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Stochastic Pricing

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Summary: In the U.S. asset/liability models have been used for more than 10 years for stress testing reserves. In Canada, universal life products are subject to a similar process with valuation technique paper #11. Some companies are also using their models for pricing and product development. This session discusses the advantages, methods, and pitfalls of stochastic pricing for life and annuity products.

MR.TIMOTHY HILL: Our first speaker is Michael Bean. Michael currently works in variable annuity pricing and valuation at Manulife Financial, where he has been for the past year. Prior to joining Manulife, he was a professor at the University of Michigan, Ann Arbor, the University of Toronto, and the University of Western Ontario. Michael is also a Course 7 instructor for the Society of Actuaries and has recently published the book *Probablity: The Science of Uncertainty with Applications to Investments, Insurance, and Engineering*, published by Brooks/Cole (www.brookscole.com).

MR. MICHAEL BEAN: My presentation will provide a general overview of stochastic pricing—what it is, when it should be used, etc. More specific aspects of the subject, such as a discussion of particular models, will be addressed by my colleagues on the panel.

When considering stochastic pricing, there are four questions that naturally come to

Note: The chart(s) referred to in the text can be found at the end of the manuscript.

mind:

1. What is stochastic pricing?
2. When should it be used?
3. What practical problems are associated with it?
4. Is it worth the effort?

Hopefully by the end of this session, you'll be convinced that for many products like variable annuities, it is indeed worth the effort.

Let's start off by considering the first question: What is stochastic pricing? There are probably a couple of definitions that you can give, but this is the one that I came up with: Stochastic pricing is an approach to pricing that acknowledges upfront that things are not going to turn out the way we expect them to. Stochastic pricing techniques enable us to quantify in a formal way the risk that experience will be unfavorable.

Before we talk in any detail about what stochastic pricing is, I think it's a good idea to go back and talk about the traditional approach to insurance pricing. The traditional approach begins with looking at averages. If you think about mortality tables, essentially you're looking at deterministic averages. That applies also if you look at, say, an assumption about long-term interest rates and need some kind of average assumption. The second stage is putting in some kind of provision for adverse deviations (PfADs), and then the third stage is testing your results under what you think are adverse scenarios.

The theory behind traditional insurance pricing is essentially the law of large numbers from probability. In figure 1, I've given the statement of the law of large numbers in theoretical form: If you have n independent losses that are denoted X_1 up to X_n and you consider the random variable Y, which is the average of those losses, then the law of large numbers asserts that if these random variables are independent and identically distributed, the mean of this average (or the sample mean) has the same mean as the individual losses, but the variance goes to zero as the number of losses tends to infinity. This is the theoretical justification for using averages in insurance pricing.

Figure 1

The Theory Behind Traditional Insurance

- Underlying risk can be eliminated (in principle) by diversification
- Based on the law of large numbers:

  Suppose that $X_1,\ldots, X_n$ are independent, identically distributed losses with mean μ and variance σ^2. Let $Y = (X_1 + \ldots + X_n)/n$ be the average loss per policy. Then

$$E[Y] = \mu ,$$
$$\mathrm{Var}(Y) = \sigma^2/n \rightarrow 0 \text{ as } n \rightarrow \infty.$$

There are several cases in which averages are insufficient, with three main ones being that losses are widely dispersed, risks are dependent in some nontrivial way, and claims occur with low frequency and high severity. I'm going to give you an example of each one of them.

The first example is losses widely dispersed. This is a very, very simple example, but I think it gets the point across in a fairly straightforward way. We have two losses. In the first case, the individual loss is either $1,100 or $900, with equal probability for an average loss of $1,000. In the second case, it's either $1,500 or $500, again with equal probability for an average loss of $1,000. So in this example, you can see that the average is the same, but clearly the risk is different.

The second example is that risks are dependent. The one that obviously comes to mind here is investment risk. There is a lot of policy guarantees out there, for instance, interest rate guarantees or equity guarantees, in which the risks are not independent because of systematic risk. While you can eliminate policy-specific risk, you can't eliminate systematic risk. That's why the risk in these types of policies can be diversified, but it can't be entirely eliminated.

The third example is low-frequency/high-severity, or catastrophic, risks. I have a very simple example here, too: Consider a $1 million loss and suppose the probability of that loss occurring is .001 percent. Then the average loss is $10. Now even if you had a large number of independent losses of this type, how many policies would you have to write to collect enough premiums just to cover one loss? This example indicates that if you're pricing just on the basis of averages, you're going to have some problems.

What are some other difficulties with the traditional approach to pricing? Well, I mentioned that you're going to be putting PfADs and you're going to be testing adverse scenarios. The first things you have to ask are, "What are appropriate levels of PfADs?" And more importantly, for testing under an adverse scenario, "What is an adverse scenario?" If you think back to a year ago, most people wouldn't have thought that an adverse scenario for technology stocks is a 90 percent decline, but that's exactly what we've experienced in a lot of individual cases. So deterministic scenario testing can actually be dangerous. Why? Your choice of scenarios is colored too much by your recent experience. And the point of this session is that stochastic pricing can help to overcome a lot of these problems.

So what is the methodology behind stochastic pricing? Well the basic idea is that rather than considering averages, you consider the entire distribution. In figure 2, I have labeled the distribution of the aggregate loss as $S = X_1 + \ldots + X_n$. We're going to then set our per-policy premium using some percentile of the distribution rather than the average. You can express this using a condition of the form "probability of aggregate loss less than aggregate premiums is at least alpha," where alpha is typically something like 95 percent or 80 percent or whatever your criterion is.

Figure 2

Stochastic Pricing: The Basic Idea

- Consider the ***distribution*** of the ***aggregate*** loss $S = X_1 + \ldots + X_n$.
- Set the per-policy premium using a condition of the form

  Pr(aggregate loss < aggregate premium) > α

  where α is typically close to one.

As a simple example, think about variable annuities. Now this isn't the only way you can price variable annuities, but this is one approach. First, you generate a large number of investment scenarios randomly. That could be 1,000, 5,000, 10,000—we won't get into exactly what the number is at this moment. Next, for each scenario, you project the cash flows. You calculate the benefits, which could be death benefits, maturity benefits, or other kinds of benefits. You look at the present value of that plus the present value of your premium if your premium is calculated on a margin basis, or percentage of account value basis. Third, you set your premiums using a condition of the form "probability that present value cost is less than the present value of the premium is high." For instance, it's at least alpha where alpha is 95 percent, 80 percent, or whatever, again depending on what your pricing criterion is.

Now some of you might be thinking that this sounds an awful lot like traditional scenario analysis, and the point that I want to make is that while it's similar, it actually is quite different. With traditional scenario analysis, all of the uncertainty is upfront. You basically say, "I'm going to examine three scenarios, five scenarios, 10 scenarios, 100 scenarios" or whatever, but you pick all the scenarios today, and all of the uncertainty is determined today. So if you look one time step ahead, you're on a particular path, and there is no uncertainty about the future that remains. With a full stochastic simulation, you're going to allow uncertainty to come into the model at each time step. So you might have 10 choices today, and then one time step from now you have another 10 choices, and one time step from there you have another 10 choices, and so on. The difference here is that when you start out and as time progresses, you don't know which of those paths you're going to eventually take.

Chart 1 shows this concept. On the left side, there is an illustration of traditional scenario analysis. You can see that in this case, there are three paths. We don't know which path we're going to be on at time zero, but at time one we know we're going to be on one of those three paths, and once we're at time one, we know which one of those paths we're going to be on.

With a stochastic simulation, which is illustrated on the right-hand side, there are several branches at each time step, so the uncertainty is not all concentrated at time zero.

