RECORD, Volume 25, No. 3*

San Francisco Annual Meeting
October 17–20, 1999

Session 12PD
A Retrospective on 50 Years of Advances in Theory and Practice of Finance

Track: Investment
Key Words: Finance, Investments

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Summary: This session will review the past half century's advances in finance, such as:

- *Immunization and hedging theory*
- *Modern portfolio theory*
- *Capital asset pricing model*
- *Option-pricing theory*
- *Term structure of interest rates models, and so on.*

Their application to asset and liability management in insurance and pensions will be highlighted and discussed.

Mr. James C. Hickman: My colleague is Elias Shiu. I have the misfortune to have to inform you that Bob Reitano is unable to join us, and we will try valiantly to complete the tour of this fascinating 50 years without his assistance.

I will be making several references to a book distributed a little over a year ago by the Actuarial Foundation, titled *Financial Economics with Applications and Investments, Insurance and Pensions*, written by a team of about a dozen distinguished scholars in finance and actuarial science. I will try to use that book any time I can. I recommend it to you as you continue your reading in these areas.

Practice: 1949–69

Elias and I have an awesome responsibility because we are covering a period of extraordinary activity in the development of financial economics and in its practice. I shall start by talking a bit about practice. I have the assignment of covering about 20 years of this period, and later we will take on some of the other years. I have chosen to start by commenting on the asset allocations of the U.S. life insurance industry. This is, I believe, basically going from 1950–1970.

Note: The chart referred to in the text can be found at the end of the manuscript.

Asset Allocation

The big thing is that government bonds shrank during this period. Coming out of World War II, virtually 45% of U.S. insurance assets were invested in government bonds, both as a matter of patriotism and because war was the principal economic activity of the U.S. between 1941 and 1945. Corporate bonds in that 20-year period basically remained stable. Stocks increased quite dramatically, but you don't see anything yet until the next part of this 50-year period.

Real estate mortgages are an interesting story during those 20 years. They were growing rapidly in the early part of the 1950s, declined, and then came back up again. Real estate was, during this period, a relatively small investment outlet.

Policy loans were also a dramatic story. As the public became more financially sophisticated and understood the value of those fixed interest policy loans, they were used more extensively. Once again, this was just a precursor of the storm of the 1970s. Of course, you always have to have, at the end, a miscellaneous category. Besides a few of the facts, we do have the basic data available. It came from the *Life Insurance Fact Book*. I chose to present beginning and ending values for those 20 years as a device for capturing some of the action of that period.

Economic and Business Environment

What was the economic and business environment of that time? We came out of World War II with very low interest rates. That was, of course, a government policy, in part, to minimize the cost of financing the war and also to promote the economy in the immediate post-World War II years. Both the Labor government in Great Britain and the Truman government in the U.S. were quite convinced that the western world faced a return to the Depression of the 1930s.

One of the actions to prevent this from occurring was to keep interest rates low, and this was done. It was done largely by the actions of the Federal Reserve to peg the federal bond market. By the early 1950s, it was clear that this pegging, that is, buying federal government bonds to keep the interest rate low, was fueling the fires of inflation by increasing bank reserves. The life insurance industry was accused in 1951 of working counter to that policy by selling U.S. bonds and making the price pegging policy more difficult to implement.

If you read the "Second Presidential Address" by Valentine Howell in *Transactions, Society of Actuaries,* you'll get a sense of those years in which officials in the administration were accusing the life insurance industry of frustrating government monetary policy by selling government bonds. You guessed it, they then were investing in government-insured mortgages at several percentages higher. That foolishness of pegging the federal bond market was in part relieved in 1951. In studying the economic history of those early days, one has to recognize what was a consistent practice of the U.S. and the U.K. Up until about 1951, it was low interest rates. In fact, you will not understand the political environment that motivated Redington's immunization rules until you understand that that's what he was worried about—low, not high interest rates.

The period that I'm talking about was a period of remarkable economic growth. True, there were ups and downs, but almost everyone looks back on the growth of productivity and gross domestic product during that 20 year period as remarkable. Toward the end of that period, as we tried to fight both the war in Vietnam and the war on poverty, the economy was plagued with inflation. Inflation was higher than we had experienced, except in a few wartime periods.

This was also a period of new insurance products. These new products largely reduced the financial guarantees that were built into them. The best example perhaps are the variable annuities introduced by TIAA college retirement equities fund (CREF) in the beginning of the 1950s. The loss of pension funds from life insurance companies to trust companies was another example of large funds being built up without specific asset guarantees.

This was also a period of change in regulation. Perhaps to the generation represented by this group, it is difficult for you to appreciate how tightly investment decisions were imposed on the insurance industry up until basically this period. Such things as loan-to-value ratios in mortgages and the percentage of assets that could be invested in common stocks were matters, not just of management judgment or even regulation, but of detailed statutes. During this period, much of this was relaxed, but not eliminated. Separate accounts, a device by which life insurance companies could maintain funds exempt from the highly detailed regulation, was a product of the late 1960s. The very low percentage of funds that could be invested in common stocks was lifted and an increase in loan-to-value ratios was permitted for mortgages.

This, of course, has been a continuing story, but if you are to understand the financial practice of the years that SOA has been around, you need to understand how much investments were regulated in that early period.

