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By Karen Rudolph and Ruijuan Wang

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The evolution of VM-20 has brought a renewed focus on statistical credibility. This article will highlight the requirements of VM-20 as it relates to measuring the credibility of a company's mortality data, remind the reader of several published references on credibility, provide an overview of Limited Fluctuation and Bühlmann Empirical Bayesian methods and highlight some of the findings of a 2009 research study on credibility sponsored by the Society of Actuaries (SOA).[1]

While statistical credibility theory is a broad topic, within this article the authors discuss credibility methods and concepts as applied specifically to the VM-20 mortality credibility requirements. The authors hope you find this a helpful refresher.

VM-20 MORTALITY CREDIBILITY REQUIREMENTS

The concept of credibility first appears in VM-20 in Section 9A(6) General Assumption Requirements. Here it is stated the company is required to "… use its own experience, if relevant and credible, to establish an anticipated experience assumption for any risk factor. … For risk factors (such as mortality) to which statistical credibility theory may be appropriately applied, the company shall establish anticipated experience assumptions for the risk factor by combining relevant company experience with industry experience data, tables, or other applicable data in a manner that is consistent with credibility theory and accepted actuarial practice." For mortality experience, the manner with which credibility is measured is prescribed later in Section 9C(4).

Specific requirements for credibility determination depend on the valuation basic table (VBT) being used to represent industry mortality. Table 1 summarizes the requirements of Section 9C(4).

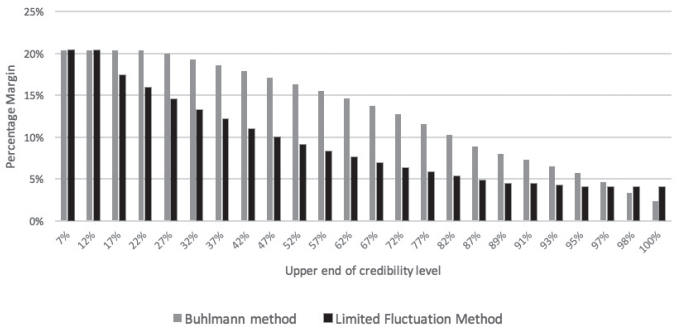
Table 1

	Industry Table	
	2008 VBT	2015 VBT
Permitted methods	A method that follows common actuarial practice as published in actuarial literature including, but not limited to, Limited Fluctuation and Bühlmann Empirical Bayesian	Limited Fluctuation by amount Or Bühlmann Empirical Bayesian by amount
Constraints	None	Limited Fluctuation method by amount must be calibrated for a minimum probability of at least 95 percent with an error margin of not more than 5 percent. Bühlmann method can use the direct approximation formula for Z provided in VM-20.
Flexibility in method after first use	No specific requirements are spelled out if a company using the 2008 VBT as industry table wants to change credibility methods.	A company seeking to change credibility methods must request and subsequently receive the approval of the commissioner. The request must include justification for the change and a demonstration of the rationale in support of the change.

IMPACT OF CREDIBILITY ON PRUDENT ESTIMATE MORTALITY

The credibility of a company's mortality data drives two very important aspects of the prudent estimate mortality assumption as specified in Section 9C of VM-20. First, the percentage margin to be added to the company experience rates depends on the credibility method used and the credibility of the company's experience based on that method. Understandably, partially credible mortality data (i.e., credibility less than 100 percent) requires a higher (or greater) margin than does fully credible data. The tables of margins appear in VM-20 Section 9C(5). There are margin tables applicable for when the 2008 VBT is the industry mortality table and margin tables for when the 2015 VBT is the industry mortality table. Chart 1 below shows the percentage margins for attained ages less than 45 for Limited Fluctuation and for Bühlmann when the 2015 VBT is the industry mortality table. Margin percentage rates decrease with increasing attained age, but only the first attained age grouping (the broadest grouping) is shown in Chart 1.

