SPECIFICATIONS

FOR

MONETARY TABLES

BASED ON

1958 CSO AND CET MORTALITY TABLES

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Specifications for Monetary Tables Based on 1958 CSO and CET Mortality Tables

Insert (to be pasted at the right edge, next to III-A-b), on page 18.

RE: Vol. III, Section A-b)

The definition of the Modified Net Continuous Yearly Premium used in the Commissioners Reserve Valuation Method (p. 18) provides that the quantity "1000 (a-b)" or the "Excess of (a) over (b)" is exactly the same as the corresponding quantity used in the definition of the Modified Net Annual Premium (p. 17), where curtate functions are used, and that this quantity is available at issue, i.e., the present value of the modified premiums at issue is equal to the present value of benefits at issue, plus the quantity "1000 (a-b)". Since the law does not contain specific language relating to the use of continuous functions, it was the judgment of the Committee that the initial expense allowance used for continuous function reserves should not exceed that specified in the law for curtate functions. The above definition is conservative in that it results in higher reserves than those that would have been produced by any higher level of expense allowances.

INTRODUCTION

This booklet has been prepared as a supplement to the Monetary Tables based on the Commissioners 1958 Standard Ordinary Mortality Table and the Commissioners 1958 Extended Insurance Table published by the Society of Actuaries. It contains the detailed specifications for the calculation of the monetary values and the formulas used for checking in the machine process. It can serve as a guide for the calculation of values for plans not included in the published volumes on a basis consistent with the published figures.

Early in the development of the tables a set of specifications was prepared by the Committee for the Preparation of Monetary Tables, for the guidance of the International Business Machines Corporation in the computation of the tables. These specifications have been reproduced in this booklet, with some modifications. It seemed desirable to delete all of the examples of format which were needed for the original job but which probably would be unnecessary for any subsequent computations. For one thing, any company desiring to compute its own tables probably would want to decide on its own format for the tables, and even if it wished to follow the same format as in the Society's tables, it could do so by referring to the published volumes.

In addition to the built-in computer validity checks and programmed mathematical checks, copies of the output data were distributed to a few companies in the vicinity of New York for further checking by sight, by general review, by spot-check desk machine calculations, and by such other means as seemed appropriate. The minimum standards established by the Committee for this checking operation appear in Appendix A. It was not deemed necessary or desirable, however, to specify in detail the full checking procedures used by the different companies, since it was assumed that each actuary would want to devise his own checking procedures for any tables which he might produce independently.

It was found during these manual and visual checking operations of the output data that in some cases the original specifications needed further clarification. Such changes or additions as were found necessary in this connection have been introduced into the specifications presented in this booklet.

Certain interest constants needed for exact reproduction of values are listed in Appendix B.

The actual programs used by the International Business Machines Corporation on the 704 Computer have not been reproduced in this booklet since it was felt that few, if any, life insurance companies would have this machine available.

General Specifications for the Monetary Tables Based on the 1958 CSO and CET Mortality Tables

The Monetary Tables published by the Society of Actuaries cover seven different interest rates, ranging from 2% to $3\frac{1}{2}$% in $\frac{1}{4}$% steps. For each of the seven interest rates, the following volumes have been compiled for non-continuous or curtate functions, i.e., on the basis of formulas assuming claims payable at the end of the year of death and premiums payable annually at the beginning of each year.

Volume I - Basic Values
Volume II - Premiums and Reserves by the Net Level Premium Method
Volume III- Premiums and Reserves by the Commissioners Reserve Valuation Method
Volume IV - Minimum Cash Values and Non-Forfeiture Benefits as defined in the Standard Non-Forfeiture Law
Part 1 - Life and Term Policies
Part 2 - Endowment Policies

In addition, volumes of monetary values based on continuous functions corresponding to Volumes I, II, and III above have been compiled for $2\frac{1}{2}$%, 3% and $3\frac{1}{2}$% interest rates. For these values it was assumed that claims are to be payable at the moment of death and premiums are payable continuously. It was not necessary to produce Volume IV for continuous functions as the minimum cash values and non-forfeiture benefits required under the Standard Non-Forfeiture Law are defined in terms of curtate functions.

The description of each of the four volumes which appears in this booklet applies to all seven interest rates used for non-continuous or curtate functions and, with the indicated exceptions, to the three interest rates used for continuous functions. Unless otherwise noted, a maximum of ten significant digits was used in the derivation of any insurance function. Any intermediate multiplication or division involved in arriving at a published value was rounded to no more than ten digits. In rounding, except where otherwise specified, all values were rounded to the nearer digit; an exact 5 was rounded up.

In the specifications of the volumes which follow, an alphabetic prefix of either an F or M before an age designation refers to the sex (female or male, respectively). Also, age 0^* indicates for a policy issued at age 0 a death benefit during the first policy year of one-fourth of the death benefit thereafter. Because of typographical difficulties, the location of the asterisk modifying age 0, when used in conjunction with a commutation function, is as follows: C_0^* , instead of C_{0^*}. When an age range is referred to in the Specifications, it is in general abbreviated; for example "F0^* through M99/F102" is used instead of "F0^*, F0 through F14, M0^*, M0 through M11, and M12/F15 through M99/F102."

The functions which were computed and published, the number of decimals to which each was carried, and the range of ages for which they were computed, are presented in the descriptions which follow. Because of the additional programming required, decimal points are usually not shown in the printed results. When absent, the decimal point is indicated by the first blank space of the monetary value.

Volume I - Basic Values

The volume of Basic Values includes the following:

I-A - 1958 CSO Mortality Table, Commutation Columns and Valuation Factors
I-B - Net Single Premiums
I-C - 1958 CET Mortality Table and Commutation Columns
I-D - Non-Forfeiture Functions
I-E - Table of Frequently Called Numbers (published in non-continuous function volumes only)

I-A - 1958 CSO Mortality Table, Commutation Columns and Valuation Factors

Except for those functions for which it is otherwise indicated, values for each of the functions listed below were computed and published for each age F0* through M99/F102. The maximum of ten significant digits was increased to twelve for the S_x, $\bar{S}_x$, R_x and $\bar{R}_x$ functions.

