Article from:

Forecasting & Futurism

January 2012 – Issue 4

When Algebra Gets Chaotic

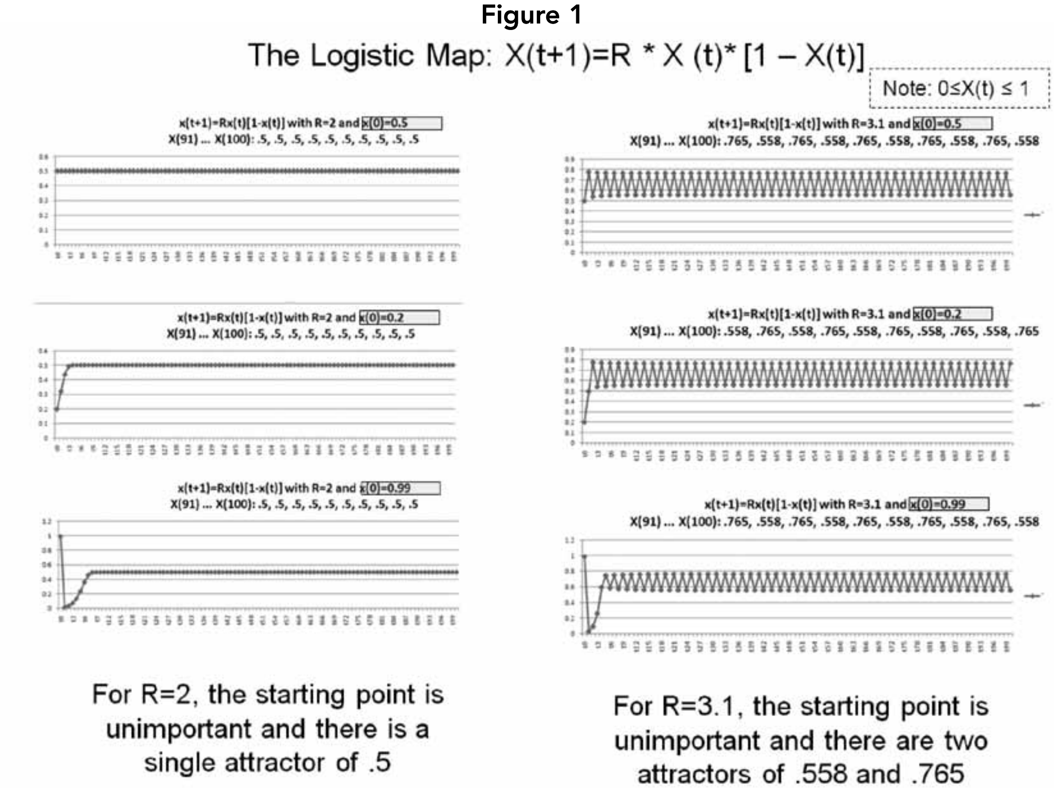
By Dave Snell

Pierre François Verhulst first published his logistic growth function in 1838 after he had read *An Essay on the Principle of Population*, by Thomas Malthus. Benjamin Gompertz, of actuarial mortality function fame, also published work developing the Malthusian growth model further.

It is a very simple equation: X(t+1)=some constant, R, times X(t) times [1-X(t)]; but it has some interesting properties. When R=2, it does not matter what starting value you choose, 0.5, 0.2 or even 0.99—the equation moves toward a single attractor of 0.5. Note that X(t) is bounded by 0 and 1. The general idea is that while population is small and resources are large, growth is fostered. When population becomes large, resources are less plentiful, and population growth is constrained.

If you increase R to 3.1, two attractors emerge, and the value oscillates between them. At slight increases in R, according to something called Feigenbaum's constant, the amount of attractors keeps doubling. Both of these cases are shown on Figure 1.