Suppose you're interested in testing pricing results under various appropriate assumptions. Under a traditional scenario analysis, you might look at three scenarios: an average expected scenario of eight percent growth; an optimistic scenario of 10 percent growth, and a pessimistic scenario of six percent growth. Then you have three different paths. On the other hand, if you were doing stochastic analysis, you would allow the growth rate to vary randomly at each time step, and the result would be a large number of independent paths.

Next I'm going to be talking about the effect of compounding on uncertainty, but before I do that, I'd like you to think about the following investment opportunity. Let's suppose that for each time step, you either gain 50 percent or you lose 40 percent and your returns are independent in each time step. Here's a concrete illustration: Let's suppose I have a coin in my pocket that's a fair coin and I flip it. If it comes up heads you get 50 percent, if it comes up tails you lose 40 percent. Is this a good investment? And why or why not?

FROM THE FLOOR: This is not a good investment because the compounded return is negative.

MR. BEAN: A lot of people make the mistake of thinking this isn't bad and that's because they'll look and say "wait a minute, up 50 percent, down 40 percent, average return five percent, okay" and then they'll use five percent to project returns going forward.

Yes, the compound return is negative. You were supposed to say that the investment is good so that I could tell you why this isn't the case!

FROM THE FLOOR: It's a real-world example about uncertainty.

MR. BEAN: We've just considered why this isn't a good investment, but are there circumstances under which it *could* be a good investment? When would this be a good investment? Would this ever be a good investment? Well if we had a large number of investments of this type and we could hold them for one day, then on half we would make 50 percent and on the other half we would lose 40 percent. So overall we would make five percent on average. In that circumstance, it would be a good investment. If we must "buy and hold," compounding our returns over time, then it is a bad investment because our principal value is changing. We don't know what our principal value is going to be looking forward. This example illustrates quite clearly a couple of things. First of all, you can't just use arithmetic averages blindly, and second, when you have uncertainty in investment returns, the arithmetic average overstates the actual return.

Now, let's talk about some practical problems with stochastic modeling. Hopefully I've convinced you that this is something that's important and this is something that you should be doing if you're not already doing it. But there are a lot of things that you have to consider:

- The first is that it's more complicated and it requires a greater level of mathematical expertise. Hopefully, the last example was a concrete illustration of this.
- The second thing is that selecting appropriate values for input parameters can be quite challenging in some models, such as the regime-switching model. (If you're not familiar with that model, it will be discussed a little bit later in this presentation.) In the regime switching log normal model for equities, the parameters do not have intuitive interpretations, and that can be a problem.
- The third item is more of a practical consideration; the results have to be interpreted with care. That's particularly important when the audiences that are getting end results are not familiar with stochastic techniques. Unlike

deterministic approaches, the results of stochastic approaches are distributions. You don’t get one number; you get a whole bunch of numbers. For end users such as accountants and auditors, this can be a little bit disconcerting and it requires education.

- A fourth consideration is that small changes in the inputs to a stochastic model can lead to large changes in the output. This can be true, of course, with a deterministic model, but it’s much more the case with a stochastic model.
- Fifth, in the hands of a novice, a stochastic model can be a dangerous tool. If you don’t know what you’re doing, a stochastic model can actually be worse than a deterministic model that could be a little bit deficient.
- Finally, when you use stochastic modeling, you have to have some kind of simulation process that involves random number generation. But is the simulation process really random? And what’s the quality of your random number generator? If you’re using some kind of software or some kind of methodology and you think that the results are random but in fact they’re not random, then you can be making faulty conclusions.

So, to end my part of the presentation, let me restate the fourth and final question that I posed at the beginning of my presentation: Is it worth the effort? The answer I’d like to leave with you is Yes, definitely. It’s worth the effort when you’re dealing with investment risks or catastrophic risks or other similar risks. On the other hand, for traditional risks that can be eliminated through diversification, it depends. I’m sure that you can get valuable information using the stochastic approach, but you may not get as much of a bang for your buck as you think.

MR. TIMOTHY E. HILL: Thank you, Michael. The next speaker is Steve Prince. Steve works for Dion Durell and Associates. He works with stochastic modeling seg funds guarantees, policy-holder behavior and reinsurance, and he has recently published an article entitled "The Securitization of Insurance" that can be found in the June issue of *The Actuary*.

MR. W. STEVEN PRINCE: My topic today is stochastic variables, including equity market models, interest rate models, advantages of various models, and some words on policy-holder behavior. The objective today is to provide insights, not necessarily details (which are available in abundance in various text books), and also some observation on what I call the disconnect between the models people use and the way they think of markets. They will build the world’s most elaborate models and they'll say it proves this, but then when you ask them what the market does, they’ll say something entirely different.

A few people have asked how anything as complex and sophisticated as the financial markets of the world can be modeled randomly—does this possibly work at all? The answer is yes, because markets are reactions to news. The analysts of the world study things to death and then something happens tomorrow and they’re surprised, and it’s the surprise they’re reacting to. If Microsoft is forecasting profits of $100 billion this year and it comes in at $105, the world reacts to the five parts, not the 100 parts. News is, by definition, unexpected, and these unexpected events are well-modeled by random behavior.

Whether you like the philosophic justification or not, there's an awful lot of empirical evidence that these models do a reasonably good job. None of these models is attempting to predict the market, and we're not economists telling you what the inflation rate will be next month. We're simply producing 1,000 scenarios that reasonably reflect 1,000 things that might happen.

You'll hear about log normal models. The reason log normal comes up is that we're saying that the percentage changes are what the world is most interested in. You're interested in percentage change generally because somebody tells you the stock market has gone up 50 points today or 10 points, but that doesn't tell you much about your return unless you know whether the market started at 100 or 1,000. I'll explain why that is when I cover the equity model.

The simplest equity model is that the value of your stock at time t is equal to the value of your stock at time t-1, multiplied by 1 + G (Figure 3), where G is some growth rate. That's mathematically equivalent to saying S(t) is equal to S(t-1) multiplied by e^r for some number r and you can take a logarithm for both sides and say the logarithm of the current stock value equals the logarithm of the previous stock value plus some r factor. R tends to be normally distributed for the random period in question.

Figure 3

Simplest Model

$$S(t) = S(t\text{-}1) * (1+g)$$

or

$$S(t) = S(t\text{-}1) * \exp(r)$$

or

$$\ln(S(t)) = \ln(S(t\text{-}1)) + r$$

$S(t)$ is equity index value (e.g. S&P 500)

r is normally distributed random return for the period

So in this simple model, if you were trying to model investment returns, you'd dig up this Standard & Poors (S&P) 500 index returns for a time period—that could be a year, 10 years, or 50 months. You calculate the logarithms of the index and you get the mean and standard deviation of the growth from period to period, and then you just generate 1,000 simulations of those index returns.

That's one simple approach. It assumes the periods are independent, and from that you conclude that markets don't bounce back from a slump and markets don't cool off after a gain. I stress this by comparison with the probability coin toss example. If a coin has been tossed heads five times in a row, what are the odds of another head? The textbooks would say the odds are 50 percent, but that ignores the real question: how many heads in a row do you need before you wonder whether it's truly an unbiased coin? If this is your model, even if you had four lousy quarters in a row, the next one could be good or it could be bad. You still don't know what's going to happen next quarter.

This simplest model does, in fact, fit the data reasonably well. If you don't have much data to begin with, you can't build a very elaborate model. If you get behind things like the Black-Scholes in pricing models, the basic assumptions are independence and lognormal distribution, and they get a bit complicated. And then they adjust the parameters daily to reflect emerging events, but the basic assumptions are the same.

Now we can go to a more refined model. There's a mean reversion model, and it says that markets go in cycles to some degree. If things are down, the world will tend to move toward the mean. This model is also called auto regression, because it tends to regress to itself.