Function	Derivation	Number of Decimal Places	Comments
$1000\,q_x$	A table of q_x values was supplied	Two	For table of 1000 q_x values see Volume I
l_x	$l_x = l_{x-1} - d_{x-1}$ for x = M1 through M99 and F15 through F102 for x = M0 and M0* l_x = 10,000,000 Values of l_x were supplied for ages F0* through F14	None	For values of l_x for ages F0* through F14 see Volume I
d_x	$d_x = q_x \cdot l_x$ for x = M0* through M99 and F15 through F102 Values of d_x were supplied for ages F0* through F14	None	For values of d_x for ages F0* through F14 see Volume I
$\mathring{e}_x$	$\mathring{e}_x = \dfrac{\sum_{z=1}^{\omega-x} l_{x+z}}{l_x} + .50$	Two	Published in continuous function volumes only

Function	Derivation	Number of Decimal Places	Comments
v^x	v^x was calculated by a continuous process, starting with 1.00 000 000 and dividing successively by $1+i$. Each quotient was rounded to ten decimals. However, the values published (and also used in the calculations of D_x and C_x) were rounded to eight decimal places. For M0* through M99/F102 the exponent in v^x is the male age; for F0* through F14, the exponent is the female age minus 3; for female ages 0, 1, and 2, v^{x-3} was taken as $(1+i)^{3-x}$ and calculated by multiplying successively by $1+i$.	Eight	Published in continuous function volumes only
D_x	$D_x = v^x \cdot l_x$	One	
$\frac{10,000,000}{D_x}$		Seven	Published so long as value did not exceed ten digits
$\bar{D}_x$	$\bar{D}_x = \frac{\delta - d}{\delta^2} D_x + \frac{i - \delta}{\delta^2} D_{x+1}$ Values of $\frac{\delta - d}{\delta^2}$ and $\frac{i-\delta}{\delta^2}$ to ten significant digits were supplied for each interest rate (see Appendix B)	One	
N_x	$N_x = \sum_{t=0}^{\omega-x} D_{x+t}$	One	
$\bar{N}_x$	$\bar{N}_x = \sum_{t=0}^{\omega-x} \bar{D}_{x+t}$	One	
S_x	$S_x = \sum_{t=0}^{\omega-x} N_{x+t}$	One	
$\bar{S}_x$	$\bar{S}_x = \sum_{t=0}^{\omega-x} \bar{N}_{x+t}$	One	
C_x	$C_x = v^{x+1} \cdot d_x$ for $x \neq 0^*$	Three	
C_0^*	$C_0^* = (.25) \cdot C_0$	Three	

Function	Derivation	Number of Decimal Places	Comments
$\bar{C}_x$	$\bar{C}_x = \frac{i}{\delta} C_x$ Values of $\frac{i}{\delta}$ to ten significant digits were supplied for each interest rate (see Appendix B)	Three	
M_x	$M_x = \sum_{t=0}^{\omega-x} C_{x+t}$ for $x \neq 0^*$	Three	
M_0^*	$M_0^* = C_0^* + M_1$	Three	
$\bar{M}_x$	$\bar{M}_x = \sum_{t=0}^{\omega-x} \bar{C}_{x+t}$ for $x \neq 0^*$	Three	
$\bar{M}_0^*$	$\bar{M}_0^* = \bar{C}_0^* + \bar{M}_1$	Three	
R_x	$R_x = \sum_{t=0}^{\omega-x} M_{x+t}$ for $x \neq 0^*$	Three	
R_0^*	$R_0^* = M_0^* + R_1$	Three	
$\bar{R}_x$	$\bar{R}_x = \sum_{t=0}^{\omega-x} \bar{M}_{x+t}$ for $x \neq 0^*$	Three	
$\bar{R}_0^*$	$\bar{R}_0^* = \bar{M}_0^* + \bar{R}_1$	Three	
$D_x - M_x$		One	
$D_x - \bar{M}_x$		One	
u_x	$u_x = \frac{D_x}{D_{x+1}}$	Seven	
$\bar{u}_x$	$\bar{u}_x = \frac{\bar{D}_x}{D_{x+1}}$	Seven	
$1000\,k_x$	$k_x = \frac{d_x}{l_{x+1}}$	Five	
$1000\,\bar{k}_x$	$\bar{k}_x = \frac{\bar{C}_x}{D_{x+1}}$	Five	

Function	Derivation	Number of Decimal Places	Comments
$1000\,k'_x$	$1000\,k'_x = u_x \cdot 1000\,{}_{19}P_{x+1}$	Five	The value of 1000 ${}_{19}P_{x+1}$ used in this calculation was rounded to seven decimal places. Published in non-continuous function volumes only and only for ages F0* through M90/F93.
$1000\,c_x$	$c_x = \dfrac{C_x}{D_x}$	Five	
$1000\,\bar{c}_x$	$\bar{c}_x = \dfrac{\bar{C}_x}{D_x}$	Five	
$1000({}_{19}P_{x+1} - c_x)$	${}_{19}P_{x+1} = \dfrac{M_{x+1}}{N_{x+1} - N_{x+20}}$	Five	The values of 1000 ${}_{19}P_{x+1}$ and 1000 c_x used in this calculation were rounded to seven decimal places. Published in non-continuous function volumes only and only for ages F0* through M90/F93.
$1000 \cdot [({}^{ADJ}_{OL}P_x)(.25)+.02] \not> 30$	${}^{ADJ}_{OL}P_x = \dfrac{A_x + .02}{\ddot{a}_x - .65}$	Five	The values of ${}^{ADJ}_{OL}P_x$ used in this calculation were rounded to seven decimal places. Published in non-continuous function volumes only and only for ages F0 through F14 and M0 through M90/F93.

I-B - Net Single Premiums

Net Single Premium values were computed for this section of the Basic Values volume using the factors listed below, and according to the following schedule:

1. Whole Life: - Life insurance and annuity values were computed for each Attained Age from F0* through M99/F102.

2. Terminal Age

   $X+t$: - A table of net single premiums was computed for each of the one hundred and thirteen Terminal Ages (Terminal Ages run from F1 through M99/F102 excluding M0 and M0*). Each table includes endowment insurance, annuity, term insurance and pure endowment values for each Attained Age from F0* through the Terminal Age minus one.

Factor	Derivation	Number of Decimal Places	Comments
$1000\,A_x$	$A_x = \frac{M_x}{D_x}$	Five	
$1000\,\bar{A}_x$	$\bar{A}_x = \frac{\bar{M}_x}{D_x}$	Five	
$\ddot{a}_x$	$\ddot{a}_x = \frac{N_x}{D_x}$	Six	
$\bar{a}_x$	$\bar{a}_x = \frac{\bar{N}_x}{D_x}$	Six	
$1000\,A_{x:\overline{t\rvert}}$	$A_{x:\overline{t\rvert}} = \frac{M_x - M_{x+t} + D_{x+t}}{D_x}$	Five	Numerator was obtained to three decimal places before division.
$1000\,\bar{A}_{x:\overline{t\rvert}}$	$\bar{A}_{x:\overline{t\rvert}} = \frac{\bar{M}_x - \bar{M}_{x+t} + D_{x+t}}{D_x}$	Five	Numerator was obtained to three decimal places before division.
$\ddot{a}_{x:\overline{t\rvert}}$	$\ddot{a}_{x:\overline{t\rvert}} = \frac{N_x - N_{x+t}}{D_x}$	Six	
$\bar{a}_{x:\overline{t\rvert}}$	$\bar{a}_{x:\overline{t\rvert}} = \frac{\bar{N}_x - \bar{N}_{x+t}}{D_x}$	Six	
$1000\,{}_tE_x$	${}_tE_x = \frac{D_{x+t}}{D_x}$	Five	
$1000\,A^{1}_{x:\overline{t\rvert}}$	$A^{1}_{x:\overline{t\rvert}} = A_{x:\overline{t\rvert}} - {}_tE_x$	Five	
$1000\,\bar{A}^{1}_{x:\overline{t\rvert}}$	$\bar{A}^{1}_{x:\overline{t\rvert}} = \bar{A}_{x:\overline{t\rvert}} - {}_tE_x$	Five	

I-C - 1958 CET Mortality Table and Commutation Columns

Values for each of the functions listed below were computed on the basis of the 1958 CET Mortality Table, for the same age ranges, and by the same methods as those described in section I-A:

$$1000\,q_x,\ l_x,\ d_x,\ D_x,\ \frac{10{,}000{,}000}{D_x},\ C_x,\ C_0^*,\ \bar{C}_x,\ M_x,\ M_0^*,\ \bar{M}_x,\ \bar{M}_0^*,\ R_x,\ R_0^*,\ \bar{R}_x,\ \bar{R}_0^*.$$

I-D - Non-Forfeiture Functions

Non-forfeiture benefits may be granted to a policyholder at any age from F0* through M99/F102. For this section of the Basic Values volume, it was therefore necessary to compute a separate table of non-forfeiture functions for each "Age Benefits Granted" from F0* through M99/F102.

