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THE PROBLEM OF THE INSTALLMENT LOAN

by David M. Good

The example presented by Robert Myers in the February issue of *The Actuary* illustrated the fact that actuaries from time to time tackle some of the nastier problems of compound interest. As Mr. Myers shows, satisfying the desire for a quite accurate result sometimes involves rather sophisticated and lengthy computations. This note presents a general solution to a similar problem of finding the effective annual interest rate commonly arising in finance. Since the solution is fairly easy to apply and gives a surprising degree of accuracy, it may be of use in upholding our reputation as experts in this field.

The Problem

The terms of a commercial loan are usually stated as an initial charge, with the loan to be repaid in installments; for example, a charge of $6 per $100 in advance, the loan to be repaid in twelve monthly installments. The effective annual interest rate is of course neither 6% nor 6.383% (from 6/94), since on the average only about half the loan is outstanding. The usual procedure of multiplying by $2n/(n+1)$ gives only a crude approximation.

The Solution

The following is an improved approximation for the interest rate on a loan to be repaid in twelve monthly installments. Let z equal the ratio of the initial charge to the amount initially received (the 6/94 of the above example). Then the better result, in form for computing,

$i = (1.8439 + 1.09139z)z$

This formula is to be used in the range of i from about 3% to about 24%,

(Continued on page 5)

SINGLE PREMIUM DECREASING TERM USING CONTINUOUS FUNCTIONS

by William H. Lewis

It is likely that most of the decreasing term insurance policies which are designed to cover a typical mortgage loan are on an annual premium or a monthly premium basis, so that the premium payments may be geared to the level mortgage loan payments and the combination considered a package type of payment by the mortgagor-policyholder.

In some instances, however, it may be desirable to use a single premium decreasing term policy for this type business. This type policy may have particular appeal to a bank or savings and loan institution which holds the mortgage and pays the single premium to the life insurance company while lending the same to the borrower. The amount of the single premium is added to the regular mortgage loan and is amortized by level monthly payments along with the regular mortgage loan payments.

Method Limited

This method of providing life insurance benefits does not work too well in a situation where the borrower is anxious to make a minimum down payment and hence obtain the maximum amount of mortgage, but may well fit a situation where the collateral is substantial as compared with the amount of mortgage loan. In such a case the fact that the amount of mortgage loan the borrower has in mind needs to be increased by a relatively small amount to accommodate the borrowed single premium is of no great concern to him.

If it is assumed that mortgage payments are made on a continuous basis, using the force of interest which corresponds with the monthly effective mort-

(Continued on page 3)

A VIEW OF PLANNING — SORCERY TO COMPUTERS

by James C. Hickman

John Maynard Keynes said that he was only interested in the short term, for in the long term we are all dead. Despite Keynes' concentration on the short term, the future and the possibility of its control have always fascinated man. The Romans diligently studied the pattern of the intestines of sacrificed animals in an attempt to lift the veil from the future. The scriptures leave no doubt but that the three kings learned of the impending birth of Jesus by a study of the stars. Alexander thought it prudent, before embarking on his memorable trip of conquest, to inquire about the probability of success from the oracle of Delphi.

Because of the successes of Alexander, the legions of Rome, and of the search of the three kings, it is not immediately apparent that scientific and rational attempts at perceiving and modifying the future are superior to sorcery. Let us acknowledge that the superiority of planning over luck is a matter of faith and not a verified fact.

Difficult to Verify

Few industries, except perhaps life insurance, have had much experience with other than short term plans. Consequently, it is difficult to empirically verify that long term planning is superior to a more myopic view (plus a mystical faith in Adam Smith's "invisible hand") in making sure that economic enterprises are guided to success. In fact such verification may be impossible. In the course of human affairs the social, political, and sometimes even the physical and moral environments in which plans are realized change so frequently that it is

(Continued on page 4)

A View of Planning

(*Continued from page 4*)

[hole]ents have been developed. Once these assignments are fixed, the vast machinery of statistics is available to help trace the possible consequences of alternative decisions and to place indices of reliance on estimated consequences.

(3) It offers more statistical data. A function of the federal government is to collect and summarize statistics which are useful in planning. For example, the Office of Business Economics of the Commerce Department estimates the components of national income; demographic facts come from the Census Bureau; price and employment indices are produced by the Bureau of Labor Statistics; and health information flows from the National Center for Health Statistics and other agencies of the Public Health Service. If this information is used to produce more rational and efficient plans, it would seem that such statistical work is a very productive investment of public funds.

