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Warm and Fuzzy ... And Real!

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(This is part one of a two-part article. Part two will be in our next issue; and both parts will be incorporated into a presentation at the 2014 SOA Annual Meeting in Orlando (October 25) and a Forecasting & Futurism Webcast in December, 2014)

Fuzzy logic is not new. It has been around for a long time.

The previous two sentences contain a few examples of fuzzy logic in our real life environment.

- New—when does new begin or end?
- Around—nearby? How near? How prevalent?
- Long—how long is long? How many years, months, hours, minutes, seconds?

We learn fuzzy logic as children, well before we enter formal schooling:

"Don't touch that! It's hot."

"We had lots of fun and good food at Grandma's kitchen."

So, why should it come as a surprise to us to learn that fuzzy logic is often a better methodology than "crisp" logic for many actuarial modeling situations?

Crisp set theory and crisp logic are more recent terms for what we used to consider set theory and Boolean logic. In crisp set theory an item is either a member of the set or not a member of the set. We can easily say that 0.96 is a member of the set {-5.7, 0.96, 7} and that -2.5 is not a member of that set. Fuzzy set theory deals with sets where membership does not have to be strictly in or out. Take Tall for example. A person 69 inches might be considered Very Tall for a 10 year old, or Tall for an adult woman; but Not Tall for a basketball player.

I'm six feet tall; and I used to consider myself a little taller than the average male. When my wife and I first started dating, I met her 6' 4" brother and her 6' 6" cousin. Her dad liked me even though he thought I was a bit "short."

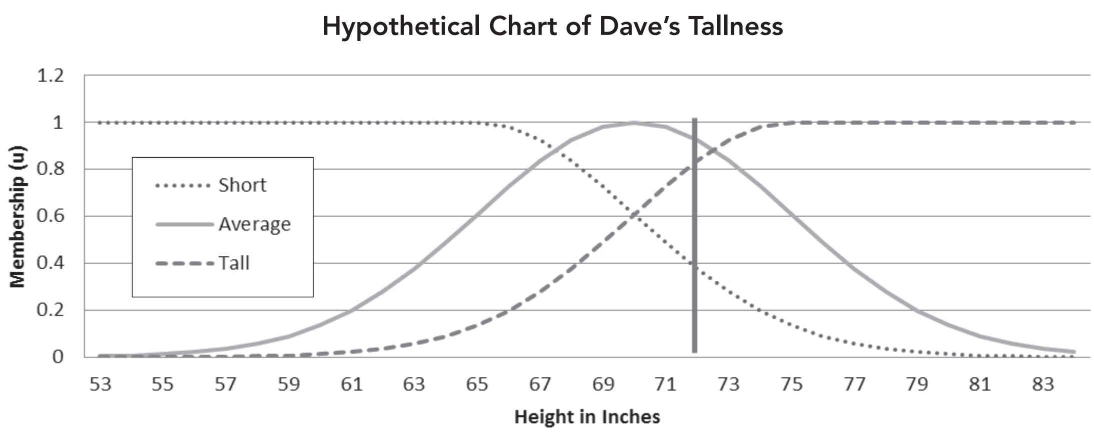

Figure 1 - A hypothetical chart of 'Tallness'. Although it is often tempting to consider membership in a fuzzy set as the probability of being in that set, that is misleading. Note that one data point can be a member of several sets, and the membership values do not have to sum to 1.

Soon after we moved from Connecticut, on the East coast of the United States, to St. Louis, in the Midwest, we were eating in a restaurant and the waitress asked if we wanted any dessert. She offered sherbet among the selections, but pronounced it sherbert [sic]. I picked up on this right away. It was the same way we mispronounced sherbet in my section of Connecticut; and I asked if she was from back East. She enthusiastically said "yes!" ... she was from East St. Louis! Our definitions of East differed by about 1,000 miles. Some of my Asian friends would consider my definition as laughable, as they think of Japan as back East.

Tall, East, Near, Hot and *Many* other adjectives are what Fuzzy Logic considers Linguistic Variables. Like the more conventional variables we use in our actuarial models, they can take on specific values (72 inches, 86 degrees longitude, 3.8 miles, 40 degrees Celsius, 7,583,278); but they usually imply a range and that range is relative to some other ranges. It is not necessary to tell your child that the food is 160.53 degrees Fahrenheit. The more important information, that you can say quickly and your child can understand immediately, is that it is hot, and might burn his tongue.

