



SOCIETY OF ACTUARIES

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# ACTUARIAL TECHNIQUES IN APL

by D. W. F. Crossley\*

APL is both a unique form of mathematical notation and a programming language. As a notation it has intrinsic appeal and gives rise to many elegant and ingenious definitions of mathematical concepts. As a programming language, or system, it also provides the user with instant evaluation of these concepts. The combination is extremely powerful, and some interesting applications may be demonstrated with respect to annuities.

Consider the annuity  $a_x$ , defined by

$$a_x = \sum_{i=1}^{n-1} v^i i p_x.$$

The quantities  $1, v, v^2, \dots$ , and  $1, p_x, {}_2p_x, \dots$ , are two vectors of equal length, which may be denoted by  $V$  and  $TPX$ , respectively. The above summation is then expressed in APL by the symbols  $+/V \times TPX$ , where  $+/\prime$  is the APL equivalent of  $\Sigma$ . There are no summation limits—these are implicitly defined by the length and content of the vectors  $V$  and  $TPX$ . There is insufficient space to describe how these two vectors may be represented in APL, but neither is difficult or involves more than a short line of coding. The appropriate expressions may be substituted for  $V$  and  $TPX$ , and  $a_x$  may then be evaluated at an APL terminal.

This is a simple enough concept, but at this point it does not have any advantage over the much easier calculation:  $N_x/D_x$ . The vector method, however, is capable of much greater flexibility.

Consider a vector of payments  $p_1, p_2, \dots$ , denoted by  $P$ , having the same number of terms as  $TPX$ . The present value of an annuity returning these amounts each year is  $+/P \times V \times TPX$ , and  $P$  can be specified at will to represent any kind of payment pattern. Missing payments, as in the case of deferred or temporary annuities, are equivalent to payments of zero. Thus annuities of any nature—increasing, decreasing, temporary, deferred, reducing after a given age, or any combination—may all be evaluated by respecifying  $P$ , and it is interesting that by generating a vector of payments increasing by, say, 5 per cent one could evaluate an annuity that offsets inflation.

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Joint-life annuities such as  $\ddot{a}_{xy}$  are represented by

$$+/V \times TPX \times TPY,$$

where  $TPY$  is the vector  $1, p_y, {}_2p_y, \dots$ . In this case the mortality basis for  $(x)$  and  $(y)$  need not be the same, implying that  $TPY$  might refer to probabilities taken either from another mortality table or from another type of decrement table. This expression need not be limited to two lives; thus joint  $n$ -life annuities may be evaluated without using joint-life commutation columns or tables of uniform seniority.

The vector  $V$  has so far been assumed to involve only one interest rate, but this is an unnecessary restriction. It is a trivial problem to create discount factors based on rates  $i_1$  for ten years,  $i_2$  for the next five years, and  $i_3$  thereafter, for example; then all the foregoing expressions become much more powerful because they have the ability to utilize varying interest rates.

Last, there is the problem of annuities payable  $m$  times per year. Given a period interest rate  $i^{(m)}/m$ , and a mortality table of  $m$  entries per year, the year loses its fundamental significance and is replaced by the period defined by  $1/m$ . A mortality table can be expanded by inserting intermediate values based on arithmetic or geometric means, or according to the Balducci hypothesis, for example. The age and all periods should now be made  $m$  times greater, and the identical APL expression will evaluate an annuity of any fundamental period. Moreover, this table may be entered at nonannual points, thereby allowing for calculations at fractional ages.

The above expressions do not rely on commutation columns, but it should be emphasized that they are by no means made redundant by this approach. APL can utilize commutation functions very effectively, and standard calculations should still be performed with them. Beyond the scope of this method is the vector or first-principles approach, which undoubtedly has great merit, and, through these combined procedures, APL will become, or already is, an invaluable aid for many actuaries.

For further information concerning the use of APL please write to D. Crossley, I. P. Sharp Associates, P.O. Box 71, Toronto Dominion Centre, Toronto, Ontario M5K 1E7.

## Letters (Continued from page 4)

### Youth

Sir:

While visiting the cancer center at the National Institutes of Health in Bethesda, Md., where unbelievable breakthroughs in medical research are being developed, I was surprised at the youth of the doctors. Most of the doctors seemed to be under 35. Also, I seem to recall a statement by a math professor of mine that most of the brilliant breakthroughs in math have been made by mathematicians who were under 25.

What about our younger actuaries?

What brilliant breakthroughs are they developing? — The actuarial exams? — Repetitive clerical work? I wonder if actuaries really need to memorize material or do they just need to know where to find that material? What would happen to our profession and to our industry if the actuarial exam syllabus were cut in half and companies were encouraged to contribute two years of their new F.S.A.s' time to centers of research? Perhaps our profession could contribute to objectives higher than larger corporate profits or larger policyholder dividends.

Jim Lewis

## Study Manuals

Sir:

I would appreciate the courtesy of your columns to advise users of the Northeastern study manuals of the following.

To expedite production of the manuals, Northeastern will prepare the basic manual without the later study notes, and then send additional pages to manual purchasers if the later notes constitute a significant part of the exam material. Because of the extra clerical work involved in this subsequent mailing, it will be imperative that order forms for the

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## Letters

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manual be accurately and completely filled out.