Theory

Present Values

I'd like to shift now to some of the developments in that 20-year period in theory. It might be an exaggeration to say that financial economics did not exist in 1949, but not much of an exaggeration. As a matter of fact, the birth of the SOA almost coincided with the beginning of a period of incredibly rapid development in thought in financial economics.

In 1949 the theory of finance was largely based on the theory of present values. Actuaries learned the basics from the books of the British actuary. George King wrote two textbooks, Textbook 1 and Textbook 2. Textbook 2 was *Life Contingencies*, and Textbook 1 was *Compound Interest*. That book by George King was basically the standard text until early in this century, when a book by Ralph Todhunter, again, a member of the British Institute, took over. I think I can look around the room and observe at least three or four people who probably studied compound interest from Todhunter.

The principle theory had to do with present values. An interesting sidelight to that has to do with the work of Frederick Macaulay, not an actuary but the son of one, in the late 1930s. Macaulay was working on a large project for the National Bureau of Economic Research that had to do with bond yields. When economists study things, they are interested in the price sensitivity to changes in a market. The idea of changes in price due to changes in demand and supply are the mother's milk of economics. Macaulay was interested in the sensitivity of bond prices to interest rate changes, a very natural idea. He took derivatives and came to the idea of what we would now call duration, but from a somewhat different motivation than actuaries use.

Immunization

One of the developments that happened early in the history of the SOA, although it did not have much of an immediate impact on either theory or practice in North America, was Frank Redington's theory of immunization. It is easy to find precursors to this theory. Macaulay is one, and there was a famous memorandum that floated about Penn Mutual written by Koopmans, later a Nobel Prize winner in economics, that clearly articulated these ideas. Nevertheless, from the viewpoint of influencing practice, Redington's 1952 paper and a whole series of papers on matching asset and liability (A/L) cash flows showed what actuaries were thinking about in the U.K.

Redington was concerned with what I suspect many thought actuaries had been concerned with from the very beginning, that is, a coordinated view of A/L valuation. I must confess to you that we weren't. These topics were pretty much independent and A/L managers didn't talk to each other much. Redington appreciated in the early 1950s that he was faced with a bunch of liabilities that had buried in them higher interest rates than were realizable in the markets at that time. How to minimize the impact on surplus of that environment was Redington's problem. He used some calculus and proposed some rules.

Redington derived two rules. The first rule had to do with an average time of cash flows from assets (the big A's) and the average time of liabilities (the big L's), all weighted by some discount factors. That is exactly the same kind of formula that Macaulay arrived at in the 1930s for a different application. Redington was trying to minimize the impact on the surplus of a financial institution due to interest rate changes. We began to call those first numbers derived by differentiation the durations of assets and liabilities.

Redington could take second derivatives. His own development had to do with series expansions, thinking of the total value of assets as the sum of cash flows due to assets times discount values, and the liabilities, the sum of the cash flows, due to liabilities times these discount factors. If you want to minimize the change because of the change of interest rate, you would look at the first and second derivatives.

$$\sum_{t>0} \frac{tA_t}{(1+i)^t} = \sum_{t>0} \frac{tL_t}{(1+i)^t},$$

Now, as is true with most new developments, the assumptions that Redington either made, explicitly or implicitly, really set the research agenda that is still with us today. Redington assumed that those cash flows were independent of the interest rate. Clearly, you know they aren't because some bonds are callable and mortgages can be paid off in advance. Interest rates do change those *A*'s. And certainly on the *L* side, there are cash flows for example, because of withdrawals that are very much sensitive to interest. The research agenda was how to modify these ideas to incorporate the fact that the *A*'s and *L*'s depended upon the interest rate.

That's a pretty deterministic bunch of equations. You don't see any probabilities. And, yet, we're actuaries, so we know the world is uncertain. In fact, probabilities were our main intellectual contribution to these studies, yet you don't see any probabilities. These cash flows look like they are certain, but you know they aren't. That became another part of the research agenda for the next few years. By only going out through those first two moments and using derivatives, you get the idea that the interest rate changes that you're talking about were small, and, in some sense, the changes in interest rates were parallel.

Even in the discussion of Redington's paper, someone pointed out that the second moment of the assets is bigger than the second moment of the liabilities, and asked if that really happens in markets. If it could happen, it would be a money machine. That is a violation of the canonical no-arbitrage assumption of modern finance, and it was pointed out in 1952 that this was unlikely to be achievable. This also became part of the research agenda that occupied researchers in finance, starting right after 1950 and clear up to our current day.

Portfolio Theory

Another development occurred virtually simultaneously, and I think many of us would look upon it as the birth of modern finance. Harry Markowitz ultimately earned a Nobel Prize in large part for this work. As an aside, Markowitz worked for our Society in the late 1970s and early 1980s. He was primarily interested in simulation at the time, and there is a monograph that you can still buy from the Society called "Adverse Deviations." John Woody is the principal author, but most of the work was done by Markowitz. The monograph is about simulating to learn more about financial reporting. Many people thought, including some distinguished economists at The University of Chicago, like Milton Friedman, that, even though Markowitz came up the economics route, his work wasn't economics. This was something new. This wasn't like studying John Maynard Keynes or Irving Fisher. There was a real question as to whether his ideas belonged intellectually in the economics department. That question, I think, has now been resolved in Markowitz's favor.

