INVITED TALK - CONFERENCE INVITEE

On a Global Education and Examination System

Jean-Louis Masse
President, International Actuarial Association

Abstract: The International Actuarial Association is studying the possibility of creating a global education and examination system, ideally with the help of universities, to accelerate the growth of the profession and to answer the needs for professional actuaries in the world. The IAA hopes to take a decision soon on this matter. The president of the IAA will talk about the great challenges of this ambitious and promising project.

A Multivariate Tweedie Family with Applications to Risk Measurement

Edward Furman and Zinoviy Landsman
University of Haifa, Israel

Abstract: In the present talk we propose a multivariate extension of the well-known exponential dispersion models. We name this class the multivariate Tweedie family (MTwF) because its univariate marginal distributions correspond to the univariate Tweedie ones. Furthermore, the proposed family possesses a dependence structure, which is reflected by its covariance structure, and it allows good modelling multivariate portfolios with dependent claims or risks. As special cases, MTwF contains, for instance, the multivariate inverse Gaussian, multivariate gamma and multivariate compound Poisson distributions in the sense that their univariate marginals are inverse Gaussian, gamma and compound Poisson respectively. We demonstrate the construction method of MTwF, derive exact density functions of multivariate Tweedie random vectors and calculate higher order moments. Some useful Chebyshev's type inequities are also produced for easy evaluations of higher and lower probabilistic bounds. Lastly, some applications to risk measurement are considered.

Bayesian Inference Resistant to Outliers, using Super Heavy-tailed Distributions, for the Calculation of Premiums

Alain Desgagne, Universite du Quebec a Montreal, Canada
Jean-Francois Angers, Universite de Montreal

Abstract: We assume that the claim sizes for several risks are conditionally independent random variables given the same scale parameter. The Bayesian predictive distribution of a next claim size is used to estimate the pure premium. Robust procedures to conflicting information (prior or outliers) depend mainly on the tail behavior of the likelihood and prior densities. For that purpose, the notion of left and right log-credence is introduced to characterize respectively the left and right tails of a density defined on the positive line. Simple conditions are established to determine the proportion of observations that can be rejected as outliers. It is shown that the posterior distribution converges in law to the posterior that would be obtained from the reduced sample, excluding the outliers, as they tend to 0 or infinity, at any given rate. An example of calculation of a pure premium is given. We compare the log-normal model with the robust super heavy-tailed (log-Pareto type) distributions model.

Extreme Behavior of Multivariate Phase-Type Distributions

Alexandru V. Asimit and Bruce L. Jones
University of Western Ontario, London, Canada

Abstract: This paper investigates the limiting distributions of the component-wise maxima and minima of suitably normalized iid multivariate phase-type random vectors. In the case of maxima, a large parametric class of multivariate extreme value (MEV) distributions is obtained. The flexibility of this new class is exemplified in the bivariate setup. For minima, it is shown that the dependence structure of the Marshall-Olkin class arises in the limit.

Fitting Combinations of Exponentials to Probability Distributions

Daniel Dufresne
University of Melbourne, Australia.

Abstract: Two techniques are described for approximating distributions on the positive half-line by combinations of exponentials. One is based on Jacobi polynomial expansions, and the other on the logbeta distribution. The techniques are applied to some well-known distributions (degenerate, uniform, Pareto, lognormal and others). In theory the techniques yield sequences of combination of exponentials that always converge to the true distribution, but their numerical performance depends on the particular distribution being approximated. An error bound is given in the case of the logbeta approximations.
Some applications are:
Risk theory. It has been known for some time that the probability of ruin is simpler to compute if the distribution of the claims, or that of the inter-arrival times of claims, is rational. The simplfications which occur in risk theory have been well-known in the literature on random walks (see, for instance, the comments in Feller (1971)) and are also related to queueing theory.
Convolutions. The distribution of the sum of independent random variables with a lognormal or Pareto distribution is not known in simple form. Therefore, a possibility is to approximate the distribution of each of the variables involved by a combination of exponentials, and then proceed with the convolution, which is a relatively straightforward affair with combinations of exponentials.
Financial mathematics. Suppose an amount of money is invested in the stock market and is used to pay an annuity to a pensioner. What is the probability that the fund will last until the pensioner dies? The answer to this question is known in closed form if the rates of return are lognormal and the lifetime of the pensioner is exponential; this is due to Yor (1992). In practice, however, the duration of human life is not exponentially distributed, and the distribution of the required stochastic life annuity can be found by expressing the future lifetime distribution as a combination of exponentials.

Modeling and Estimating Individual and Firm Effects with Panel Data

Jean-Francois Angers, Denise Desjardins Georges Dionne and Francois Guertin
Centre de recherche sur les transports, University of Montreal and HEC
Montreal, Canada

Abstract: In this article, we propose a detailed analysis of the modeling and estimation of distributions of vehicle accidents. This analysis uses panel data to account simultaneously for individual as well as fleet effects. The distribution of accidents can be affected by both observable and non-observable factors. Non-observable factors are modeled as random factors.