Chart 1
VM-20 Mortality Margins
Attained ages <45



The second aspect of the company's prudent estimate mortality assumption that is dependent upon a company's mortality credibility measure is the number of years of its own mortality experience which can be recognized, and the schedule for **grading** this experience into the industry table. Here is where the VM-20 requirements diverge from the classical treatment of credibility theory. Most traditional applications that use credibility theory result in a **blending** of the company experience weighted by its credibility measure, Z, with industry experience multiplied by (1 – Z). Within the VM-20 requirements, the Z factor is used, as noted above, for establishing margins, and for determining how many years the company data may be used before linearly grading this data into the industry mortality rates.

The VM-20 grading rules establish the duration at which grading must begin and the duration at which the prudent estimate mortality rates must be fully graded into the industry mortality rates. In fact, there are two grading tables: one for valuations before Jan. 1, 2017 (e.g., for AG 48) and one for valuations after. Unlike the mortality margins, the grading rules do not vary by the credibility method used. Also, the credibility categories are broader for grading than for the margins. For valuations as of Jan. 1, 2017 and later, companies with less than 20 percent credibility may not use their own experience, only industry experience. For credibility of 20 percent or greater, the number of policy years that company data may be used before beginning the linear grading to the industry table rates increases with increasing credibility. The linear grading also depends upon the company's sufficient data period. A sufficient data period ends at the last policy duration that has 50 or more claims. The grading table also sets the duration at which the prudent estimate mortality assumption must be fully graded to 100 percent of the industry table. There are additional considerations related to the grading process applicable for higher attained ages.[2]

RESOURCES IN ACTUARIAL LITERATURE

Many resources exist that provide background and relevant formulas on the topic of credibility. A small sampling of these are listed here.

- American Academy of Actuaries Credibility Practice Note, July 2008.
- Actuarial Standard of Practice No. 25 Credibility Procedures Applicable to Accident and Health, Group Term Life, and Property/Casualty Coverages, October 1996.
- Expected Mortality: Fully Underwritten Canadian Individual Life Insurance Policies. Committee on Life Insurance Financial Reporting, July 2002. Canadian Institute of Actuaries.
- A Credible Theory of Credibility by Drew Tindall and Jess Mast. Contingencies September/October 2003.
- Credibility Theory Practices Report, Sponsored by the Committee on Life Insurance Research, the Financial Reporting Section, and the Product development Section of the Society of Actuaries. Prepared by Stuart Klugman, Tom Rhodes, Marianne Purushotham, and Stacy Gill of MIB Solutions. 2009 Society of Actuaries.

The reader should note the last reference above provides a comprehensive annotated bibliography. It should also be noted that, while the credibility calculations within VM-20 follow the generally accepted principles, the exact formulas within VM-20 are unique and must be followed to be in compliance.

OVERVIEW OF LIMITED FLUCTUATION AND BÜHLMANN EMPIRICAL BAYESIAN

Limited Fluctuation and Bühlmann Empirical Bayesian are two well-established statistical credibility methods used to adjust ex-

perience-based estimates. Both are accepted methods in VM-20 to meet the mortality credibility requirements. This section provides a brief introduction to the principles, data requirements, and advantages and disadvantages of the two methods. In this discussion, the value for which credibility is being measured is the company's actual-to-expected mortality ratio (A/E ratio), as is typically produced through a mortality experience study.

The Credibility Theory Practices Report provides a concise summary of the fundamental differences of these two methods.

> In both the Limited Fluctuation and the Bühlmann Empirical Bayesian methods, the results are calculated with respect to a mean (A/E ratio) and incorporate a variance. The methods differ in the treatment of the components of the variance (σ^2). The total variance of the observations is the sum over all companies of two different sources of variation, which are:
>
> 1. For each company, the variation of a company's observations about that company's mean, and
> 2. The variation between each company's mean and the overall mean.
>
> Limited Fluctuation credibility uses only the first source while the Bühlmann Empirical Bayesian method uses both. Thus, Limited Fluctuation credibility requires only data from the company being studied. For the Bühlmann Empirical Bayesian approach, data is needed for all companies under study.[3]

Within VM-20 values are provided for use with the Bühlmann Empirical Bayesian approach to approximate the second variance component above. This is explained in more detail below.