Likewise, a life insurance underwriter has neither the time nor the data to determine that an applicant for this $5,000,000 policy will live for another 17.45 years with a standard deviation of 5.6 years. She is under time (and data) constraints; and must quickly decide if this person is a preferred, standard, substandard, or uninsurable risk.

Her decision may be based on a glycohemoglobin blood test (aka A1c - longer term sugar level) result in the normal range, a body mass index (BMI) of overweight, but not obese, and a family history (one or more close family members) of diabetes but a blood pressure only slightly elevated over normal for the applicant's age and gender.

She may have results from several blood tests for this applicant, and she compares them to the 'reference range' values such as shown in Figure 2.

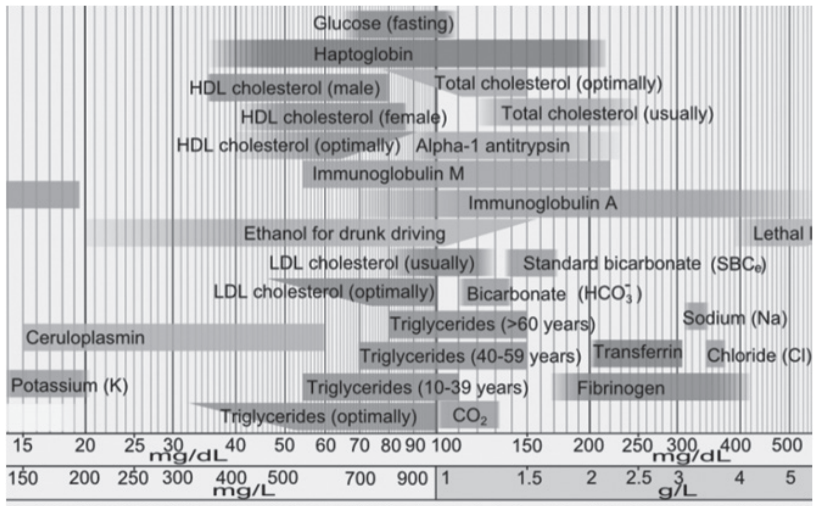

Figure 2 - Subset of common blood reference ranges—source: author's subset of excellent (award winning) Wikipedia image //upload.wikimedia.org/wikipedia/commons/thumb/c/cb/Blood_values_sorted_by_mass_and_molar_concentration.png contributed by Mikael Häggström, MD and released under the Attribution-Share Alike 3.0 Unported license

Fuzzy logic provides a way to work with these linguistic variables and reach a quantitative (if desired) answer.

According to the Mayo Clinic, the normal fasting blood sugar range for an individual without diabetes is 70-100 mg/dL (3.9-5.6 mmol/L).[1] Does that mean that every "normal" person without diabetes will have a fasting blood sugar level in that range? If you have 69.9 mg/dL or 100.1 mg/dL does that automatically make you less healthy than an individual with 70 or 100 milligrams/deciliter? Is the range truly that crisp as in Figure 3?

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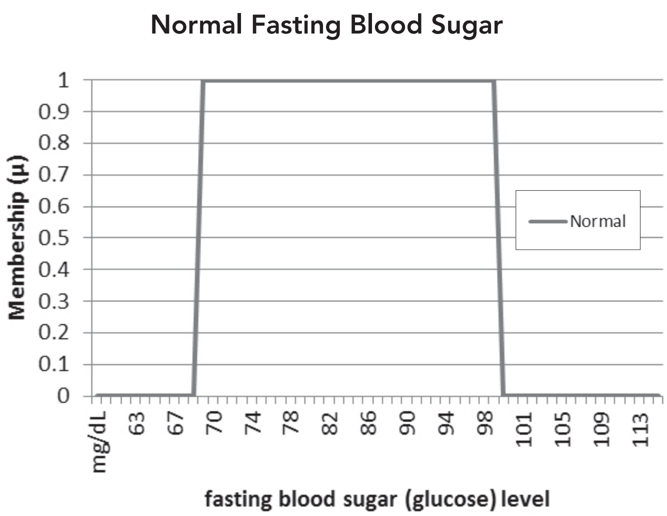