Using Expert Opinion in Actuarial Science

Michel Jacques and Mathieu Pigeon
Universite Laval, Canada

Abstract: Actuaries may have to model costs for which data are not available, because they are either scarce or confidential. Third party liability coverage can be an example of such a situation. An alternative avenue to data collection, often used in engineering and policy analysis, is to ask human experts to make statements about the possible costs. The presentation will outline how this information could be used to obtain a cost distribution. Traditional ad hoc approaches to combine expert opinion will be described. Bayesian models will be reviewed and the all important subject of expert calibration will be discussed in detail. A calibrated model introduced by Mendel and Sheridan (1989) will be presented and implemented in the S language. We then consider an actuarial application by fitting a parametric distribution to the output of the calibrated model.

Actuar: an R package for Actuarial Science

Vincent Goulet
Universite Laval, Quebec, Canada

Abstract: R is a free software environment for statistical computing and graphics not unlike S-Plus. More than just another statistical environment, R is also a complete, vector based, programming language. For actuaries with an APL background, moving to R should be natural and they gain sophisticated statistical and graphical tools as a bonus. In the hope to promote the use of R in Actuarial Science, both in practice and in research, we started the actuar project in 2005. This consists in building a comprehensive package of actuarial functions for R. Here, we present the functions already in the package and forthcoming additions.

Ascertainment Bias in Estimating Rates of Early Onset Alzheimer's Disease: a Critical Illness Insurance Application

Angus Macdonald, Carolina Espinosa
Heriot-Watt University, Edinburgh, UK

Abstract: Estimation of rates of onset of rare, late-onset dominantly inherited genetic disorders is complicated by: (a) probable ascertainment bias resulting from the recruitment of strongly affected families into studies; and (b) inability to identify the true at risk population of mutation carriers. To deal with the latter, Gui and Macdonald (2002a) proposed a non-parametric (Nelson-Aalen) estimate $\Lambda(x)$ of a simple function $\Lambda(x)$ of the rate of onset at age x. $\Lambda(x)$ had a finite bound, which was an increasing function of the probability p that a child of an affected parent inherits the mutation, unfortunately the estimation procedure fails if $\Lambda(x)$

exceeds this bound, which can happen at quite low ages. We show that such failure may in fact be a useful measure of ascertainment bias. Usually we would assume p =1/2, but in the presence of ascertainment bias p> 1/2 in the sample, so the maximum value attained by $\Lambda(x)$ allows us to estimate p, and therefore the degree of ascertainment bias that may be present, leading to bias-corrected estimates of rates of onset. We apply these to early-onset Alzheimer's disease associated with mutations in the Presenilin-1 gene. We apply our new estimates to the actuarial questions of extra premiums, given genetic information, and adverse selection, if genetic information need not be disclosed to insurers writing critical illness (CI) insurance. CI insurance premium increases are in the main high. The possible cost of adverse selection in respect of PSEN-1 gene mutations under various moratoria appears to be negligible except in the case of small markets and severe adverse selection.

Optimal Retention Levels in Dynamic Reinsurance Markets

Enrico Biffs
City University, UK

Abstract: We consider the problem of determining optimal retention levels for insurers willing to mitigate their risk exposure by purchasing proportional reinsurance. We revisit De Finetti's classical results in continuous-time and allow reinsurance premiums and retention levels to change dynamically in response to claims experience and market performance. We also take up some ideas from dynamic reinsurance markets to intertwine De Finetti s work and Markowitz's mean-variance portfolio theory.

Short Period Non-Catastrophic Rainfall Options

Barry Turner
McGill University, Canada

Abstract: While a single severe storm with a return period of many years may cause extensive damage, the financial impact of more numerous non-catastrophic precipitation events is also significant. Many outdoor activities are adversely affected by even light rainfall, and can be sensitive to the timing of intermittent showers or thunderstorms at specific locations on scales as short as hours. However, financial products related to rainfall more commonly tied to a daily, monthly or seasonal index. We will explore the use of high-resolution rainfall measurements and modeling for the design and pricing of short-period rainfall options.

Stochastic Analysis of Life Insurance Surplus

Natalia Lysenko
Simon Fraser University, Canada

Abstract: The behaviour of insurance surplus over time for a portfolio of homogeneous life policies in an environment of stochastic mortality and rates of return is examined. We distinguish between stochastic and accounting surplus and derive their first two moments. A recursive formula is proposed for calculating the distribution function of the accounting surplus. We then examine

the probability that the surplus remains positive in every insurance year. Numerical examples illustrate the results for portfolios of temporary and endowment life policies assuming an AR(1) process for the rates of return.

Broken-Heart Mortality

Lily Yunsui Li (with Mary Hardy)
University of Waterloo, Canada

Abstract: Although actuaries often assume independence in joint life and last survivor insurance, it is generally accepted that mortality of married lives worsens after the death of their spouse. Earlier work proposed a Frank's copula to manage this dependence. In this work we propose a multiple state approach, which will provide a more direct measure of dependency. We will explore data on deaths by marital status and look at the impact of this model on annuity and insurance pricing.

