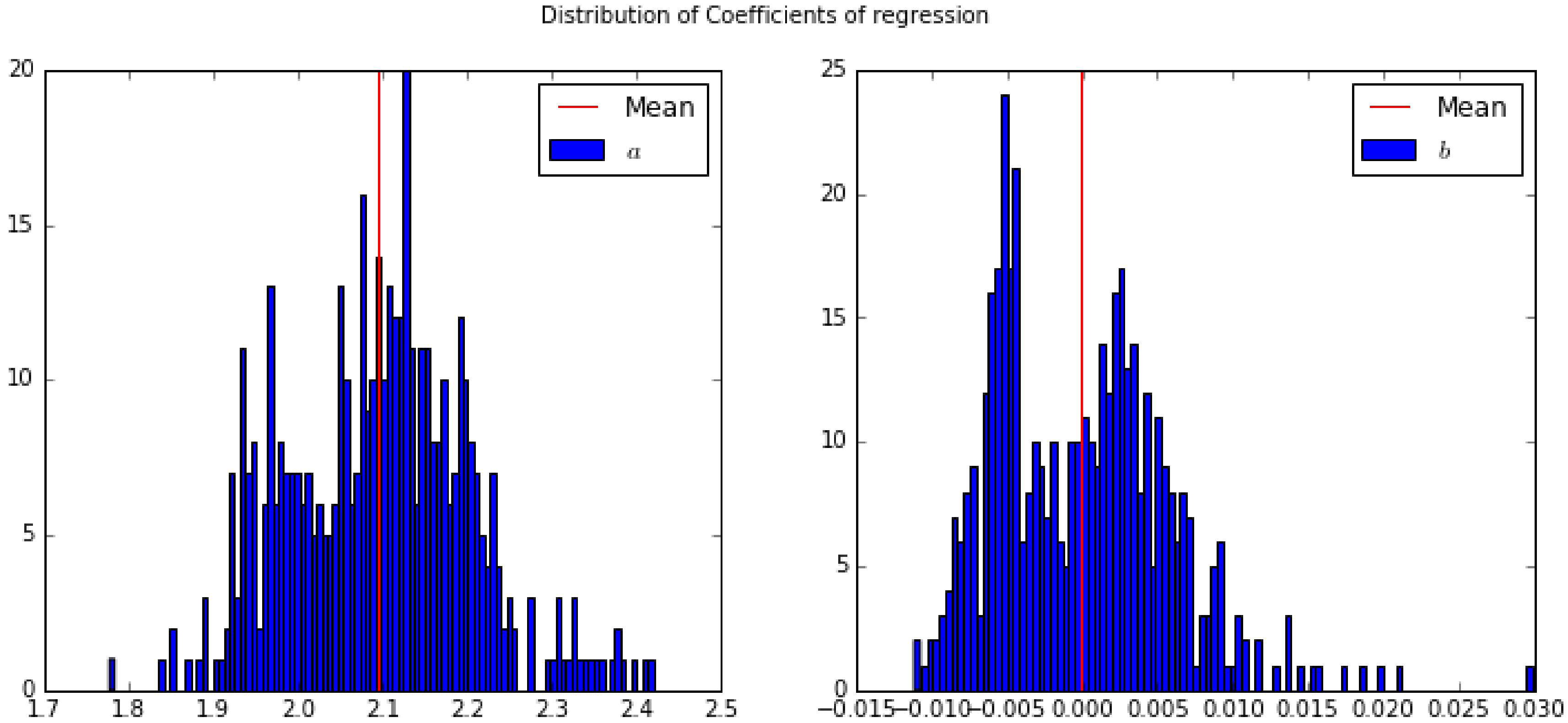
$$Price_{PTP}(cap_{PTP}) = Price_{Cliquet}(cap_{Cliquet})$$

