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MORTALITY TABLES AND BENFORD'S LAW OF FIRST DIGITS*

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It has been shown that the distribution of the first digits of numbers selected from various published tables and directories violates our intuitive hypothesis that each should appear with a frequency of approximately .111.

In 1938, the physicist Frank Benford hypothesized that the first digit p would appear with the frequency $\log_{10}(p+1) - \log_{10}(p)$. Thus, the first digit 1 should appear with the frequency $\log_{10}2$ or .301 while the digit 9 should appear with the frequency $\log_{10}10 - \log_{10}9$ or .046.

It is interesting to see to what extent mortality tables obey Benford's Law. The traditional method of testing goodness of fit is the chi-square test. Values of chi-square for various mortality tables are shown below:

Table	*Chi-Square*
Standard Industrial	1.932
59-61 U.S. Life White Male	6.089
55-60 Basic Ultimate	8.572
GA-51 Male	8.674
59-61 U.S. Life White Female	8.810
1961 CSI	9.898
1941 Standard Industrial	10.068
Northampton	10.309
Combined Annuity	10.920
1937 Standard Annuity	11.071
American Annuitants	11.971
1960 CSG	12.164
Carlisle	12.670
1941 CSO	14.131
1958 CSO	14.337
A-49 Male	14.785
McClintock's Annuitant's	28.857
American Men	29.116
Actuaries	33.377
American Experience	62.662

Critical Values of Chi-Square for 8 Degrees of Freedom

P=.98	.30
2.032	9.524
.05	.01
15.507	20.090

Thus, all the tables down through the A-49 table fall within the first 95 per cent of the chi-square distribution. The Standard Industrial table shows surprisingly good fit.

One is tempted to draw conclusions from this table. Some of the tables seem to obey Benford's Law very well indeed. Certain types of tables appear to be grouped in the ranking. Experience tables show a better fit than tables used for life insurance reserves. Not shown above is a decided preference for the first digit 1. For example, the 1958 CSO table does not show a remarkable overall fit but has 1's appearing as the first digit with a frequency of .45.

Graduation methods, loading, the type and age of the table undoubtedly have an important effect on the degree of fit.

It may very well be that there is nothing of significance in all of this beyond the fact that some series of mortality rates obey Benford's Law. Yet the results are provocative and I would offer the following hypothesis:

> *If there is a "law" of mortality and if that law is mathematically expressible, then it belongs to the class of functions that obey Benford's Law.*

The fact remains that some tables do conform rather well to Benford's Law. □

*"The Law of Anomalous First Digits," by Frank Benford. *Proceedings of the American Philosophical Society*, March 31, 1938, Vol. 74. No. 4, pp. 551-572.

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sire to keep on being a student *after* he has met our formal requirements. The student, they insist, reaches that coveted Fellowship of the Society of Actuaries with his system full of tired blood that no dosing with tonic can alleviate.

"Fourth, it is said that we are equipping each generation of actuarial soldiers to fight the battles of tomorrow with the weapons of yesterday. This reminds us, if reminder is needed, how difficult it is to keep our text material from falling woefully behind. We teach, mostly, that which has been—somewhat, that which is—hardly at all, that which is to be.

"Fifth, somehow, they tell us, we infect our successful candidate with an unpleasing sense of his own mental eminence. Such intellectual arrogance results in his advice being shunned or taken like physic rather than nourishment —to the detriment alike of the advisor and the advised.

"Have I overstated the gravity of all this? Perhaps so; perhaps my colleague feel moved to protest.

"For how many of these ailments can universities, now or later, help to develop the human side of a budding actuary; to stimulate his thought processes; to implant a notion that actuarial exploration can and should be one of life's joys; to insist (and assist) that prescribed learning material not drift into obsolescence; and to instill some but not too much humility into our future craftsmen?"

The address concluded with references to the Society's new effort in the field of Continuing Education, and to existing discussions about the possibility of qualifying through an "alternate route." □

Black Actuarial Recruitment

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classes and all majors may attend. In the presentation one tries to look alive, to act interested and interesting, and to develop in the audience some consciousness that the field exists and might be worth looking into.

A second program segment is an out-and-out recruiting visit to the campus—preferably by the "lecturer." Our feeling was that a member of the Society might be more effective in telling the actuarial story than an all-purpose recruiter.

The third aspect of the work has developed as a Summer Institute. This is a six-week, summer-school (with scholarships) for about 15 participating college sophomores. The first one will be held in a month or so at Lincoln. The emphasis will be on the first actuarial exam. Our hope is that the Institute will develop interest in the profession, will help the interested students over the hurdle of the first exam, and will prepare participants for conventional junior-year summer programs.

By early 1971, the Committee hopes to be able to evaluate these program pieces, and expects to seek a substantially broadened support for those segments that are worth continuing. It will not be easy to measure the impact of a program with long-range objectives after only one year. Hopefully there will be further ideas and improvements. □