Entropy, Longevity and Annuities

M. Khalaf-Allah, S. Haberman and R. Verrall
City University, UK

Abstract: In this paper the entropy measure applied in population biology by Demetrious (1976) is extended to measure the effect of any changes in the force of mortality on the cost of life annuity for different interest rate scenarios and levels of mortality improvements. This allows different sources of risk in a life annuity contract to be summarized in a one figure index. Numerical values for the entropy measure are derived using an approach which extends that of Keyfitz (1977). Results are illustrated using English life tables over the period from 1851 till 1991 and also by applying different mathematical models for mortality projections such as the Gompertz and the Sithole et al (2000) mortality projection models for both males and females aged 60 as an attempt to get a better understanding regarding the properties of the entropy measure. This is followed by testing the sensitivity of the results obtained with regard to the different factors that are likely to affect the value of the entropy measure. In this study the effect of gender, age, assumed interest rate and the level of mortality improvement are investigated.

Method to Develop a Provision for Adverse Deviation (PAD) for the Longevity Risk for Impaired Lives

Sudath Ranasinghe
University of Connecticut

Abstract: This paper examines longevity risk and its implications for impaired lives. Life annuities and other long-term living benefits are the most important insurance products concerned with longevity risk. There are no standard mortality tables for impaired annuitants, especially for those with chronic medical and acute conditions. Therefore, it is difficult to capture longevity risk using traditional mortality projection methods.

This study recognizes the main components of longevity risk and treats them separately to develop an analytical method to calculate the provision for adverse

deviation (PAD). The study also considers implications of this PAD model to the other insurance products such as Life Settlement and Structured Settlements.

Mortality Improvement Scales for the Canadian Insured Lives

Johnny Siu-Hang Li, Mary Hardy and Ken Seng Tan
University of Waterloo, Canada

Abstract: In the valuation of products involving life contingencies, actuaries often rely on life tables with a forecast of future trends, typically summarized by multiplicative mortality improvement scales. In this study we derive mortality improvement scales for the Canadian insured lives by means of a joint model that consists of multiple fitting stages. In the first stage we project the mortality experience of the whole Canadian population using an extended version of the Lee-Carter model in which special attention is paid to the measure of uncertainty and to the possible recurrence of outliers. In the second stage we relate the experience of the insured lives and the whole population by the so-called Brass-type approach. We then summarize the projection by a small number of parameters to obtain the improvement scales. Finally, we discuss the effects of selection and smoker-status.

Pandemic Influenza Claims Risk in the U.S.

Tom Edwalds
Munich American Reassurance Company, Chicago, IL, USA

Abstract: The Society of Actuaries (SoA) is sponsoring a study of the potential effect on U.S. life and health insurers of an influenza pandemic. Will insurers be able to pay all the claims if a new deadly and transmissible strain of the influenza virus appears? We will explain the model developed for the SoA by Jim Toole, FSA, of MBA Actuaries, and discuss the results of this model for a severe pandemic scenario.

Threshold Life Tables and their Applications

Johnny Siu-Hang Li
University of Waterloo, Canada

Abstract: The rapid emergence of super-centenarians has highlighted the importance of the tail of the survival distribution, and motivated researchers to look for alternative ways for closing off the life tables instead of the prevailing practice of using the value of one at an arbitrarily chosen age. Based on the asymptotic results in the extreme value theory, we propose a model -the threshold life table -that allows practitioners to extrapolate the survival distribution to the extreme ages and to determine the appropriate end point of the life table. The model is further extended to a dynamic version which takes account of the non-diversifiable longevity risk, which originates from the uncertainty in future trends. The theoretical results are finally applied to the stochastic valuation of a life annuity portfolio and to the prediction of highest attained age for various cohorts.

Pension Plan Evaluation using Conditional Tail Expectation (CTE)

Claude Pichet, Ren´e Delsanne and Carole Turcotte
UQAM, Canada

Abstract: We present a new method to calculate the liabilities of a defined benefit pension plan. The interest rate used will be obtained through the calculation of the conditional tail expectation of the 20th percentile of the lowest returns of the simulated portfolios.

Policyholder Behavior Study in Variable Annuity with Guaranteed Minimum Withdrawal Benefit

Yan Liu
University of Waterloo, Canada

Abstract: The Guaranteed Minimum Withdrawal Benefit (GMWB) guarantees a certain minimum monthly (quarterly) withdrawal amount to the policyholder equal to the total investment even if the account value reaches zero. Policyholder Behavior is hard to estimate and predict, but it's a critical part in pricing and hedging the Product. Factors affecting withdrawal and lapse decisions may include economic variables, in-the-moneyness of the benefit, personal situations (age, other income etc.), and product features (policy duration, benefit features). Assumptions here seem to be subjective since there is no published data. We will use a multinomial logistic regression model to estimate the effects of some of these factors. We model withdrawal and lapse behavior as discrete-time stochastic processes.