Cap relationship between P2P and Cliquet



Cap relationship between P2P and Cliquet

Regression for: $cap_{PTP} = a \cdot cap_{Cliquet} + b$



Cap relationship between P2P and Cliquet

$$cap_{PTP} \sim 2.1 \cdot cap_{Cliquet}$$

With that knowledge we continue with historical performance

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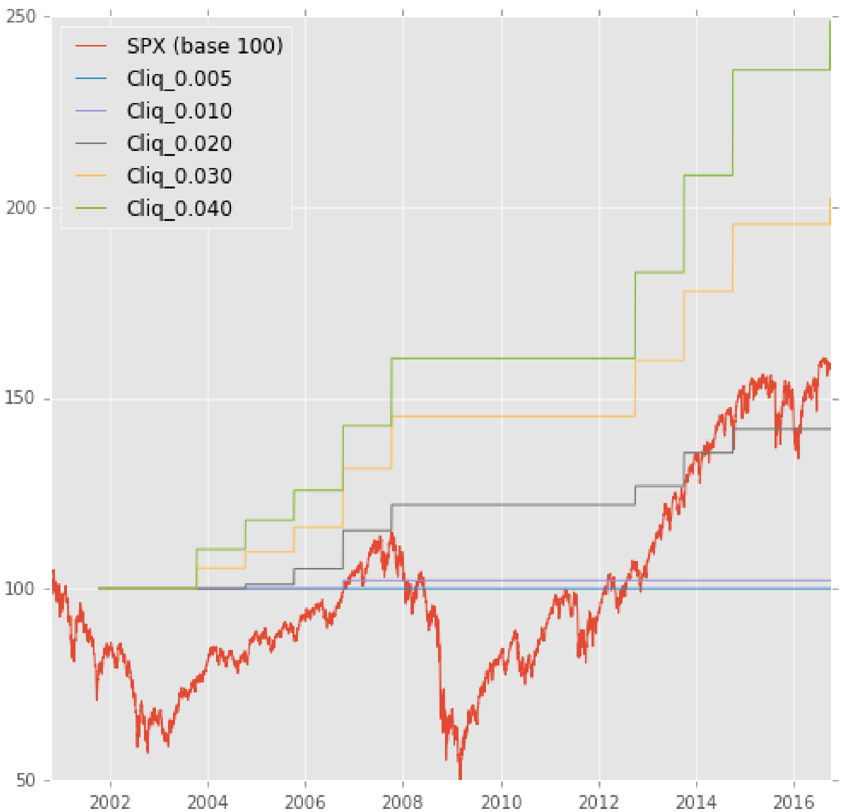
Historical Performance

