Article from:

Health Section News

April 2000 – No. 37

Credibility Theory for the Health Actuary: The Need for an Inter-Company Experience Study

by James M. Robinson

As health actuaries, we must frequently assess the credibility of data upon which we base pricing, valuation, and other product management decisions. The importance of evaluating data credibility is clearly indicated in Standard of Practice 25 as well as several insurance laws and regulations relating to the use of company-specific claims experience. While there is extensive literature relating to modern credibility theory and methodology, we too often resort to unnecessarily simplistic or arbitrary methods in assigning weight to a block's distinguishing characteristics and observed experience.

The Society of Actuaries Credibility for Health Coverages Task Force has taken steps to provide health actuaries with the tools to properly apply modern credibility theory, including (1) a two-day seminar to present and demonstrate credibility formulas and (2) efforts to specify the inter-company experience data needed to calibrate those formulas. This article summarizes key aspects of both of these steps, especially the need for industry-wide claims data to properly apply credibility theory.

Competing Estimators

Modern credibility theory seeks to assess the relative reliabilities of two or more sources of information relating to a parameter of interest, such as next year's expected per-member-per-month (PMPM) claim cost (pure premium) for a particular insured group. While older approaches to credibility theory might ask, "Is this data source credible?" we now ask, "Which data source is more credible?" or "How can we combine estimates from two or more sources to maximize the reliability of the resulting blended estimate?" Answering these questions requires that we consider the sources of estimation error associated with each data source.

Suppose we are interested in estimating Group A's true underlying PMPM claim rate, hereafter denoted as μ_A. This estimate of μ_A might be used in an experience return calculation for Group A or, after inflation-adjustment, as the basis for rerating Group A. We consider two reasonable estimators. One estimator is taken from a hypothetical industry claim table which, after considering Group A's age/sex distribution, benefit structure and underwriting method, yields a PMPM estimate of $M_A = \$234.44$. The second estimator is Group A's average PMPM claims, $X_A = \$278.14$, observed during the most recent accounting period.

Modern credibility theory suggests an optimal weighted average of M_A and X_A,

$$Y_A = Z_A X_A + (1 - Z_A) M_A,$$

where Z_A, Group A's credibility factor, is determined to minimize the mean squared error (MSE) in using Y_A to estimate μ_A. This optimal credibility factor is inversely proportional to the MSE in using X_A alone relative to the MSE in using M_A alone. That is,

$$Z_A = MSE_X^{-1} / (MSE_X^{-1} + MSE_M^{-1}).$$

How much more reliable is Y_A than X_A or M_A as an estimate of Group A's true PMPM claim rate? It is easily shown that the MSE in using Y, MSE_Y, is given by the equation:

$$MSE_Y = 1 / (MSE_X^{-1} + MSE_M^{-1}) = Z_A\, MSE_X = (1 - Z_A)\, MSE_M.$$

Since Z_A and $1-Z_A$ are fractions, we know MSE_Y is smaller than both MSE_X and MSE_M. Note also that this result implies that MSE_Y is no less than half the lesser of MSE_X and MSE_Y. That is, we cannot expect this simple blending to reduce estimation error by more than 50%. Y_A will not produce results that are an order of magnitude better than are available from X_A or M_A separately.

To compute Z_A we must first estimate MSE_X and MSE_M. MSE_X arises from the variation of the average of individual claims within Group A about μ_A. MSE_M arises from two sources, the variation of inter-company tabulated rates from the true industry-wide PMPM rates, and, the variation of Group A's true PMPM rate, μ_A, from the true industry-wide rate. For convenience, let's call the underlying true industry-wide PMPM rate as it relates to Group A as α_A. Quantifying these sources of error requires that we formulate and fit a statistical model to the underlying claim process.

Mixed Effect Models

The previous example represents the simplest application of credibility modeling. More complex situations involve multiple sources of information regarding a group's expected claim experience. For example, the group-specific variation about an industry-wide risk-adjusted average might be composed of insurer-level effects, group-level effects, and insured-level effects. If we again let X_A denote Group A's observed average claim rate from recent experience, then the previous example assumes a model of the form,

$$X_A = \text{"fixed effect"} + \text{"group effect"} + \text{"sampling error"} = \alpha_A + (\mu_A - \alpha_A) + \varepsilon_A.$$

A more elaborate model might look like,

$$X_A = \text{"fixed effect"} + \text{"insurer effect"} + \text{"group effect"} + \text{"insured effect"} + \text{"sampling error"}$$

The "fixed effect" represents the impact of observed risk factors, such as age, sex, benefit type and underwriting, on the tabular PMPM estimate obtained from the inter-company study. In other words, α_A is the true industry-wide PMPM rate for groups sharing Group A's observed risk profile. The myriad of other factors (observed and unobserved) that influence Group A's true PMPM claim rate are grouped by source in the remaining "random effect" components.