Figure 1



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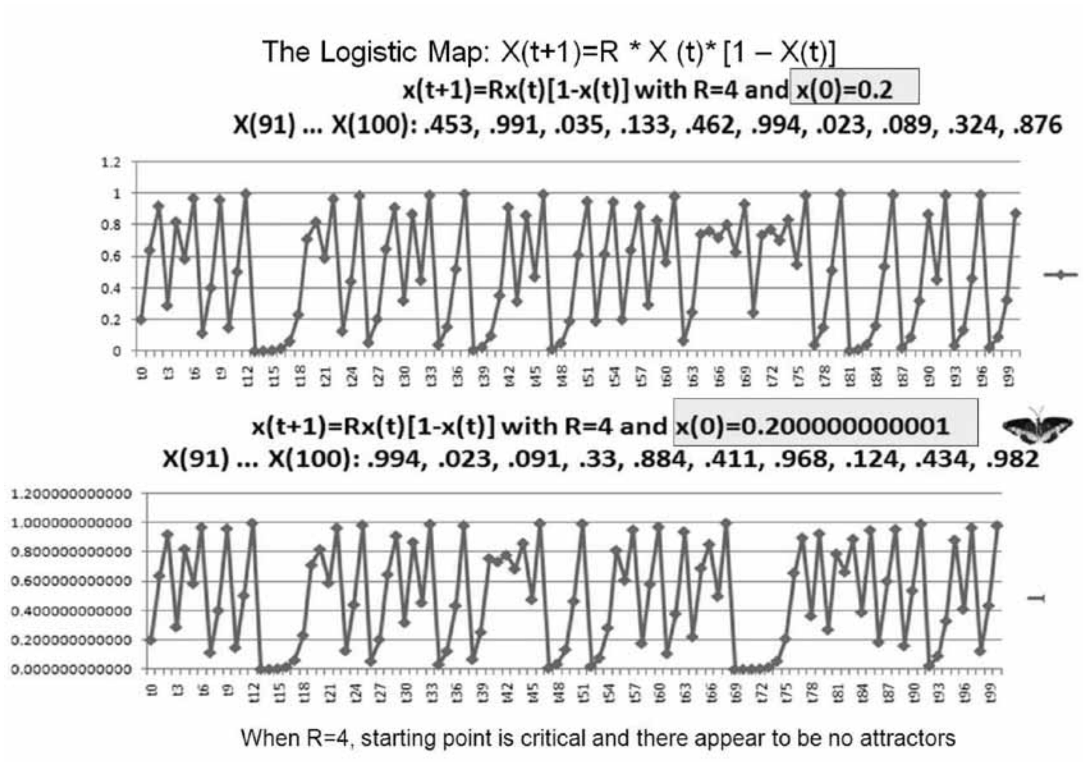
By the way, Feigenbaum's constant is around 4.6692016, and it shows up in a lot of different scientific applications. Read about it in *Chaos: Making a New Science*, by James Gleick. Do you know how Feigenbaum derived his constant? He used a calculator. This is an example of what is called experimental mathematics. Use a computer, calculator or other means to find the answer, then go back and develop a formal proof for it. The term was coined in 1972, but I think the Egyptians used it way back when they were building the pyramids.

Now, here is the cool part!

Looking at Figure 2, we see that once you reach R=4, just a tiny change in one of your assumptions may cause an undetermined effect on the validity of your model. The two graphs are somewhat similar; but there are definite differences in some areas.

Figure 2
DETERMINISTIC CHAOS



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Everyone has heard of the butterfly effect. Here is the butterfly effect in action in very basic algebra. Keep in mind the only thing that caused the two graphs to differ so noticeably at the later durations is a starting assumption difference beyond the trillion decimal place. That's 0.2 versus 0.2000 blah, blah, blah, 001.

Some of you may be thinking: So what! So what if some trick equation can do this? My equations are not so vulnerable. I deal with more clear boundaries in my pricing or valuation or modeling work. Do you?

What about pricing for a last-survivor policy? In computing the probability of survival for two individuals X and Y in a last-survivor situation, we often assume that:

$$p_{\overline{xy}} = p_x + p_y - p_x\, p_y$$

In English, the probability of the last survivor of X and Y living through the coming year is the probability of X living, plus the probability of Y living, minus the double count if both live through the year.

This equation implicitly assumes that the survival of X is completely independent of the survival of Y. Yet, don't we all know of several instances where a parent or grandparent died and the spouse died soon afterwards because of what medical professionals call "losing the will to live"? Is that in our equation? Do you really think the death of one spouse has no impact on the survivorship of the lifelong mate?

In modeling, we make assumptions about mortality, morbidity, investment returns, tax rates, etc., and we project these forward for 60, 80 or even 100 years. Then, just to be safe actuaries (and because regulations sometimes encourage it), we vary the initial assumptions slightly and execute 10,000 stochastic runs to come up with conditional tail expectation (CTE) results to assure ourselves we are in complete control. But is that true if the tail of one assumption will more likely than not precipitate the tail of others and then cause the whole set of assumption dominoes to start falling down? And even if by some stretch of the imagination our theory is correct, do 10,000 stochastic runs using starting assumptions correct to four but not 40 decimal places actually guarantee accurate results on the other end of a model far more complicated than the logistic equation?

My wife's cousin, Dan Nolan, was a male in his early 30s. An avid runner and health enthusiast, Danny had a great job as a high-priced consultant working and living in Chicago. Back in 2001, Danny's company sent him to a meeting in New York City at the World Trade Center. Danny, along with about 3,000 other supposedly independent individuals, all perished together in the tragedy of 9/11.

Are we all independent from a mortality perspective; or does our complex network of interrelationships introduce a covariance that can rock our world of accurate calculations … like it rocked Danny Nolan's world … and that of his wife … and his two children?

Do many of us drink the same brand of soda, or fly the same airline, or attend the same actuarial meetings?

Are we sure we are correct that our basic modeling assumptions of independence of key and obscure variables are accurate, or are we just drinking the same marketing Kool-Aid?

Deterministic chaos sounds like something that actuaries, sensible people that we are, would never have to deal with in our financial calculations. Yet, as we see, it can even happen in simple algebra … and life is not always as simple as algebra. ▼