Perhaps the most important conceptual notion is that Markowitz started with the idea that investment returns are random variables. They are uncertain. They have a distribution. It may be hard to find the distribution, but this was the way to think about them. The equations below summarize Markowitz's idea. He set out a mathematical programming idea to minimize the variance of portfolio returns. Great idea. You do that in statistical estimation and in the design of experiments.

All those ideas were already out there. Can we use the same ideas in finance? Let's minimize the variance of a portfolio:

minimize $s_p^2 = X^T \sum X$ subject to the following constraints:

1. $\sum_{i=1}^{N} x_i = 1$
2. $\sum_{i=1}^{N} x_i m_i = m_p$
3. $\sum_{i \in C} x_i \leq 0.10$
4. $x_i \geq 0, \text{for all } j \in J.$

Here you see a big X with a T on it, meaning transpose. You see a big sigma, meaning a variance/covariance matrix. Then you see another X. This expression is the variance of a portfolio where those Xs are vectors of numbers that add up to one, and are the fraction of investible funds that you put into each of the possible investments indicated by N. And let's set a goal, a portfolio mean that we want to achieve with small variance.

The third requirement was not fixed by Markowitz. It is in the book to show that one could modify this programming problem to take into consideration, for example, either management or regulatory considerations. For example, at one time, life insurance companies were bounded by 10% of their investments that they could put in common stocks. That might be such a programming requirement. At least originally, we were going to make all of these Xs non-negative. Random variables, minimization of variance, ideas from probability and statistics, and mathematical programming—all of those ideas were afoot at the time.

Now, Markowitz was a man of his age, and there were some things that he didn't solve directly. His model was basically short-term. He was looking at achieving an investment goal subject to the minimization of risk, and he was looking basically one year ahead. Could you modify this or jazz it up in some way to make it more long term? Although utility theory in the sense of von Neuman and Morganstern or Jimmy Savage's *Foundations of Statistics* were out there at the time, Markowitz didn't either quite believe it or thought it might be too controversial to defend at the time.

One of the criticisms is that he did not directly use utility theory. He summarized preferences by simply saying big means are good and variances are bad, and let's go out and make portfolio management a mathematical programming problem. Trying to both make this model longer term and directly using utility theory is part of the work after Markowitz, much of which is reviewed in that textbook.

Markowitz did make diversification—which had always been a buzzword in finance—measurable. This was a great idea. He based this idea on what statisticians had been talking about for years. But he did not relate it to a general economic theory. Once again, that was a subject of study for the next few years. Was there a more general economic theory? Economists love to think about equilibrium or moving towards some kind of equilibrium. Markowitz didn't do that.

Another requirement, which at that time that seemed quite awesome, was the massive amount of inputs. If you even let that N be the total stocks in the New York stock market, the idea of minimizing a quadratic form that had that many variables in it staggered us in 1952.

These were some of the problems that were apparent right off the bat and, in fact, have constituted part of the research agenda in recent years. Most of you are well aware of the fact that these ideas have gone not just into investments but to parts of actuarial theory. If you are talking about the allocation of capital into various lines of business, you can find lots of papers on maximizing the rate of return in auto insurance, health insurance, property insurance, or life insurance, once again using mean-variance ideas. That is, create a multiline insurance company that, subject to an earning rate goal, will minimize variance.

Efficient Markets Hypothesis

The efficient markets hypothesis in some ways had been around a long time. It's buried into the very theory and folklore of capitalism. This theory states that markets are engines by which we make allocations. Markets work when there is complete information, and self-serving operators in the markets will optimize using all that information. In a certain sense, that was all the efficient markets hypothesis said. I've given two references—Eugene Fama (1965) and Paul Samuelson (1965)—because they are historically very important, but it is false to think that these ideas had not been around a long time.

Ask both the man and woman on the street, as well as the investment manager, and the majority are absolutely convinced that there are certain cycles and deterministic signals built into the stock market, and, in fact, other commodity markets. One cannot read the literature of the 1950s and 1960s without sensing the astonishment when people started to analyze the real data from cotton markets and stock markets, and they couldn't find any patterns. The autocorrelations were zero or just about zero. It dumfounded them. Up until that time they did not have the computing power to do all that analysis. The model below is certainly not the only model in the efficient markets hypothesis, but it helps illustrate some of the ideas.

$$\log \mathrm{P}(t+h) = \log P(t) + mh + s\sqrt{h}e_{t+h}$$

The idea was that the price of this asset at this time, *t + h* (we'll take the logarithm to make it well-behaved), is equal to the logarithm of the price in the previous period, plus perhaps a mean *Mu* and *H.* The *Mu* could be zero, it is kind of deterministic uplift factor. And there comes the important point—that random part at the end with those little epsilons, which, as usual, are independent random shock terms. These are canonical statistical assumptions. That variance, of course, would get bigger as you stretched out the time horizon.

This fairly simple equation gave rise to a host of statistical studies to see if, in fact, it corresponded to market data. The efficient markets hypothesis in these years was tested, not just by statistical tests, but by what were called trading or filter rules—that is, a rule, whether it's buy low (and you have to define what low is) and sell high or some other set of technical trading rules.

