



SOCIETY OF ACTUARIES

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AN INTERESTING DILEMMA

by John C. Fraser

In interest theory it is always assumed that money received from an investment is reinvested at the same interest rate as that at which the original investment was made. The rapidly changing interest rates of the past few years have led some people to try to measure the effect of reinvesting funds at a rate different from that at which the investment is made.

Consider a \$1,000 five-year bond purchased at par with annual coupons of \$50. Assuming that the reinvestment rate is 10%, what is the true rate of return?

One method that I have seen for solving this problem would first find the total funds at maturity from

$$\$50 S_{\frac{10\%}{51}} + \$1,000$$

to be equal to \$1,305.26 and would then find that a single sum of \$1,000 accumulated at a single interest rate of 5.47% is equal to this amount after five years.

Unfortunately, the use of the foregoing method leads to problems. Consider a second \$1,000 bond purchased at par with annual coupons of \$44.13 and with ten years to run. The method would first determine the total funds at maturity to be

$$\$44.13 S_{\frac{10\%}{101}} + \$1,000 = \$1,703.32$$

and would then find that a single sum of \$1,000 accumulated at an interest rate of 5.4% is equal to this amount after ten years.

Note that the two bonds have the same effective interest rate of 5.47%. It would seem logical, therefore, that if they are combined into one package deal, the 5.47% interest rate would be unchanged. Such a package deal would involve an investment of \$2,000 in return for coupons of \$94.13 for five years and of \$44.13 for another five years and lump sum payments of \$1,000 at the end of five years and another \$1,000 at the end of ten years. The total funds at the end of ten years under this package deal amount to

$$\$94.13 S_{\frac{10\%}{51}} (1.10)^5 + \$44.13 S_{\frac{10\%}{51}} + \$1,000 (1.10)^5 + \$1,000 = \$3,805.44$$

and an interest rate of 6.64% will accumulate a single sum of \$2,000 to this amount after ten years.

Why did the 5.47% increase to 6.64% when the bonds were combined? The reason is that we changed the equivalence point of the five-year bond from five years to ten years when we combined it with the ten-year bond. At a reinvestment rate of 10% the funds arising from the five-year bond at the end of ten years amount to

$$\$50 S_{\frac{10\%}{51}} (1.10)^5 + \$1,000 (1.10)^5 \\ = \$2,102.13$$

and an interest rate of 7.71% is required to accumulate \$1,000 to this amount in ten years.

The difficulty with this method is that the result is a function of the point in time where the equivalence is being measured. It is easily shown, for example, that the interest rate developed by this method approaches the reinvestment rate as the equivalence point approaches infinity. As the equivalence point approaches zero, the interest rate approaches infinity, either positive or negative depending on whether the reinvestment rate is lower or higher than the investment rate. Perhaps some method involving an average duration of investment could be devised so that standards could be set for comparing one investment with another in cases where the reinvestment rate differs from the rate at which the investment is made.

One also gets the vague feeling that somehow the use of a reinvestment rate different from that at which the investment is made is creating an interdependence between investments that may lead us into deep waters—e.g., is the interest on a reinvestment with a high yield being reflected more than once?

We have not yet been able to come up with a solution to this problem but feel that it is a legitimate one that is becoming of increasing interest to investment men in these days of rapidly changing interest rates. □

Black Actuarial Recruitment

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summer employment in the summer actuarial programs of the sponsoring companies. So far no permanent students have been hired as a direct result of this program. The Committee members are aware that the program is long-term and that results cannot be expected to emerge immediately. The Committee intends to review each phase of the work

continually and will drop or revise ineffective phases.

The program is attracting considerable notice within the profession and, in the meantime, the original sponsors decided to attempt to enlist broader nationwide support for the program.

To this end a letter asking for extended sponsorship was sent to certain life insurance companies and consulting actuaries throughout the United States. The arbitrary requirement for receipt of this letter was employment of ten or more members of the Society of Actuaries. More than fifty letters were sent out last November and the response has been good. Readers of this article can help by bringing the program to the notice of their employers who were not on the November list. Any employer or individual actuary wishing to support the program should get in touch with the author. □

Letters

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Promoting the Profession

Sir:

I enjoyed Mr. Boeckner's letter on "Promoting the Profession" in the October issue of *The Actuary*. In particular, I would like to state that I, too, advocate including some sort of Management training in the examination syllabus.

I suggested at the Concurrent Session on Education and Examination of Actuaries at the recent Annual Meeting in Denver that students be permitted to write papers in lieu of up to two Fellowship examinations. While this is not Management training *per se*, I feel it would provide much needed training in a particular Management area—communicating through the written word. It also has other benefits—e.g., it would permit a qualified individual to pass a sometimes unjustified obstacle to Fellowship and it could promote useful up-to-date text material for future (and past) students.

I would also like to suggest to Mr. Boeckner (and others) another means through which we might interest students at the high school level in actuarial work—expand actuarial summer student programs to include promising high school students. This would be an inexpensive way to educate tomorrow's generation as to what the insurance business is all about.

Steve Cooperstein

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