The formula for this is not as complex as it might seem (Figure 4). It says that the return in period r is equal to some basic mean value plus some other factor based on how far the last period's return was from the mean, plus some volatility for the period. ε is the normal number; K is usually in a range of −1 to +1. Your return this period depends on the return last period. If r_{t-1} was above the mean, then K is greater than zero, and this factor in the middle K X(R_{t-1} mean) is positive so if r was above the mean last period, then r is likely to be above the mean this period. If returns are below the mean, they're likely to stay there. That same factor, if it's less than the mean and K is positive, tends to pull the mean down. The larger K is, the longer it tends to stay down.

Figure 4

Mean Reversion

Formula is:

$$r_t = \text{mean} + k * (r_{t-1} - \text{mean}) + \text{volatility} * \varepsilon$$

ε is normal(0,1)

k is usually in the range (-1,1)

mean, *volatility* are parameters

So now you have some kind of cycle. If things are down, they're probably going to be down next time. If things are up, they're probably going to be up next time. If the return was below the mean last period and K is negative, then that central factor tends to add to the return this period, and you'll more likely get some kind of bounce-back. Conversely, if the return is above the mean, you'll probably have some cooling off. Similar formulas are easily developed and look back for two or three or four or five periods. You can fit parameters to the curve using curve fitting and parameter fitting least-square types of formulas. Some note that the mean and volatility in that formula are not simply the mean and standard deviation of your sample size. You have to choose the mean and volatility so that when you apply the K factor, the end result has the desired mean and volatility.

Mean reversion also fits the data reasonably well. There is evidence of cycles in financial markets. There's a lot of debate about whether they're real cycles or just noise that happens to follow a cyclical pattern at times, and I don't have the answer to that. In practical terms, if there isn't really a cycle and you get lots of data, your parameter-fitting algorithms will give you a K factor of close to zero. You can plug this into the formula and if it comes out close to zero, you might simply ignore the fact.

We have touched on regime-switching models, which is another approach to going beyond the one-period models. There is evidence that the market goes through periods of low volatility and high volatility, and if you simply combine all that and take one mean and one standard deviation, you're missing something. So the regime-switching models have two or more regimes—typically two. You have a low-volatility period with a mean return of μ and a standard deviation of σ_1 and a high-volatility period with a mean deviation of M_2 and a standard deviation of σ_2 and two probabilities: ρ_{12} and ρ_{21}. The model will switch from one regime to another. This is not a correlation coefficient; it's a probability of switching from one regime to another or visa versa. So if you set that against the data for the last 30 years or so, you find that the world has been in a low-volatility regime about 80 percent of time. Typically σ_1 is a low-volatility period less than the market as a whole, and σ_2 is a high-volatility period greater than the market as a whole. That's no surprise, but pretty consistently, the mean return when things are volatile is less than the mean return when things are not volatile.

So then when the markets are volatile, they generally aren't gaining, and if that affects anybody's investment strategy, it's worth taking note. There's nothing in what I just said that says regimes have to be log normal, but evidence shows that this is a fairly good choice, hence the name regime switching, log normal. Academic papers that have tried more than two regimes say that log normal works as well as anything else and sometimes better. Another attraction of regime-switching log normal is that it lets you get the tail volatility more broadly without also affecting the volatility near the mean of the distribution. That's significant because the world is moving toward stochastic capital models and stochastic reserve models, and if you're only using these models to set your capital levels, then you can just increase the volatility to give yourself a safety margin. If you're using the same model to set your reserve and you've increased the volatility, now you have also increased your reserve, which was not the objective. There are ways to compensate for that. Basically, when in doubt, pick the model that fits the data and run with it.

There are other equity models out there. Some of the models have been criticized because the parameters do not change over time. It is possible to make models in which the parameters shift over time, but while such models fit specific data sets better, they don't seem to fit all data better.

At a session two years ago that was put on by the CIA, we heard several models of ever-increasing refinement, and several of the proponents said, "You know, if you pick this particular set of data and study this to death and choose your parameters just so, that does actually fit this particular model better than the other model," but they will also generally admit that if you simply go to another period, the model doesn't work any better than something else. If you have enough terms and study the same data long enough, you can get a better fit, but you haven't really added any insight.

Models have been criticized for ignoring that things are different now. There's a new economy and the world has globalized. The reply to this is that the world has been changing all along. We did not wake up one morning and realize we've gone global. The world has been becoming more globalized for the last 30 years, and these models fit that data. . The world tends to view things through fairly short time horizons, and anyone who tells you things are different is usually surprised the next day.

Interest rate models work reasonably well with equity returns. Either the simple model or the mean reversion model works reasonably well with interest rates, but you apply the math to the logarithm of the interest rate rather than the accumulated fund value and then you get a pattern of future interest rates. Algorithms tend to give you a single interest rate, such as the 90-day T-bill rate or the one-year T-bill rate or the 20-year bond rate. They do not on their own give you a yield curve. You could create a number of independent models to give you the yield or the interest rate at different terms, but there's extremely little economic justification because there's generally some relationship between short- and long-term interest rates.

One approach is to have a mean reversion model for the long-term interest rate and a second mean reversion series for the short-term rate as a percentage of the long-term rate. There's significant empirical evidence that this works, and I've tested it myself, although I've never seen it in one of the finance textbooks.

Another approach is to generate a series of short-term interest rates and then calculate the implicit long-term rate by multiplying out those future short-term rates. Doing this calculation rests on the assumption that interest rates are arbitrage-free, which is a point of some debate. If you want to make arbitrage-free your assumption, you determine your three-year rate today by running your model for three years with the short-term rate (in figure 5, the rates are five, six, and seven percent). Then, if the world is truly arbitrage-free, you could invest for three years at the same effect as three one-year rates, in theory, so multiplying the note and taking a cube root, the three-year rate today must be 5.9 percent.

Figure 5

Yield Curve Example

- Your mean reversion algorithm says that the first three 1 year rates are:

  $i_1 = 5\%$, $i_2 = 6\%$, $i_3 = 7\%$
- Then, implied three year rate at time 1 is

  $(1.05 * 1.06 * 1.07)^{(1/3)} - 1 = 5.9\%$

That only gives you the interest rate itself; it doesn't give you the returns on your investment portfolio. To get a portfolio return, you have to model the interest rates, and then you have to generate a portfolio of bond holdings. These might be one-year bonds or 20-year bonds or any combination thereof. Then you have to model the shift in bond values as interest rates change. If you're simply modeling one-year investments, then the simple i interest rate is all you need. If you're modeling a 20-year investment, you have to make a 20-year investment choice and then a year from now work out the value of a 19-year to maturity bond at whatever interest scenario you've got. That gives you a return for the period, and if you got a mix of different term bonds, you've got a fairly complex modeling exercise. But that's the only way to have the portfolio return accurately reflect the interest rates and your choice of investment.

Of some note, significant testing has shown that the models I've shown you do not work well if you simply take your total bond portfolio just like you would take your total stock return and model the returns in your total bond portfolio. That approach tends to produce periods of implied negative interest rates, and we just haven't seen enough of those to believe it.

There aren't many ways to coherently model interest rates and equity returns in the same model, and this is particularly important in the seg fund guarantee, where to some extent you're hoping that your equity returns are counterbalanced by bond returns going in the other direction. It's also important if you're trying to forecast hedge prices. Hedge prices reflect the underlying or assumed underlying behavior of the stock. Another important point in hedge point calculation is what you think interest rates are doing on the day you're trying to calculate the future price for your hedge. You have to have some kind correlation between your interest rates and your equity return. One of my favorites is the Wilkie model, which has relationships among inflation, interests rates, and equity returns. In practical terms, the model does have a reasonably good fit, and Wilkie and his colleagues have produced a few papers showing it will work surprisingly well in a number of countries. However, there are 30 parameters per country to fit, and if you're doing more than one country, that's 30 times however many countries you're doing. The parameters are very difficult to fit over time.