The Effects on the Funding and Contribution Variance using the Modified Spreading Model

Steven Haberman, M. Iqbal Owadally and D. Gomez
City University, London, UK

Abstract: The present work analyses the effects on the funding and the contribution rate when the only source of unpredictable experience is through volatile rates of return. The funding of a defined benefit scheme has been one of the main concerns of the sponsors of a pension plan. A volatile fund is not desirable, as it might imply volatile contributions. It is shown that the modified spreading model, developed by Owadally (2003), eliminates gains and losses, arisen by favourable or unfavourable experience, by paying a specific amount of unfunded liabilities through time. A comparison of the modified spreading model is made with the spreading model developed by Dufresne (1988). The modified spreading model is shown to be more efficient than the spreading model, as it minimises the variance of the fund and the contribution and as it leads to a smoother fund and contribution rate. Real investment rates of return of the pension fund, are assumed to be represented by two stochastic models: bootstrap sampling method by using historical data and the IID special case of the autoregressive model. The bootstrap sampling analysis considers two different assets, a high-risk asset given by UK equities and a low-risk asset given by gilts. Also, six different asset allocations, three different periods of time to project the value of the fund, and three scenarios for the actuarial assumptions

on the rates of return are considered and analysed. The basis of our work is found mainly in Owadally and Haberman (2004) and Owadally (2003).

The Risk Management of a DB Underpin Pension Plan

Kai Chen
University of Waterloo, Canada

Abstract: Hybrid pension plans offer employees the best features of both defined benefit and defined contribution plans. In this work, we consider a hybrid design offering a defined contribution benefit with a defined benefit guaranteed minimum underpin. We consider valuation and risk management for this hybrid pension plan. A hedging portfolio is constructed using inflation-linked bonds, and a stock index. The payoff of the guarantee of the hybrid pension plan is similar to the payoff of an exchange option, which was developed by Margrabe (1976). We separate the pension guarantee by month and implement delta hedging to each part. We also calculate the monthly hedging cost and test the sensitivity to the crediting rate and the contribution rate.

A Bias Reduction Technique for Monte Carlo Pricing of Early Exercise Options

Tyson L. Whitehead, Matt Davison and R. Mark Reesor
The University of Western Ontario, London, Canada

Abstract: We present a new method for reducing the bias in Monte Carlo estimators of the price of American-style contingent claims. At each exercise opportunity (in a time discretization), we assume there is an unbiased estimator of the claim value at the next exercise opportunity. We approximate the distribution of this statistic using the Central Limit Theorem and use this to derive an expression for the bias. This expression is easily estimated in the context of a simulation, which allows for the straightforward computation of bias-reduced estimators of the claim value. We conclude by presenting a well-studied multivariate pricing example to show that this method offers significant improvements over the vanilla stochastic mesh technique and that it is a much more computationally effcient approach to reducing bias than nonparametric bootstrapping.

Decomposing Loan Portfolio Value-at-Risk and Expected Shortfall

Tingting Fan and Zhongfei Li
University of Waterloo, Canada
and Lingnan College (University), Sun Yat-Sen University, Guangzhou, PRC

Abstract: A variety of models have been proposed with the objective of calculating the loss distribution of a credit portfolio. From the loss distribution, one can obtain useful risk measures such as the Values-at-Risk (VaR) and Expected Shortfall (ES). While these risk measures are important, it is also of significant interest of obtaining information about attributing fair VaR contributions and ES contributions to loans underlying the portfolio. This paper presents a new and efficient tool of decomposing loan portfolio's VaR and ES using the probability generating function corresponding to the portfolio loss distribution. Our

objective is to assign appropriate risk contributions to the parts of a loan portfolio with dependent default risks. We consider a factor copula approach which has the capability of modeling the default dependent structure while maintaining a balance between tractability and parsimony. Using our proposed approach, we derive semi-explicit expressions of credit risk contributions to VaR and ES, which in turn allow us to calculate the risk contributions quickly and exactly. We illustrate our proposed technique via some numerical examples.

Econometric Models for Interest Rates

R. Keith Freeland and Mary R. Hardy
University of Waterloo, Canada

Abstract: In this paper, we fit numerous econometric models to short-term and long-term UK interest rates. Most of the models considered are regime switching models, where the distribution depends on a latent regime. These models are routinely used to capture the dynamics of financial returns. We compare both regime switching models and single state models. In particular we consider the CIR and GARCH models. In addition to considering Gaussian returns we also consider the more complex non-central chi-square distribution. The models are all estimated via maximum likelihood estimation. The models are evaluated based on their complexity, likelihood value and residuals.

Fuzzy Volatility Forecasts and Fuzzy Option Values

K. Ranee Thiagarajah
Illinois State University, Normal, USA

Abstract: Many financial time series, such as returns on stocks and foreign exchange rates, exhibit leptokurtosis and volatility varying in time. Decision-making problems, in general, are not well defined as their model parameters are not precisely known. As a result there has been growing interest in using fuzzy models in such cases. Fuzzy probability theory is a simple and potentially a useful way to propagate impreciseness through a cascade of calculations. The use of fuzzy probability theory, introduced by Buckley (2004) as a methodology for modeling and analyzing certain financial problems, is of particular interest to a number of researchers due to fuzzy probability theory's ability to quantitatively and qualitatively model those problems which involve vagueness and imprecision. In this paper, we summarize the basics of fuzzy random variables and introduce a class of fuzzy random coefficient (FRC) volatility models. Fuzzy option values and fuzzy forecasts are also discussed in some details.