Limited Fluctuation

The Limited Fluctuation method is a classic statistical method based on confidence intervals. It only requires the subject company's seriatim experience data, and the underlying distribution of the data is assumed to be approximated by the normal distribution. Under a normal distribution assumption, the credibility factor (Z) can be derived from the following form.

$$\text{Limited Fluctuation } Z = \min\left\{1, \frac{r * \hat{m}}{z * \hat{\sigma}}\right\}$$

where

- z (in the denominator) is the z-value from the normal distribution table with selected probability value p. This z-value is provided by any standard two-sided normal distribution table. VM-20 explicitly requires the relative error in the estimate be no more than 5 percent with a probability (p) of at least 95 percent. If p = 95 percent confidence, the z-value is 1.96.
- r is the error margin in the confidence interval development. In practice, VM-20 has specified an error margin of no more than 5 percent, therefore r = .05.
- $\hat{m}$ is a company's estimated value (i.e., the estimate) and is calculated from the company's experience mortality data, the A/E ratio.
- $\hat{\sigma}$ is the standard deviation of the estimate.
- $\hat{m}$ and $\hat{\sigma}$ are derived from the company experience mortality study data. In most mortality studies, the number of observed lives, the fraction of the year each life was observed, the face amount insured, and the number and amount of claims are all involved in the calculation of these two quantities.

The formula is re-stated below with the constraints required by VM-20.

$$\text{Limited Fluctuation } Z = \min\left\{1, \frac{.05 * \hat{m}}{1.96 * \hat{\sigma}}\right\}$$

When there is full credibility (Limited Fluctuation Z=1), the credibility factor with VM-20 constraints means there is more than 95 percent probability that the estimation is no more than 5 percent in error of the true value. As can be seen from the formula, when $\hat{\sigma}$ is small enough, the credibility factor will be 1. If the conditions of full credibility are not met, the experience data are considered partially credible with the calculated credibility factor value of Limited Fluctuation Z.

Bühlmann Empirical Bayesian

Bühlmann Empirical Bayesian method is also called Greatest Accuracy Credibility method. It is based on a linear Bayesian model and requires past experience data from more than one company. As stated above, this method recognizes the variation of a company's observations about that company's mean observation and variation between each company's mean and the overall mean for the entire data set.

The credibility factor takes the form of Z = n/(n+k), where n is a measurement using exposures and k requires both the variance of observations in each company and the variance from one company to another. As each company's detailed policy data are confidential, a statistical agent is required to collect the data and perform the calculations for a pure Bühlmann Empirical Bayesian method. Since this is not practical in the context of regulatory reporting, VM-20 specifies a formula to approximate the credibility factor.

$$\text{Bühlmann Z} = \frac{A}{A + \frac{109\% * B - 120.4\% * C}{0.019604 * A}}$$

where

A = ∑(amount insured)∗(exposure)∗(mortality)

B = ∑(amount insured)^2∗(exposure)∗(mortality)

$$C = \sum(\text{amount insured})^2 * (\text{exposure})^2 * (\text{mortality})^2$$

Also note that the Bühlmann Empirical Bayesian method will never result in a credibility factor value equal to 1. The VM-20 margin tables take this into account, since the highest credibility category for the Bühlmann approach is 99 percent plus.

Comparison of the two approaches

There are advantages and disadvantages to each approach.

As Limited Fluctuation only requires data from one company and has straightforward inputs, the resulting formulas are relatively easy to understand, implement and interpret. However, it does not specify any procedures to estimate parameters p and r, so they can only be selected arbitrarily. In VM-20 these values are specified. The Limited Fluctuation method only considers the accuracy of the single company's experience and makes no consideration for the relationship of the company's experience to industry experience. Finally, this method makes an a priori assumption that the data has a normal distribution. As a result, no quantities can be optimized in the calculation.

As a comparison, Bühlmann Empirical Bayesian is a systematic modeling approach with assumptions and optimizations defined. No parameters need to be selected arbitrarily. Also, it reflects the accuracy in both single company and industry data through two variance calculations. On the other hand, it is hard to interpret and the calculation process is hard to explain. Finally, in its pure form, use of the Bühlmann approach requires the company to rely on statistical agents for the calculation. This impracticality is overcome by the formulaic approximation found in VM-20 Section 9C(4)a.

