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The Calculus of DAC

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Understanding the Deferred Acquisition Cost (DAC) asset balance movements from one reporting period to the next can be a challenge when reporting under FAS 97. The mechanics of FAS 97 call for an amortization ratio (i.e., k factor) that forces the amortization of the DAC asset to be matched to the earnings received over the life of the product. However, the fact that the k factor is dynamically set results in "unlocking" components of the DAC rollforward. This article shows how the basic equation for the amortization ratio can be used to derive equations that provide insights into what the DAC balance represents. Furthermore, partial derivatives of these equations are used to show how unexpected changes in the Estimated Gross Profit (EGP) stream impact the amortization and unlocking components of the DAC rollforward. The concepts explored here apply not only to the DAC balance, but also to the sales inducement cost (SIC) asset, the front end load (FEL) liability, and SOP 03-1 reserve for excess claims. The equations derived here complement the analysis provided in Steve Malerich's "Simply Unlocking" article in the June 2015 edition of *The Financial Reporter*.[1]

DAC ROLLFORWARD

The typical way of analyzing DAC is to rollforward the balance from one period to the next. An example DAC rollforward exhibit is provided below.

Beginning DAC	500,000
+ New Deferrable Expenses	25,000
+ Interest on DAC	10,000
- Amortization (k*EGP)	(45,000)
+ Unlocking	8,000
Ending DAC	498,000

Beginning with the prior period's balance, the asset is increased for new deferrable expenses. The asset earns interest at the discount rate used in the calculation of the amortization ratio (k). Amortization equal to k multiplied by EGPs causes the asset to decrease. As you are all aware, the k factor is calculated as the ratio of the present value of deferrable expenses to EGPs over the life of the product:

$$(1)\quad k = \frac{PV(Def)}{PV(EGP)}$$

This simple equation will be the basis for deriving the remaining equations in this article.

An unlocking term is included in the rollforward and is needed to arrive at the final DAC balance for the reporting period. Far from being simply a plug in the rollforward, the unlocking row has a significance of its own. The row is needed to handle changes in the k factor that occur when the stream of deferrable expenses and/or EGPs anticipated at the beginning of the reporting period are replaced with a new stream of cash flows. By adding the unlocking component to the beginning DAC balance, one arrives at what the prior period balance would have been if it had been calculated using the most recent stream of EGPs and deferrable expenses. In other words, it answers the question of "How would my prior period DAC balance have changed if I knew then what I know now?" In this regard, it might make more sense to move the unlocking row to the top of the rollforward making it the first item to change the prior period balance. Another interpretation is that the unlocking row reflects the revised amortization of all prior periods using the most recently calculated k factor.

There are two primary reasons why the amortization schedule would have changed. First, a quarter's worth of projected EGPs and deferrables are replaced with actual values. This component is often referred to as a "true up." Second, the projection of cash flows beyond the current valuation date has likely changed due to assumption changes or an updated policy inventory. This second component is often referred to as "prospective unlocking." Stated another way, the unlocking component comes about if our crystal ball used to project cash flows at the beginning of the reporting period was broken. True up occurs when our prior period DAC model did not accurately predict what we now know to have happened this quarter. Prospective unlocking occurs when our prior period DAC model had a different prediction of what we now project for what lies ahead.

While this high level description of the unlocking line is helpful, one often finds the need to answer more detailed questions. For example, if a positive \$3M variance to my EGP stream occurs, how will the DAC balance change? While the additional EGPs can be expected to cause k*3M more DAC amortization, how much unlocking will also occur? Is it possible the unlocking effect could outweigh the amortization effect? A further breakdown of the DAC equations is needed to dig deeper.

BREAKING DOWN THE K FACTOR EQUATION

A first step in achieving greater insights from equation (1) is to break the deferrable and EGP streams into historical cash flows (occurring before the valuation date) and future cash flows (occurring after the valuation date) as shown in equations (2) and (3).