Insurance Planning

American actuarial literature already contains a delightful essay on planning. I recommend to your re-reading John Hogan's provocative paper. (3)

Actuaries were the first long range business planners and, at least until the present, have probably been the most successful. When most businesses were formulating quarterly plans, actuaries were setting prices for decades in advance. Asset share computations have for generations been an actuarial tool for checking the long term consistency of price and benefit structure decisions. Model office computations of great complexity have been used in setting agency development goals. Gross premium valuations have been a tool in setting surplus and reserve objectives.

Actuaries have traditionally been interested in expected values. Model office, pension fund projections, and asset shares have usually been presented as a single number or set of numbers with no indication of the probable range of variation of these results. This was almost the only possible way to report actuarial results before the advent of cheap computation, and it seemed to some actuaries that to present anything other than a single answer to an actuarial question was unprofessional hedging.

Yet the results of the operations of the financial systems that actuaries guide are not known with certainty, and greater predictive reliance can be placed in certain expected results than on others. Even a crude measure of this reliance can be helpful to a decision maker. Today, by simulation techniques, it is entirely possible to report an approximate distribution for prospective financial results. These distributions will be based on probable lifetimes and other cost variables, all part of the basic assumptions. (4)

Modern technology does more than provide simply another dimension to visualize in considering traditional actuarial problems. It is now possible, and probably it is imperative, to expand the application of actuarial techniques to other aspects of insurance operations. But a few guide posts should be kept in mind.

Guideposts

- Much of the work in planning involves reducing to an operational formula vaguely stated corporate goals. In a word, performance indices are required. In life insurance management, questions such as the following arise: How do you measure the performance of an agent or an agency? How do you measure the comparative performance of an equity fund?
- Do not attempt a total company plan as your first effort. Experience is a great teacher in planning as in most endeavors. By working on local problems before tackling global problems, one can gain such needed experience. Concentrate on an area which has not already been intensively studied in order to achieve the satisfaction of some success in your early efforts.
- Plans are not static but are dynamic guides for action. Robert Burns wrote that, "The best-laid schemes of mice and men gang aft a-gley". The modern business planner should try to improve on the success ratio predicted by the romantic Scot by continually reviewing his plans as the corporate environment and goals shift. Planning is a dynamic operation for achieving defined corporate goals, and it is not a sterile machine for squeezing the human juice from business management. □

Installment Loan

(*Continued from page 1*)

within which its error is no greater than 0.00004 (4/1000 of 1%). It is *not* to be used outside of this range under any circumstances, since the error increases markedly on *either* side. The formula overstates the rate in the center of the range and understates it at each end.

This formula was developed specifically for the range $i^{(12)} = 3\%$ to $i^{(12)} = 24\%$, approximately the range cited above. I chose to express i in terms of this quantity z since the coefficients in the corresponding Taylor series were smaller than in other related expansions. The Taylor series itself was not used, since its error is far greater at the right end of this range than at the left end, and furthermore is of the same sign throughout. Both of these facts indicate that the Taylor series is wasteful of effort. The formula given here is one of "greatest economy" or "minimax" type.

The formula was obtained by fitting a straight line to the function i/z in such a way that the errors were positive at each end of the range and negative in the middle. The straight line was determined by requiring that the errors in i at $i^{(12)} = 3\%$, $i^{(12)} = 18\%$, and $i^{(12)} = 24\%$ be equal in size and alternating in sign. There are several ways of performing the mechanics; a good discussion is given by Cecil Hastings in *Approximations for Digital Computers*, (Princeton University Press).

Consider the example given above. The usual rule gives i as (24/13) (6/94) = 11.78%. The formula of this note gives i as 12.21%, correct to the digits shown (with a possible ± 1 in the last digit due to rounding), a difference of almost one-half a percentage point.

Corresponding formulas for 24 and 36-month loans and the range of 24% to 48% have also been developed.

Since the subject of consumer loan charges is discussed frequently in the press and even is a minor political issue, perhaps such formulas will provide more accurate illustrations of the true effective annual interest rate. And until a universal Truth-In-Lending bill is passed,* they may assist actuaries in calculating the cost of automobile loans. □

**S. 5, Consumer Credit Protection Act, was signed by the President, May 29, 1968.*