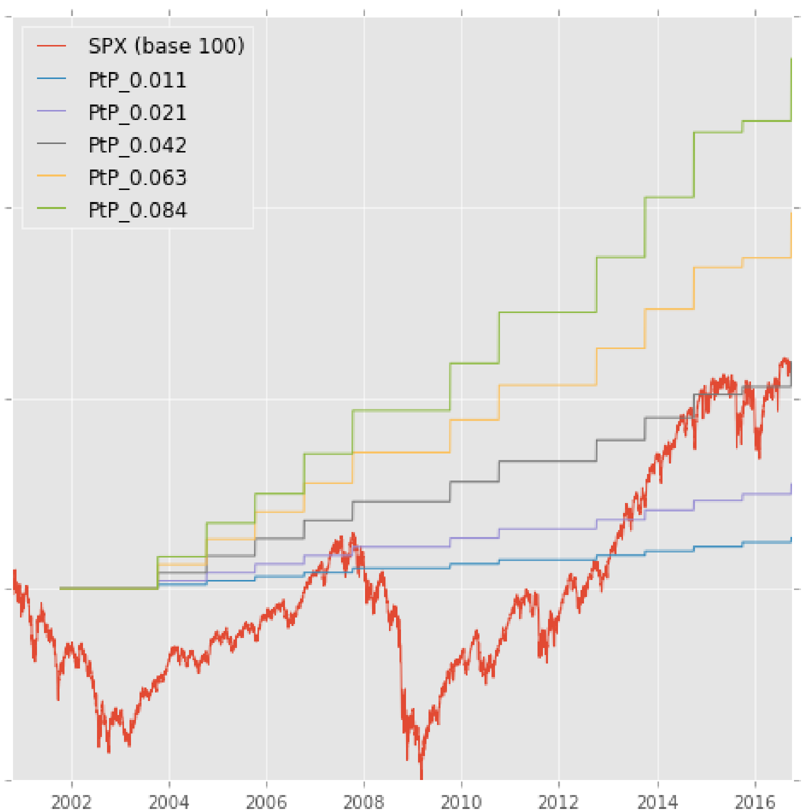
In this section we assume

$$cap_{PTP} \sim 2.1 \cdot cap_{Cliquet}$$

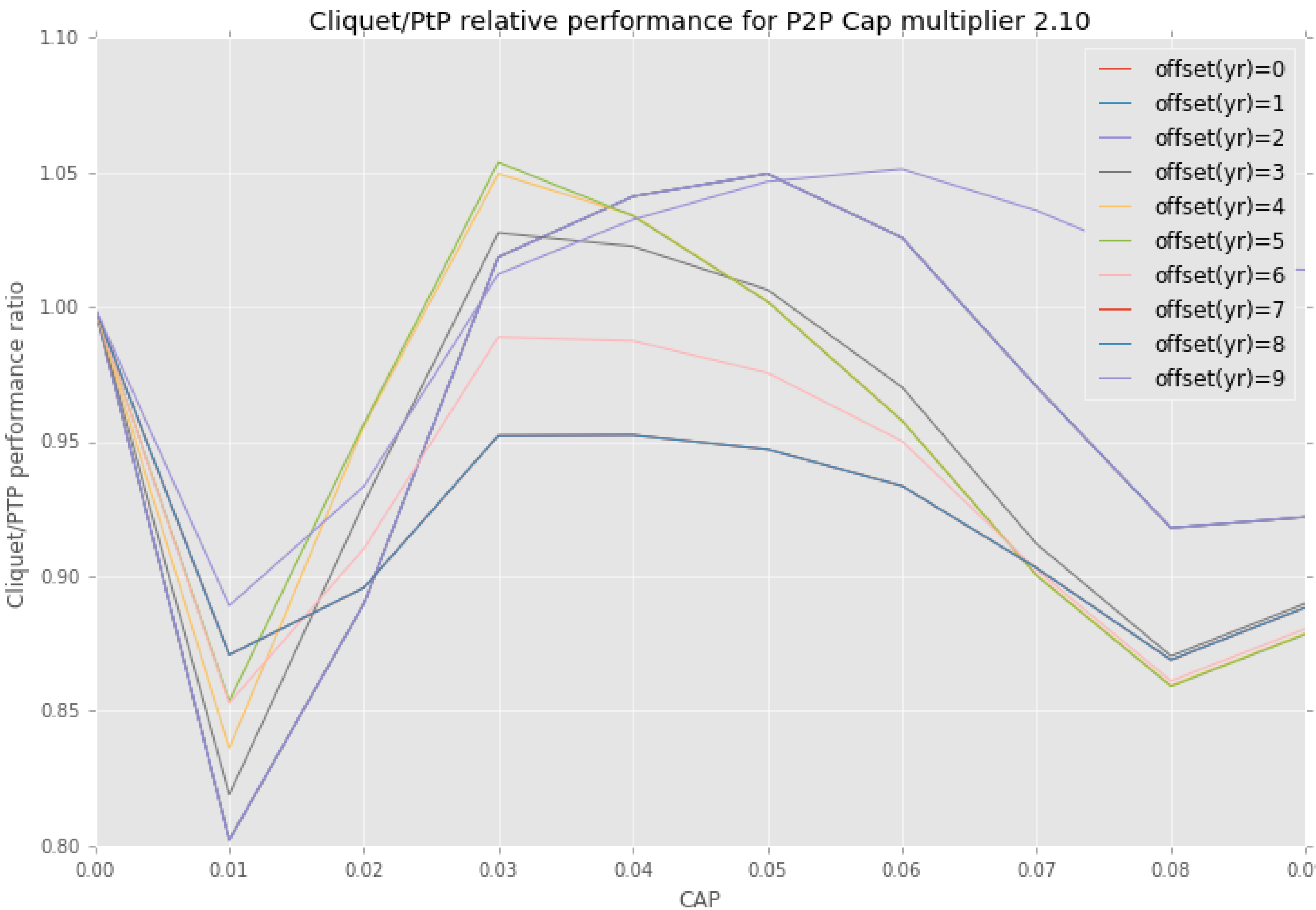
and there are **no fees**!