Figure 3 - Crisp set theory representation of "Normal" fasting blood sugar

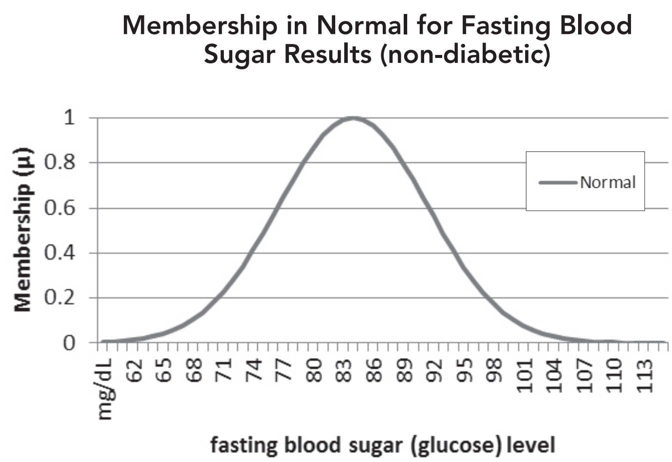

Figure 4 - Fasting Blood Sugar (Glucose) Results (assuming a normal distribution with mean 85 mg/dL and standard deviation of 7.5 mg/dL)

Actually, no! One drawback of this binary classification approach is the conflict between sensitivity and specificity. Sensitivity measures the proportion of actual positives which are correctly identified as such. Also called a True Positive, this measures the percentage of people tested for dread disease X who actually have dread disease X. Specificity measures the True Negative rate—those people who do not have dread disease X and are correctly diagnosed as not having it. In general, laboratory testing attempts to maximize specificity, even if it means missing a few positives.[2]

A reference range is usually a set of values 95 percent of the normal population falls within. A better view of this might be that of Figure 4:

ONE DRAWBACK OF THIS BINARY CLASSIFICATION APPROACH IS THE CONFLICT BETWEEN SENSITIVITY AND SPECIFICITY.

Now that we know the potential advantages of fuzzy logic, how do we apply it?

It's as simple as one, two, three:

1. Fuzzification – convert your input and output to linguistic values, utilizing ranges and membership functions.
2. Apply rules (from your experience or knowledge base) using fuzzy logic.
3. Defuzzification – convert your results to the form you want (often a numeric result).

OK, that's probably not apparent, so let's look at a very simple example in order to better understand this.

Let's assume that Applicant James, age 25, has applied for a \$20,000 life insurance policy. James lives in a state considered "medium" for cocaine usage; but he works five miles away in a state classified as "high" for such usage. Assume also, that we are back in 1996, when the following chart may have applied to the situation.

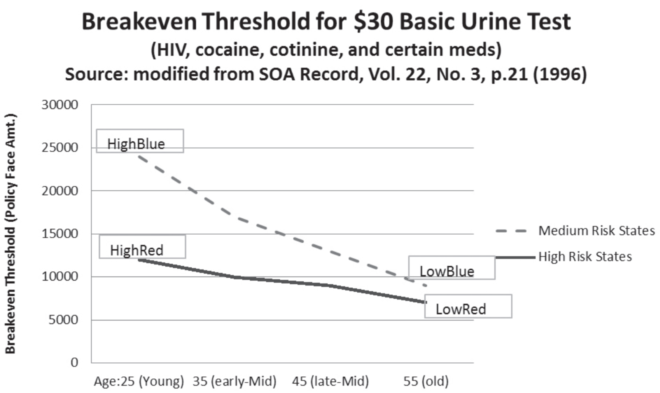

Figure 5 – Breakeven threshold (point at which test becomes cost effective) based upon applicant age and state of residence. Some states have much higher incidence of cocaine usage. Note that prices and state characteristics may have changed significantly since 1996. Chart built from Table in SOA Record, Vol. 22, p.21 and modified for this example.

Let's assume that an underwriting rules table has been developed for this condition. Here is a portion of the rules that were developed:

1. If Age is Young and HighStateActivity is Significant then Threshold is HighRed
2. If Age is Old and HighStateActivity is Significant then Threshold is LowRed
3. If age is Young and HighStateActivity is Not Significant then Threshold is HighBlue
4. If age is Old and HighStateActivity is Not Significant then Threshold is LowBlue

Obviously, a real situation would have more rules, since I have not even covered the two mid-age groups and we would normally have more information and criteria.