The "insurer effect" represents the impact on the expected group claim rate of factors associated with the insurer (e.g., marketing strategy, underwriting expertise and methodology), that are not completely reflected in the risk-adjusted "fixed effect." All groups within a specific insurer would share the same "insurer effect" value. This value would vary from insurer to insurer throughout the industry, with the average effect being zero.

The "group effect" represents the impact of characteristics of the group, such as geographic location and industry type that are not reflected in the "fixed effect" or the "insurer effect." The "group effect" is shared by all insureds within the same group. The effect varies from group to group, but has an average value of zero.

The "insured effect" represents the impact of aspects of the individuals within a group not already reflected in fixed or the other random effects, such as athletic habits and generic disposition. This effect would be unique to each insured in the group. New entrants to the group would share the same insurer and group effects, but would introduce new insured-level effects to the group's expected claim rate. Repeated observations over time from the same insured would share the same "insured effect."

The "sampling error" is associated with random fluctuation of actual average claims rate about the true PMPM value for Group A.

Fitting these "mixed effect" models involves estimating fixed effect parameters and the variances of the random effect components. This requires individual insured claim data from several companies and groups. Fixed effect parameter estimation is similar to conventional regression analysis. Such analysis provides best estimates of the parameters, as well as assessments of the reliability of the parameter estimates. Our focus with the random effect components is on variance estimates. Temporary estimates of the random effects for each contributing company, group and individual are employed to impute the variances of the random effect terms. Absent modern computing technology, the volume of calculations would be prohibitive. Luckily, the computing hardware and software (e.g., the SAS MIXED procedure) exist to allow the authors of industry tables to fit these mixed effect models to inter-company claim data. In fact, the task force was able to fit such a model to data provided by one of its members. While the results were encouraging, it was clear that a much larger volume of claim data was needed to reasonably estimate the random effect variances.

Once fit, these models can be used by actuaries at large to estimate the unobserved random effects (insurer, group and individual), which are used as adjustments to M_A, the fixed effect estimate. This process involves blending industry-wide, insurer-level, group-level and insured-level claim rate observations. The mathematics expands from a simple weighted average of M_A and X_A, to a matrix weighted average of vectors of candidate estimators, but remains within the reach of a company actuary with access to spreadsheet software.

Use of Hypothetical Inter-Company Results

In this section, we demonstrate how the results of a hypothetical inter-company study might be employed by a health actuary to blend industry-wide, company-wide, and group-specific data to estimate the true PMPM claim costs for Group A. To simplify the presentation, we assume only a few fixed effects (underwriting / benefit type, age group and sex) and only two random effects, an insurer effect and a group effect. So, the model form is:

$$X_A = \alpha_A + \beta_C + \gamma_A + \varepsilon_A, \text{ where,}$$

α_A denotes the true average fixed effect for Group A,

β_C is the realized value of the insurer effect for Company C, the insurer of Group A,

γ_A is the realized value of the group effect for Group A, and,
ε_A is the sampling error for Group A.

We wish to estimate the realized value of $\mu_A = \alpha_A + \beta_C + \gamma_A$; i.e., the true PMPM claim rate for Group A. In this example, assume that we have four observed values that can be used to estimate μ_A:

X_A, the observed PMPM average claim rate for Group A

M_A, the inter-company table estimate of α_A, derived as the sum of the tabular PMPM claim cost values for each member of Group A

X_C, the observed PMPM company-wide average claim rate for Company C, and

M_C, the inter-company table estimate of α_C, the company-wide fixed effect for Company C, derived as the sum of the tabular PMPM claim cost values for each member covered by Company C.

From these observed values, we can construct three reasonable estimators of μ_A:

X_A, since X_A varies about μ_A,

M_A, since μ_A varies about α_A and M_A is an estimator of α_A, and,

$X_C - M_C + M_A$, since μ_A varies about $\alpha_A + \beta_C$, M_A is an estimator of α_A, and $X_C - M_C$ is an estimator of β_C.