The non-forfeiture values for each "Age Benefits Granted" depend on "t", the period-in-years coverage. This period-in-years will vary from 0 to (100 minus "Age Benefits Granted") for males, or from 0 to (103 minus "Age Benefits Granted") for females, for each table.

Depending upon the underlying mortality table, the following non-forfeiture functions were computed:

1. Based on 1958 CSO Mortality Table

   a. Extended term insurance - cost for period.
   b. Extended term insurance - additional days, per $1
   c. Pure endowment, if living at end of period, per $1
   d. Paid-up policy, payable at death or end of period, per $1

2. Based on 1958 CET Mortality Table

   a. Extended term insurance - cost for period.
   b. Extended term insurance - additional days, per $1
   c. Pure endowment, if living at end of period, per $1

Function	Definition	Number of Decimal Places	Comments
Cost for Period t	$\frac{1000(M_x - M_{x+t})}{D_x}$, or $\frac{1000(\bar{M}_x - \bar{M}_{x+t})}{D_x}$	Two	
Additional Days per $1	$\frac{365}{(\text{Cost per Period } t+1) - (\text{Cost per Period } t)}$	Three	Numbers of additional days per $1 were published only if less than or equal to 999.999. Cost per period used in this calculation was to four decimal places.
Pure Endowment per $1	$\frac{D_x}{D_{x+t}}$	Five	Amounts were published so long as age at end of period was less than or equal to M90/F93.
Paid-up Policy per $1	$\frac{D_x}{M_x - M_{x+t} + D_{x+t}}$, or $\frac{D_x}{\bar{M}_x - \bar{M}_{x+t} + D_{x+t}}$	Five	Published for 1958 CSO Mortality Table only.

I-E - Table of Frequently Called Numbers

A table of "Frequently Called Numbers" was calculated and published for each of the seven non-continuous or curtate function basic values volumes only. This table consists of net single premium values per $1,000, based on the 1958 CSO Mortality Table, for Whole Life and a variety of both male and female fixed Maturity Age Endowment insurance plans.

A single premium value was published under each insurance plan for each attained age ranging from 0* through the maturity age less one (F0* through M99/F102 for the Whole Life insurance plan). The following formulas were used for these calculations:

$$\text{Whole Life} = \frac{1000\, M_x}{D_x}$$

$$\text{Endowment at Age } x+n = \frac{1000(M_x - M_{x+n} + D_{x+n})}{D_x}$$

where x = Attained Age

x+n = Maturity Age

All net single premium values were rounded to five decimals.

Programmed Validity Checks

The following internal machine procedures were used to check the accuracy of the published values of Volume I:

Non-Continuous Checks

1) $\frac{10{,}000{,}000}{D_x} \cdot D_x = 10{,}000{,}000$

2) $C_x = v D_x - D_{x+1}$ for x = F0 through M99/F102 excluding MO*

Separate Sums for Male and Female:

3) $\sum_{x=0}^{\omega} N_x = \sum_{x=0}^{\omega} (x+1) \cdot D_x$

4) $\sum_{x=0}^{\omega} M_x = \sum_{x=0}^{\omega} (x+1) \cdot C_x$

5) $\sum_{x=0}^{\omega} R_x = \sum_{x=0}^{\omega} \frac{(x+1)(x+2)}{2} \cdot C_x$

6) $\sum_{x=0}^{\omega} S_x = \sum_{x=0}^{\omega} \frac{(x+1)(x+2)}{2} \cdot D_x$

7) $N_x \cdot 1000\,{}^{ADJ}_{OL}P_x = 1000\,M_x + D_x(650\,{}^{ADJ}_{OL}P_x + 20)$ for x = F0 through M99/F102 excluding MO*

8) $A_x + d\,\ddot{a}_x = 1$ for x = F0 through M99/F102 excluding MO*

9) $1000\,k_x = 1000\,c_x(u_x)$ for x = F0 through M89/F92 excluding MO*

10) $A_{x:\overline{t}|} + d\,\ddot{a}_{x:\overline{t}|} = 1$ for x = F0 through M89/F92 excluding MO*

11) $\sum_{x+t=k} ({}_tE_x \cdot D_x) = m \cdot D_{x+t}$

12) $\sum_{x+t=k} A_{x:\overline{t}|} = \sum_{x+t=k} (A^{1}_{x:\overline{t}|} + {}_tE_x)$

for x+t = F1 through M99/F102 excluding Mo and MO*, where m equals the number of terms in the summation.

(The summation was performed down a diagonal keeping x+t constant.)

Continuous Checks

Separate Sums for Male and Female:

1) $$\sum_{x=0}^{\omega} l_x \cdot \mathring{e}_x = .5\sum_{x=0}^{\omega} l_x + \sum_{x=0}^{\omega} x \cdot l_x$$

2) $$\sum_{x=0}^{\omega} \bar{N}_x = \sum_{x=0}^{\omega} (x+1)\,\bar{D}_x$$

3) $$\sum_{x=0}^{\omega} \bar{M}_x = \sum_{x=0}^{\omega} (x+1)\,\bar{C}_x$$

4) $$\sum_{x=0}^{\omega} \bar{R}_x = \sum_{x=0}^{\omega} \frac{(x+1)(x+2)}{2} \cdot \bar{C}_x$$

5) $$\sum_{x=0}^{\omega} \bar{S}_x = \sum_{x=0}^{\omega} \frac{(x+1)(x+2)}{2} \cdot \bar{D}_x$$

6) $$\frac{\delta - d}{\delta^2}\, 1000\, \bar{k}_x + \frac{i - \delta}{\delta^2}\, 1000\, \bar{c}_x = 1000\, \bar{c}_x \cdot \bar{u}_x$$

for x = F0 through M89/F92 excluding M0*

7) $$\bar{A}_x + \delta\, \bar{a}_x = 1$$

for x = F0 through M89/F92 excluding M0*

8) $$\bar{A}_{x:\overline{t|}} + \delta\, \bar{a}_{x:\overline{t|}} = 1$$

for x = F0 through M89/F92 excluding M0*

9) $$\sum_{x+t=k} \left({}_tE_x \cdot D_x\right) = m \cdot D_{x+t}$$

10) $$\sum_{x+t=k} \bar{A}_{x:\overline{t|}} = \sum_{x+t=k} \left(\bar{A}^{1}_{x:\overline{t|}} + {}_tE_x\right)$$

for x+t = F1 through M99/F102 excluding M0 and M0*, where m equals the number of terms in the summation. (The summation was performed down a diagonal keeping x+t constant.)

