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THE PROBLEM OF THE INSTALLMENT LOAN

by David M. Good

The example presented by Robert Myers in the February issue of *The Actuary* illustrated the fact that actuaries from time to time tackle some of the nastier problems of compound interest. As Mr. Myers shows, satisfying the desire for a quite accurate result sometimes involves rather sophisticated and lengthy computations. This note presents a general solution to a similar problem of finding the effective annual interest rate commonly arising in finance. Since the solution is fairly easy to apply and gives a surprising degree of accuracy, it may be of use in upholding our reputation as experts in this field.

The Problem

The terms of a commercial loan are usually stated as an initial charge, with the loan to be repaid in installments; for example, a charge of \$6 per \$100 in advance, the loan to be repaid in twelve monthly installments. The effective annual interest rate is of course neither 6% nor 6.383% (from 6/94), since on the average only about half the loan is outstanding. The usual procedure of multiplying by $2n/(n+1)$ gives only a crude approximation.

The Solution

The following is an improved approximation for the interest rate on a loan to be repaid in twelve monthly installments. Let z equal the ratio of the initial charge to the amount initially received (the 6/94 of the above example). Then the better result, in form for computing,

$$i = (1.8439 + 1.09139z)z$$

This formula is to be used in the range of i from about 3% to about 24%,

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SINGLE PREMIUM DECREASING TERM USING CONTINUOUS FUNCTIONS

by William H. Lewis

It is likely that most of the decreasing term insurance policies which are designed to cover a typical mortgage loan are on an annual premium or a monthly premium basis, so that the premium payments may be geared to the level mortgage loan payments and the combination considered a package type of payment by the mortgagor-policyholder.

In some instances, however, it may be desirable to use a single premium decreasing term policy for this type business. This type policy may have particular appeal to a bank or savings and loan institution which holds the mortgage and pays the single premium to the life insurance company while lending the same to the borrower. The amount of the single premium is added to the regular mortgage loan and is amortized by level monthly payments along with the regular mortgage loan payments.

Method Limited

This method of providing life insurance benefits does not work too well in a situation where the borrower is anxious to make a minimum down payment and hence obtain the maximum amount of mortgage, but may well fit a situation where the collateral is substantial as compared with the amount of mortgage loan. In such a case the fact that the amount of mortgage loan the borrower has in mind needs to be increased by a relatively small amount to accommodate the borrowed single premium is of no great concern to him.

If it is assumed that mortgage payments are made on a continuous basis, using the force of interest which corresponds with the monthly effective mort-

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A VIEW OF PLANNING — SORcery TO COMPUTERS

by James C. Hickman

John Maynard Keynes said that he was only interested in the short term, for in the long term we are all dead. Despite Keynes' concentration on the short term, the future and the possibility of its control have always fascinated man. The Romans diligently studied the pattern of the intestines of sacrificed animals in an attempt to lift the veil from the future. The scriptures leave no doubt but that the three kings learned of the impending birth of Jesus by a study of the stars. Alexander thought it prudent, before embarking on his memorable trip of conquest, to inquire about the probability of success from the oracle of Delphi.

Because of the successes of Alexander, the legions of Rome, and of the search of the three kings, it is not immediately apparent that scientific and rational attempts at perceiving and modifying the future are superior to sorcery. Let us acknowledge that the superiority of planning over luck is a matter of faith and not a verified fact.

Difficult to Verify

Few industries, except perhaps life insurance, have had much experience with other than short term plans. Consequently, it is difficult to empirically verify that long term planning is superior to a more myopic view (plus a mystical faith in Adam Smith's "invisible hand") in making sure that economic enterprises are guided to success. In fact such verification may be impossible. In the course of human affairs the social, political, and sometimes even the physical and moral environments in which plans are realized change so frequently that it is

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not make their discovery public.

However, in the course of the action described in the book, news of the discovery eventually leaks out, and very soon after the first rumours start to fly, the Threadneedle and Western Assurance Company declares a moratorium on the payment of annuities and guaranteed incomes until further notice. They describe the step as "a purely temporary measure undertaken with regret pending legal opinion upon the obligations of the company in cases where means have been employed to extend the normal expectation of life."