(2) PV(Def) = PV(HistDef) + PV(FutDef)

(3) PV(EGP)= PV(HistEGP)+PV(FutEGP)

Substituting these equations into (1), we arrive at:

$$(4)\quad k = \frac{PV(HistDef) + PV(FutDef)}{PV(HistEGP) + PV(FutEGP)}$$

Often either the cohort inception date or the valuation date are used as defining the "present" time when calculating the present values in the k factor equations of (1) or (4). One can switch between the two dates by either multiplying or dividing the top and bottom of equation (1) or (4) by the discount factor between the valuation date and cohort inception date. Because the discount factor is applied to both the top and bottom of the k factor ratio, either choice of "present" time reference results in the same value for k. However, for the DAC balance equations that follow to hold, the present values must be calculated relative to the valuation date. When using the valuation date as the "present" time reference, the PV(HistDef) and PV(HistEGP) terms can be interpreted as cash flows accumulated forward with interest (interest rate equals the discount rate). **For this article, we will choose to calculate all present values using the valuation date as the reference point of time.**

After multiplying both sides of equation (4) by the denominator of the right side, one arrives at the following identity:

$$(5)\quad PV(HistDef) - k * PV(HistEGP) = k * PV(FutEGP) - PV(FutDef)$$

While not proven here, both sides of the equation (5) are also equal to the DAC balance when the PVs are calculated relative to the valuation date. Further insights can be gleaned by looking at each half.

$$(6)\quad DAC = PV(HistDef) - k * PV(HistEGP)$$

Equation (6) focuses on the historical cash flows. The DAC balance can be interpreted as the present value of all historical deferrable expenses minus the present value of historical amortization (k*PV(HistEGP)).

$$(7)\quad DAC = k * PV(FutEGP) - PV(FutDef)$$

Equation (7) focuses on the future cash flows. Often the PV(FutDef) term is negligible or nonexistent because the vast majority of deferrable expenses occur early in the cohort's life. In that case, the DAC balance carries the interpretation of being equal to the present value of all future DAC amortization. Substituting the k factor definition from equation (1) into equation (7) results in the following equation:

While [the] high level description of the unlocking line is helpful, one often finds the need to answer more detailed questions.

$$(8)\quad DAC = \frac{PV(FutEGP)}{PV(EGP)} * PV(Def) - PV(FutDef)$$

Equation (8) shows the DAC balance is driven by the percentage of EGPs that occur in the future period of the amortization schedule (PV(FutEGP)/PV(EGP)). For newer DAC cohorts,

this percentage is near 100 percent and the DAC balance is close to the PV(Def).

The opposite is true for older cohorts that are nearing the end of their amortization period. Equation (8) is also useful for predicting how a change to the EGP stream would affect the DAC balance:

- Increases in future EGPs always result in increases to the DAC balance since they increase the ratio of PV(FutEGP)/PV(EGP).
- Increases in historical EGPs (without increases in future EGPs) always result in decreases to the DAC balance since they decrease the ratio of PV(FutEGP)/PV(EGP). In other words, it is not possible for the unlocking effect of a positive historical EGP variance to outweigh the amortization effect. For historical EGP variances, amortization always beats unlocking.

More insights into the sensitivity of the DAC balance to changes in the deferrable expenses and EGPs can be found by taking partial derivatives.

PARTIAL DERIVATIVES OF THE DAC EQUATIONS

Equation (8) can be written out in long form by substituting the definitions of equations (2) and (3) to produce the following equation:

$$DAC = \frac{PV(FutEGP)}{PV(HistEGP) + PV(FutEGP)} * (PV(HistDef) + PV(FutDef)) - PV(FutDef) \quad (9)$$

This equation represents the DAC balance as being a function of four variables: PV(HistEGP), PV(FutEGP), PV(HistDef), and PV(FutDef). The following equations can be found by taking the partial derivative of the DAC balance with respect to each of those four variables:

$$(10)\quad \frac{\partial DAC}{\partial PV(HistEGP)} = -k * \frac{PV(FutEGP)}{PV(EGP)} = -k + k * \frac{PV(HistEGP)}{PV(EGP)}$$

$$(11)\quad \frac{\partial DAC}{\partial PV(FutEGP)} = k * \frac{PV(HistEGP)}{PV(EGP)}$$

$$(12)\quad \frac{\partial DAC}{\partial PV(HistDef)} = 1 - \frac{PV(HistEGP)}{PV(EGP)} = \frac{PV(FutEGP)}{PV(EGP)}$$

$$(13)\quad \frac{\partial DAC}{\partial PV(FutDef)} = -1 + \frac{PV(FutEGP)}{PV(EGP)} = \frac{-PV(HistEGP)}{PV(EGP)}$$