Volume II - Premiums and Reserves by the Net Level Premium Method

The volume of Premiums and Reserves by the Net Level Premium Method includes the following:

II-A - Net Level Annual Premiums
The continuous function volumes contain Net Level Continuous Yearly Premiums and also Net Level Annual Premiums (Discounted Continuous Yearly Premiums)
II-B - Terminal Reserves for Premium Paying Insurance Plans
II-C - Terminal Reserves for Paid-up Life and Endowment Plans
II-D - Mean Reserves for Premium Paying Insurance Plans
II-E - Mean Reserves for Paid-up Life and Endowment Plans

II-A - Net Level Annual Premiums

Net Level Annual Premiums were calculated for each of the insurance plans appearing in the table of contents of the non-continuous volumes of Volume II. For the continuous function volumes, Net Level Continuous Yearly Premiums and Net Level Annual Premiums (Discounted Continuous Yearly Premiums) were calculated and published for the same insurance plans. A value was published under each plan for each applicable issue age. For those plans whose lower limiting issue age is zero, an Issue Age 0* premium value was also calculated.

Generalized Formulas for the Calculation of Net Level Premiums

a) Net Level Annual Premiums:

$$\text{Premium Value} = \frac{1000\,(M_x - M_{x+n} + D_{x+n})}{N_x - N_{x+m}}$$

where x = Age at Issue

x+n = Age at End of Coverage

x+m = Age at End of Premium Paying Period

D_{x+n} = 0 for all non-endowment plans

All Net Level Annual Premiums were rounded to seven decimals for use in later calculations, but the premiums were rounded to five decimals for publication.

b) Net Level Continuous Yearly Premiums:

$$\text{Premium Value} = \frac{1000\,(\bar{M}_x - \bar{M}_{x+n} + D_{x+n})}{\bar{N}_x - \bar{N}_{x+m}}$$

where x, x+n, x+m and D_{x+n} are defined as above.

All Net Level Continuous Yearly Premiums were rounded to seven decimals for use in later calculations, but the premiums were rounded to five decimals for publication.

c) Net Level Annual Premiums (Discounted Continuous Yearly Premiums):

Premium Value = $\frac{d}{\delta}$ (Net Level Continuous Yearly Premium).

Values of $\frac{d}{\delta}$ to six significant digits were supplied for each interest rate (see Appendix B). A Net Level Continuous Yearly Premium rounded to seven decimal places was used for this calculation.

All Net Level Annual Premiums (Discounted Continuous Yearly Premiums) were rounded to seven decimals for use in later calculations, but the premiums were rounded to two decimals for publication.

II-B - Terminal Reserves for Premium Paying Insurance Plans

A table of terminal reserve values was calculated, by the net level premium method, for each of the insurance plans for which net level premium values were calculated. Reserve values were published, under each applicable issue age, for every premium paying duration. Also, terminal reserves for endowment plans for the year of maturity were shown as $1000 whereas for term plans, terminal reserves in the year of expiry were not shown. For those plans whose lower limiting issue age is zero, terminal reserve values were also published for Issue Age 0*, through all applicable durations.

Generalized Formulas for the Calculation of Terminal Reserve Values for Premium Paying Insurance Plans

a) Non-Continuous Functions:

Terminal Reserve at Duration t = $\frac{1000(M_{x+t} - M_{x+n} + D_{x+n}) - P(N_{x+t} - N_{x+m})}{D_{x+t}}$

where x, x+n, x+m and D_{x+n} are defined in section II-A.

P is the Net Level Annual Premium premium value defined in Section II-A and depends upon the insurance plan and the issue age. A premium value rounded to seven decimals was used in the calculation of terminal reserves.

b) Continuous Functions:

Terminal Reserve at Duration t = $\frac{1000(\bar{M}_{x+t} - \bar{M}_{x+n} + D_{x+n}) - P(\bar{N}_{x+t} - \bar{N}_{x+m})}{D_{x+t}}$

where x, x+n, x+m and D_{x+n} are defined in section II-A.

P is the Net Level Continuous Yearly Premium premium value defined in section II-A and depends upon the insurance plan and the issue age. A premium value rounded to seven decimals was used in the calculation of terminal reserves.

In deriving terminal reserve values, the calculation was performed as follows:

(i) $1000\,(M_{x+t} - M_{x+n} + D_{x+n})$ was calculated without rounding.

(ii) $(N_{x+t} - N_{x+m})$ was calculated without rounding.

(iii) "Premium Value" $\cdot\,(N_{x+t} - N_{x+m})$ was calculated using a seven decimal place rounded premium value, and the product was rounded to the nearer integer.

(iv) The terminal reserve was obtained by dividing the resulting numerator by D_{x+t} and was rounded to two decimals for publication.

The above calculation procedure was also followed for the continuous function volumes, substituting, of course, the necessary continuous function values where appropriate.

When the terminal reserve value was negative, the published amount was followed by a minus sign.

II-C - Terminal Reserves for Paid-up Life and Endowment Plans

A table of terminal reserves was calculated by the net level premium method, for the Paid-up Life and Endowment plans which are included in Volume II. Values were published, under each plan, for each attained age applicable.

Generalized Formulas for the Calculation of Terminal Reserve Values for Paid-up Life and Endowment Plans

a) Non-continuous functions

Terminal Reserve at Attained Age x = $\dfrac{1000\,(M_x - M_y + D_y)}{D_x}$

where x = Attained Age
y = Terminal Age

For Paid-up Life, M_y and D_y were taken as zero.

b) Continuous functions:

Terminal Reserve at Attained Age x = $\dfrac{1000\,(\bar{M}_x - \bar{M}_y + D_y)}{D_x}$

where x and y are defined as above.
For Paid-up Life, $\bar{M}_y$ and D_y were taken as zero.

All Paid-up Life and Endowment terminal reserve values were rounded to two decimals for publication.

II-D - Mean Reserves for Premium Paying Insurance Plans

A table of mean reserve values was calculated, by the net level premium method, for each of the insurance plans for which net level premiums and terminal reserve values were obtained. The mean reserve values were published in the same order, and according to the same formats, as were the terminal reserve values.

Generalized Formula for the Calculation of Mean Reserve Values for Premium Paying Insurance Plans

Mean Reserve for Policy Year t =

$$\frac{1}{2}\Big[\text{(Terminal Reserve at Duration t-1)} + \text{(Terminal Reserve at Duration t)} + P\Big]$$

where: The Terminal Reserve at Duration 0 = 0.

P is either the Net Level Annual Premium or the Net Level Annual Premium (Discounted Continuous Yearly Premium) "premium value" defined in section II-A and depends upon the insurance plan and the issue age. A premium value rounded to seven decimals and terminal reserves to two decimals were used in the calculation of mean reserves for premium paying plans.

For 10-Pay Life and Whole Life a mean reserve for attained age M100/F103 (i.e. policy year t for which x+t = M100/F103) was published, assuming a terminal reserve of $1000 at attained age M100/F103; the $1000 terminal reserves were not published. Whenever a negative terminal reserve value was used in the calculation of a mean reserve, the mean reserve value was published with an asterisk to the right.