Simple performance comparison



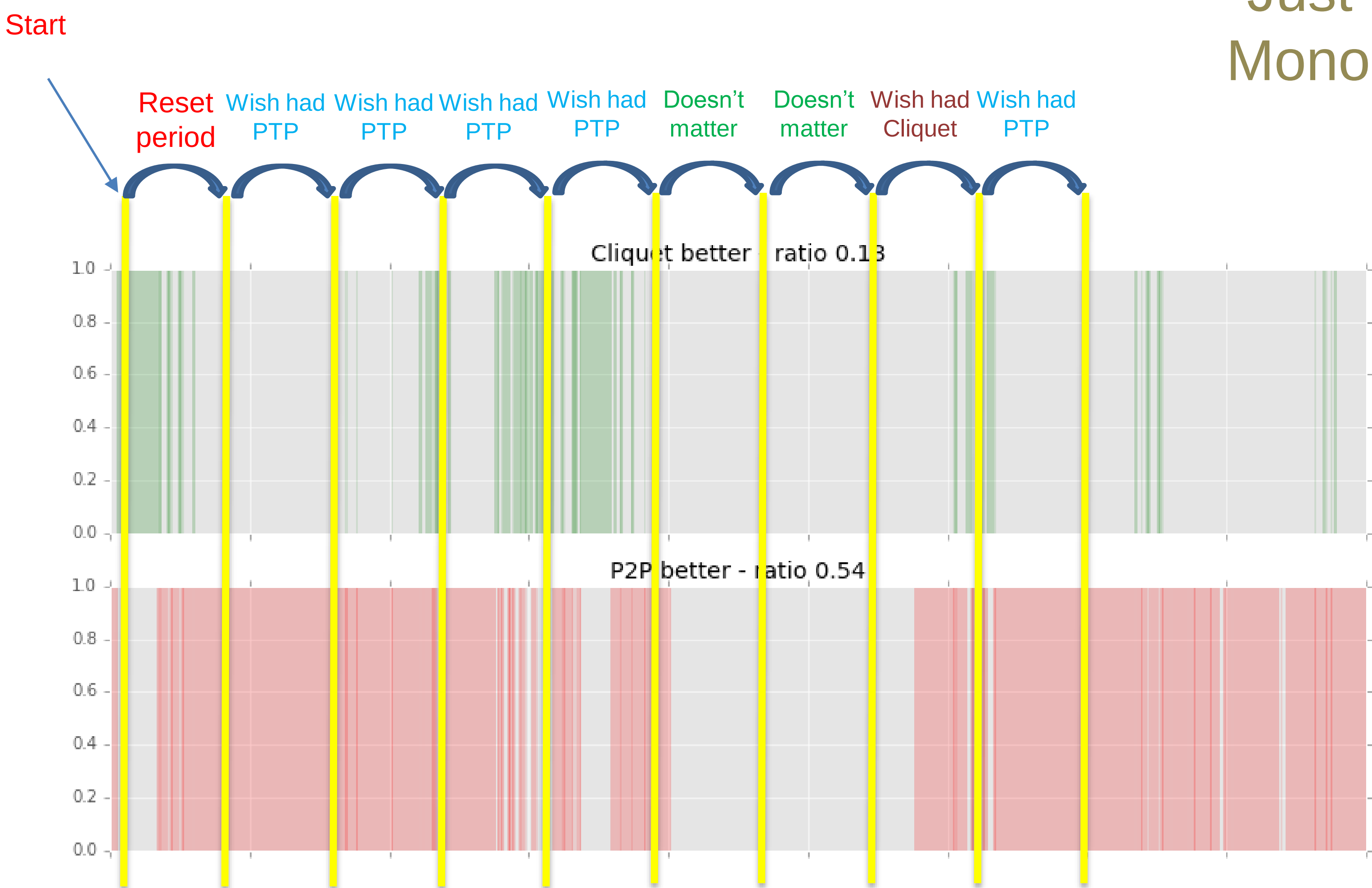


Simple performance comparison – with year offset



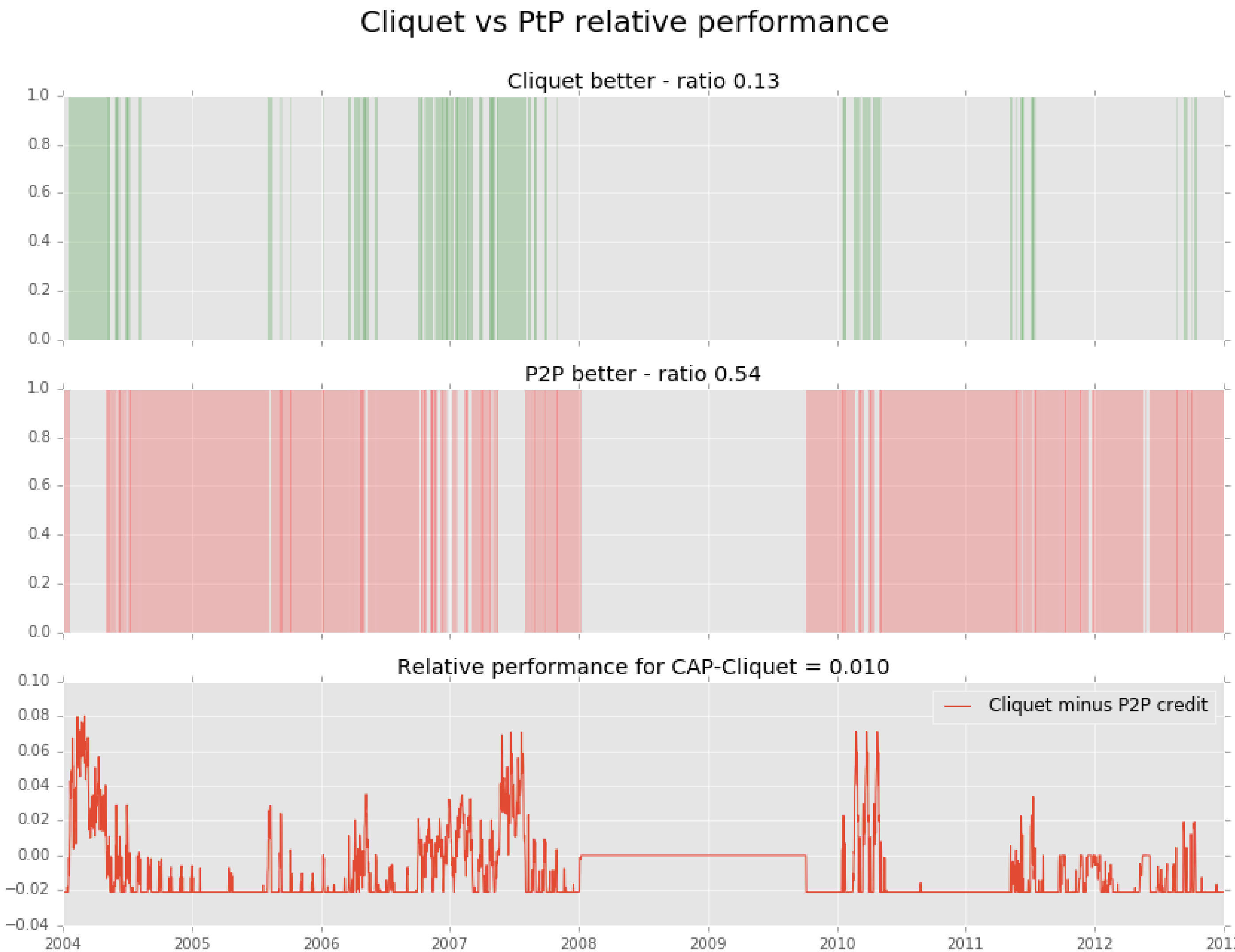
Heads up – interpretation of graphs

Just like Monopoly