We consider a linear blend of these estimates,

$$Y_A = Z_A X_A + Z_C (X_C + M_A - M_C) + (1 - Z_A - Z_C) M_A.$$

The weights, Z_A and Z_C, are computed to minimize MSE_Y, the mean squared in using Y_A to estimate μ_A. MSE_Y is a quadratic function of Z_A and Z_C in which the coefficients are functions of the variances and covariances of M_A, M_C, and the random effect terms of the model. The quadratic function can be differentiated with respect to Z_A and Z_C to obtain z-values that minimize MSE_Y.

(continued on page 8, column 1)

Credibility Theory for the Health Actuary: The Need for an Inter-Company Experience Study

continued from page 7

Assume that our inter-company table includes estimates of the random effect variances, i.e. $Var(\beta) = 20^2$, $Var(\gamma) = 30^2$, and $Var(\varepsilon) = 2{,}000^2$. In the following table, fixed effect M-values and standard deviations are shown for 48 combinations of underwriting / benefit type, sex, and age group. Starting with these tabulated values, the health actuary can derive a credibility-adjusted estimate of Group A's expected monthly claim cost. The table below shows several of the necessary intermediate calculations.

For each of the 48 fixed effect cells, company-wide and Group A insured counts are shown, along with weighted average fixed effect estimates of $M_C = 197.34$ and $M_A = 234.44$, respectively. The corresponding variances are $Var(M_C) = 11.08$ and $Var(M_A) = 27.62$ and the covariance is $Cov(M_A,M_C) = 11.08$.

The observed company-wide and Group A average PMPM claim costs are $X_C = 221.52$ and $X_A = 278.14$, both somewhat greater than the tabular fixed effect estimates. X_A alone is a reasonable estimate of Group A's expected average claim cost. We need to adjust the company-wide average, however, to reflect the difference between company-wide and Group A fixed effect factors. A reasonable adjusted value is $X_C + M_A - M_C = 258.63$. Finally, the tabular estimate of Group A's fixed effect, $M_A = 234.44$, is a third reasonable estimate of Group A's expected average monthly claim cost.

There are n = 36,096 company-wide insureds and $n_A = 1{,}000$ Group A insureds. We also see that there are 20 insured groups comprising the company-wide data.

Fixed Effects					Company		Group A	
UW/Benefit	Sex	Age Grp	Average	Std. Dev.	#	%	#	%
A	M	<25	100	20.0	1,000	2.8%	55	5.5%
A	M	25-34	130	14.1	2,000	5.5%	105	10.5%
A	M	35-44	185	12.6	2,500	6.9%	160	16.0%
A	M	45-54	260	14.1	2,000	5.5%	150	15.0%
A	M	55-64	375	16.3	1,500	4.2%	110	11.0%
A	M	65+	515	31.6	400	1.1%	45	4.5%
A	F	<25	90	25.8	600	1.7%	33	3.3%
A	F	25-34	117	18.3	1,200	3.3%	63	6.3%
A	F	35-44	167	16.3	1,500	4.2%	96	9.6%
A	F	45-54	234	18.3	1,200	3.3%	90	9.0%
A	F	55-64	338	21.1	900	2.5%	66	6.6%
A	F	65+	464	40.8	240	0.7%	27	2.7%
B	M	<25	85	16.9	1,400	3.9%	0	0.0%
B	M	25-34	111	12.0	2,800	7.8%	0	0.0%
B	M	35-44	157	10.7	3,500	9.7%	0	0.0%
B	M	45-54	221	12.0	2,800	7.8%	0	0.0%
B	M	55-64	319	13.8	2,100	5.8%	0	0.0%
B	M	65+	438	26.7	560	1.6%	0	0.0%
B	F	<25	77	21.8	840	2.3%	0	0.0%
B	F	25-34	99	15.4	1,680	4.7%	0	0.0%
B	F	35-44	142	13.8	2,100	5.8%	0	0.0%
B	F	45-54	199	15.4	1,680	4.7%	0	0.0%
B	F	55-64	287	17.8	1,260	3.5%	0	0.0%
B	F	65+	394	34.5	336	0.9%	0	0.0%
Total			197.34		36,096	100.0%	1,000	100.0%
				Fixed Effect Estimates	197.34		234.44	
				Estimate Variances	11.08		27.62	
				Estimate Covariance	11.08			
				Observed Average Claims	221.52		278.14	
				Number of Groups in Company	20			