Can you perform better than the market average by using those filter rules or trading rules? The literature, particularly in the 1950s and 1960s, is full of attempts to find those trading rules and to test them. As you might guess, by and large, the tests said, no, they do not beat market averages. All of the added support to the efficiency of the commodity and equity markets is captured in current prices by active market trades processing the information.

There were, of course, a lot of unsettled issues in this work. In many ways, the theoretical and empirical work was already done; actually testing asset prices to see if the theory worked was required. There are a lot of issues in this effort that occupy us clear up to today. For example, what distribution should we use? Does that little epsilon come from a nice normal distribution or from one that has fat tails that's skewed? Then there is the reality of what Alan Greenspan called "irrational exuberance." Does the variance remain the same, or every now and then does some kind of foolishness push it up or down? These are questions from the 1950s, and they remain research issues.

Then there are what I call "pesky deviations" from these models. The January effect, the fact that stock prices go up in January, and the small cap effect, which we observed at one time, are two examples. These made you think that maybe the market wasn't as efficient as we thought. All of these ideas were out there in the late 1950s, and they constituted the birth of what we would call modern financial economics.

E. Capital Asset Pricing Model

The capital asset pricing model (CAPM) is a model of the mid-1960s and, in many ways, it has multiple roots. It depends on your taste for theory which of those routes appeal to you. Almost as soon as Markowitz finished, efficient frontier charts appeared, both in textbooks as well as in presentations, where the *y* axis represented an expected rate of return on the portfolio, and the *x* axis was its standard deviation. The boundary was called the efficient frontier, and almost from the beginning someone said, "If there is a riskless asset, that is, one with 0 standard deviation, you could plot it and then draw a straight line between that point and the tangent point to that efficient set." By that action, I have reduced all investment problems to how much you want to put in the riskless asset and how much into this optimal portfolio.

This got fancified in the late 1950s and early 1960s with such words as the "separation theorems." Clearly, when scholars did it, they used a lot more axioms than I have, but the idea that you could separate your investment decision into those two questions, deciding how much to put in the riskless investment and then figuring out exactly where the tangent with the efficient frontier is. This was one of the roots of the CAPM. The idea that you could simplify the Markowitz portfolio model, not by looking at all of the points on that efficient frontier but by looking at only one of them, had great appeal.

There were also people who thought of the same idea with respect to rating your mutual fund. That is, if you took a broad market index, the *x*, the regressor variable, and the *y* being the rate of return of your mutual fund, you could learn about your mutual fund by seeing whether the regression line slope was bigger than 1 or less than 1 and about the deviations from that slope. So before we even had a CAPM we had people like Jack Treynor talking about doing things like that, without all the theory, to rate your mutual funds.

The complete theory, a theory that did involve equilibrium ideas, filling in some of the things that Markowitz had not done, was done almost simultaneously by two people, Sharpe and Lintner. Sharpe also received a Nobel Prize for his work. This created not only a new theory, but also a cottage industry. We created jobs for people that computed betas, that is, the slope regression line in the CAPM. Some of you may have been employed in that industry.

There are many paths to this result, equilibrium pricing, extensions, and summaries of portfolio theory, and the model below is basically a nice linear model where *R sub i*, a random variable, is the rate of return on this investment *i*. There is a real rate of return, or *r*. There's the usual slope coefficient, the beta, and the constant alpha *i.* We multiply the slope coefficient times the difference between a market rate of return and the real rate of return and then, as usual, have the error term on the end. In fact, from the mid-1960s up until the present day, this is part of the standard education of finance students and one of the standard things that actuaries need to know.

$$R_i - r = a_i + b_i (R^M - r) + e_i$$

From the beginning, there were a lot of issues about this model. Among the issues were, how big is that market portfolio? Is it the New York Stock Market? Does it also involve commodities? Does it involve the world markets? Does it involve investments in human capital? How big is that market portfolio? If you're going to talk about equilibrium, it's cheating just to talk about equilibrium in some kind of subset market. And there also have been a series of anomalies that were observed in this model.

This, I hope, has brought you toward the end of this 20-year period. There is, however, what I will call a bridge to the next period. In the next period, shortly after 1970, we began to apply the theory of stochastic processes to open markets. We take great pride in these developments, but, in fact, it was an old idea. In the early part of this century, Louis Bachelier, a Ph.D. student in France, wrote a remarkable dissertation which has been translated. You can find it in Paul Cootner's book about random characteristics of the stock market. It was one of the most remarkable dissertations of our century, and it barely made it into the current century. Bachelier not only applied stochastic processes, but also invented a good deal of the theory in order to understand the operations of speculative markets. Although I call this a bridge to the next period, it was a rather incomplete bridge, although we didn't know it in 1970.

The second point I want to make is that, as we went into the next period, many of the ideas were very closely related to what those actuaries already knew. There was an enormous overlap with reinsurance theory, an overlap that many of us did not recognize for a long time. The standard investment derivative contracts were easily interpretable as stop-loss agreements, and, as a matter of fact, any actuarial student who stopped and thought about it for a little while would have written down the pricing formula or very close to it. They might have argued about what distribution to use or hassled about what interest rate should be in the formula, but it was basically very natural, because it was a risk management idea.

