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Investment Year Method

EQUIVALENCE OF FIXED INDEX AND DECLINING INDEX SYSTEMS

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With the introduction of the Investment Year Method of allocation of general portfolio investment results among insurance company lines of business and among participants within particular lines of business, two different systems of performing the allocation within particular lines of business have emerged.

Under either system the funds, or asset shares, of the participants are analyzed by "year of investment" and investment return rates corresponding to these investment years are applied to the analyzed funds to obtain the investment results allocable to the participants.

Under the "declining index system," the funds associated with a particular investment year decline with the passage of time as the investments of that year are repaid (through maturity, sales in advance of maturity, adjustments in asset carrying value, etc.), so that the funds remaining identified with a particular investment year correspond to the assets acquired in that year that remain on the books of the company. The investment return rates reflect the investment performance of these remaining assets.

Under the "fixed index system", the funds associated with a particular investment year normally remain fixed in amount, regardless of the repayment of assets. Repayments are retained with the original investment year. The investment return rates associated with a particular investment year reflect the investment results, not only of the remainder of the original investments, but also of the reinvestments arising from both repayments of the original investments and repayments of re-investment repayments.

While these two systems are quite different in appearance, they will, *subject to certain restrictions,* produce identical financial results. Although this may seem intuitively obvious, it has not previously been subjected to detailed mathematical analysis. This article is based on rigorous proof of the statement, the length and complexity of which prevents presentation here, although copies are available for any who care to have them.

The statement means that the investment year method would allocate the same investment return to each participant, whether a fixed index system or a declining index system is used, *provided the restrictions are satisfied.*

The following outlines the proof, with a statement of the conditions under which the proof will hold. Some of the conditions which would make financial equivalence impossible are also indicated.

The Steps

The steps are as follows:

(1) Definition of notation for each system.

(2) Algebraic expressions for the accumulation of funds with interest under the respective methods are developed, reflecting the common understanding of how the systems work. The *a priori* expressions for the early durations are generalized to all durations, and the resulting mathematical expressions are taken as definitions of the *fixed index* and *declining index* methods, respectively.

(3) The maintenance of equality, under the two systems, of total funds originally comprising a given generation, is observed.

(4) A statement, and proof by mathematical induction, of formulas for expressing, in each system, investment income and total funds in terms of the year-by-year net changes in the total funds. The statement of these formulas becomes quite complicated under the declining index system, but is relatively simple under the fixed index system. To simplify the notation, a function "$\Pi\,\Theta$" is introduced, which denotes an ordered product of turnover (or repayment) rates and retention (or non-repayment) rates, such that the successive factors apply to successive experience years. Only the last factor may be an interest rate, and each factor is identified with an investment year (the investment year being the experience year of the preceding factor, if the preceding factor is a retention rate). Θ denotes the form of the last of these ordered factors.

The function $\Sigma\,\Pi\,\Theta$, which denotes the sum of all distinct configurations of $\Pi\,\Theta$ for a given initial investment year, number of factors, and form Θ is then introduced. It is necessary to the proof to count the number of such configurations.

(5) Finally, induction is used to prove the proposition that if each interest rate under the fixed index system is equal to the appropriate $\Sigma\,\Pi\,\Theta$ under the declining index system (where Θ is an interest rate), then investment incomes under the two systems will be identical, and funds under the two systems will be identical.

While it is theoretically possible to continue the investment year method indefinitely in accordance with the formulas discussed above, it is generally judged impractical to do so. It seems pointless, one hundred years after the establishment of the method, to be concerned with the inconsequential volume of investments remaining in force from among the investments made quite a long time previously (if the declining index system is used) . . . or to be concerned with the minor differences in investment return rates that would be observed on very old, adjacent investment year funds that have had almost identical experience on repayments (if the fixed index system is used). The expense of retaining these inconsequential investment year distinctions must exceed their value by a wide margin.

Consequently, companies have either established so-called "select periods" or have made plans to establish select periods when it appears reasonable to do so. This differs from a select period in the conventional sense as it is applied to mortality, since it refers to the number of years during which experience is differentiated rather than the period after inception.

The select periods that have been established range from 10 to 25 years. All funds otherwise attributable to investment years during the period before the select period, measured from the present, are combined to form in effect a single investment period or generation.

If such a select period is established, then the formulas on which the proofs were constructed will no longer apply, and it becomes possible — in fact, it becomes quite likely — that the equivalence of the fixed index system and the declining index system cannot be maintained. It is not generally possible to define investment rates under the fixed index system so as to preserve equivalence, once a select period is introduced.

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Social Security

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(7) The coverage basis of ministers is revised by making it compulsory unless the minister opts out on grounds of conscience or religious principle.