Specific information will be sent to all members of the Society preceding each exam period. We expect to be mailing this information by July 1 this year, with order forms, price, and availability date for all of our publications.

Richard L. London  
Assistant Professor  
Northeastern  
University

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## Survivor Benefits for the Uniformed Services

Sir:

Mr. Kriss Cloniger, III, gives an interesting description of the new legislation on Survivor Benefits for the Uniformed Services in the April issue. This plan, replaces the previous Retired Serviceman's Family Protection Plan. This last was intended to be on an actuarial-equivalent basis, fully paid for by the retired members, but was not widely used—probably because of the lack of a "barrier" to the participants. The cost of the new plan is more than implied by the Survivor Benefits since the Government has, in addition, to pay for non-contributory retirement and disability pensions and for the employer's share of Social Security.

The new SBP closely parallels the provisions for survivor benefits for pensioners under the Civil Service Retirement Act. Mr. Cloniger cites the reason for the automatic adjustment of benefits, the CPI increase plus 1%, as being "meant to recognize the lag time in the adjustment process." Careful analysis indicates that this is not so. To accomplish this result would require such an increase to be made for each pensioner only the first time that an adjustment is applicable to him.

The same fallacious reasoning was given by the House of Representatives when it enacted this provision for Civil Service Retirement. When the error was pointed out in the Senate consideration of the legislation, the same basis was retained, but the "rationale" was changed—the 1% supplement was "justified" as letting the pensioners have a share in the increased productivity of the nation (as reflected by wages generally rising more rapidly than prices).

Robert J. Myers

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## Committee on Review

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- We should coordinate closely with the editor of *The Actuary*, in providing good book review coverage, on a more timely basis in *The Actuary* and of a more permanent comprehensive character in the *Transactions*.
- Another important objective is to extend the usefulness of the Society's library. Hopefully the recent addition of the computer bibliography prepared by the Society's Committee on Computer Science will further this aim.

The committee includes ten members who each follow a category of actuarial interest for new texts and for good reviewers. One member of the Committee is responsible for liaison with the Society library. It is not necessarily the job of a member of the Committee to prepare a review himself; a more usual procedure is for him to find an able reviewer with a special interest in the text. We want to share the burdens and pleasures of reviewing among as many members of our Society as possible.

One of the more difficult tasks of the Chairman of the committee is to define the kinds of books for which we will prepare reviews. Several examples may illustrate the kind of judgment that is involved.

- We have decided in general not to review the vast outpouring of texts on consumer issues in life insurance. This decision was reached with some reluctance since clearly an actuarial viewpoint might be useful in evaluating some of the more radical statements and proposals. Nevertheless, we felt that we would overwhelm the *Transactions* if all texts related to life insurance and in particular to the price disclosure and other consumer issues were covered.
- We did decide to review a few carefully chosen books in the literature on ecology. In the Fall 1972 *Transactions* we included two reviews on books related to pollution and population. They were the Club of Rome book, *Limits to Growth* and the important text of the Presidential Commission on *Population and the American Future*.

The Editor of *The Actuary* has also included some texts on the environment and has also reviewed some consumerist items. We hope through the combination

## ARCH

The list of contents for the latest issues (1973.2 and 1973.3) is given below.

### Issue 1973.2

Comments: on G. Berger's "Asset Shares and Anderson's Concept" Richard W. Ziock

Financial Projections and the Pricing of Life Insurance Frank Irish

A Stochastic Model of Bond Prices, Richard W. Ziock

Transcripts from the Waterloo Actuarial Research Conference on Time Series Analysis, 1972 September 28-30

- On the Relevance to Actuaries of Research on the Long Term Interest Rate, James C. Hickman
- Discussion of Fama's paper, Risk Return, and Equilibrium Empirical Tests, Paul M. Kahn
- Box-Jenkins Models for Hospital Insurance, Donald A. Jones
- Overview, J. L. Lewis, Jr.
- Overview, Newton L. Bowers, Jr.

### Issue 1973.3

This is a Special Issue of the written material for Hilary Seal's *SHORT COURSE IN RISK THEORY* for the Actuaries' Club of New York, March, 1973.

### Contents

- Risk Theory: Input
  - Risk Theory: Output
  - Practical forms for  $p_n(t)$  and  $B(\cdot)$
  - Numerical Calculation of  $F(x,t)$ 
    - Computer Output
  - Numerical Calculation of  $U(w,t)$ 
    - Computer Output
- Annotated Bibliography

Subscriptions can still be sent to David G. Halmstad, Area 222, Metropolitan Life Insurance Company, One Madison Avenue, New York, N. Y. 10010. □

of our two publications we will satisfy the tastes of most actuaries.

The Chairman of the Committee on Review welcomes suggestions of texts from all members of the Society and also welcomes any volunteers who wish to prepare reviews. Assignments can easily be made to the proper individual on the Committee on Review. We hope our work provides an important service to the Society's membership, and we think broad participation in reviewing can best assure fulfillment of that hope. □