One advantage of this type of rule set is that it uses a more natural language. Underwriters are used to using natural language terms such as *Overweight, Obese, Hypertension, Diabetic,* etc. versus a series of numbers. Plus, using these as parameters, the definition of terms like *Obese* and *Hypertension* can be refined (and they have been as standards have been changing to reflect new medical study results) and the same rules can apply.

Here we shall define HighStateActivity as the membership in the High risk state; and we'll say that it is *Significant* if that membership is greater than 0.50, and not *Significant* it is 0.50 or less.

Our definitions of *HighBlue, LowBlue, HighRed, and LowRed* here are going to be very simple. We shall make them the endpoint values for the Blue line (the dashed line if you are not seeing this in color) and the Red line (the solid line). According to the underlying table from the SOA Record, this would mean that they would be as follows: *HighBlue*=$24,000; *HighRed*=$12,000; *LowBlue*=$9,000; *LowRed*=$7,000.

Since there are no mandates for how you choose your membership functions,[3] we can go ahead here and say that James is Young with a membership of 1.00 and Old with a membership of 0.0 and then we'll just have to deal with one linguistic variable, the HighStateActivity, for our example. We will assume a distribution of membership (μ) in the two states according to the carefully prepared proximity study (Not!) below in Figure 6 and the similarly prepared time chart of Figure 7 as follows:

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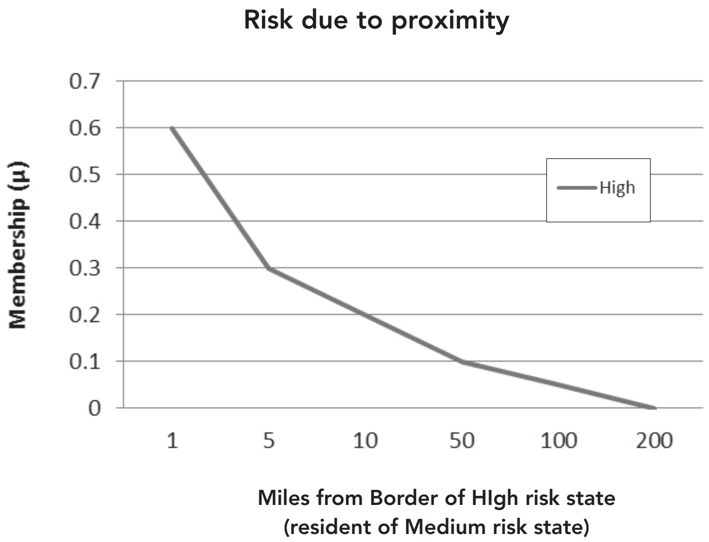

Figure 6 - Assumed memberships for resident of Medium risk state living near High risk state (hypothetical values)

Distance to Border (miles)	μ^{High}_{miles}
1	0.60
5	0.30
10	0.20
50	0.10
100	0.05
200	0

Membership due to proximity to the High state

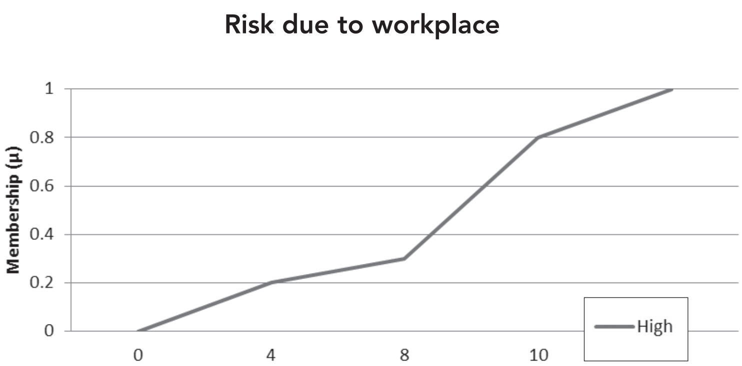

Figure 7 - Assumed memberships for resident of Medium risk state who works in nearby High risk state (hypothetical values)

Time at work (hours)	μ^{High}_{miles}
0	0
4	0.2
8	0.3
10	0.7
12+	1.0

Membership due to work in the High state

We know that James resides in the Medium risk state so his membership in that state is

μ^{Medium} = 1.0 (see note[4] for discussion)

I am going to assume here that James works 9 hours per day at his job in the High risk state, just so that we can use some fuzzy set theory here (and even do an interpolation … but no fancy stuff). What do we have going here? Essentially, we have a union of two sets. The two sets have some overlap (James must spend some time in the High risk state in order to go to work there) but he may also spend time there after work. Since he lives only five miles away, he might go there on weekends or evenings; but this is not required.