All mean reserve values were rounded to two decimals.

II-E - Mean Reserves for Paid-up Life and Endowment Plans

A table of mean reserve values was calculated by the net level premium method, for each of the Paid-up Life and Endowment plans for which corresponding terminal reserves were obtained. The mean reserve values were published in the same order, and according to the same format, as were the corresponding terminal reserve values.

Generalized Formulas for the Calculation of Mean Reserves for Paid-up Life and Endowment Plans

a) For Attained Ages F0 through M99/F102, Excluding 0*

Mean Reserve at Attained Age x =

$$\frac{1}{2}\Big[\text{(Terminal Reserve at Attained Age x)} + \text{(Terminal Reserve at Attained Age x+1)}\Big]$$

where: Terminal Reserve at Attained Age M100/F103 for Paid-up Life and at maturity ages for Paid-up Endowment plans equals $1,000.

b) For Attained Age 0* only:

Mean Reserve at Attained Age 0* =

$$\frac{1}{2}\left[(\text{Terminal Reserve at Attained Age } 0^{*}) + (\text{Terminal Reserve at Attained Age } 1)\right]$$

Terminal reserve values to two decimals were used in the calculation and all mean reserve values obtained were rounded to two decimals.

Programmed Validity Checks

The following internal machine procedures were used to check the accuracy of the published values of Volume II:

Premiums and Mean Reserves

Calculations were performed twice. Two separate routines were used, and the results were compared.

Terminal Reserves for Premium Paying Insurance Plans

For a given plan, for a set of r consecutive issue ages, where $r \leq 10$, the following check was made at each constant attained age $x+t$:

$$D_{x+t} \cdot \sum_{x,t} (\text{Terminal Reserve for issue age x at duration t}) =$$
$$\sum_{x}\left[(\text{"Premium Value"}) \cdot N_x - 1000 M_x\right] - N_{x+t} \sum_{x} (\text{"Premium Value"}) + r \cdot 1000 M_{x+t}$$

This same checking procedure was used for the continuous function volumes substituting, of course, the appropriate continuous function values.

Volume III-Premiums and Reserves by the Commissioners Reserve Valuation Method

The volume of Premiums and Reserves by the Commissioners Reserve Valuation Method includes the following:

III-A - Modified Net Annual Premiums
The continuous function volumes contain Modified Net Continuous Yearly Premiums and also Modified Net Annual Premiums (Discounted Continuous Yearly Premiums)
III-B - Terminal Reserves for Premium Paying Insurance Plans
III-C - Terminal Reserves for Paid-up Life and Endowment Plans
III-D - Mean Reserves for Premium Paying Insurance Plans
III-E - Mean Reserves for Paid-up Life and Endowment Plans

The organization of this volume is identical to that of the volume of Premiums and Reserves by the Net Level Premium Method (Volume II). Premiums and reserve values in Volume III were calculated for the same insurance plans, were published in the same order and, with a single exception, according to the same formats as those in Volume II. The format exception occurs in section III-A, where there are two columns for each issue age under each insurance plan. This change in format necessitated an expansion in the number of published pages in Volume III as compared with Volume II.

III-A - Generalized Formulas for the Calculation of Modified Net Premiums:

a) Modified Net Annual Premiums

$$\text{Modified Premium Value} = P + \frac{1000(a-b)\,D_x}{(N_x - N_{x+m})}$$

where 1000 (a-b) = "Excess of (a) over (b)" = the smaller of

$$\frac{1000(M_{x+1} - M_{x+n} + D_{x+n})}{N_{x+1} - N_{x+m}} - \frac{1000\,C_x}{D_x} \quad \text{or}$$

$$\frac{1000\,M_{x+1}}{N_{x+1} - N_{x+20}} - \frac{1000\,C_x}{D_x}$$

but not less than zero,

and x = Age at Issue

$x+n$ = Age at End of Coverage

$x+m$ = Age at End of Premium Paying Period

D_{x+n} = 0 for all non-endowment plans

P = Net Level Annual Premium per $1000 for the plan and issue age involved and is defined in section II-A of Volume II

Values of P and 1000 (a-b) rounded to seven decimal places were used in this calculation. The Modified Net Annual Premium, as well as such premium less "Excess of (a) over (b)", were rounded to seven decimals for use in later calculations, but were rounded to five decimals for publication.

b) Modified Net Continuous Yearly Premiums

$$\text{Modified Premium Value} = P + \frac{1000(a-b)\, D_x}{(\bar{N}_x - \bar{N}_{x+m})}$$

where 1000 (a-b), x, x+n, x+m and D_{x+n} are defined as above;

P = Net Level Continuous Yearly Premium per $1,000 for the plan and issue age involved and is defined in section II-A of Volume II.

Values of P and 1000 (a-b) rounded to seven decimal places were used in this calculation. The Modified Net Continuous Yearly Premium, as well as such premium less "Excess of (a) over (b)", were rounded to seven decimals for use in later calculations, but were rounded to five decimals for publication.

c) Modified Net Annual Premiums (Discounted Continuous Yearly Premiums)

$$\text{Modified Premium Value} = \frac{d}{\delta} \text{ (Modified Net Continuous Yearly Premium)}$$

Values of $\frac{d}{\delta}$ to six significant digits were supplied for each interest rate. (See Appendix B).

A Modified Net Continuous Yearly Premium rounded to seven decimal places was used in this calculation.

The Modified Net Annual Premium (Discounted Continuous Yearly Premium), as well as such premium less "Excess of (a) over (b)", were rounded to seven decimals for use in later calculations, but were rounded to two decimals for publication.

III-B - Generalized Formulas for the Calculation of Terminal Reserves for Premium Paying Insurance Plans

a) Non-continuous functions:

$$\text{Terminal Reserve at Duration } t = \frac{1000(M_{x+t} - M_{x+n} + D_{x+n}) - P^{MOD}(N_{x+t} - N_{x+m})}{D_{x+t}}$$

where x, x+n, x+m, and D_{x+n} are defined as in section III-A.
P^{MOD} is the Modified Net Annual Premium value defined in section III-A and depends upon the insurance plan and issue age. A premium value rounded to seven decimals was used in the calculation of terminal reserves.

b) Continuous functions:

$$\text{Terminal Reserve at Duration } t = \frac{1000(\bar{M}_{x+t} - \bar{M}_{x+n} + D_{x+n}) - P^{MOD}(\bar{N}_{x+t} - \bar{N}_{x+m})}{D_{x+t}}$$

where x, x+n, x+m and D_{x+n} are defined as above.
P^{MOD} is the Modified Net Continuous Yearly Premium value defined in section III-A and depends upon the insurance plan and issue age. A premium value rounded to seven decimals was used in the calculation of terminal reserves.

In deriving terminal reserve values, the calculation was performed in the same way as in section II-B.

When the terminal reserve value was negative, the published amount was followed by a minus sign.

III-C - Generalized Formulas for the Calculation of Terminal Reserves for Paid-up Life and Endowment Plans

Since no premium value is involved, the same formulas as defined in section II-C of Volume II were used.