(8) The maximum taxable and creditable earnings base is increased to $7,800 for both OASDI and Hospital Insurance (HI).

(9) The contribution schedule is revised, making it slightly lower until 1971 and then somewhat higher. The ultimate combined employer-employee rate will be 10.0%, or .3% higher than previous law.

(10) The allocation to the Disability Insurance (DI) Trust Fund is increased from .70% of taxable payroll (with respect to the combined employer-employee rate) to .95%.

(11) The pay of persons in military service is, in essence, deemed $100 per month higher than the basic pay on which they contribute. The cost of additional benefits will be borne by general revenues as incurred.

(12) The effective date for the general benefit increase and certain other benefit changes is February, 1968, payable at the beginning of March.

The following are the most important changes in the HI program:

The outpatient diagnostic benefit is moved to Supplementary Medical Insurance (SMI).

An additional "lifetime reserve" of 60 days of hospital benefits is provided, subject to cost-sharing of $20 per day initially.

The contribution rate is increased after 1967 by .1% for each party (employers, employees, and self-employed).

The following are the most important changes in the SMI program:

The outpatient diagnostic benefits are transferred from HI.

The cost-sharing provisions are no longer applicable to the professional component of pathology and radiology services furnished to inpatients.

The standard premium rate for persons enrolling in the earliest possible period, which was $3 for July, 1966, through March, 1968, is to be determined annually on a standing basis — namely, for April, 1968, through June, 1969, and then for 12-month periods.

The cost estimates indicate that both OASDI and HI are in very close actuarial balance. The benefit changes made in OASDI are financed, in part, from the sizeable favorable actuarial balance for the previous law shown by the latest estimates and, in part, by the additional financing provided in the amendments. The higher allocation to DI was necessary primarily because of the unfavorable actuarial balance indicated by the latest estimates. This was far more than offset by the favorable situation for OASI.

The increased financing for HI — the higher taxable earnings base and contribution rates — was necessitated by the more conservative assumptions made for the future trend of hospital costs and usage of the extended care facility benefits (the changes in the benefit provisions had a relatively small cost effect). The changes made in SMI resulted in a relative increase in cost of about 6%, which is reflected in the higher premium rate ($4) promulgated for the period April, 1968, through June, 1969. The higher cost estimate includes an allowance for the effects of anticipated future increases in physician's fees and in utilization, as well as the approximate 5% deficiency in the premium rate for the initial period. □

Index Systems

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As an example, suppose that in one year an insurance company purchased only 3% bonds and holds them continuously, and that in the next year the company purchases only 5% one-year bonds which then mature at the beginning of the year after that, the proceeds being re-invested in 6% bonds which are then held continuously.

Suppose there is a two-year select period. For financial equivalence of the two systems, the rate credited in the fourth year on funds received in the first year would need to be 3%, while the rate in the fourth year on funds received in the second year would need to be 6%. With a two-year select period, this is impossible.

As a result, we conclude that it is possible to define a fixed index system and a declining index system so that, if the rates under the two systems are appropriately related and a fixed select period is not used, the two systems will produce identical financial results. Generally, the introduction of a fixed select period will destroy this possibility of equivalence. □

Rejuvenation

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medical history reaching back for thousands of years.

The first was to a work on aging by Alex Comfort, an authority on gerontology, where the provision of the fair damsel to cherish the King is called *Shunamitism* after the young lady, Abishag, the Shunammite (i.e. from the city of Shunem).

The second reference was in a biography of Dr. Thomas Sydenham (1624-1689), who has been called the English Hippocrates and the greatest physician England has ever produced.

The writer's opinion has been that in medicine anything prior to a century ago was uniform in its reminiscence of the dark ages. Yet here he read of a medical man of three centuries ago who seemed ever so modern.

Sydenham was a great believer in fresh air, exercise and dietary moderation. He abhorred "bleeding" in a period when, as one reads what was done to the patient, one concludes that if the patient survived the bleeding treatment he would survive anything!

Among Sydenham's many innovations in medicine was the pioneering of quinine in the treatment of malaria and other fevers.

Sydenham was a great believer in the benefit of long rides on horseback. This was related to another of his remedies, *Accubitis* — a method of warming devitalized elderly patients with either a live puppy or a young person. Revitalizing elderly sick persons by close physical contact with a young person or animal is evidently a mode of treatment of great antiquity.

It is made clear in Verse 4, *Kings* I that there was no sexual significance in the usual sense of the term. However, in the Freudian sense, Shunamitism must touch the vital origins of man's being. It is well known that in an Elderly Citizens' Home the visit of a bevy of young ladies from a School of Nurses revitalizes the whole population.

Unfortunately, the effect is, more or less, temporary. □