Another point that Elias will make is that for this theory to work, for the efficient market to work, and for the random nature of these markets to work, open markets are required, and the more complete it is, the better.

Mr. Elias S. Shiu: I'll talk about the next half of the 50-year period. I'll start by giving you a Web site, www.kva.se from the Royal Swedish Academy of Sciences. It includes the announcement of the 1997 Nobel Prize in economics given to Robert Merton and Myron Scholes for their work on option-pricing theory. They were given a prize for new insights into how to measure economic variations and how to facilitate more efficient risk management. I should mention that because Fischer Black died, he never got the Nobel Prize. If you read his obituaries, you'll find that people expected him to win a Nobel Prize. He died in 1997. Had he lived, he would surely have won.

In their famous paper, "The Pricing of Options and Corporate Liability," Black and Scholes were thinking about how to price corporate liability. We actuaries always think about how to price insurance liability. I also want to point out that this paper was published in *The Journal of Political Economy*. This is a reflection of a story that Jim told you. The Markowitz thesis had a hard time getting accepted by the department of economics at the University of Chicago because it was so new. He had a hard time getting the paper published in *The Journal of Finance*. They would not accept this paper because it was so new, so different from the old paradigms.

Merton's 1973 paper was titled "Theory of Rational Option Pricing." He put forth a lot of ideas that go beyond the paper of Black and Scholes. In fact, there are so many elegant results, it is really a classic.

When the Swedish Academy announced the Nobel Prize, they explained the problem: to price options, one has to come up with a spread, so what kind of spread should we use to value an option?

A call option is simply an instrument that gives its holder the right, but not the obligation, to buy the underlying asset by a certain date for a certain price denoted by *K*. If the underlying asset price is below *K*, then the payoff is zero. If it's above *K*, then you have this one-to-one correspondence.

$$\text{Payoff} = (S(t) - K)_t = (S(t) - K) \cdot I(S(t) > K)$$
$$= s(t) \cdot I(\cdot) - K \cdot I(\cdot)$$

where I (true) = 1, I(False) = 0.

If I define a little bit more notation, in this *X*+ function, if *X* is positive, it's zero. If *X* is negative, then the call option payoffs are simply positive values of the stock price minus the exercise price *K*. When it's negative, because you don't have to purchase the asset, the payoff is zero. Only when the stock price is above the exercise price *K* would exercise the option. As Jim pointed out earlier, an actuarial student will recognize that this is like a stop loss. If you think of *S*, the stock price, as claims, and *K* as the retention limit, this would be like a stop loss.

Now, when you talk about this spread, what they meant was this. Let's have *r* equal the risk-free force of interest. Then the price is simply the expectation of the payoff. We average out a payoff and discount at a risk-free rate plus a risk premium. The problem is, what should this risk premium be?

But what Black and Scholes showed was that you don't need to talk about risk premium. What you need to do is to change the expectation. You change the underlying probability measure of the stock, so you can stick to the risk-free force of interest. Therefore, we do need to verify the risk premium. But you would change the probability measure to build in the risk premium. It is still an expectation, but the underlying probability measure changed. You say, "What is the probability measure?" Well, the probability measure is such that, when you estimate a stock price at time *t* by taking on an expectation, and you discount it at the risk-free rate, it gives you back the original stock price. You'll find a probability measure such that the discounted expected value is the observed current stock price. Then you will use the probability measure to calculate the option price.

For certain stochastic processes and assumptions on the stock price, this change in measure is unique. You know there is only one price. And this result now has a name. We call this the Fundamental Theorem of Asset Pricing. This says that, if you have a frictionless market, if there are no free lunches, which is the absence of arbitrage, then all prices of all securities are expected present values. The word "expectation" in this special form is taken with respect to a changed measure. It means that there is a probability measure we call a risk-neutral probability measure or equivalent martingale measure and, with respect to which, the price of a stochastic payment is the expected discounted value. Actuaries use the formula that the price is an expected present value, but here expectation has changed. This is not the actual probability measure but a change probability measure.

The Royal Swedish Academy of Science's Web site shows the Black and Scholes formula. You can download this using your computer. The one difference is that they use *L* where I used *K* as the exercised price. *K* is, I think, the more standard notation. The Academy also tries to explain the formula in words, there is a problem. It turns out that the explanation is not quite correct. And this error becomes worse when it was translated or expanded by the *New York Times*. In my talk, I would like to give at least one proof of something. I will derive a form of the Black-Scholes formula, and point out why this is not correct.

$$\text{Black-Scholes Price}$$
$$= e^{-rt} \left[F^{*} \left[(S(t) - K)_{t} \right] \right.$$
$$= e^{-rt} \left(\left[F^{*} \left[S(t) \cdot I(\,) \right] \right] - K \left[F^{*} \left[I(s(t) > K) \right] \right] \cdot \right)$$
$$\Pr^{*}(S(t) > K)$$
$$\left[F * \left[(S(t) \cdot I(\,) \right] = \left[F^{*} \left[S(t) \right] \right] \frac{\left[F^{*} \left[S(t) \cdot I(\,) \right] \right]}{\left[F^{*} \left[S(t) \right] \right]} \right.$$
$$= S(o) e^{rt} \left[F^{**} \left[I(\,) \right] \right]$$
$$= S(o) e^{rt} \Pr^{**}(S(t) > K)$$
$$\text{Black-Scholes' Option Price} = S(o) \Pr^{**}(S(t) > k) - e^{-rt} K \Pr^{*}(S(t) > K)$$