III-D - Generalized Formula for the Calculation of Mean Reserves for Premium Paying Insurance Plans

Mean Reserve for Policy Year t =

$$\frac{1}{2}\left[(\text{Terminal Reserve at Duration } t\text{-}1) + (\text{Terminal Reserve at Duration } t) + P^{MOD} \right]$$

where the Terminal Reserve at Duration 0 = 0.

P^{MOD} is either the Modified Net Annual Premium or the Modified Net Annual Premium (Discounted Continuous Yearly Premium) premium value defined in section III-A and depends upon the insurance plan and issue age. A premium value rounded to seven decimals and terminal reserves to two decimals were used in the calculation of mean reserves for premium paying plans.

P^{MOD} less "Excess of (a) over (b)" was substituted for P^{MOD} at mean duration 1.

For 10 Pay Life and Whole Life a mean reserve for attained age M100/F103 (i.e. policy year t for which x+t = M100/F103) was published, assuming a terminal reserve of $1000 at attained age M100/F103; the $1000 terminal reserves were not published. Whenever a negative terminal reserve value was used in the calculation of a mean reserve, the mean reserve value was published with an asterisk to the right.

All mean reserve values were rounded to two decimals.

III-E - Generalized Formulas for the Calculation of Mean Reserves for Paid-up Life and Endowment Plans

Here again, since no premium value is involved, the same formulas as defined in section II-E of Volume II were used.

Programmed Validity Checks

The following internal machine procedures were used to check the accuracy of the published values of Volume III.

Premiums and Mean Reserves

Calculations were performed twice. Two separate routines were used, and the results compared.

Terminal Reserves for Premium Paying Insurance Plans

For a given plan, for a set of λ consecutive issue ages, where $\lambda \leq 10$, the following check was made at each constant attained age x+t:

$$D_{x+t} \cdot \sum_{x,t} (\text{Terminal Reserve for issue age x at duration t}) = \sum_{x} \left[(\text{"Modified Premium Value"}) \cdot N_x - 1000\, M_x \right] - N_{x+t} \sum_{x} (\text{"Modified Premium Value"}) + \lambda \cdot 1000\, M_{x+t} - \sum_{x} 1000 (a-b) \cdot D_x$$

This same checking procedure was used for the continuous function volumes substituting, of course, the appropriate continuous function values.

Volume IV-Minimum Cash Values and Non-Forfeiture Benefits as Defined in the Standard Non-Forfeiture Law

The final volume shows tables of adjusted annual premiums and minimum cash and non-forfeiture values which were calculated for various insurance plans on a non-continuous basis only. For publication purposes this book was separated into two parts as follows:

Part 1	Life and Term Plans
Part 2	Endowment Plans

The format of the adjusted annual premium table followed that of net level annual premiums shown in Volume II.

The schedule below lists the non-forfeiture values which were calculated in addition to the cash values:

1. Amount of Reduced Paid-up Insurance
2. Extended Insurance based on the 1958 CET Mortality Table
3. Pure Endowment (Endowment Plans only)

A separate table of values was calculated and published for each insurance plan, under each applicable issue age, for durations 1 through 20 and for attained ages M55, M60, M65, F55, F60 and F65. However, no values were calculated for those durations or attained ages beyond the premium paying period, and a cash value for duration zero was published. Negative cash values published under each issue age were followed by a minus sign.

In this Volume, values are shown for an issue age 0^{**} as well as issue age 0^{*}. While for both these ages the death benefit during the first policy year is one-fourth of the death benefit thereafter, the distinction lies in the definition of the "equivalent uniform amount," which for age 0^{*} is that defined by the Hooker Committee, and for age 0^{**} is equal to the death benefit after the first policy year.

Formulas for the Calculation of Adjusted Annual Premiums and Minimum Cash and Non-Forfeiture Benefits

I. Adjusted Annual Premiums

1. Term Plans

a. $$P_x^{ADJ} < 40; \quad P_x^{ADJ} = \frac{\frac{1000(M_x - M_{x+n})}{D_x} + 20}{\frac{(N_x - N_{x+m})}{D_x} - .65}$$

b. $$P_x^{ADJ} \geq 40; \quad P_x^{ADJ} = \frac{\frac{1000(M_x - M_{x+n})}{D_x} + 46}{\frac{(N_x - N_{x+m})}{D_x}}$$

2. Non-Term Plans, except for Issue Age 0* and 0**

a. $P_x^{ADJ} < 40; \quad P_x^{ADJ} = \dfrac{\dfrac{1000(M_x - M_{x+n} + D_{x+n})}{D_x} + (20 + 250\,{}_{OL}^{ADJ}P_x)}{\dfrac{N_x - N_{x+m}}{D_x} - .4}$

b. $P_x^{ADJ} \geq 40; \quad P_x^{ADJ} = \dfrac{\dfrac{1000(M_x - M_{x+n} + D_{x+n})}{D_x} + (36 + 250\,{}_{OL}^{ADJ}P_x)}{\dfrac{N_x - N_{x+m}}{D_x}}$

3. Non-Term Plans, for Issue Age 0* and 0**

a. $P_x^{ADJ} < 40(1-k); \quad P_x^{ADJ} = \dfrac{\dfrac{1000(M_0^* - M_n + D_n)}{D_0} + (1-k)(20 + 250\,{}_{OL}^{ADJ}P_0)}{\dfrac{(N_0 - N_m)}{D_0} - .4}$

b. $P_x^{ADJ} \geq 40(1-k); \quad P_x^{ADJ} = \dfrac{\dfrac{1000(M_0^* - M_n + D_n)}{D_0} + (1-k)(36 + 250\,{}_{OL}^{ADJ}P_0)}{\dfrac{N_0 - N_m}{D_0}}$

where $\quad {}_{OL}^{ADJ}P_x = \dfrac{A_x + .02}{\ddot{a}_x - .65}$ $\quad$ except in no event was a value greater than .04 used

for Issue Age 0*, $1-k$ = "equivalent uniform amount" and is equal to $\dfrac{M_0^* - M_n}{M_0 - M_n}$

for Issue Age 0**, $1-k = 1$

In calculating adjusted annual premiums, ${}_{OL}^{ADJ}P_x$ values to seven decimals were used, and in general, a numerator and denominator evaluated to nine decimals were used in the calculations. For Issue Age 0* a value of $1-k$ to six decimals was used.

All adjusted annual premiums were calculated to seven decimals without rounding for further computations but were rounded to five decimals for publication.

II. <u>Cash Values</u>

$$CV = \frac{1000\,(M_{x+t} - M_{x+n} + D_{x+n}) - P_x^{ADJ}(N_{x+t} - N_{x+m})}{D_{x+t}}$$

In evaluating cash values, the calculations were performed as follows:

i) $1000\,(M_{x+t} - M_{x+n} + D_{x+n})$ was calculated without rounding.
ii) $(N_{x+t} - N_{x+m})$ was calculated without rounding.
iii) $P_x^{ADJ} \cdot (N_{x+t} - N_{x+m})$ was calculated using a seven decimal place truncated adjusted premium and the product was truncated to an integer.
iv) The cash value was obtained by dividing the resulting numerator by D_{x+t} and was truncated to four decimals for further computation but rounded to two decimals for publication.