In fuzzy logic, there are several ways to handle membership in the union of two fuzzy sets. A very popular (and simple) one, that Lotfi Zadeh (founder of fuzzy logic) proposed, is to take the maximum of the two set memberships.

Thus, $\mu^{High} = \max(\mu^{High}_{miles}, \mu^{High}_{work}) = \max(0.3, (.3+.8)/2) = .55$ which is *Significant*, so we will use our *HighBlue* and *HighRed* result values in the defuzzification process.

and $\mu^{Medium} = 1.0$ since James lives in the Medium risk state

James applied for $20,000 of coverage.

The threshold for his age (25) would be

$$\frac{\mu^{High}}{\left(\mu^{High}+\mu^{Medium}\right)} \times HighRed + \frac{\mu^{Medium}}{\left(\mu^{High}+\mu^{Medium}\right)} \times HighBlue$$
$$= \frac{.55}{(1+.55)} \times HighRed + \frac{1.0}{(1+.55)} \times HighBlue$$
$$= \frac{1.0}{(1+.55)} \times \$12{,}000 + \frac{1.0}{(1+.55)} \times \$24{,}000 = \$19{,}742$$

so a urine test would be cost effective (but just barely).[5] Fuzzy logic has given us an alternative way of addressing a problem with incomplete data.

Recap:

1. Fuzzification—We converted our input and output to linguistic values (*Young, Old, HighStateActivity, Significant* for input; *HighRed, LowRed, HighBlue, LowBlue* for output), utilizing ranges and membership functions.

2. We applied rules (based on *age, stateactivity, significance*) using four fuzzy logic rules we defined to determine our outcome (threshold).

3. Defuzzification—We converted our results to the form wanted (in this case, we just took the endpoint values and computed an average, weighted by membership).

This was a contrived example where I tried to avoid nearly all mathematics and programming. In practice, *HighBlue* and the other linguistic values and variables would be shapes where you would use centroids, matrices, and various types of distribution functions for memberships.

Yes, you could have done this example with crisp logic. Most destinations can be reached by several paths. Fuzzy

FUZZY LOGIC OFFERS A MORE NATURAL LANGUAGE, A WAY TO DEAL WITH IMPRECISE OR INCOMPLETE DATA, AND A WAY TO GROUP ITEMS TOGETHER SO THAT COMPLEXITY IS REDUCED.

logic offers a more natural language, a way to deal with imprecise or incomplete data, and a way to group items together so that complexity is reduced, rule sets can be smaller, and speed of solution can be increased. Consider it as one more arrow in your quiver of actuarial tools.

More sophisticated examples would also be likely to employ hedging. If *Tall* has a membership value in the range from 0 to 1.0, then *Very Tall* could be defined as the square of this value; and *Nearly Tall* might be the square root of the *Tall* membership value. In this way, we keep a consistent relationship, where the *Very Tall* is more selective than *Tall*, which in turn is more selective than *Nearly Tall*.

The logical question (fuzzy or crisp) you may be asking is "why isn't fuzzy logic in wider use in the actuarial profession?" In the actuarial area, fuzzy logic is still a relatively new paradigm. It is a shift from old ways of thinking; and that results in initial resistance from those more comfortable with their older toolset.

George Klir and Bo Yuan stated this eloquently in their book, *Fuzzy Sets and Fuzzy Logic – Theory and Applications*:[6]

"Each paradigm, when proposed, is initially rejected in various forms (it is ignored, ridiculed, attacked, etc.) by most scientists in the given field. Those who support the new paradigm are either very young or very new to the field and, consequently, not very influential. Since the paradigm is initially not well-developed, the position of its proponents is weak. The paradigm eventually gains its status on pragmatic grounds by demonstrating that it is more successful than the existing paradigm in dealing with problems that are generally recognized as acute. As a rule, the greater the scope of a

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paradigm shift, the longer it takes for the new paradigm to be generally accepted."

