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DR. BELTH'S "PRICE" THEORY

by William Gould

Mrs. Julia Oldenkamp's letter (*The Actuary*, January 1969) refers to the example given at the New York Actuaries Club workshop discussion of the "Belth" price theory (as reported in *The Actuary*, November 1968). She says that the example "is a bit shallow, since Dr. Belth did not propose to use a single year's cost as the sole criterion in comparing policies."

As the author of the example, I would like to explain why I consider it to be quite significant and instructive.

The Washington replacement regulation requires a comparison of the "yearly prices per $1000 of protection" for the old and new policies, for selected individual years ("current policy year," "5 years hence," and "10 years hence"). One of my purposes in concocting my "horrible example" was precisely to demonstrate that such comparisons for selected single years are quite devoid of significance.

About the Example

The example was presented at the workshop as a comparison of the cost figures for two policies for a *single* policy year. There is no warrant for Mrs. Oldenkamp's supposition that the cash values of the two policies differ by a constant amount at all durations. Actually the example compares the cost figures for the 6th policy year for two policies issued on the 10-Year Endowment plan at age 50 (last birthday) with premiums equal to the net premiums based on the 1958 CSO Table at 3% with immediate payment of claims. The policies differ only in their cash values, those for Policy A being equal to the net level premium reserves on the premium basis while the cash values for Policy B are somewhat less.

The true cost per $1000 of protection should be, under the stated assumptions, approximately equal to the tabular rate of mortality increased by half a year's interest at the tabular rate multiplied by $1000, in this case $13.80. The Belth "yearly price per $1000 of protection" is $13.97 for Policy A, which is reasonably accurate, but only $11.84 for Policy B, which is actually less than the tabular cost. The Belth "yearly price of protection" is $6.26 for Policy A, and $5.58 for Policy B; the difference between these two figures is equal to the difference in interest on the cash values at the beginning of the policy year. (Note: For reference, these Belth formulae are shown in Appendix 1.)

APPENDIX 1

Dr. Belth's "yearly price" formulae may be expressed as follows:

"Yearly Price of Protection":

$$KP_t = ({}_{t-1}CV+GP)\ (1+i^B) - (D_t+{}_tCV)$$

where ${}_tCV$=Cash Value at end of policy year t

GP=Gross Annual Premium

D_t=Annual Dividend at end of year t

i^B=Some arbitrary rate of interest

"Yearly Amount of Protection":

$$AP_t = 1000-({}_{t-1}CV+GP)\ (1+\tfrac{1}{2}\, i^B)$$

"Yearly Price per $1000 of Protection":

$$KPP_t = \frac{KP_t}{(AP_t)\ (.001)}$$

The further assumption in the example that the two policies would have the same increase in cash values for that policy year was an incidental touch, to show the effects for two policies having the same total net cost for the policy year.

A most significant point brought out by this example is the fact that the Belth "yearly price" formulae, which purport to measure the cost of the protection element in a life insurance policy, produce figures that are different for these two policies. There is no reason for the cost of protection for these two policies to be different. The example points to a major defect in the Belth formulae, namely, that the formulae do not take proper account of significant differences in the savings elements of the two policies.

Another serious defect in the Belth formulae is that the "yearly price of protection" quite arbitrarily includes the entire yearly expense of the policy (plus interest), i.e., the expense on the savings portion of the policy as well as the expense on the risk portion. Since the protection element in a policy is inseparable from the savings element in that policy, the Belth "price of protection" formulae are inherently objectionable as providing incomplete comparisons. When "yearly price" figures can be so obviously fallacious, as in this example, it would be irresponsible to accept them as valid or meaningful indices of cost.

In addition to the material on "yearly prices," the published report on the workshop discussion contained a brief reference to Dr. Belth's method of calculating "level prices," which I would like to amplify. The "level price per $1000 of protection" for a period of years is an average of the yearly prices during that period. It is calculated as the present value of the "yearly prices per $1000 of protection" weighted by the yearly amounts of protection, divided by the present value of the yearly amounts of protection, using discount factors involving interest, mortality and lapse. Clearly, the "level price" has no more validity than the "yearly prices" it contains. But even if it were possible to devise a method of calculating valid "yearly prices," it should be recognized that the "level prices" calculated by Dr. Belth's method are very strongly affected by the particular choices of assumed rates of interest, mortality and lapse.