These partial derivatives can be used in understanding how the DAC balance is impacted by differences in the EGP and deferrable streams when comparing the prior period and current period schedules. Numerous insights can be gleaned from these equations:

- Equation (10) shows how the DAC balance is affected by changes in historical EGPs. This equation has an amortization (-k) and unlocking (k*PV(HistEGP)/PV(EGP)) effect. Again we see that the amortization effect must outweigh the unlocking effect because the PV(HistEGP)/PV(EGP) ratio (referred to as the historical ratio in the "Simply Unlocking" article[1]) must be less than 1.
- The DAC impact of historical adjustments are often approximated as DAC_Adj=-k*HistAdj. Equation (10) shows that this approximation holds well for young cohorts where the historical ratio is small, but is inaccurate for older cohorts when the unlocking piece of the equation has more weight.
- The historical ratio shows up again in equation (11) showing that older cohorts are more susceptible to DAC unlocking than younger cohorts.
- Equations (12) and (13) deal with the change in DAC due to variances in the stream of deferrables. This contributor to DAC unlocking is often overlooked because there are not typically significant levels of projected deferrables. Increases in historical deferrables serve to increase the DAC balance while increases in projected deferrables push down the DAC balance.

	PV (HistEGP)	PV(FutEGP)	PV(HistDef)	PV(FutDef)	k	Balance
Original	20	30	35	5	80.0%	19.000
Partial Derivs	-48.0%	32.0%	60.0%	-40.0%		

	PV(HistEGP)	PV(FutEGP)	PV(HistDef)	PV(FutDef)	k	Balance
Revised	21	32	35	6	77.4%	18.755
Variance from Original	1	2	0	1		
Estimated DAC Impact= Variance*PartialDeriv	-0.48	0.64	0	-0.4		18.760

EXAMPLE

An example might help to show how this theory can be used in practice. You are the financial reporting actuary for a small block of Universal Life policies sold five years ago. All the policies are grouped into one cohort and the DAC balance for this cohort is 19M (calculated using equation (6), (7) or (8) above) as shown in the top table above.

Note that the k factor is 80 percent and that the PV of historical EGPs is 20M. Because the PV of Future EGPs is 30M, 40 percent of the EGPs occurred in the past. The four partial derivatives can be calculated using equations (10)-(13).

After quarter end has completed, three changes to our UL model occur that will change the DAC balance calculation:

- We were informed of a 1M gain on our investments that was not included in our original calculation. This changed the PV(HistEGP) by 1M.
- A change to our recurring premium assumption will result in an increase of future EGPs of 2M.
- The recurring premium assumption change also causes a 1M increase to our PV of future deferrables calculation. (See bottom table above.)

Putting these changes through the DAC model causes the DAC balance to drop to 18.755M. This change of -0.245M will appear in the unlocking row when rolling the DAC balance forward to the next reporting period. Using the partial derivatives, an estimate of the DAC impact of each of these changes can be calculated. Adding the three estimated DAC impacts to the original DAC balance, produces a revised DAC balance estimate of 18.76M. While only incurring a modest approximation error, the partial derivative technique allows the attribution of the DAC balance unlocking to be split between the three changes without requiring three separate DAC calculations.

CONCLUSION

Starting with the basic equation for the DAC amortization ratio, one can derive a variety of equations that provide insight into the DAC balance. The DAC balance can be simplified to a function of four variables: PV(HistEGP), PV(FutEGP), PV(HistDef), and PV(FutDef). Partial derivatives were calculated to show the sensitivity of the DAC balance to the movement of each of these variables. By using these equations, we can better understand how unexpected changes to EGPs or deferrable expenses drive movements in the DAC balance. In addition to improving our intuition, the partial derivative equations can be used to attribute the total change in DAC among the different drivers of two DAC runs. This ability to tease out multiple attributions from only two sets of runs saves time during the financial close and enhances the ability to do ad-hoc analysis. ■

ENDNOTES

[1] https://www.soa.org/Library/Newsletters/Financial-Reporter/2015/june/fr-2015-1ss-101.pdf

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