The Wilkie model doesn't model individual funds versus the overall market. It gives you risk-free interest rates and total equity market returns and a few other things, but it doesn't tell you how the XYZ fund is going to do relative to the market as a whole. Even if you do all that work, it still has some significant shortcomings. The equity models I gave you a minute ago work equally well if you take the return for the XYZ fund or the market as a whole or a foreign country or any of those things.

Arbitrage-free models relate to the setting of parameters rather than the mathematical structure of the model itself. Arbitrage-free asserts that there are no opportunities to gain by swapping what are essentially the same risks in arbitrage because the market will force such prices to equilibrium. That's the basic assumption they make. If that's the same as this, then their returns and prices ought to be the same or whatever the corresponding mirror image is.

It's been elevated to a religion in some circles, but if you read the finance text books, it's really introduced simply as the way to get to the other side of some equations. Well, I've studied this side of it and I've got a formula, but I can't solve for the price or I can't solve for the volatility because I need something on the other side of the equation. Well, there is some justification that if the market is reasonably efficient, you can look at some other aspects of the market by a similar equation and get the other side, and if this equals that, you can now solve for alpha or beta or whatever it is. That's a reasonable assumption in many quarters, but that's far short of a religious justification for saying it's necessarily true in all quarters.

Black-Scholes and many option pricing models use arbitrage-free to get prices. There's evidence that it works fairly well in the parts of the world or the parts of the economy where it's used, but that does not necessarily make it true everywhere else. Arbitrage-free models are generally used in short-terms options, typically less than six months. The assumed long-term growth rate in the arbitrage-free model tends not to matter. They are literally worried about minute-to-minute, hour-to-hour, day-to-day volatility in whatever it is they're talking about, so typically they adjust the parameters daily and do extensive back testing.

Over a period of many years, though, my understanding of this is that it's essentially assuming the markets are giving you a risk-free rate of return, and there are probably finance professors in the room that will disagree with me. Whether you've assumed that or not, that seems to be the result. In much actuarial work, such as 20- or 30-year segregated fund guarantees, the long-term return is far more important than the day-to-day volatility because you're guaranteed an amount based on the amount of money the person gave you, and if you've had a couple of years of good returns, that gives you a very nice cushion to absorb a couple of days of bad volatility. So, in most actuarial work, it's far more important to get the long-term average right than it is to get the day-to-day volatility right. Whatever your assumption, the stock market does out perform the risk-free rate over the long term, and if it didn't, you could reasonably wonder why anyone puts any money in the stock market. Whatever your mean returns are, they have to be higher than the risk-free rates.

My final topic is policyholder behavior. There's very little theoretical basis and even less factual data for modeling policyholder behavior. Two years ago, the accepted wisdom was that policyholders were rational—that they had the segregated fund guarantee that said I'll get my money back no matter what this fund does. You would think that when the market went down, people would hang on to their funds and keep their policies when they were most valuable. All the evidence over the last couple of years is completely the reverse. When funds do badly, people think they're smarter than the market and should dump their funds while they still can and go somewhere else. So just when the guarantees would cost an insurer, the most policyholders tend to leave and let the company off the hook. This is good news for companies, but whether you want to count on that continuing to happen is another question.

In terms of modeling, you build a model with two probabilities. If funds are doing poorly, there's a probability of x that people will leave; if funds are doing well, there's a probably of y that they'll leave. Your model must dynamically assess how the fund is doing. Has this been a good return this period, or the last two periods? You assess in the model that things are doing well, and then you pick x or y and randomly lapse the policy. Some of the models out there are taking single deterministic scenarios and say you're losing people at a certain aggregate rate. Some of the models I've seen actually model a number of people, such as 10,000 individuals, and have those individuals randomly lapse or die. That gives you perhaps a better model, but I'm not sure it gives you a different answer.

There's also significant evidence that policyholders are not homogeneous. Do people on average do this? Do people on average do that? There tends to be two types of policy-holders: the ones that move every time the market does anything and reset on a whim and 10 times a week want to know how their funds are doing, and the ones who don't seem to react to anything for any reason at any time in any case. It's probably a mistake to model this as one group with average behavior. Probably that second group will never cost you a cent no matter what happens, while the first group is very active and could cost you a heck of a lot more. In that case, it's important to model what the first group is costing you and whether that group is 10 percent of your customers or 90 percent of your customers.

Similar logic can be applied to assess whether or not policyholders move between funds. If funds are up, people either say, "I'm going to stay with it since it's doing well" or "I'm going to cash it now while I'm ahead." There's not much evidence as to which behavior is dominant, but it affects your behavior as a company. Of course, all of this doesn't tell you much without a fairly sophisticated pricing model to see what the related benefit costs will be.

To recap, there are models out there from the very simple to the very complex. The empirical data so far says that the moderate complexity seems to work as well as anything else, and a number of academic papers have deliberately made the models more complicated to see if that helps, although it doesn't beyond a certain limit. How can you model anything as complicated as the world economy? Well, we're not trying to make economic predictions; we're simply trying to generate a very large number of scenarios to let us get some sense of what our cost of loss or our risk of loss is.

MR. HILL: Our last speaker will be Eric Von Shiling. Eric has worked for MMC Enterprise Risk Consulting for about eight years. His primary areas of focus include guarantees on variable annuities and segregated funds, economic capital calculations, and enterprise risk management.

MR. ERIC VON SHILING: I am going to discuss stochastic pricing of variable annuity guarantees: guaranteed minimum death benefits (GMDB) and guaranteed minimum income benefits (GMIB).

Stochastic modeling generates a large amount of information that we have to digest and draw conclusions from. It's a complex exercise. We've heard a lot of information about whether this is a worthwhile event, and I think the key thing for me is that it actually goes and tries to find a likelihood of future events, and with that we can build a future picture of what our outcomes could be; we could assess with variability and come up with a price that we feel comfortable taking on. In this situation, we're taking on risk. We need to be willing to take the up sides and the down sides for that price.

Our variable annuities are highly skewed and so our most probable outcome is of little or no cost to us. If you have a very poor scenario, you can run a deterministic one and say, "if this happens, this will probably be my cost." Or how likely is that scenario? How can we relate that to our other scenarios, such as the most probable one of no cost at all?

With this stochastic pricing framework, we can integrate that and draw a picture of our future outcomes, and we can really assess the price that we feel comfortable in taking. Today I'm just to going to introduce the stochastic modeling with variable annuities. We're going to look at base benefit costs, just looking at basically the cash flow cost of the component alone, and we're going to extend this and try to incorporate a capital element and the associated cost of capital. And then once we have this framework with cost of capital, future balance sheets, and income statements, we can then apply other pricing objectives that are more common to our normal measurement.

The core part of variable annuity GMIB and GMDB benefits is the embedded option that the insurance company is guaranteeing a minimum value of the policyholder's fund. So the embedded option is asymmetrical in a sense. As markets go up relative to the minimum value, there's no payment to be made; as markets go down, then the insurance companies have to pay the benefits. An additional characteristic of the risk profile is that at the same time our costs are going up because markets are going down, our revenue stream is also going down because our revenues are generally expressed as a basis point on the fund balance. The other thing to note is the risk of the case element to these benefits. To a certain extent, you can achieve diversification across various ages, times, different maturities, etc., but underlying is still a strong systematic element. It was discussed earlier as well. What you can't really get rid of are the underlying connections to this market movement. So we're combining an asymmetric cost profile with strong nondiversifiable risk.