Outcomes from 2009 Credibility Theory Practices Report

Table 2 shows an excerpt of research results from the "Credibility Theory Practices Report." The purpose of this is to provide a better understanding of the difference between the two credibility approaches through demonstration.

Credibility results from nonsmoker mortality data for various types of life insurance products of 10 companies are shown in Table 2. Credibility Z-factors by count and by amount for both methods are provided. The "by count" result is included in the table to demonstrate how the by count measure can be materially higher than the "by amount" measure. In VM-20, for valuations in which the industry table is the 2015 VBT, one must use either the Limited Fluctuation or Bühlmann Empirical Bayesian

Table 2

			Limited Fluctuation Z		Bühlmann Z		Bühlmann Z – LF Z	
Overall A/E by Amount	Company	Company A/E Ratio by Amount	By Count	By Amount	By Count	By Amount	By Count	By Amount
77%	A	106%	.972	.708	.962	.935	-.010	.227
77%	B	118.5%	.830	.285	.945	.678	.115	.393
77%	C	63.5%	.664	.254	.949	.757	.285	.503
77%	D	89.2%	.387	.219	.843	.623	.456	.404
77%	E	61.4%	1.000	1.000	.997	.986	-.003	-.014
77%	F	71.6%	1.000	.236	.979	.704	-.021	.468
77%	G	36.8%	.044	.020	.106	.033	.062	.013
77%	H	81.2%	1.000	.409	.996	.863	-.004	.454
77%	I	82.8%	1.000	.833	.988	.963	-.012	.130
77%	J	97.9%	.952	.453	.965	.865	.013	.412

Taken from Appendix A, C and E of Credibility Theory Practices Report

by **amount**, not by count. If a company has historically measured credibility by policy count, it may find that the by amount measure is materially different. VM-20 is silent regarding whether the appropriate basis for exposure is direct or retained insurance amounts. The overall A/E for all companies combined is 77 percent. In Table 2, the Bühlmann calculation does not use the VM-20 approximation, but rather it is the Bühlmann Empirical Bayesian in pure form. Note the differences in outcomes between the two methods, particularly for companies B, C, D, F, H and J, where the Limited Fluctuation method produces a Z value which is 0.4 to 0.5 lower than the Bühlmann Z.

CONCLUSION

In VM-20, requirements around determining credibility of the company's mortality data vary by the VBT the company uses to establish its mortality assumption. For valuations using the 2015 VBT, two methods are allowed: the Limited Fluctuation method and the Bühlmann Empirical Bayesian method. Both must be determined based on face amount. Companies accustomed to measuring credibility using policy count may find the by amount requirement produces a different outcome. In VM-20 requirements, the Bühlmann Empirical Bayesian method is applied differently from the pure classic approach, primarily due to the impracticality of one company having access to mortality data from a number of other companies. A 2009 paper reveals that, for a given company, credibility results between the Limited Fluctuation and Bühlmann methods can vary materially. Each method has its own advantages and disadvantages.

In conclusion, the choice of credibility methodology for VM-20 may not be an easy one and what is best for one company may not be best for another. Since any future change in choice needs to be approved by the Commissioner and requires justification, choosing wisely upfront is the best approach. We hope this article provides you with some insights that will help you with that choice. ■

ENDNOTES

1. Credibility Theory Practices Report: Sponsored by the Committee on Life Insurance Research, the Financial Reporting Section, and the Product Development Section. Prepared by Stuart Klugman, Tom Rhodes, Marianne Purushotham, and Stacy Gill of MIB Solutions. See *https://www.soa.org/research/research-projects/life-insurance/research-credibility-theory-pract.aspx*
2. These constraints are applicable for valuations as of Jan. 1, 2017 and later.
3. Credibility Theory Practices Report, Page I.21

Karen Rudolph, FSA, MAAA, is a consulting actuary at Milliman Inc. She can be reached at Karen.rudolph@milliman.com.

Ruijuan Wang, ASA, MAAA, is an associate actuary at Milliman Inc. She can be reached at Ruijuan.wang@milliman.com.