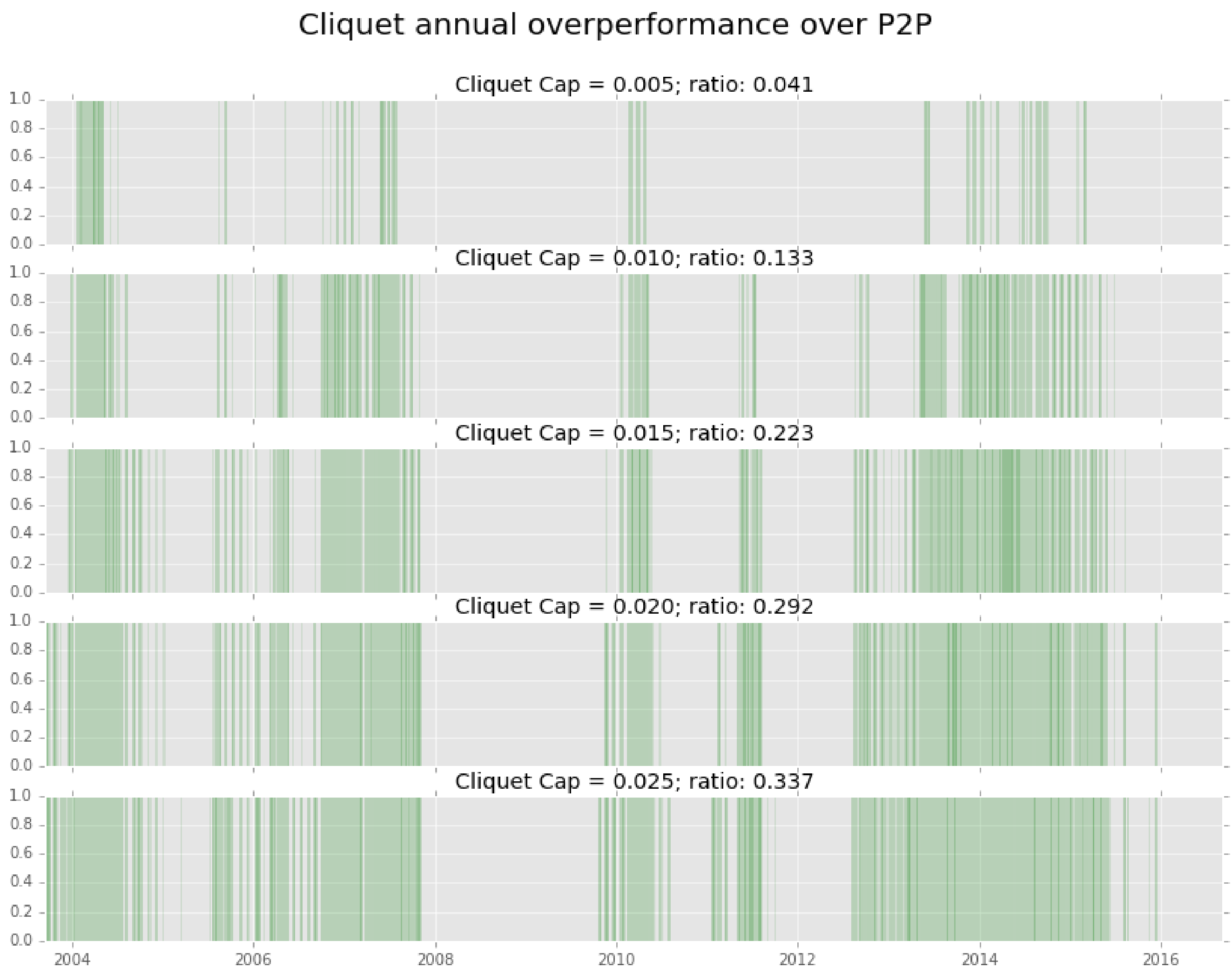
Historical Performance

How annual crediting performs for a fixed cap=0.01?



Historical Performance – Cliquet vs PtP

How annual crediting performs for different caps?