Discounting Operation

The discounting operation involved in Dr. Belth's method of calculating "level prices" in effect assigns relatively greater weights to the "yearly prices" at the early durations and lesser weights at the later durations. Several examples of "level price" calculations for policies differing only in their cash value structure were presented at the workshop. One was a comparison of two policies—10-Year Endowment for $1000, issue age 50 (nearest birthday), gross annual premium of $95.24 equal to the adjusted annual premium according to the Standard Nonforfeiture Law, based on the 1958 CSO Table at 3%—with the cash values for Policy C equal to the full reserves, and the cash values for Policy D equal to the statutory minimum values. Table 1 shows the results of this comparison.

An examination of Table 1 is instruc-

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tive. It shows that the Belth "yearly price per $1000 of protection" is higher for the policy with the higher cash values (Policy C) for every year except the first. Although Policy C is obviously preferable from the policyholder's viewpoint, since it has higher cash values than Policy D with no difference in premiums, the "level prices" calculated by methods (a), (b) and (c) are *higher* for Policy C than for Policy D.

Method (a) involves weighting the yearly figures by the yearly amounts of protection, without discounting; method (b) involves discounting for interest; method (c) involves discounting for interest and mortality. It is only when lapse rates are also introduced into the calculation by method (d)—which is Dr. Belth's method—that the "level price" for Policy C becomes less than for Policy D. Thus, although the "level prices" by method (d) are not obviously wrong in this example (i.e. the figure for Policy C is not higher than that for Policy D), this result is merely an accident of arithmetic.

The last sentence of the report on the workshop as published in *The Actuary* stated, in reference to Dr. Belth's method of calculating "level prices," that:

> A "price" reflecting probabilities of survivorship and persistency could be more meaningful than a "price" based on the assumption that the policyholder will survive to the end of the policy year.

This statement is not too clear and differs from the text originally submitted for publication; I am sure that the published statement does not represent the views expressed by the workshop discussants. In my own view, a "level price" involving probabilities of survivorship and persistency is a very technical concept at best and cannot be particularly meaningful to the individual policyholder. I think that a measure of prospective cost that is determined as an average of the prospective costs for different categories of policyholders (e.g., those who will survive to the end of the designated period and those who will not) would surely be less meaningful *to the individual policyholder* than would a measure that pertains directly to the specific category of those who will survive to the end of the period. □

TABLE 1
"Level Price" Calculations per Belth Formulae

10 Year Endowment for $1000, Issue Age 50
Cash Values Based on 1958 CSO Table at 3%
Gross Annual Premium = $95.24

	Policy C		Policy D	
Policy Year	*Cash Value (Full Reserve)*	*"Yearly Price per $1000 Protection"*	*Cash Value (Statutory Minimum)*	*"Yearly Price per $1000 Protection"*
1	$ 85.04	$ 14.46	$ 44.46	$ 59.38
2	172.71	15.89	136.01	9.18
3	263.20	17.57	230.52	10.02
4	356.75	19.55	328.22	10.92
5	453.60	22.08	429.36	11.93
6	554.06	25.40	534.28	12.96
7	658.48	30.21	643.34	14.04
8	767.27	38.56	756.95	15.14
9	880.92	59.98	875.64	15.70
10	1000.00	591.30	1000.00	0.69

Method	*"Level Price per $1000 Protection"* (Policy C)	*"Level Price per $1000 Protection"* (Policy D)
(a) Simple Weighting	$ 22.34	$ 20.09
(b) Discounted, using only interest (3%)	21.65	20.74
(c) Discounted, using Interest and Mortality (1958 CSO)	21.38	20.97
(d) Discounted, using Interest, Mortality and Lapse (½ Linton's A rates)	20.90	21.77

Note: *The interest and lapse assumptions indicated above are those specified by Dr. Belth in his book, "The Retail Price Structure in American Life Insurance." The mortality basis specified by Dr. Belth is the X_{18} Table with Buck's select modification, but the basis used above was the* 1958 *CSO Table, for convenience.*

Summer Institutes

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portunity to talk with actuaries and actuarial students, to learn something about an actuarial career, and to see inside an insurance company or consulting office: 99% felt that the programs should be continued; 65% were interested in having a speaker visit their high school. Most teachers were surprised to find another career opportunity for their students, and actuaries once again discovered how few have ever heard of their profession.

These programs should have the participation of Society members at all levels. The most important ingredient is the personal contact between individual actuaries and teachers.

The NSFSI Subcommittee has now appointed 10 area Chairmen to implement plans for 1969 and future years. Three to five Institute visits will be made each summer for each region and follow-up contacts with teachers in their home area high schools will be made to answer any questions or to provide an actuary-speaker for student groups.

The present members of the Subcommittee and the actuaries who participated in the pilot program are confident that these meetings can help to attract new recruits to the profession. However, they feel strongly that their ultimate success will depend on the contributions of individual actuaries as they work with the Summer Institutes. □