Multivariate Modeling of Asset Returns for Investment Guarantees Valuation

Mathieu Boudreault, HEC Montreal, Canada
Christian-Marc Panneton, Industrial Alliance

Abstract: Stochastic models have been used for valuing the CTE provision of segregated fund guarantees in Canada since 2002. Most publications on this topic have used a univariate estimation technique and applied it to a multivariate context. In this talk, we will analyze some of these approaches and compare

them with a true multivariate estimation using the Canadian, U.S., U.K. and Japanese markets. Such a multivariate estimation requires the modeling of both the volatility and correlation structures. We therefore discuss the use of the regime-switching model in a multivariate setting. Many issues need to be considered when using a Markovian environment for multiple indices such as how the regimes and transitions are defined. Numerous multivariate GARCH models are also considered (VECH, BEKK, CCC, FARCH), notably, Engle's DCC class of GARCH models, which provides for a 2-step consistent estimation of volatilities and correlations. This approach is very useful to represent the dynamics of large correlation structures. Copula techniques are also explored to build multivariate distributions. As a final exercise, we have conducted a Monte Carlo experiment in which we have supposed the data come from a multivariate stochastic volatility model having leverage and/or fat tails with known parameters. This helps determine how the CTE computed with either the multivariate GARCH or regime-switching models compare with the true CTE. Our analysis shows that the model selection process is not the same in a multivariate setting than in a univariate framework. With the four markets used in our analysis, we have come up with the following conclusions: (1) the constrained univariate regime-switching model used by many practitioners provides the worst fit to historical data; (2) the multivariate GARCH models provide a better fit than multivariate regime-switching models; (3) there is significant variability in the 10-year CTE provisions computed with the multivariate models. As a result, the most appropriate model will depend on the specific portfolio that needs to be modeled.

Optimal Mean-Variance Investment for an Insurer

Wenjing Guo
Nanjing University of Finance and Economics, PRC
and University of Waterloo, Canada

Abstract: How to select the optimal investment strategy turns into a difficult problem faced by insurance corporations now. The traditional investment model considers that insurer invests all its surplus in risky assets. Anna finds that the ruin probability of insurance business with a risky investment equals 1. To reduce the ruin probability insurer should invests in both risky and non-risky asset and set part of its wealth aside to cope with random claims. In this paper, a finite interval investment of insurer is studied. The insurer is permitted to invest in risky assets and non-risky assets. The wealth of the insurer is divided into two parts according to a fixed proportion, one is retained to meet the random claims and the other is used to invest. Variance is applied to measure insurer's risk. The relationship between underwriting risk and investment risk is considered in the model. The optimal investment strategy is defined to minimize the investment risk and the insurance risk for a given expected eventual wealth. By solving the model, the explicit expressions of the optimal strategy and the efficient frontier are derived. In addition, the dynamical properties of the optimal strategy and the efficient frontier are discussed. We find that the amount invested in risky asset is positively relative to the insurer's expected eventual wealth and negatively relative to its safe load.

Quality Control of Risk Measures: Backtesting Var Models

Victor H. de la Pena, Columbia University

Ricardo Rivera, State of New York Banking Department and NYU
Jesus Ruiz-Mata, Lehmann Brothers

Abstract: This paper introduces a statistical approach to assess the quality of risk measures (QCRM). The approach is applied to the problem of evaluating the accuracy of Value-at-Risk (VaR) models used to predict the maximum losses in a bank's portfolio with a given confidence level. The current value VaR back-testing method developed by the Basel Committee for Banking Supervision controls the probability of rejecting the VaR when it is correct. However, the test has limited power to distinguish an accurate model from an inaccurate one and its power is not an appropriate measure of the model's validity (the probability of accepting the model when it is correct). Using the same information and changing the role of the null and alternative hypotheses, QCRM exploits the binomial structure of the testing problem to: a) provide a uniform reduction (over Bassell's) of the probability of accepting the model when it is incorrect and b) provides a uniformly most powerful test which measures the probability of accepting the model when it is correct. These improvements are partially offset by a small increase in the probability of rejecting the model when it is correct. The test is based on correct coverage one-sided confidence intervals for the probability of an exception using the technique of pivoting the cumulative distribution function. The test results in new acceptance and rejection regions that are a complementary standard to the current regulatory regions. We compare the proposed confidence intervals to the ones used in financial literature and show their comparative strength.

Quantifying and Correcting the Bias in Estimated Risk Measures

Joseph H. T. Kim and Mary Hardy
University of Waterloo, Canada

Abstract: In this paper we explore the bias in the estimation of the Value at Risk and Conditional Tail Expectation risk measures using Monte Carlo simulation. We assess the use of bootstrap techniques to correct the bias for a number of different examples. In the case of the Conditional Tail Expectation, we show that application of the exact bootstrap can improve estimates, and we develop a practical guideline for assessing when to use the exact bootstrap.