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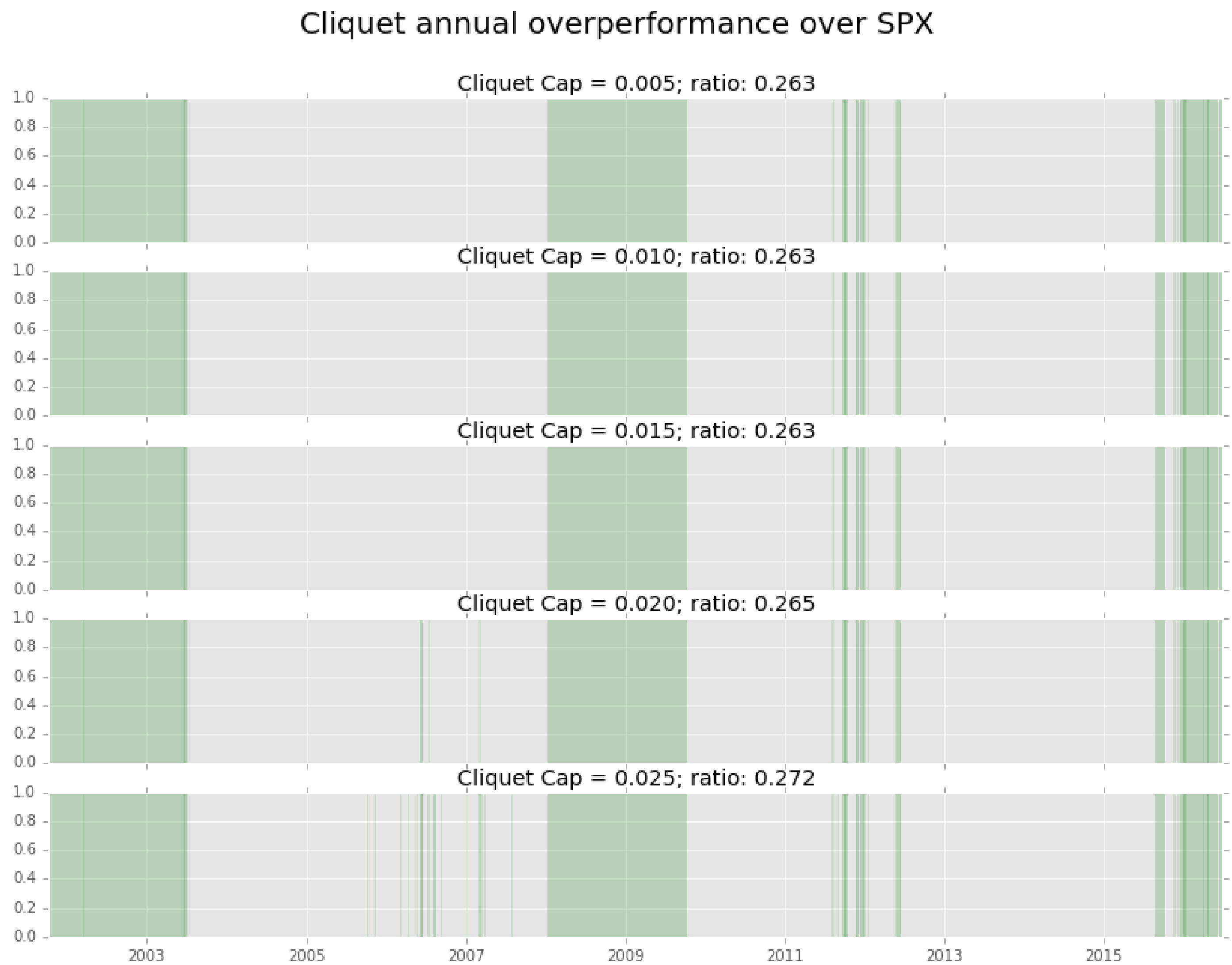
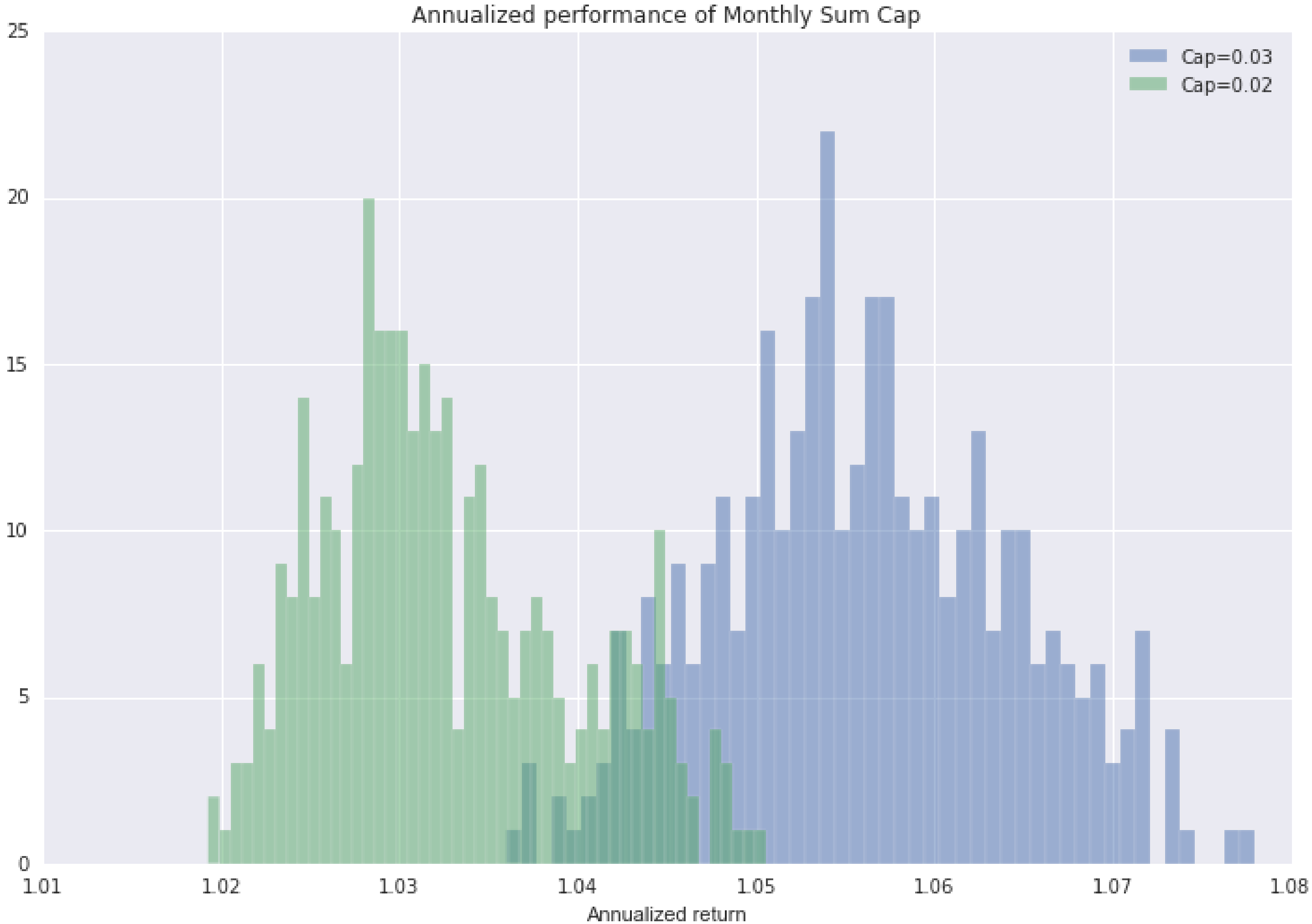
Historical Performance – Cliquet vs SPX