Table 1 gives some numbers, and next we'll try to implement these models we're talking about. This one is a sample pricing cell of a single deposit of $1,000 based on the S&P 500 or a diversified U.S. equity fund, with an issue age of 55. I gave this thing a GMDB that's sort of an annual ratchet; it resets annually on the anniversaries at minimum value. The GMIB has a minimum value rolled up at 5 percent per annum, and you're allowed to annuitize between 65 and 75. Overall, the guarantee terminates at age 85.

Table 1

Example Product

- Sample Pricing Product
  - Single Deposit of $1,000
  - S&P500 Based Fund
  - Issue Age 55
  - GMDB - Net Deposit with Annual Reset on anniversary
  - GMIB - Net Deposit rolled at 5% pa, annuitization 65-75
  - Guarantee terminates at age 85
- Assumptions
  - S&P index
  - mortality Annuity 2000
  - lapse 10%pa
  - asset earned rate = 6%
  - hurdle rate = 15%

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The important thing here is that we have to create a stochastic model from which we can do our pricing exercise. As was mentioned earlier, if you don't know what you're doing or if you're building a crazy model, you're going to get crazy results at the end. You can think of it perhaps as if you're trying to use a mirror to see what your possible future outcomes could be. If you build yourself a nice straight mirror, you're going to see an accurate reflection of yourself in that mirror. If you're looking at the back of a spoon, then you're going to have a very big nose and a small ear. You might go and get a nose job to shrink your nose, when really it's the ear that's the problem. So I can't stress enough that this is a critical component, and you have to be comfortable with your underlying stochastic model before you take the next steps to do the pricing exercise and to draw conclusions. This way you can be confident about some of the conclusions that you're drawing. Obviously not all models are perfect, and every model contains model error, so you have to recognize and remember that when you're drawing a conclusion, but hopefully you can create a reasonable representation of what the future is going to be.

Let's build our model. In Table 2, I've separated the embedded option from the underlying variable annuity contract. The approach here is that you might as well use stochastic modeling when you can benefit from it most; adding all kinds of random elements has a risk of confusing things and makes it difficult to draw conclusions. So at least we'll be beginning with as simple a base concept as possible.

Table 2

Stochastic Pricing Model

- ❑ Price Embedded Option separate from base VA product
  - – model elements which benefit from stochastic simulation
  - – fixed total expense charges (investment, M&E, etc)
- ❑ Stochastic model of investment returns
  - – Desired stochastic process of underlying risk factor
  - – Regime switching lognormal - for S&P500 index
- ❑ Cash flow / Product model
  - – projects associated cash flow for each stochastic scenario
    - • survivorship
    - • policyholder behavior (dynamic lapses, GMIB election rates, etc.)

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Our next step is to choose a stochastic model for an underlying risk factor. In this case, our future investment return is our critical component, and I'm just modeling a U.S. equity fund here. In this case, I selected a regime switching, log normal model calibrated from the S&P 500 index.

All right, so now we've generated 1,000 scenarios, or you could choose another number if you like more precision. We can go on, and we need a product or a cash flow model that will generate the cash flows along each of these scenarios. So at this stage we say, for the other uncertain elements that there may be, like mortality and lapse rates and things, we still maintain a deterministic approach using our normal actuarial decrement model.

The other element you can reflect at this point is policy-holder behavior, but perhaps we may build a case for some dynamic lapse rates depending on the movement of the market. The GMIB election rates depend on how in-the-money this guarantee is

Before I move on, I'm gong to get to this issue of realistic versus risk neutral as the basis for your scenario (Table 3). Which one should we use for this exercise? We're pricing a financial contract here, so it's difficult to know if we should be using a capital markets approach or a realistic framework. Actually, the way I see it, it could just be a simplistic view, but when you're going to be bearing a risk—in this case it's our variable annuity against uncertainties of the future—you need to understand what the future outcomes could be so you can evaluate the uncertainty and how comfortable you are with this risk. You can only do that in a realistic framework.

Table 3

Realistic vs Risk Neutral Basis

- ❑ Realistic Scenarios (P-measure)
  - – un-hedged liability
  - – policyholder behavior
  - – future balance sheet provisions
- ❑ Risk Neutral (Q-measure)
  - – assumes can build a replicating portfolio
  - – useful benchmark
  - – useful if included a hedge / risk management program

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The opposite is risk neutral, also known as Q-measures or arbitrage-free. It is often used obviously for pricing and options in the capital market, but underlying this is really the assumption that you can build a replicating arbitrage-free portfolio and that the price of this replicating portfolio must be the same as your option. However, you have to be able to fully hedge this risk and deal with all the other constraints—continuous rebalancing, no transaction costs—and everyone should know what's on this list.

I think, especially for variable annuities guarantees, most companies in general are bearing most of this risk through diversified basis. Perhaps you have some hedging in place, but there's a considerable amount of basis risk. Maybe you can use an index fund to help you hedge, but you have a fund that may or may not track that. So I think we need to view this from a realistic perspective so that we can really appreciate the risk return tradeoffs and see this picture and get comfortable with it.

All right, let's put out a few numbers. The first thing we're going to look at is just the base benefit cash flow cost (Table 4). Let's say we projected a cash flow and discounted these benefit costs and we have present value (PV) benefit costs and we divide by the present value of the fund at each of those points in time. We end up with an equivalent level spread or base point measure as a price for that particular scenario. We can see from the summary statistics what we have. The average spread is, say, 14 basic points, with a maximum of 265. So the interesting measures are percentile of distribution and conditional tail expectation. I think we've heard about this a lot these last few days, but it's essentially the average of the tail of your distribution. If your threshold is 80, we're averaging the top 20 percentile of the scenario. In Chart 2, we graph the equivalent level spread. Beyond the 90th percentile, it really starts to take off, and that's this tail risk that we've been talking about. This is where you may want to say, "I can take the 96th percentile or the 98th percentile or the 99th percentile as my worst result." At which point do you draw this line? It's a difficult question.

Table 4

Base Benefit Costs

- ❑ Equivalent Level Spread
  - – PV Benefit Costs / PV Fund Value
  - – for each scenario
- ❑ Summary Statistics
  - – average
  - – percentile of distribution
  - – Conditional Tail Expectation (CTE)
- ❑ Risk Neutral Cost
  - – theoretical price
  - – if could perfectly hedge the risk
  - – no transaction cost, continuous rebalance, no basis risk, etc. etc.

Base Benefit Cost (bps)	
Average	14.3
Min	0.8
Max	265.0
Percentile (x)	
80	15
90	23
95	54
97.5	73
99	141
99.5	183
CTE (x)	
80	43
90	67
95	101
99	196
Risk Neutral	46
RN Percentile	94.3%

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Now there's another line here labeled risk neutral. I generated a number of risk neutral scenarios as well and just determined the price that would be based on that basis, and it came out as 46 basic points, which is, I think, the 94th percentile. So that's quite a theoretical price and quite an interesting benchmark. So I think if I can perfectly hedge this risk with no transaction cost, continuous rebalancing, no basis risk, and shorting of all securities, then this could be the return that I could achieve, or this would be the price that I would have to charge.

And what question do I answer with this type of measure that I have so far? If I charge X basis points of margin, I can accumulate this with interest, less the cost that I incur over the scenario. I can basically access a Y percent likelihood of covering my base benefit cost over the lifetime of the product. This is a useful measure; it gives you a sense of the risk you might be taking. However, it doesn't answer other questions that we're generally concerned with in terms of normal operations measurement. How much capital is this product going to require? What is the return on capital at this pricing level? How volatile are my earnings, or what kind of earning patterns could I get from this price?