This is, I think, a simple derivation of the Black Scholes formula. *I* is the indicator random variable. If something is true, it takes the value 1, and when something is false, *I* takes the value zero. The payoff of the option is the (stock price minus $K)_{+}$. Here, you're saying the payoff is the stock price minus *K* times the indicator function, which is 1 when the stock price at time *t* exceeds K. We are writing a payoff using the indicator function. But now with this I can write the payoff in two parts. I use the fundamental theorem of asset pricing. The actual price is the discounted value of the expectation of the payoff, but the expectation is taken with respect to the risk-neutral measure. Now, because the payoff is written in two parts, what I have now is really two expectations. The second expectation is quite easy because the expectation of an indicator random variable is just a probability of that event.

The first expectation is a little bit harder. You can solve it using this trick. What you do is you multiply by the expectation of the stock price at time *t* and divide by the same thing. Then after you divide, this ratio defines what we call a change of probability measure. This ratio becomes what I call a changed measure. You have changed the probability measure of the indicator function. The first expectation is for a simple indicator function, because, remember, we found the risk-neutral measure, the equivalent martingale measure. It is the one such that the discounted expected value of a future stock price is the current stock price. This one is the martingale equivalent measure. Then the other expectation is simple, again, because the expectation of the indicator function is just the probability with respect to the change measure.

The final formula is the famous Black-Scholes formula, which is simply the original stock price times the probability that a stock price will exceed the exercise price at time *t* but with respect to this so-called change or double star measure minus the discounted exercise price times the probability that the stock price exceed the exercise price by using the risk-neutral measure. This is the famous Black-Scholes formula. If you can determine the probability measure, then you have a Black-Scholes formula. The rest is homework, but this is the only mathematics that I will do in this session.

I want to discuss why actuaries need to know about option-pricing theory. I'll start by reading a quote by a famous professor of economics and finance, Stephen Ross, who was at Yale but is now at MIT. He wrote: "Despite such gaps [so there are gaps in option-pricing theory], when judged by its ability to explain the empirical data, option-pricing theory is the most successful theory not only in finance, but in all of economics. It is now widely employed by the financial industry, and its impact on economics has been far ranging. At a theoretical level, we now understand that option-pricing theory is a manifestation of the force of arbitrage and that this is the same force that underlies much of neoclassical finance." This is one reason that we find option-pricing theory so useful. It's a basis of finance.

The second reason is that we have all kinds of options in our assets and liabilities. A book by Alfred Weinberger includes an "Options Balance Sheet" table. On the left side of the table, you have the assets and on the right side the liabilities. It also shows the options held by insurance companies and the options granted by insurance companies. Of course, these are the options that we need to worry about because these are options granted by insurance companies. Some of the options in the assets are the call options in bonds and the prepayment options in mortgages. In the liabilities are the surrenders, such as single-premium deferred annuity cash-ins, asset surrenders, and higher returns in the equity-indexed annuities that we are selling. We have guarantees, such as minimum death-benefit guarantees and all kinds of variable insurance products.

I started to look up some numbers on surrenders in the *Insurance Fact Book* and found two tables. One lists the payments to life insurance beneficiaries. In 1977, the total payments were $43 billion. For annuities in 1977, it was $55 billion. But for surrender, it was $140 billion. Surrender is a big problem. We need to

understand how to value the options given to our customers and how to manage that. The surrender value is more than annuity payments and insurance payments added together, according to the latest *Insurance Fact Book*.
Another reason actuaries need to study option-pricing theory is that, these days, we have all kind of insurance derivatives. We have catastrophe bonds. We have insurance futures. A few years ago, there was a symposium in Atlanta, and the Society published a monograph titled "Securitization of Insurance Risk."

Another reason we need to study option-pricing theory is because we want to do valuations. A brand-new monograph published by the Financial Reporting Section has many papers on valuation. One, by Dave Becker, is titled "The Value of the Firm: The Option Adjusted Value of Distributable Earnings." In this paper, he uses option-pricing theory to come up with a way to value insurance companies. He proposes using methods of A/L management. This is one of about 20 papers in that monograph.

Another paper I want to discuss is by Bob Reitano who, unfortunately, cannot be here today. His paper, "Two Paradigms for the Market Value of Liabilities," appeared in the first volume of the *North American Actuarial Journal*. Bob discussed two paradigms for market values of liabilities. One uses option-pricing theory for valuing insurance liability.

The third paper I want to recommend is "Market Value of Insurance Liabilities: Reconciling the Actuarial Appraisal and Option Pricing Methods," which will appear next year in the *North American Actuarial Journal.* It's a paper by Luke Girard of Lincoln Investment who tried to reconcile the two methods of getting market values of insurance liabilities. One is an option-pricing method and the other he calls the actuarial appraisal method.