How annual crediting performs compared to SPX for different caps?



Historical Performance – Cliquet

How annual crediting performs with respect to anniversary date?



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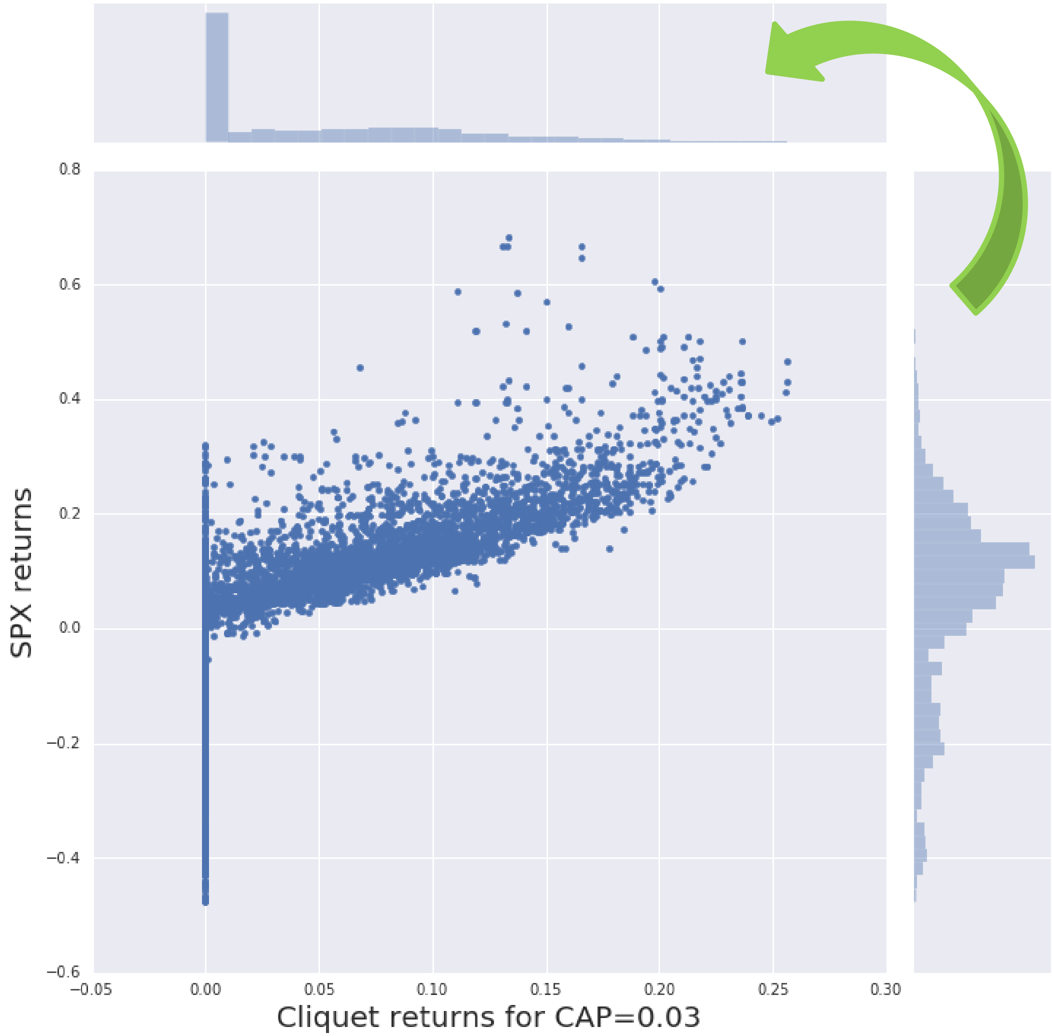
Historical Performance – Cliquet vs SPX