It's important for us to extend our pricing structures to give us more information than we normally deal with in our course of managing the business, and a key part of that is projecting the future cost of balance sheet provisions. In general, balancing provisions takes two forms: your liability or reserve and the capital. The combination of the two is your total balance sheet requirement. Now you can do this by an economic or statutory basis, depending on the rules that are out there. I think if you're going to be taking on this risk, there's a certain amount of capital that's required to support this risk. So I think it's important to pursue what the economic capital would be and that you would need a return on that economic capital. Unfortunately, this builds in a bit of complexity, because basically you have an option here, and the amount of money that you're going to require will be value based, and I use the term loosely. It's not necessarily a market value, so to speak, but it could be a more conservative value measure that you might use for your reserve and your total balance that appears in the capital.

Now the implication of this is that it's going to be state dependent; it's going to be dependent upon the relationships of the contract at the point in time that you do this valuation. You know from option theory you have the strike price and then the security price. The more in-the-money in the guarantee, the higher the value of the option. So again, with our embedded option here, as we move out on along a particular path, we're going to experience that the value or the capital requirements that we're going to need are going to fluctuate depending on this relationship of our market value to guarantee value.

As a first example, I'm just going to use the Canadian GAAP requirements. You can use other ones—perhaps U.S. GAAP market value accounting—or you could come up with another economic framework you might feel more comfortable using. Now, a few other sessions have gone over these bases, but the key is that it is sort of an economic base calculation. It does do evaluations, so I'm just going to do a quick recap. As part of a stochastic valuation, you do, say, 1,000 or 2,000 scenarios and you calculate for each scenario your present value net cost, with the present value benefits, less the present value risk margins. This is your price that you can incorporate in your total balance sheet. You set your total balance sheet requirements at a high threshold and the stochastic modeling basis in this environment will be, let's say, CTE (95), and you figure your liability at perhaps a lower threshold—maybe CTE (80) or a little less, and the capital represents the difference between the total balance sheet requirement and the liability, and you may include a operating multiple.

So we get back to our problem here, which is projecting balance sheet provisions. As I mentioned before, it's state dependent and so theoretically requires stochastic analysis within a stochastic analysis. You can imagine the practical problem this causes. I have my original 1,000 scenarios and I'd like to project capital along each of those 1,000 scenarios. So I move out one time point after the other valuation and another time point and have to do another valuation. By the thousandth scenario, let's say I'm doing them quarterly for 30 years, 120 time points, I'm talking about theoretically 120,000 valuations to do this correctly.

How can we get around this? We could use perhaps a factor-based approach so it's already the Canadian basis for setting the total balance sheet requirements and it's based on a factor, which is a percentage of your fund value, and it relates to different products that you have. The problem with this is that it's difficult to reflect the changing evolutions of your product—high immaturity and all the nuances that you may have. We could go on perhaps and develop an auction pricing format closed form. There have been a few put out there that you could essentially calculate a closed form solution, a CTE (95) based on the state of your option value. Indeed try out some deterministic scenarios and just get a measure for where the value is. I'm set on one called the neighboring path approximation, which we used effectively, or you could just go with the full stochastic within stochastic projection.

What is the essence of the neighboring path approximation? We'll use the information that we already have. We have our initial set of stochastic projections, and let's say among that first scenario and they're moving it to time one, and I would like to measure this valuation; I could look for other scenarios that happen to be in a similar state to where I'm at. So let's say my MV/GV ratio has gone off to 1.1; if I scan across all my other scenarios at the same time point, there could be a number of similar ones. Let's say I can collect them and pull them all together, and I can do a mini valuation based on this subset of scenarios that is in a similar state. Let's say I can extend my original projection and generate a large number of these 10,000, I might be able to find in a given point in time 300 that are very similar and try to estimate this valuation. A smaller number is good perhaps for producing a standard error in your measurements and a little more difficult to get to the edge of your projection. There aren't many that are similar, but what we've used is actually quite effective, and it allows us to overcome this probability of the problem; it allows us to project future capital and reserves.

So what can we do now? With the future capital reserves, we have future financial statements, so for a given risk charge, we can project those cash flows doing income statements, and we have our after-tax net income, and we can also select a charge on this capital. So now we can generate perhaps an embedded value for each scenario.

When we come to our pricing objective, we now have much more information that is in our more common operating measures. We have information about income and how much capital we need, and we can measure the return on capital. So this is now in the framework of how we're normally doing our business, we can set ourselves a pricing objective and try to solve for a spread that satisfies that. In my example (Table 5), I'm just going to try to get the return on capital to meet the hurdle rate on averaging all across my scenario.

Table 5

Set Pricing Objective

- New Possibilities in setting Pricing Objective
  - Average ROC = 15%
  - Likelihood / Severity of negative scenarios (how bad could it get?)
  - Incorporate Earnings Volatility
- Example - Pricing Objective
  - Average ROC = 15%
- Solved Margin = 58 bps
  - 96th percentile in base cost distribution

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Once we're doing the embedded value, we actually assign capital that varies depending on the riskiness of this, and we've included a charge for this. First, we take an average of the return. We use a solved margin of 58 base points and a 96th percentile in base cost distribution.

Chart 3 shows embedded values as a percentage of deposit, and it's a cumulative distribution. So we see that for balanced distribution, you just need the base benefit costs; however, it does have a bit of a longer tail on the far left, and we see that in this case about 45 percent of the time we're negative and the remainder would be positive.

At this stage, we have to take a step back and decide how comfortable we are with this downside. We may have to go back and add more constraints to the formula or perhaps introduce a change in the product structure, or possibly even exit the market.

Charts 4 and 5 show the total balance sheet requirement as a percentage of fund value at each point in time. The blue line is the average at each point in time across all the scenarios. We see that it starts at around 1.5 percent or so and trends up to seven or eight percent and then it declines over time. However, if you look at a single particular scenario and you find it's plucked out number 444, you see that it bounces around quite a bit, and this is a reflection that we have a sort of value-based approach to setting this capital environment. As the scenario is bouncing all around, so is our capital requirement. This shows that we aren't hedging liability and we're exposing ourselves to volatility. This is just a single policy. We are not including diversification across the block, and we don't have any potential smoothing elements to this. So this is what we would experience if we were just to do a pure value approach for setting our total balance sheet requirement.

And then we can look at our earnings volatility; the blue line is consistent and steady. However, along a particular scenario, we have a significant amount of volatility. This again reflects the fact that we have a single policy and that our reserve element is also value-based. It's not focused on an emergence of income at all. As a consequence, we'll see this bouncing around considerably, but the benefit from this is that we now have taken our pricing exercise; we've assigned likelihood to our future events, which allows us to paint this picture of what our future outcomes could be. We're going to have to bear this risk unless we do some hedging, and even with hedging, we'll still have a considerable amount of basis risk. So we're setting a price, and we need to be comfortable with it.

With an embedded option, we can project base benefit costs, but it's important to extend them, I think, to the other operating measures that we're normally used to dealing with, and we can make better decisions about the risk we're taking on.

MR. HILL: I just had one quick note on what he was talking about and it was the complexities. If you start doing some type of stochastic capital within a model of stochastic, that gets really difficult, but I think one problem in the U.S. has been that we're allowed a guaranteed living benefit—the GMIBs, the GMDBs. The typical pricing is present value/future benefits divided by present value/future account value to get a basis points cost, and, you know, you pick an 80th percentile or a 90th percentile, and that's the charge, and you're ignoring any type of a balance sheet piece. As he was pointing out, it's very important that you look at the cost of your capital and any reserve that you have to set up. I think often times that is being ignored.