It seems that the actuaries of the Threadneedle and Western Assurance Company decided to take this unusual step when they discovered that the wife of the Chairman of the Board of the company had been taking the antigerone for a number of years, and that the process of aging certainly appeared to have slowed down in her case.

The reaction of a prominent lawyer to this whole affair was that "neither God, nor the law, was aware of any obligation to justify an actuary's figures for him." And while the term "his natural life" might raise some speculation regarding the nature of "unnatural life", life continued, for the reasonable man, to mean that life had not been terminated by death.

All of which raises sobering thoughts for us young actuaries who are busy working out rates for annuities and pension plans! However, it is at least comforting to know that here is one author who does seem to have some understanding of actuaries and their problems.

B. L. Burnell

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Sir:

I wondered if you were aware that the expression "actuary" occurs in Gibbons' *Decline and Fall of the Roman Empire*? Rather than have you read the entire Gibbons work in order to find it, I will tip you off that it occurs in a footnote in Chapter XI where it is reported that one Victorinus died at the hands of jealous husbands. The footnote states that, among other things, "He ravished the wife of Attitianus an actuary or army agent."

Robert G. Espie

Single Premium

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gage interest rate, it is possible with the aid of calculus to arrive at a fairly compact formula for net single premiums and cash values. Cash values are simply dependent on attained age and remaining term for each \$1,000 initial amount of indebtedness. The formula for the net single premium is as follows:

$$\frac{1,000}{1-u^{12n}} \cdot \bar{A}_{x:\overline{n}|} - \frac{1,000u^{12n}}{1-u^{12n}} \cdot \bar{A}'_{x:\overline{n}|}$$

Where the first \bar{A} is based on 58 CSO at the valuation interest rate, the second \bar{A}' is based on 58 CSO at a special interest rate which is a function of the valuation interest rate and the monthly mortgage interest rate,

x = the issue age

n = the original term of the mortgage in years

j = the nominal annual interest rate of the mortgage and

$$u = \frac{1}{1 + \frac{j}{12}}$$

Gross single premiums may be obtained by an appropriate loading formula applied to the net single premium.

A Comparison

It is interesting to compare net single premiums covering a mortgage based on continuous functions with net single premiums using various modifications of curtate functions. The various modifications used by the author are as follows:

(A) Payment of the death benefit at

Sir:

Further on the subject of the actuary in literature, there was a short story by Kipling entitled *The Janeites*, laid in World War I, in which the narrator casually mentions that one of the characters had been an actuary in civil life. This fact had no bearing on the plot of the story.

I also remember a movie (the name of which I have forgotten — it might have been *Double Indemnity*) in which a claim investigator posed as an actuary, exhibiting his credentials by quoting expectancies at the drop of every hat.

James E. Hoskins

the end of the policy year of death for the amount of mortgage at the end of the policy year. This method is not desirable, particularly because the amount of cash value at the beginning of the final policy year is zero.

(B) Payment of the death benefit at the end of the policy year of death for the amount of mortgage at the end of the policy month of death. This method assumes a uniform distribution of deaths and is analogous to the method applied in Jordan's *Life Contingencies* to single premium cash refund annuities (see page 144).

(C) Payment of the death benefit at the end of the policy month of death for the amount of mortgage at the end of the policy month of death. This method appears to be more realistic than the one in (B) above, but the author does not recall having seen it used in practice.

(D) Immediate payment of the death benefit for the amount of mortgage at the beginning of the policy month of death increased by one-half month's interest at the mortgage interest rate. The method used here involves multiplying the mortgage amount at the month-end prior to death by

$$\left(1 + \frac{j}{24}\right)$$

and discounting the payment by

$$v^{\frac{2m-1}{24}}$$

where m is the policy month of death.

The numerical results for 5 and 20-year term mortgages at issue ages 20 and 50 with a nominal annual mortgage interest rate of 6.6% and 58 CSO at 3% interest are as follows. The values denoted by K were obtained by using the method of continuous functions covered in this paper.

NET SINGLE PREMIUMS

Method	Age 20		Age 50	
	5-Yr.	20-Yr.	5-Yr.	20-Yr.
A	3.66	18.60	18.19	130.51
B	4.44	19.33	22.37	136.83
C	4.51	19.60	22.70	138.73
K	4.59	19.69	23.11	139.48
D	4.67	19.81	23.56	140.44

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