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## TAXATION OF

## (ARIABLE ANNUITIES

by William H. Crosson.

The U.S. Treasury Regulations provide that whenever the amount received under a variable annuity is less than the amount excludable, the taxpayer has the option of redetermining the excludable amount. The result of the election of the option is to spread the deficiency as an addition to the excludable amount even. ly over the remaining duration of the annuity. The incidence of the additions will depend on the timing of the election and the final results may differ.

When the amount received is less than the excludable amount for two or more successive years, should the taxpayer make the election following each such year, or make the election only after the end of the succession of such years? It is easy to demonstrate that the election should be made each year (for a life-contingent annuity), and this demonstration rests on the fact that a life expectancy at any given age is less than one plus the life expectancy at the next following age.
The regulations give an example of a tuation where the annuity amount falls short of the excludable amount in two successive years, with the election being made after the second year. Shown herewith are the calculations for this example as given by the regulations, and if the election is made each year.

## SHORT SALES AND INTERNAL rate of return calculations <br> by Pctcr A. Marlss \& J. L. Dake

In the January issue of The Actuary Mason Hess cites an interesting case of return on investment determination. The problem he develops revolves around specifying when the cash flows take place.


Let the period between the short sale and the end of the transaction be one time unit. Furthermore, let the period between the instant that the short sale is

Example: Variable Life Annuity, Annual payment in arrears, purchased by a male, aged 64, for $\$ 20,000.00$. Payments received are:

| At the end of the |  |
| :---: | :---: |
| lst year | $\$ 1,000.00$ |
| 2nd year | 0 |
| 3rd year | $1,500.00$ |

Expected Return Multiples, adjusted for annual payment in arrears are:

| Male Age | Years |
| :---: | :--- |
| 64 | 15.1 |
| 65 | 14.5 |
| 66 | 13.9 |


| First Year | Excludable Limit $\left(\frac{\$ 20,000}{15.1}\right)$ | $\begin{gathered} \text { One } \\ \text { Election } \\ \$ 1,324.50 \end{gathered}$ | Successive <br> Elections <br> \$1,324.50 |
| :---: | :---: | :---: | :---: |
|  | Amount Received | 1,000.00 | 1,000.00 |
|  | Amount Excludable | 1,000.00 | 1,000.00 |
|  | Excludable Carry-forward | 324.50 | 324.50 |
| Second Year | Addition to Excludable Limit $\left(\frac{\$ 324.50}{14.5}\right)$ | - | 22.38 |
|  | Excludable Limit | 1,324.50 | 1,346.88 |
|  | Amount Received | 0 | 0 |
|  | Amount Excludable | 0 | 0 |
| (A) | Excludable Carry-forward | 1,649.00 | 1,346.88 |
| Third Year | Addition to Excludable Limit $\left(\frac{\mathrm{A}}{13.9}\right)$ | 118.63 | 96.90 |
|  | Excludable Limit | 1,443.13 | 1,443.78 |
|  | Amount Received | 1,500.00 | 1,500.00 |
|  | Amount Excludable | 1,443.13 | 1,443.78 |
|  | Amount Inclüdablë -- .-. .. ... | ${ }^{\cdot} 56.87$ | - ${ }^{-} 56.22$ |

Admittedly, the regulations stipulate that the election may only be made when some amount is received under the annuity during the year. The receipt of $\$ 1$ in the second ycar would satisfy this requirement and would not materially change the situation.
covered (C) and the end of the transaction ( P ) be $n$ units, $n<l$. In the case presented by Mr. Hess, he cites that if Tom sells 100 shares of stock $Y$ short on $12 / 31 / 66$ at $\$ 10$ a share and covers on $12 / 31 / 67$ at $\$ 8$ a share, the interest rate cannot be found by solving the equation $-100(1+i)+800=0$.

This seems obvious, since the $\$ 1000$ cash inflow and the $\$ 800$ cash outflow both occur at $12 / 31 / 67$. What we intend to show is that if a short sale is actually covered at any instant before the time at which the proceeds are received, a reasonable interest rate can be determined. Let $n$ be the period between the cover and the receipt of the proceeds. If $\mathbf{P}$ is the proceeds from the transactions and $C$ is the cost of covering, we get the following equation defining the present value of the transactions, at the time of the initial commitment.

$$
\frac{\mathrm{C}}{(1+i)^{1-n}}=\frac{\mathrm{P}}{(1+i)^{i}}
$$

$$
\frac{\mathrm{P}}{(1+i)}-\frac{\mathrm{C}}{(1+i)^{1-n}}=0
$$

The problem cited by Mr. Hess arises when we take the limit of this expression as $n \rightarrow 0$ then

$$
\begin{aligned}
& \operatorname{Limit}_{n \rightarrow 0} \frac{\mathrm{P}(1+i)^{1-n}-\mathrm{C}(1+i)}{(1+i)^{2-n}} \\
&=0 \\
&=\frac{\mathrm{P}(1+i)-\mathrm{C}(1+i)}{(1+i)^{2}}=0 \\
&=\frac{(\mathrm{P}-\mathrm{C})}{(1+i)}=0
\end{aligned}
$$

Thus as $n$ approaches zero, the solution to our equation, $i$, approaches positive infinity. By the proper choice of our time horizon and definition of the cover and inflow we can determine the proper rate of return on a short sale. It does not seem inconsistent that the rate of return is infinite if the cover and inflow occur at the same point in time since return on investment is a time concept, i.e. a rate.