I want to say a few words about A/L management. This is an extension of Redington's idea for a case where the A/L cash flows have interest-sensitive options. Chart 1 is a yield curve price diagram. This is a bond and if yield goes up, the price goes down. If yield goes down, prices go up. If you have a callable bond and use option-pricing theory to price it, because the bond is callable, its price must be lower than the same bond that is noncallable. You expect the callable bond price to be lower. However, as interest rate goes down more and more, then there's a higher and higher probability that the bond will get called. Therefore, the price of the callable bond comes down. It moves down in this way, and this is what we call price compression. You'll find a curvature like this if your assets are callable.

In contrast, the liabilities we sell have surrender options. This would be like a noncallable bond plus a put option. If a bond has put option, then it's more valuable than a bond without the option. You would expect a putable bond to pay something like this and, as interest rates go higher and higher, the options become more and more valuable.

What I want you to understand is that if the assets have call options in them, they will behave in a curvature like this. If the liabilities have surrender options, they would behave this way. Then you can combine these two and look at your assets and liabilities. If you value your assets and liabilities using the same option-pricing model, then you can expect your asset to have a curve like this because your assets are callable, and your liabilities would have a curve something like this because your liabilities have surrender options.

Of course, what is important to us is the difference, or the surplus, which is assets minus liability. You have a curve like this, which is what we call a short straddle.

Before I switch topics, there is something I want to mention to you. Remember earlier I talked about changing measures to get at this risk-neutral measure. In the August 1999 issue of *Risks & Rewards,* there is this article by Mark Tenney and Luke Girard, who discuss the two measures—the risk-neutral measure and the actual probability measure—and when you should and should not be using them. I think this is a well-written article and it's only two pages. You may want to look at it because it discusses the problems of using risk-neutral scenarios.

I want to change topics to talk about another important idea in finance in the last 30 years, but before I do that, I want to read you a paragraph from a book titled *Investment Science*, by David Luenberger: "Conclusions about multiperiod investment situations are not mere variations of single-period conclusions. Rather, they often reverse those early conclusions. This makes a subject exciting both intellectually and in practice. Once the subtleties of multiperiod investments are understood, the reward in terms of enhancement, enhanced investment performance, can be substantial."

Jim mentioned the problem that Markowitz and CAPM are one-period models, but what we are facing in insurance companies or in pension plan management are multiperiods. It's not just one period. The solution provided to us by Markowitz and CAPM may not be suitable at all.

Luenberger is not a professor of finance, and that's why this is a very nice book. He is in the department of engineering and economic systems, and I think this book is quite suitable for actuarial students because he goes through the mathematics and doesn't have too much fluff in his book.

The multiperiod portfolio optimization solution comes from a paper by Robert Merton, who won the Nobel Prize in 1997. His paper was published 30 years ago, in 1969. The title of the paper he won the prize for is "Lifetime Selections Under Uncertainty: A Continuous-Time Case." In it, he says you are allowed to rebalance as time passes.

From the Floor: Wasn't he the one involved in long-term capital management?

Mr. Shiu: That's right. Even a brilliant person can lose money. One of the latest papers in this area is titled "Optimal Portfolio Strategy for Outperforming a Stochastic Benchmark." Suppose we issue a product with a lot of liability, which is stochastic. How should we invest to beat the stochastic benchmark? This paper

just came out a few months ago in *Finance and Stochastic*. The authors discuss, assuming that the benchmark is the S&P 500, we need to get to at least 80% of the index. How will you invest your money to beat that benchmark?
I think this is a very important subject. Unfortunately, it is still not in our actuarial syllabus. This is the course 6 syllabus, the Investment and Finance Track's investment finance exam, and we do not have this multiperiod optimization yet. Even in course 8 we still do not have multiperiod optimization in the syllabus. The problem is that the mathematics are very difficult. They use something called the Hamilton-Jacoby-Bellman equation, and that is something very messy; it's very technical mathematics. Hopefully, one day the theory can be simplified, and we can put some of this stuff into our actuarial syllabus because I think we are dealing with multiperiod portfolio problems.

Finally, let me give you an example of results in this area. Some of you may know about TIAA-CREF. Jim mentioned that it started out in variable annuities. One of TIAA–CREF's accounts is a social choice account. In this account, the assets are in two groups, stocks and bonds, and what the company does is rebalance it at each point of time. It's not just at time zero, but at each point of time, it has 60% of the money in stocks and 40% of the money in bonds. It rebalances the portfolio continuously, maybe once a week or once a month, but the strategy is to always have 60% in stocks and 40% in bonds.

This result or this strategy can be justified by this continuous-time optimization that I mentioned. Suppose you have the following situation.

Risk - free asset, risk - free force of interest r.

Risky asset: $S(t) = S(o)e^{X(t)}, t \geq 0$

$X(t)$ Normal $(mt, s^2 t)$

Assume $s = 25\%$ and $m = \underbrace{5\%}_{\text{risk premium}} + \left(r - \frac{s^2}{2}\right)$.

If the investor's utility function is

$U(X) = -X^{-1/3}, \ X > 0,$

then at each point of time,

he will put 60% of his wealth in the Risky asset

and 40% in the Risk - free asset

There are only two kind of assets in which you can invest. One is a risk-free asset that uses a risk-free force of interest rate *r.* The other is a risky asset, and it follows a so-called geometric Brownian motion. What it means is this: The value at time *t* is the value at times zero times $e^x(t)$, where $x(t)$ is a normal random variable with mean *mu* times *T* and variance sigma squared, times *T*.