PANELIST: The point that I was trying to make is that in some circumstances, arithmetic averages are the correct thing to look at, and in other circumstances, geometric averages are the correct thing to look at. When you have compounding and uncertainty, then the geometric average is the thing you want to look at. But I also did mention that whether or not the investment is a good one really depends on how you play it. If you buy and hold, then it's not a good investment. On the other hand, if you've got a large number of investments that are independent, then your expected return is five percent across them. So it's not necessarily a black and white issue. It depends on the circumstances. You need to take care and not just do the obvious thing, because the obvious thing might get you into trouble.

MS. MARY HARDY: I have some comments mainly for Steve Prince. I enjoyed your presentation, and thank you for mentioning the regime switch model, which is close to my heart, but I did have a few disagreements when you got on to discussing arbitrage-free models. For stock return models, any model that has a positive probability of either positive or negative outcome, is arbitrage–free, because you can't make a risk-free profit on it. When you use the term arbitrage-free model, I think what you really mean is a risk-neutral model, which is actually something different. What I would call a risk-neutral model is what you call arbitrage–free. The arithmetic turns out when you're valuing under Q-measure or risk neutral that the drift term of the P- measure doesn't matter. In other words, you made a point that we're discounting out the risk-free rate of interest, but that absolutely does not mean that that's what we're assuming happens in the market, which was the implication of one of your slides. It falls out of the mathematics, but it is not part of the assumption that stocks will not outperform bonds. In fact, the whole theory says that stock has to outperform bonds, in the sense that you have to have extra returns to compensate for the extra volatility.

I also had a point for Michael Bean, and I think it relates to the point that someone else was making. I am absolutely sure that if I have independent returns, and my bank distributes its returns each year and the expected return each year is five percent, and I've accumulated over n years, the expected value of my accumulation is 1.05^n. I can personally prove my result, and I can actually disprove your result.

MR. BEAN: In the example that I gave, the arithmetic expected return per period is 5 percent, which means that the expected per-period accumulation factor is 1.05. Since returns in distinct periods are assumed to be independent, it follows that the arithmetic expected value of a $1 investment after n periods is $(1.05)^n$, which tends to infinity as n becomes arbitrarily large. On the basis of this observation, many people erroneously conclude that following a buy-and-hold strategy with this investment is a good one. However, consider the probability of actually having a gain after n periods, i.e., $Pr(P_n>1)$, where P_n is the value of a $1 investment after n periods. For large values of n, the distribution of P_n is approximately log normal, and one can show that $Pr(P_n>1)$ is approximately zero, which means that there is virtually no chance of coming out ahead with this investment over the long term. This demonstrates that the arithmetic expected value is an inappropriate statistic to use when analyzing an investment in which uncertainty compounds over time. The appropriate statistic to consider is the geometric expected value, which in this case is $(0.9)^{n/2}$ after n periods.

PANELIST: Steve Mitchell made some comments about a professor in the audience who was going to have an issue. I don't disagree with Mary. I would caution that some other finance professors have different points of view with which I was disagreeing, and thank you for making that clarification, Mary.

MR. RANDY WRIGHT: When you calculate variable annuity GMDB costs and you send those costs over 30 years and then pull them back at a present value to consider basis point equivalent or something, what interest rate would be the right theoretical rate used to pull back that present value? Would it be the interest rate you used for setting reserves, or the hurdle interest rate that your company is going to hold you to, or the interest rate that you originally priced as your goal, or perhaps the interest rate that you've assumed as your mean market return?

PANELIST: I guess the answer is none of the above. The point is you have some cash flows; what are the reserves today against those cash flows? One approach is to say it's whatever I do my reserves on, because that's what I'm discounting. The second approach is that somewhere in your modeling, you have an implicit return on whatever it was you were going to be investing in when you set up those reserves, and if your model can extract that effective interest rate, I think that the theoretically more precise basis is the discount factor. If we hold X dollars today and it earns interest in the meantime, then what are the odds that that money will be sufficient to cover the cash flow claims we've also modeled? But try to explain that to a regulator or an auditor.

FROM THE FLOOR: My question is for Mr. Bean. Do I understand correctly that if I play your game assuming the corporate account model—that is, if I earn the $0.50, then I transfer the extra $0.50 to my corporate account and if I lose the 40 I get the 40 from the corporate account, then on average I'd be winning? Is that right?

MR. BEAN: Yes.

FROM THE FLOOR: But if you play one consecutively, then you'd lose. The math works that way. However, if you played the one consecutively with the corporate account model where if you win the extra $0.50 you transfer to the corporate account and if you lose the $0.40 you transfer it back from the corporate account to a processing center and you play consecutively, then you'd win.

MR. BEAN: Yes, that's right because in that circumstance you'd be keeping your principal value constant.

FROM THE FLOOR: I just wanted to make sure that the corporate accounts model works.

MR. BEAN: The point is that if your principal value changes over time in an uncertain way and your gains and losses are compounded, then you might not get the results that you think. That's the point that I was making.

FROM THE FLOOR: I just had a quick question for Steve. You mentioned in your presentation with respect to policyholder behavior that the little available data shows that the policyholders are tending to leave policies that are actually in the money and they're guaranteed with more frequency, which is the opposite of what the rational assumption might have been, but clearly the data that's available is from the first, second, or third years of 10-year maturity guarantees. Would you think that in the seventh, eighth, and ninth years of the maturity guarantees, that trend might reverse itself, and people who are in the money might actually tend to stick around?

MR. PRINCE: Yes, I'd expect that. What you're saying is people aren't going to wait eight or 10 years or something, but they might very well behave differently when it's a matter of waiting one or two years.

FROM THE FLOOR: Because as the maturity date became closer to today, you would expect that behavior to reverse.

MR. PRINCE: I would expect so, but this is simply based on the data one way or the other.

PANELIST: And just to add to that, when we would do GMIB modeling, for instance, it would be something similar. We have an increase in lapses if the market's down, but then we'd have another parameter that would show the value of the guarantee, and the value gets greater as it gets closer to the end of the waiting period, and that would pull lapses back down.

MR. DAVID SHURIK: This is for Steve. Do you have any testing to see what the percentile of the returns of the last 18 months has been against any of the models that you mentioned?

MR. PRINCE: Not I personally, but the working group of the Canadian Institute of Actuaries has gone to significant lengths to add to that very question, and when they talk about people adopting other models, they have established what we call calibration criteria because you have to track that question fairly well. That report is available on the Canadian Institute of Actuaries Web site.

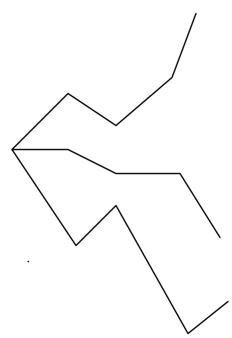
MR. ALLAN BENDER: I have a question for Eric. Everything about the pricing seems to be dependent upon the model, so one of the questions is, if the market really moves significantly, what kind of move does it take to get you to reprice? Are you always locked in or are you continually repricing?

MR. VON SHILING: I guess I wasn't envisioning repricing as you go along; it was more starting out with the money and taking the chance of being locked into your pricing structure.