When the cash value was negative, the published amount was followed by a minus sign.

III. <u>Extended Insurance (All Commutation Function Values Were Taken From the CET Table)</u>

$$M_{x+t+s} = M_{x+t} - \frac{CV}{1000} D_{x+t}$$

Where CV is the four decimal cash value for the applicable issue age, plan and duration.

For endowment plans only, a comparison was made of the value M_{x+t+s} with the tabular value of M_{x+n}, where x+n is the age at maturity. If M_{x+t+s} was less than M_{x+n}, the period of term extension was set equal to the number of years remaining until maturity, or (x+n) - (x+t). The amount of pure endowment was then calculated as follows:

$$\text{Pure Endowment} = \left[CV - \frac{1000\,(M_{x+t} - M_{x+n})}{D_{x+t}}\right] \cdot \frac{D_{x+t}}{D_{x+n}}$$

Each of the two components in the above brackets was used to four decimal places. A pure endowment value to two decimals was obtained for publication, by rounding to the next higher cent.

When M_{x+t+s} was greater than M_{x+n}, there was no pure endowment, and the length of term extension was calculated as described below.

For non-endowment plans and endowment plans with no pure endowment, a search of the M_x table was performed to locate the two values bracketing M_{x+t+s}. These bracketing values were denoted as M_p and M_{p+1}. The years of term extension were then set equal to $p-(x+t)$. The days of term extension were obtained as follows:

$$\text{Days of ET} = \frac{365\,(M_p - M_{x+t+s})}{M_p - M_{p+1}}$$

A result rounded to the next higher integer was obtained for publication and a comparison with 365 was performed in order to determine if the years of term extension should be increased by 1 and the days of term extension changed to zero.

IV. Paid-up Values

$$PV = CV\left(\frac{D_{x+t}}{M_{x+t} - M_{x+n} + D_{x+n}}\right)$$

A four decimal cash value was used and a result rounded to the next higher cent was obtained for publication.

In all of the above equations, the definition of the notation is as follows:

x = Age at Issue
t = Duration
$x+n$ = Age at End of Coverage
$x+m$ = Age at End of Premium Paying Period
N_{x+m} = 0 for ordinary life plan
D_{x+n} = 0 for non-endowment plans
M_{x+n} = 0 for ordinary life plan

Programmed Validity Checks

The following internal machine procedures were used to check the accuracy of the published values of Volume IV.

1) Values of θ_x were obtained by the formula

$$\theta_x = (20 + .4\alpha_x + .25\beta_x)\cdot D_x$$

$$\text{WHERE } \alpha_x = \begin{cases} P_x^{ADJ} & \text{IF } P_x^{ADJ} \le 40 \\ 40 & \text{IF } P_x^{ADJ} > 40 \end{cases}$$

$$\beta_x = \begin{cases} 1000\,{}_{OL}^{ADJ}P_x & \text{IF } 1000\,{}_{OL}^{ADJ}P_x \le \alpha_x \\ \alpha_x & \text{IF } 1000\,{}_{OL}^{ADJ}P_x > \alpha_x \end{cases}$$

2) Adjusted premiums (P_x^{ADJ}) were checked by the following formula:

$$P_x^{ADJ}\cdot(N_x - N_{x+m}) = 1000(M_x - M_{x+n} + D_{x+n}) + \theta_x$$

3) Cash Values (CV) were checked by the following formula:

$$CV\cdot(N_{x+t} - N_{x+t+1}) = P_x^{ADJ}(N_x - N_{x+t}) - [1000(M_x - M_{x+t}) + \theta_x]$$

4) Paid-up Values (PV) were checked by the following formula:

$$CV\cdot D_{x+t} = PV(M_{x+t} - M_{x+n} + D_{x+n})$$

5) Extended insurance values were checked as follows (using commutation functions based on the 1958 CET Mortality Table)

a) $$\frac{365\cdot CV\cdot D_y}{1000} = d(M_y - M_{y+r+1}) + (365 - d)(M_y - M_{y+r})$$

b) $$CV\cdot D_y = 1000(M_y - M_{y+r}) + P.E.\cdot D_{y+r}$$

Where r = years of term extension
d = days of term extension
y = age at issue plus duration (i.e. $x+t$)
PE = Pure Endowment

APPENDIX A

Specifications for Checkout of IBM Tabulations of Monetary Values Based on 1958 CSO and CET Tables

Each of the "checking" companies will checkout the IBM tabulations in accordance with the following specifications, as applicable to the functions being checked.

1. Completeness and Format Check. The tabulations will be checked against the specifications to determine whether all the required values are included and that they are in the prescribed format. This will involve inspection of plans and ages, their sequence, policy durations, etc.

2. Sample Check Values. The following network of check values will be calculated by hand and compared with the values calculated by IBM.

   (a) Monetary Values Based on 2½% Interest.

      Premiums - four values for each plan, one of which will be issue age 0* and the other three for a low, middle, and high issue age.

      Terminal Reserves and Minimum Values - 10 to 15 values for each plan, comprised of from three to five durations for three issue ages. For each of the 10 to 15 minimum cash values, it will be necessary to calculate corresponding paid-up and extended term values.

      Mean Reserves - the same network of check values for terminal reserves will be used for checking mean reserves on the assumption that the IBM terminal reserve for the next duration is correct. In addition, the sum of the mean reserves will be checked against the appropriate sum of terminal reserves and net premiums.

      Net Single Premiums and Nonforfeiture Factors - three check values for each function for each of 8 terminal ages (age benefits granted in the case of nonforfeiture factors).

   (b) Monetary Values Based on Other Rates of Interest.

      Premiums - one value for each plan.

      Terminal Reserves and Minimum Values - three values for one issue age.

      Mean Reserves - three values for one issue age, corresponding to the check values for the terminal reserves.

      Net Single Premium and Nonforfeiture Factors - three values for each of 8 terminal ages (or age benefits granted) spread fairly evenly over the various functions for which values are calculated.

3. Grading. The tabulations will be reviewed for a reasonable gradation in values between issue ages, terminal ages or age benefits granted, as the case may be. For most functions, this grading check will be in two directions, i.e., by duration for a given issue age and cross issue ages for the same duration.

4. Check of IBM Printer. As a check on the IBM printer and as an additional check on the calculations, column sums will be taken and compared with the IBM independently calculated sum for the column, as follows:

   (a) Monetary Values Based on $2\frac{1}{2}$% Interest. One column for every three pages with the columns being varied to check the 119 print positions to the fullest extent possible.

   (b) Monetary Values Based on Other Rates of Interest. One column for every 15 pages to be varied as above.

5. Progression Across Interest Rates. To check that the values at the various interest rates are reasonably related to each other, the ratio (or the difference, or both) of the corresponding column totals will be compared. The extent of this check will be left to the discretion of the checking company.

6. Commutation Columns. The commutation columns will be checked out by a number for number comparison with values independently computed and furnished to the checking companies. In addition, each column will be added and the sum compared with the IBM calculated value.

7. Overall Summation Check. The checking companies will, to the extent feasible, devise overall summation checks which will be compared against the summation of the IBM calculated column totals.