Strategic Valuation

Steve Craighead, Towers Perrin - Tillinghast Software Solutions
Greg Slone, Nationwide Insurance

Abstract: In the process of assigning interest-crediting rates for accumulation of net premium deposits, there is competition between insurance companies. One company may set their rates based on new money rates, whereas another may set theirs based upon current portfolio rates, or any combination of the two. The different companies also use different metrics to determine their rate-setting practices. These metrics may be designed to measure competitive information, expected customer behavior, business line and/or company profitability, or other possible indicators. The determination of a competitive advantage must also account for regulatory control as well as the economic impact on the competitive and regulatory environment. The modeling of competitive rate setting must

address all of these issues. In our initial research, we attempted to implement the above strategic situation within a traditional game theoric environment. However, due to the complexity of our industry, any attempt to recast the problem therein was frustrated. This has led us to develop the strategic valuation system Simian, which allows for the intricate inter-relationships required. We outline the fundamental C++ classes that allow us to model different rate-setting processes in the presence of competitors, regulators, and various states of the economy. We have found that these classes are very flexible and can be easily modified to account for an array of strategic questions.

The Theoretical Problem of Managing a fund by entering into independent bets

Matthieu Dufour
Universite du Quebec a Montreal, Canada

Abstract: Let us define a bet (of an amount M) as a random experiment where the gain is M with probability p and $-M$ with probability $1 - p$. A necessary condition for a risk-averse person to enter this bet is that $p > 0{:}50$. Suppose that a fund manager can, at the end of each period, gamble a proportion x_i of his portfolio in a bet with $p_i > 0{:}50$, for a finite choice of bets $i = 1, 2, \ldots, n$. It is easily seen that a strategy that aims to maximise the expected gain yields to ruin. However, there exists a strategy that maximises the expected return. Determining it, quickly becomes more complex as the number n of possible bets increases, especially if the outcomes are not independent. We shall study this problem, which has surprising links with Shannons's information theory.

An Integro-differential equation for a Sparre Andersen Model with investments

Corina D. Constantinescu and Enrique A. Thomann
Oregon State University, Corvallis, USA

Abstract: This talk considers one of the classical problems in the actuarial mathematics literature, the collective risk model. The claim number process $N(t)$ is assumed to be a renewal process, the resulting model being referred as the Sparre Andersen risk model. The inter-claim times form a sequence of independent identical distributed random variables with distribution Erlang(n, β). The additional non-classical feature is that the company invests in a risky asset with returns modeled by a diffusion. The analysis is focused on the probability of ruin $\psi(u)$, where u is the initial surplus. It is shown that $\psi(u)$ satisfies a certain integro-differential equation. As an example the equation for an Erlang (2,β) inter-arrival times distribution and a geometric Brownian motion for the returns from investments is considered and the asymptotic decay of the probability of ruin is investigated.

Bounds on the Ruin Probability in a Controlled Risk Model

Maikol Diasparra and Rosario Romera
Universidad Carlos III de Madrid, Spain

Abstract: We consider a discrete risk process modeled by a Markov Decision Process. The surplus could be invested in stock market assets. We adopt a realistic point of view and we let the investment return process to be statistically dependent over time. We assume that follows a Markov Chain model. To minimize the risk there is a possibility to reinsure a part or the whole reserve. We consider proportional reinsurance. Recursive and integral equations for the ruin probability are given. Generalized Lundberg inequalities for the ruin probabilities are derived. Stochastic optimal control theory is used to determine the optimal stationary policy which minimizes the ruin probability. To illustrate these results numerical examples are included considering claim distribution of the PH type.

Extreme Value Analysis for Partitioned Insurance Losses

Ping-Hung Hsieh and John B. Henry III
Oregon State University, Corvallis, USA

Abstract: Extreme value theory has become increasingly important in the actuarial literature for modeling the distribution of large claims in the situation when individual losses have been recorded. However, it is often the case that losses are available only in a partitioned form, i.e., that only the frequencies of losses residing in certain loss intervals are recorded. The objective of our project is to expand on our prior study of extreme partitioned data so as to offer a comprehensive extreme value analysis of losses in the partitioned data setting. By assuming only that the underlying survivor function of losses is regularly varying at infinity, we have derived a maximum likelihood estimator for the tail-index and investigated its performance through a simulation study. A necessary next step in this project is to propose a threshold selection procedure for determining a threshold above which the assumption of regular variation is valid. Estimators of important quantities such as expected loss above a high threshold, extreme quantiles, and tail probabilities will also be derived. We illustrate the proposed techniques in an example using actual partitioned loss data obtained from the Insurance Services Office.

Lundberg-type Approximations for Defective Renewal Equations: a Heavy-traffic Perspective

Jose Blanchet
Harvard University, USA

Abstract: Renewal equations for which the renewal time distribution, $F(\cdot)$, is defective (that is $F(\infty) < 1$) arise often in many applied probability settings, such as insurance risk theory, queueing theory, reliability and branching processes among others. The text by Willmot and Lin (2001) discusses a great variety of applications to insurance and connections between defective renewal equations (DRE's) and compound geometric sums. Our focus is on DRE's that are close to be proper, that is, $F(\infty) \backsimeq 1$. In the queueing context, this situation is often related to systems operating under heavy-traffic and it also arises naturally in risk theory applications that involve low net profit environments (for instance, low safety loading or small interest rates). Motivated by these applications, we develop asymptotic expansions for the solution of DRE's in powers of $(1 - F(\infty))$.