Now, suppose its sigma is 25% and *mu* is 5%. This 5% is the risk premium. Your risky asset earns 5% beyond the risk-free asset. Now, there's *r*. So, you have 5% plus *r*, the risk-free force of interest, minus 1/2 sigma squared. The 1/2 sigma squared is just a mathematical technicality, so don't worry about that. Just think that there is a 5% risk premium. Your risky asset earns more than a risk-free asset, 5% on the average.

Now, if the investor has a power utility function $-X^{-1/3}$ —Jim mentioned that Markowitz never really used a utility function—then his or her optimal investment strategy is to always put 60% of the money in the risky asset and 40% in the risk-free asset. At each point of time, he or she will rebalance this portfolio so that it's always 60/40, i.e., 60% in stocks and 40% in bonds.

Mr. Hickman: I want to ask a question about the risk-neutral measures. Most of the utility theories learned in a decision theory class represent an individualistic idea. This helps you, with your attitude towards risk and your resources, to make an optimal decision. The risk adjustment that you're making here is a market-driven one, isn't it?

Mr. Shiu: That's right.

Mr. Hickman: In some way, it's kind of an aggregate of everybody's attitude towards risk?

Mr. Shiu: That's right, but here in asset allocation, there is an individual.

Mr. Hickman: I agree.

Mr. Shiu: In Black-Scholes, that is the market aggregate. Black-Scholes did not need to talk about the utility function because they could synthetically replicate to produce their options. Therefore, the value of the initial portfolio must be the price. This self-replicating portfolio is an important concept.

Mr. Hickman: We have not talked about self-replicating portfolios. I do want to stress that this is no way to avoid utility in individual decisions. It's a way to elicit from the market the kind of aggregate or macro-utility of the market.

Mr. Shiu: I think David Li's point is that we don't have a market for insurance products. We do not buy and sell insurance policies. Therefore, can we really use Black-Scholes or option-pricing theory? And the answer is no. I think there are problems. I can show you three papers, and my feeling is that each of these three have theoretical problems. But that's all we can do at the moment. That's how much we understand now. Insurance is an incomplete market. Therefore, that change measure is not unique. We don't have a unique price. We do have a problem applying option-pricing theory to value insurance.

Mr. Hickman: I want to underline what Elias just said. I'm not going to change it one bit, but underline that the insurance market is incomplete. That is, the completeness of the market is an important characteristic to consider in the mathematics that we'd like to do. But, because the insurance market does not have every conceivable contract or have open buying and selling, it is an incomplete market, and, therefore, the applications of some of these ideas are problematic.

Mr. Shiu: Yes.

Mr. Hickman: That market is becoming more complete every day.

Mr. David Li: Yes it is, in some areas.

Mr. Hickman: Elias, would you respond to that so the audience understands what his excellent comment was about?

Mr. Shiu: Okay. What David Li is saying is that, as time passes, there will be a market for insurance. In those times, I think then we can apply option-pricing theory, but now it's just one-sided. Insurance companies just sell insurance products. They don't buy insurance products.

From the Floor: What does the future hold for new theories in financial economics and their application in the pension area or life insurance area?

Mr. Hickman: This is probably an ill-informed reply. I think that, in many ways, on the theory side, the last 50 years have been extraordinary. I doubt if we'll see another 50 years that change that much. I think the biggest changes are going to be in the market. In the sense that we just had this conversation about the incompleteness of the insurance market, it's difficult to be a buyer and seller of any quantity at any time you want to. I think you're going to see more of the market converging towards the mathematics rather than mathematics encompassing the market.

Markets in general are becoming much more efficient, that is, some of the axioms about efficient markets and no arbitrage that were just mathematical niceties are being realized. Every time you talk to a banker, a stock broker, or an insurance person, each talks about how much tougher it is. The margins are getting thinner. That's not bad news. That's the efficient market at work. I expect more theoretical advances in financial economics. I'm not smart enough to predict them, but I think the market responses and the convergence of the market to the realization of no arbitrage and completeness is going to dominate the practical lives of those of you who will be living during the next 50 years.

From the Floor: Although I'm an actuary, I work exclusively on the asset side, where we commonly use multiple factor analysis. For example, there was recently an excellent National Bureau of Economics research paper that expresses it more articulately than I can. We start off looking at the factors at work, i.e., towards lower interest rates and credit spreads moving in a certain direction. Then you take a look at the risk versus per unit of nondiversifiable risk from all the factors that are available, and only then do you start your portfolio. I know that insurance companies often do simulations, but how sophisticated are insurance companies today at looking at the various different factors other than just interest-rate risk and stock market risk?

Mr. Hickman: Shall we turn that back to the audience? How sophisticated are insurance companies in the development of their models? Rather than simply sitting in the armchair and concocting them, are we, in fact, trying to look at more basic economic variables as we build them?

I'm sure many of you are aware of those who have built on yield curves as summarizing almost all current views of the future market. Gross domestic product, interest rates, unemployment, and everything else can be summarized in there. But that begs the question of what tomorrow's yield curves will be. You can generate them at random and be consistent, but in the sense of utilizing all available information, you may not be, so I don't know the answer to your question.

CHART 1
YIELD CURVE PRICE