8. Checking of Proofs. The checking companies will review the proofs for appropriate headings, ages and page number and for readability.

In addition to the above, the checking company will review the tabulations to see that negative values are appropriately indicated, that mean reserves based on negative terminal reserves are indicated by an asterisk and that negative mean reserves are indicated by both a minus sign and asterisk.

APPENDIX B

Interest Constants Developed for Use in Computing Functions for New Mortality Table

	$2\frac{1}{2}\%$	3%	$3\frac{1}{2}\%$
	10 significant figures		
$\frac{i}{\delta}$	1.012448558	1.014926104	1.017399664
$\frac{\delta - d}{\delta^2}$	.4959098448	.4951097239	.4943154023
$\frac{i - \delta}{\delta^2}$	.5041409666	.5049630884	.5057832231
	6 significant figures		
$\frac{d}{\delta}$	.987755	.985365	.982995

SPECIFICATIONS FOR THE CALCULATION OF 4% MONETARY VALUES PUBLISHED IN 1976 BY THE SOCIETY OF ACTUARIES

SPECIFICATIONS FOR THE CALCULATION OF 4% MONETARY VALUES PUBLISHED IN 1976 BY THE SOCIETY OF ACTUARIES

THE SPECIFICATIONS FOR MONETARY TABLES BASED ON 1958 CSO AND CET MORTALITY TABLES, compiled and published by the Society of Actuaries, copyright 1961 which applied to the $2\frac{1}{2}\%$ to $3\frac{1}{2}\%$ tables formed the basis for the calculations of the 4% tables published in 1976.

It should be noted that in 1961 the specifications were written for and the values were calculated on an IBM 704 computer. In 1976 with advanced computer technology and current programming techniques, certain modifications in the specifications were deemed appropriate. It also became apparent that some items could be interpreted in more than one way.

The specifications as published by the Society of Actuaries in 1961 were used with the following clarifications and/or modifications.

1. Where a commutation column value was indicated in the specifications, it was the published value that was used, i.e., D_x to 1 decimal place, M_x to 3 decimal places, etc.

2. Any time a predefined term was used in a calculation, the value printed for that term was used unless specific notation stated to do otherwise.

3. The program validity checks listed on pages 10, 11, 16, 20 and 24 were not required and thus deleted from the specifications.

4. Page 2 - negative numbers were rounded by rounding the absolute value of the number.

5. Page 2 - the limitation on significant digits: "unless otherwise noted, a maximum of ten significant digits was used in the derivation of any insurance function" was not imposed but rather the number of significant digits implied by the number of decimal places was carried throughout the calculations. Where calculations were carried to a certain number of decimals and it was not specified whether truncation or rounding was to occur, the calculation was rounded.

6. Page 4 - v^x was not double rounded. That is, in each iteration the calculated value was independently rounded to ten decimals for the next iteration and to eight decimals for calculating D_x and C_x.

7. Page 5 - $\overline{C}_0^*$ was calculated as $\frac{i}{\delta} \cdot C_0^*$ using the rounded values for C_0^*.

8. Page 5 - k_0^* was calculated as $.25k_0$ using the rounded value for k_0.

9. Page 5 - $\overline{k}_0^*$ was calculated using the rounded value for $\overline{C}_0^*$.

10. Page 6 - The value $1000c_x$ was not double rounded for printing, but was independently rounded to seven places for the calculation of $1000({}_{19}P_{x+1} - c_x)$ and to five places for printing.

11. Page 6 - c_0^* was calculated using the rounded value for C_0^*.

12. Page 6 - $\overline{c}_0^*$ was calculated using the rounded value for $\overline{C}_0^*$.

13. Page 6 - The value $1000({}_{19}P_{x+1} - c_x)$ was not printed when the result was negative, but rather the notation "negative" was used with an explanatory note.

14. Page 6 - ${}_{ol}^{adj}P_x$ was rounded to ten decimals. This modification was made in order to be consistent with the previous calculations made in 1961. Apparently, the actual calculations made in 1961 were not consistent with the specifications.

15. Page 9 - The cost of period t was not doubled rounded but was rounded to four decimals for the calculation of additional days per dollar and rounded to two decimals for printing.

16. Page 9 - The pure endowment, per dollar "comment" was changed to the following: Amounts were published so long as age at end of period was less than or equal to M 90/F 93 and so long as such values did not equal or exceed $1,000.

17. Page 9 1-E - Table of Frequently Called Numbers. These Tables of Frequently Called Numbers were eliminated from the 4% volumes.

18. Page 12 and 18 - Net premiums were not double rounded as indicated in the specifications; but rather the calculated value was independently rounded to seven or five decimals as needed. This was done to be consistent with what apparently was done in 1961.

19. Page 18 - Interpret the insert as follows for continuous plans:

    a. The "excess of (a) over (b)" is calculated exactly as for the curtate plan.

    b. The Modified Net Continuous Yearly Premium is calculated as shown, assuming this is available at issue. Note that the "P" in this formula is continuous even though 1000(a-b) is not.

c. The Modified Net Continuous Yearly Premium less the excess of (a) over (b) is calculated by simple subtraction although this is neither of the first year continuous premiums; this number is not used in further calculations.

d. The Modified Net Annual Premium is calculated from the Modified Net Continuous Yearly Premium and the Modified Net Annual Premium less the excess of (a) over (b) is the first year Modified Net Annual Premium which is used in calculating the first year mean reserve.

20. Page 21 and 22 - Clarification of rounding rules for the calculation of the adjusted premium:

a. The present value of benefits is calculated and rounded to nine decimal places.

b. ${}^{adj}_{ol}P_x$ is calculated and rounded to ten decimal places using A_x rounded to eight decimal places and $\ddot{a}_x$ rounded to six decimal places.

c. If appropriate, (1 - k) is calculated to six decimal places.

d. The balance of the numerator is then calculated and rounded to nine decimal places.

e. The annuity is calculated and rounded to nine decimal places.

f. P_x^{adj} is then calculated and truncated to seven decimal places.

21. Page 22 - Where ${}^{adj}_{ol}P_x$ is greater than ${}^{adj}P_x$, the appropriate lower value was substituted in the equations in 3. on page 22.

22. Page 23 - The zero year cash value was calculated using a prospective formula identical to that of the other years rather than by deriving the expense amounts directly.

23. Page 23 - The pure endowment value was obtained by truncating the calculation to five decimals, then rounding to the next higher cent.

24. Page 23 - Days of ET was obtained by truncating the calculation to three decimals, then rounding to the next higher day.

25. Page 24 - Paid-up values were obtained by truncating the calculation to five decimals, then rounding to the next higher cent.

26. Page 25 - Appendix A was deleted and new checking specifications developed to assure accuracy of the printed figures in the 4% tables as well as consistency with the earlier published tables. In general these checks were similar to those indicated in the 1961 checking specifications.

27. Page 27 - Interest constants developed for use in computing functions at 4% interest were as follows:

	4%
	10 significant figures
$\frac{i}{\delta}$	1.019869268
$\frac{\delta - d}{\delta^2}$	.4935268093
$\frac{i - \delta}{\delta^2}$	.5066013859
	6 significant figures
$\frac{d}{\delta}$	.980644