In the presence of light-tailed claims, our expansions can be connected naturally to so-called Lundberg approximations (popular in the analysis of DRE's). However, our expansions can also be developed in the context of heavy-tailed claims; hence, we are able to develop Lundberg-type approximations in the presence heavy-tailed claims. Connections to more standard heavy-tailed asymptotics for ruin probabilities and applications to ruin models perturbed by a diffusion will also be discussed.

Multivariate Phase Variables and Recursion Principles

Karl-Theodor Eisele
Universite Louis Pasteur, Strasbourg, France

Abstract: We present discrete and continuous multivariate phase variables. We show the rationality of their generating functions, resp. their Laplace transforms and calculate the moments in the bivariate case. Finally, a recursion principle is given for compound multivariate phase variables. Examples, calculated by a VB-program, demonstrate how the results are easily applicable.

The Dividends-Penalty Identity and the Optimal Dividend Barrier

Sheldon Lin
University of Toronto, Canada

Abstract: For a general class of risk models, the dividends-penalty identity is derived by probabilistic reasoning. This identity is the key for understanding and determining the optimal dividend barrier, which maximizes the difference between the expected present value of all dividends until ruin and the expected discounted value of a penalty at ruin (which is typically a function of the deficit at ruin). As an illustration, the optimal barrier is calculated in two classical models, for different penalty functions and a variety of parameter values. Joint work with Hans Gerber and Hailiang Yang.

A Multivariate Tweedie Family with Applications to Risk Measurement

Edward Furman and Zinoviy Landsman
University of Haifa, Israel

Abstract: In the present talk we propose a multivariate extension of the well-known exponential dispersion models. We name this class the multivariate Tweedie family (MTwF) because its univariate marginal distributions correspond to the univariate Tweedie ones. Furthermore, the proposed family possesses a dependence structure, which is reflected by its covariance structure, and it allows good modelling multivariate portfolios with dependent claims or risks. As special cases, MTwF contains, for instance, the multivariate inverse Gaussian, multivariate gamma and multivariate compound Poisson distributions in the sense that their univariate marginals are inverse Gaussian, gamma and compound Poisson respectively. We demonstrate the construction method of MTwF, derive exact density functions of multivariate Tweedie random vectors and calculate higher order moments. Some useful Chebyshev's type inequities are also produced for easy evaluations of higher and lower probabilistic bounds. Lastly, some applications to risk measurement are considered.

Bayesian Inference Resistant to Outliers, using Super Heavy-tailed Distributions, for the Calculation of Premiums

Alain Desgagne, Universite du Quebec a Montreal, Canada
Jean-Francois Angers, Universite de Montreal

Abstract: We assume that the claim sizes for several risks are conditionally independent random variables given the same scale parameter. The Bayesian predictive distribution of a next claim size is used to estimate the pure premium. Robust procedures to conflicting information (prior or outliers) depend mainly on the tail behavior of the likelihood and prior densities. For that purpose, the notion of left and right log-credence is introduced to characterize respectively the left and right tails of a density defined on the positive line. Simple conditions are established to determine the proportion of observations that can be rejected as outliers. It is shown that the posterior distribution converges in law to the posterior that would be obtained from the reduced sample, excluding the outliers, as they tend to 0 or infinity, at any given rate. An example of calculation of a pure premium is given. We compare the log-normal model with the robust super heavy-tailed (log-Pareto type) distributions model.

Extreme Behavior of Multivariate Phase-Type Distributions

Alexandru V. Asimit and Bruce L. Jones
University of Western Ontario, London, Canada

Abstract: This paper investigates the limiting distributions of the component-wise maxima and minima of suitably normalized iid multivariate phase-type random vectors. In the case of maxima, a large parametric class of multivariate extreme value (MEV) distributions is obtained. The flexibility of this new class is exemplified in the bivariate setup. For minima, it is shown that the dependence

structure of the Marshall-Olkin class arises in the limit.

Fitting Combinations of Exponentials to Probability Distributions

Daniel Dufresne
University of Melbourne, Australia.

Abstract: Two techniques are described for approximating distributions on the positive half-line by combinations of exponentials. One is based on Jacobi polynomial expansions, and the other on the logbeta distribution. The techniques are applied to some well-known distributions (degenerate, uniform, Pareto, lognormal and others). In theory the techniques yield sequences of combination of exponentials that always converge to the true distribution, but their numerical performance depends on the particular distribution being approximated. An error bound is given in the case of the logbeta approximations.

Some applications are:

Risk theory. It has been known for some time that the probability of ruin is simpler to compute if the distribution of the claims, or that of the inter-arrival times of claims, is rational. The simplfications which occur in risk theory have been well-known in the literature on random walks (see, for instance, the comments in Feller (1971)) and are also related to queueing theory.