Chart 1

Graphical Illustration:

- Scenario Analysis:
- Stochastic Simulation:



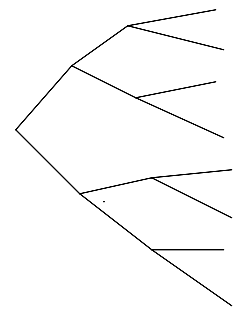


Chart 2

Base Benefit Cost Distribution

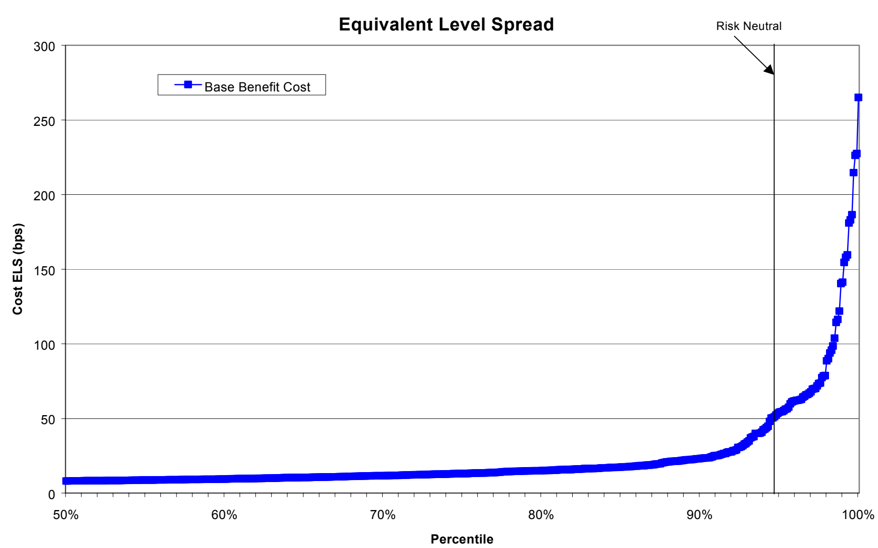


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Chart 3

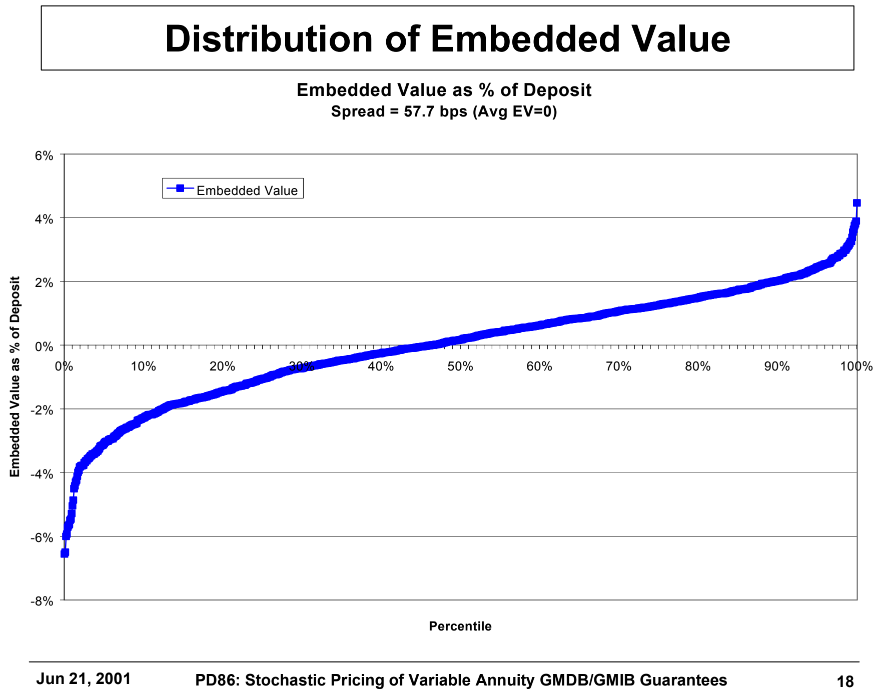


Chart 4

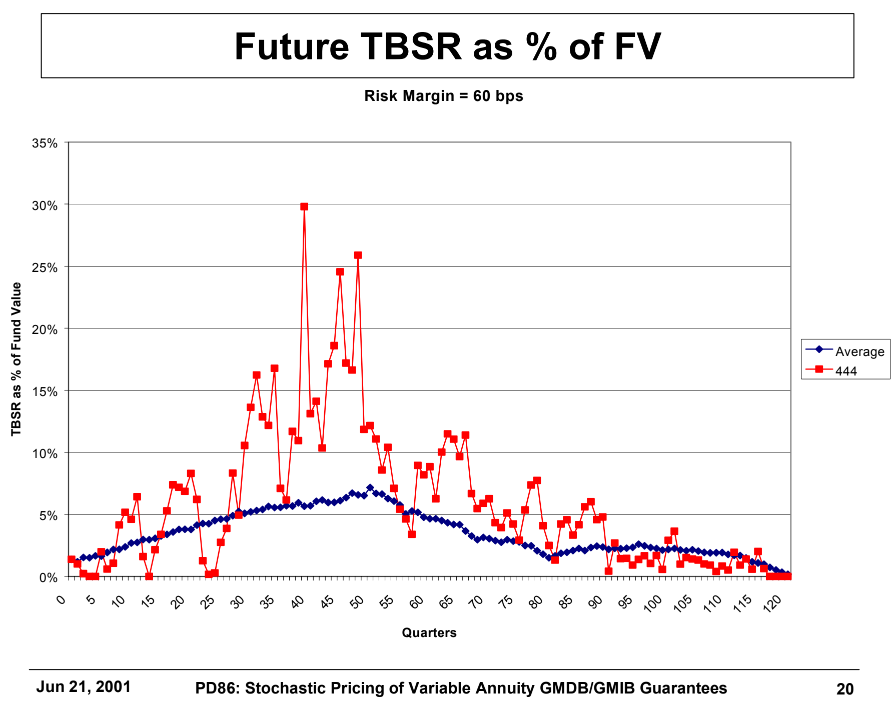
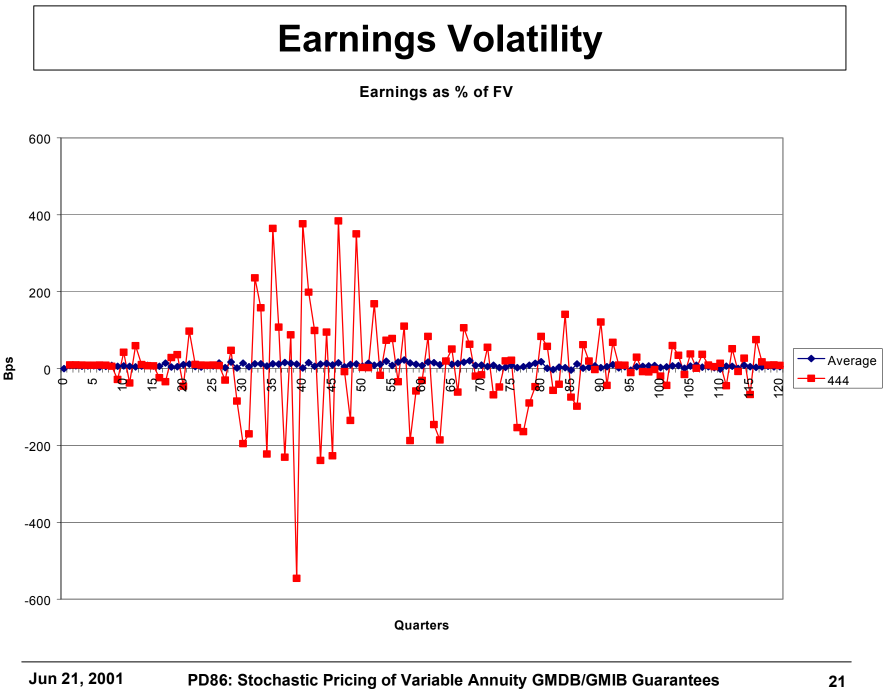


Chart 5


Earnings Volatility
Earnings as % of FV
600
400
200
0
-200
-400
-600
Bps
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
105
110
115
120
Quarters
Average
444
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