



SOCIETY OF ACTUARIES

Article from:

The Actuary

April 1970 – volume 4 - Issue 4

A NEW DERIVATION OF THE TABULAR INTEREST FORMULAE

by Richard G. Driskell

In this article, we will derive two troublesome formulae in the Gain and Loss Exhibit of the Statement using techniques developed in Measurement of Mortality. The formulae in question are those for Tabular Interest.

In each case, we will add two different representations of the denominator ("exposure"); one is net units = potential units less cancelled units while the other is net units = possible units less impossible units. For practice, here's how to show $i = 2I / (A + B - I)$. First we disperse entrants (N) and withdrawals (W) to the end points. Then:

- 1) Potential less cancelled is:
 $(A + N/2)$ less $(W/2)$
- 2) Possible less impossible is:
 $(B + W/2)$ less $(I + N/2)$
- 3) Hence, twice the denominator is:
 $A + B - I$; twice the numerator is $2I$ and the ratio is as given above.

Let's move on to tabular interest for life insurance. The following assumes deaths at the end of the (calendar) year and other events at the middle of the year. RROT and RROD refer to reserve released on termination and on death respectively.

- 1) Potential less cancelled is:
 $(\text{Reserve last year} + \text{Premium}/2)$
less $(\text{RROT}/2)$
- 2) Possible less impossible is:
 $(\text{Reserve this year} + \text{RROD} + \text{RROT}/2)$ less $(I - C + \text{Premium}/2)$
- 3) Hence, twice the denominator is:
 $\text{Reserve this year} + \text{Reserve last year} + \text{RROD} + C - I$

The rest follows easily.

Now we direct our attention to required interest for life annuities. The fund equation is $\text{Reserve this year} = \text{Reserve last year} + \text{Premium} + I - \text{Pay'ts} + (T - A)$. Assuming mid-year payments we have:

- 1) Potential less cancelled is:
 $(\text{Reserve last year} + \text{Premium}/2)$
less $(\text{Pay'ts}/2)$

- 2) Possible less impossible is:
 $(\text{Reserve this year} + \text{Pay'ts}/2)$
less $(T - A + I + \text{Premium}/2)$.
- 3) Hence, twice the denominator is:
 $\text{Reserve last year} + \text{Reserve this year} - (T - A + I)$.

Of course, $(T - A)$ can now be computed by deducting I from: $(T - A + I) = (\text{Reserve this year} + \text{Pay'ts}) - (\text{Reserve last year} + \text{Premium})$.

The above discussion has yielded no new results but merely was intended to draw upon actuarial theory learned before studying life insurance accounting, viz., exposure formulas, to derive and hopefully help to remember the Annual Statement instruction formulas for Tabular Interest. As a teacher of actuarial science, I feel that the more interrelationships we can discover among the sometimes seemingly disjointed syllabus for the Society's examinations, the more unified and comprehensible we can make the study of actuarial science to the student. \square

AIFA Report

(Continued from page 4)

ings are computed might influence the method of obtaining such adjusted earnings. The purpose of the financial analysts is to make recommendations of "buy," "sell" or "hold" based on earnings performance related to the earnings of other life companies and to the earnings of companies in other industries. The analysts make subjective judgments as well as mathematical adjustments.

Many actuaries object to a number of the Committee's proposals. The major criticisms are: (1) the use of the moving average interest rate has been demonstrated to be theoretically deficient, (2) the expense items selected from the Convention Blank are arbitrary and may give misleading results among companies due to different accounting practices, (3) the use of the A. M. Best Company lapse formula for determining the number of years over which to amortize expenses is subject to question, and (4) not recognizing the reserve method in determining the amount of expenses to capitalize and amortize will distort the adjusted earnings considerably. \square

Capitalizing acquisition expenses as defined could result in distorting a company's underlying earnings if more is actually paid for the business than was assumed would be paid. Generally speaking, it would be preferable to capitalize "formula" acquisition expenses in order to relate the expense to the "allowable" provided for in the calculation of gross premiums.

The use of the A. M. Best Company lapse ratio would cause a fast growing company to amortize expenses over a much longer period than may be justified. This is the result of including the insurance issued during the preceding year in the denominator of the lapse ratio formula.

Failure to take into account differences in reserve methods among companies may give rise to a major distortion in the adjustments.

The investment community does recognize the need for a standard, reliable method for adjusting statutory earnings since the investing public (institutional investors, at least) are becoming aware of the fact that reported statutory earnings do not always reflect true underlying earnings of a life insurance company. The complex problem of adjusting life insurance "statutory earnings" has not escaped the notice of actuaries, accountants and the life insurance industry, and various groups have been seeking a "consensus" solution. There appears to be a need for even more cooperation among all interested parties, if the adjustment to reported earnings is to be generally accepted. \square

Campus Contact

(Continued from page 1)

lege and university students who pass either of the Preliminary Examinations encouraging them to give serious thought to actuarial careers.

There are a great many colleges and universities in the U. S. and Canada—far more than the subcommittee will ever be able to reach effectively. Success in this work is dependent on the volunteer activities of many individual actuaries—and the more the better. Any Society member or any actuarial club interesting in helping is invited to contact Russ Smith, Subcommittee Chairman. \square