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A THIRD SOLUTION TO MULTIPLE INTEREST RATES

by William D. Smith

In the September issue of *The Actuary*, Mr. Kellison gave an interesting interest rate problem which produced answers of both 10% and 15%. Such interest rate equations are really n degree polynomials which have n solutions. Most of the time there is only one positive solution for i, the rest being negative or involving $\sqrt{-1}$. What follows is a not very rigorous investigation of the conditions under which multiple positive solutions may occur. The investigation produced yet a third solution to Mr. Kellison's problem—84.6%—and it is interesting that this third solution appears to be the "correct" one.

Descartes's rule of signs is: "Let $f(v)=0$ be an algebraic equation with real coefficients which is arranged in descending powers of v. The number of positive roots of the equation is either equal to the number of variations of sign presented by the coefficients of $f(v)$ or less than the number of variations by a positive even integer (a root of multiplicity m is to be counted as m roots). In particular, there is exactly one positive root if the coefficients present only one variation of sign."

Mr. Kellison's equation can be restated as follows:

$$v^9 + v^8 + v^7 + v^6 - 6.98v^5 + v^4 + v^3 + v^2 + v - .805 = 0$$

There are three changes of sign in the above equation. Therefore, only 3 or 1 positive solutions in v are possible. (Note that only solutions where $0<v<1$ produce positive values of i). Since we know there are two solutions, there must be one other, which is at $i=84.6\%$.

Normally, in profit testing a "yield rate", "i", as defined by Mr. Anderson, is calculated for each policy year, the yield rate being that which equates the profits to that policy year with the investment of the early years. Normally no attempt is made to calculate "i" until the sum of profits is greater than the initial investment. From that point on, the yield rate is likely to increase, rapidly at first, but increasing slowly after 15 or 20 years.

The general family of equations can be written as:

$$a_n v^n + a_{n-1} v^{n-1} + \ldots a\,v + a_o = 0$$

Normally a_o and sometimes a, is negative while the rest of the coefficients are positive and normally decreasing slowly such that $a_n < a_{n-1} < a_{n-2}$ etc. Whether or not the coefficients decrease, so long as there is only one change of sign in the coefficients there must be one and only one positive solution for v.

Assuming that the combination of profits and persistency continue to produce $2 of emerging profits after 10 years, the year by year solutions to Mr. Kellison's example are:

Policy Year	*Profit*	*Yield Rate*
1	$—1.61	Undefined
2	2.00	24%
3	2.00	90%
4	2.00	111%
5	2.00	119%
6	—13.96	*
7	2.00	*
8	2.00	*
9	2.00	*
10	2.00	84.6%, 15%, 10%
11	2.00	85.4%
12	2.00	85.5%
—	—	—
—	—	—
20	2.00	85.828%
—	—	—
—	—	—
30	2.00	85.830%

*During these years the sum of the profits has not yet equalled the total investment, but there are positive roots for years 7, 8, 9.

Since the yield rate should provide a smooth pattern from year to year, the 10% and 15% solutions for year 10 seem to be spurious.

In the general case, even if there is a negative coefficient after positive profits have begun to emerge, multiple solutions are unlikely unless the coefficient is very much larger in absolute value than those surrounding it. In Mr. Kellison's example, the negative coefficient is almost seven times as large as the surrounding coefficients. At 6.9 times and below, the multiple solutions disappear. Also, as that ratio increases, multiple roots disappear again—there are none when that ratio is 9. Thus, there appears to be a narrow range for this ratio which will cause multiple roots. Even if multiple roots occur, it appears unlikely they will occur in more than one year.

Thus, we can summarize that: (1) only one positive solution for i will occur if no negative profits occur in years after the first positive profit; (2) only an isolated negative profit year can cause multiple solutions and then the loss must be many times the profits in the surrounding years; and (3) even if such a result does occur it would very likely affect only one year.

It is evident that the possibility of multiple roots has little practical significance. □

Letters

(*Continued from page 3*)

in the Actuaries' Club of the Southwest to the extent of their Texas membership.

These additional contributions are welcomed and appreciated and this note is intended to give recognition to such contributions.

John D. Morrison

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A Premium of Actuaries

Sir:

One of the most delightful little books to grace my summer reading was James Lipton's "*An Exaltation of Larks, or the Venereal Game.*" In it, Mr. Lipton describes the historical and poetic meanings of collective terms such as "a school of fish," "a cry of players" and "a pride of lions."

For assemblages in our own profession, my wife feels that the proper term is "a premium of actuaries." This is, I think, correct. Yet there are many actuarial meetings (in which information and discussion is sought but not developed) which seems to involve a different group; for these I would suggest "a reserve of actuaries."

David G. Halmstad

Change of Address

Because of mailing requirements it is not possible to give immediate effect to a change of address. The address labels are prepared at least two months in advance. We hope our readers will understand the delay that arises from processing these changes.