How annual crediting performs compared to SPX - distribution



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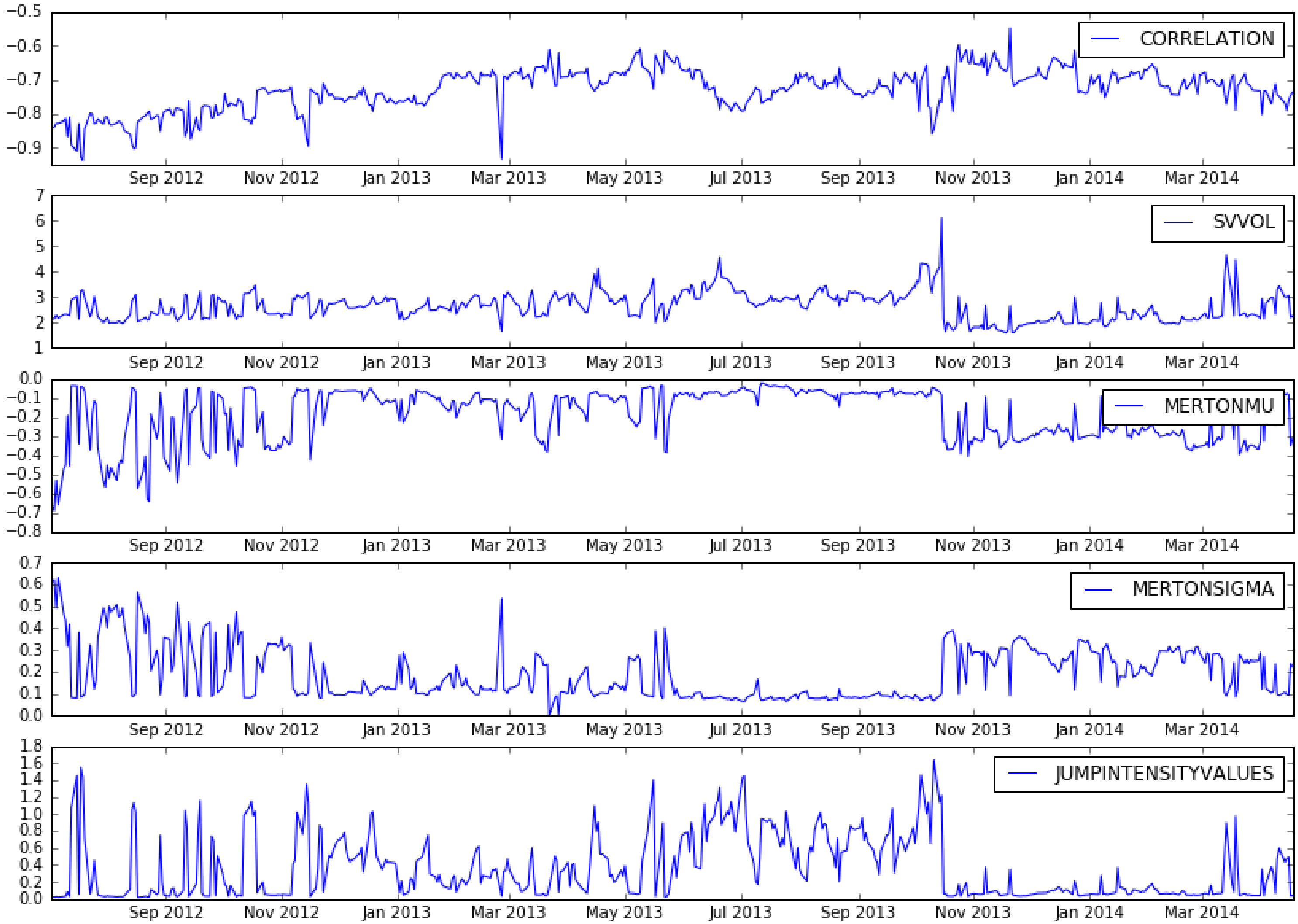
Hedging difficulties

- Hedging of exotics is difficult because:
  - They may be very sensitive and may have non-trivial greek profile
  - Market models used for pricing (e.g. Bates) are non-trivial to calibrate
    - Calibration space may have many local calibration minima
    - Stability of calibration parameters

Hedging difficulties

- Model parameters through time, example



Hedging difficulties

- Stability of model parameters can be improved:
  - Choosing the right starting point (for example yesterday's calibration)
  - Global optimization (slow)
  - Linear Algebra tricks (infer parameter values from day-to-day changes in volatility surface and sensitivity of the volatility surface to model parameters)
  - Use of a simpler model

Summary

- We have shown:
  - Exotics are sensitive to the shape of the distribution of returns, which in turn is determined by the chosen market model
  - Historical pricing should be used to determine equivalence and useful relationships between different offered products
  - The discovered relationship could be used to assess potential performance differences (watch out for DOL!)
  - The discovered relationship could be used to simplify calculations (for example for projections)
  - Stability of model parameters is a non-trivial problem

Thank you!