Surprisingly, although fuzzy logic was first proposed in the United States,[7] it was most enthusiastically accepted in Asia. Today, your Canon or Minolta camera probably has a fuzzy logic control circuit to stabilize the pictures you take. Your Honda or Nissan auto transmission selects the optimal gear ratio for your driving style and the engine load conditions; and my Toyota Prius even knows when to switch to the electrical motor or gasoline engine, or both, for the best mix of power and fuel economy. Your Sharp refrigerator decides when to turn on defrost or cooling cycles based on your needs. The newer washing machines from Korea and Japan adjust their strategy based upon the level of dirt, the water level, the fabric type and the size of the load. In Japan, "Fuzzy" has become a sort of quality seal proudly displayed on consumer products. One theory about the difference between Asian embracement of fuzzy logic is that it more closely fits with the concept of yin-yang, where contrary forces interact to various degrees in the natural world. Whatever the reason, it appears that the West was slower to adopt fuzzy logic.

Figure 8 - Yin-Yang nature of the natural world is better exemplified by fuzzy, rather than crisp set theory.

An encouraging recent exception in the actuarial area is an excellent research paper jointly sponsored by the CAS/CIA/SOA Joint Risk Management Section.[8]

As actuaries, we have a natural inclination towards precision. Yet, as Matisse so aptly reminded us, "Precision is not truth."[9] Reality is a bit more fuzzy, and fuzzy logic is better suited for the cases where we have imprecise data and incomplete subject matter expertise.

Next issue, we'll go into more depth and examples of the mechanics involved with fuzzy logic. You can get very sophisticated with matrix algebra, exotic distribution functions for the fuzzification and a host of defuzzification techniques. In the meantime, please read Jeff Heaton's article ""Fuzzy Logic in R" in this issue. We tried to coordinate in this issue so that this article could focus on the "Why" and some theory, and his on the "How" for a jump start. Jeff shows how to use the host of packages available for plug-and-play processing of fuzzy logic in the programming language R.

As Lotfi Zadeh, the founder of fuzzy logic said in 1973,

"We must exploit our tolerance for imprecision.[10]

Enjoy being less crisp, and more real!

Dave Snell

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END NOTES

1 http://www.mayoclinic.org/diseases-conditions/diabetes/expert-blog/blood-glucose-target-range/bgp-20056575

2 ALU101 Textbook – 5th Edition, p. 115, Association of Home Office Underwriters.

3 Develop your membership function to fit your problem. Sometimes it is determined heuristically and sometimes it is a subjective decision based on your experience or intuition. The fuzzy logic literature shows a lot of triangular, trapezoidal, Gaussian and bell-shaped functions. We'll investigate some of them next time; but the focus of this article was to keep the mathematics very simple.

4 You might argue that the membership in the Medium risk state should decrease as some threshold is passed of membership in the High risk state; and you may be correct! There is much subjectivity in the choice of membership function distributions. One answer or standard does not fit all situations.

5 Obviously, we are applying group methods to a single individual; and we might be wrong. In general though, we expect James to be a representative sample of the group.

6 Klir, George and Yuan, Bo [1995], Fuzzy Sets and Fuzzy Logic – Theory and Applications, Prentice Hall P T R, Upper Saddle River, New Jersey,1995, p.30

7 Lotfi Zadeh, a Professor at University of California, Berkley, is considered the founder of fuzzy mathematics, fuzzy set theory, and fuzzy logic. He published his seminal work, "Fuzzy sets", in 1965

5 Shang, Kailan and Hossen, Zakir [2013] "Applying Fuzzy Logic to Risk Assessment and Decision-Making", CAS/CIA/SOA Joint Risk Management Section. Note: Arnold Shapiro and others have also written research papers on the utilization of fuzzy logic. Search for fuzzy logic on the SOA website for a current list of actuarial papers.

6 Henri E. B. Matisse, 1869-1954, as quoted in Ross, Timothy [2010] "Fuzzy Logic with Engineering Applications, Third Edition, John Wiley and Sons, Ltd., UK.

7 L.A. Zadeh,"Outline of a new approach to the analysis of complex systems and decision processes",IEEE Trans. Syst., Man, Cybernetics, SMC-3 (1973), pp. 28–44