The actuary is now ready to compute Z_A and Z_C. The two-by-two system of equations resulting from setting the derivatives of MSE_Y with respect to Z_A and Z_C equal to zero is summarized as:

$$\begin{bmatrix} 566.90 & 546.83 \\ 546.83 & 5327.62 \end{bmatrix} \cdot \begin{bmatrix} Z_C \\ Z_A \end{bmatrix} = \begin{bmatrix} 436.02 \\ 1327.62 \end{bmatrix} \Rightarrow \begin{bmatrix} Z_C \\ Z_A \end{bmatrix} = \begin{bmatrix} 566.90 & 546.83 \\ 546.83 & 5327.62 \end{bmatrix}^{-1} \begin{bmatrix} 436.02 \\ 1327.62 \end{bmatrix} = \begin{bmatrix} 58.7\% \\ 18.9\% \end{bmatrix}$$

So, the optimal weighting of the three estimates is 18.9% of the Group A average, 58.7% of the company-wide adjusted average, and 22.4% of the inter-company fixed estimate for Group A. The following table shows the results of applying these weights.

Source	**Estimate**	**weight**	**Stdev**
Tabular Estimate for Group A	234.44	22.4%	36.44
Observed Group A	278.14	18.9%	63.25
Observed Company (adjusted)	258.63	58.7%	31.98
Blended	256.89	100.0%	28.65

Also shown in the previous table are the standard deviations of each of the separate estimators, as well as the optimal blended estimator. You can see that the blended estimate is more reliable than any of the separate estimators.

The actuary can also extract the optimal value of Z_A subject to $Z_C = 0$ and the optimal value of Z_C subject to $Z_A = 0$ by setting the off-diagonal entries in the matrix to zero and resolving the system. This produces values of 24.9% and 76.9% of Z_A and Z_C, respectively. Application of these restricted cases is shown in the following tables.

Using Only Group A Average Observation

Source	Estimate	weight	Stdev
Tabular Estimate for Group A	234.44	75.1%	36.44
Observed Group A	278.14	24.9%	63.25
Blended	245.33	100.0%	31.57

Using Only Company-Wide Average Observation

Source	Estimate	weight	Stdev
Tabular Estimate for Group A	234.44	23.1%	36.44
Observed Company (adjusted)	258.63	76.9%	31.98
Blended	253.04	100.0%	31.50

So, if only the inter-company tabular estimates and the Group A average are employed (the original situation in the original example), most of the weight (75.1%) should be given to the inter-company estimate. If only the inter-company study and the company-wide claim cost estimate are employed, then most of the weight (76.9%) is applied to the company-wide estimate.

While purely hypothetical, the previous example shows how an actuary can use inter-company estimates of fixed effect parameters and random effect variances to determine appropriate weighting factors. No elaborate statistical analysis package is needed. All that is required is an understanding of the methodology and a spreadsheet-level computational assistance.

Conclusions

Computational technology has advanced to the point that it is now practical to apply modern credibility methods to everyday problems faced by health actuaries. The Task Force has demonstrated the feasibility of these calculations in several seminar case studies, including the analysis of a large block of medical expense insurance data. In this process, it has become clear that an inter-company experience study is needed to reliably estimate the variance and covariance parameters of the medical expense claim process.

- How much variation is there in expected claim costs from insurer to insurer throughout the industry?
- How much variation exists between expected claim costs from group to group for a given insurer?
- How much variation is there from insured to insured within a group?
- For a specific insured, what is the variance of actual claims about the insured's expected claims for a period?
- How are these variations correlated within a given time period and across time periods?
- What characteristics of the insured, the group, the insurer, and the industry influence these variances and covariances?

These "parameters" are the last critical pieces needed to compute optimal blending weights when combining claim cost estimates from multiple sources, such as group-specific, company-wide, or industry-wide average claim costs, allowing the actuary to make the most effective use of available data and, equally important, to assess the reliability of the resulting estimates.

The SOA distributed a recent survey to Chief Actuaries of all health insurance companies asking for their capacity and interest in contributing data to build the first ever industry table of major medical rates as discussed in this article. Certainly, the pragmatic value of this article is only measured by the success of making such a table available. We don't want the efforts by the SOA's Credibility for Health Coverages Task Force to simply end with just another academic discussion. We need a large contribution to bring real value to our profession in this area of health insurance education for the actuary. It was identified as the number one need by actuaries practicing in health insurance. If you haven't seen the request, ask your chief actuary if you work in a health insurance company or plan. Encourage him/her to respond favorably.

(The above article was edited by Thomas J. Stoiber, FSA, MAAA).

James M. Robinson, FSA, MAAA, is Senior Scientist at the Center for Health Systems Research & Analysis at the University of Wisconsin at Madison. He can be reached at jim@chsra.wisc.edu.