Convolutions. The distribution of the sum of independent random variables with a lognormal or Pareto distribution is not known in simple form. Therefore, a possibility is to approximate the distribution of each of the variables involved by a combination of exponentials, and then proceed with the convolution, which is a relatively straightforward affair with combinations of exponentials.

Financial mathematics. Suppose an amount of money is invested in the stock market and is used to pay an annuity to a pensioner. What is the probability that the fund will last until the pensioner dies? The answer to this question is known in closed form if the rates of return are lognormal and the lifetime of the pensioner is exponential; this is due to Yor (1992). In practice, however, the duration of human life is not exponentially distributed, and the distribution of the required stochastic life annuity can be found by expressing the future lifetime distribution as a combination of exponentials.

Modeling and Estimating Individual and Firm Effects with Panel Data

Jean-Francois Angers, Denise Desjardins Georges Dionne and Francois Guertin
Centre de recherche sur les transports, University of Montreal and HEC
Montreal, Canada

Abstract: In this article, we propose a detailed analysis of the modeling and estimation of distributions of vehicle accidents. This analysis uses panel data to account simultaneously for individual as well as fleet effects. The distribution of accidents can be affected by both observable and non-observable factors. Non-observable factors are modeled as random factors.

Using Expert Opinion in Actuarial Science

Michel Jacques and Mathieu Pigeon
Universite Laval, Canada

Abstract: Actuaries may have to model costs for which data are not available, because they are either scarce or confidential. Third party liability coverage can be an example of such a situation. An alternative avenue to data collection, often used in engineering and policy analysis, is to ask human experts to make statements about the possible costs. The presentation will outline how this information could be used to obtain a cost distribution. Traditional ad hoc approaches to combine expert opinion will be described. Bayesian models will be reviewed and the all important subject of expert calibration will be discussed in detail. A calibrated model introduced by Mendel and Sheridan (1989) will be presented and implemented in the S language. We then consider an actuarial application by fitting a parametric distribution to the output of the calibrated model.

Actuar: an R package for Actuarial Science

Vincent Goulet
Universite Laval, Quebec, Canada

Abstract: R is a free software environment for statistical computing and graphics not unlike S-Plus. More than just another statistical environment, R is also a complete, vector based, programming language. For actuaries with an APL background, moving to R should be natural and they gain sophisticated statistical and graphical tools as a bonus. In the hope to promote the use of R in Actuarial Science, both in practice and in research, we started the actuar project in 2005. This consists in building a comprehensive package of actuarial functions for R. Here, we present the functions already in the package and forthcoming additions.

Ascertainment Bias in Estimating Rates of Early Onset Alzheimer's Disease: a Critical Illness Insurance Application

Angus Macdonald, Carolina Espinosa
Heriot-Watt University, Edinburgh, UK

Abstract: Estimation of rates of onset of rare, late-onset dominantly inherited genetic disorders is complicated by: (a) probable ascertainment bias resulting from the recruitment of strongly affected families into studies; and (b) inability to identify the true at risk population of mutation carriers. To deal with the latter, Gui and Macdonald (2002a) proposed a non-parametric (Nelson-Aalen) estimate $\Lambda(x)$ of a simple function $\Lambda(x)$ of the rate of onset at age x. $\Lambda(x)$ had a finite bound, which was an increasing function of the probability p that a child of an affected parent inherits the mutation, unfortunately the estimation procedure fails if $\Lambda(x)$ exceeds this bound, which can happen at quite low ages. We show that such failure may in fact be a useful measure of ascertainment bias. Usually we would assume p =1/2, but in the presence of ascertainment bias p> 1/2 in the sample, so the maximum value attained by $\Lambda(x)$ allows us to estimate p, and therefore the degree of ascertainment bias that may be present, leading to bias-corrected estimates of rates of onset. We apply these to early-onset Alzheimer's disease associated with mutations in the Presenilin-1 gene. We apply our new estimates to the actuarial questions of extra premiums, given genetic information, and adverse selection, if genetic information need not be disclosed to insurers writing critical illness (CI) insurance. CI insurance premium increases are in the main

high. The possible cost of adverse selection in respect of PSEN-1 gene mutations under various moratoria appears to be negligible except in the case of small markets and severe adverse selection.

Optimal Retention Levels in Dynamic Reinsurance Markets

Enrico Biffs
City University, UK

Abstract: We consider the problem of determining optimal retention levels for insurers willing to mitigate their risk exposure by purchasing proportional reinsurance. We revisit De Finetti's classical results in continuous-time and allow reinsurance premiums and retention levels to change dynamically in response to claims experience and market performance. We also take up some ideas from dynamic reinsurance markets to intertwine De Finetti s work and Markowitz's mean-variance portfolio theory.

Short Period Non-Catastrophic Rainfall Options

Barry Turner
McGill University, Canada

Abstract: While a single severe storm with a return period of many years may cause extensive damage, the financial impact of more numerous non-catastrophic precipitation events is also significant. Many outdoor activities are adversely affected by even light rainfall, and can be sensitive to the timing of intermittent showers or thunderstorms at specific locations on scales as short as hours. However, financial products related to rainfall more commonly tied to a daily, monthly or seasonal index. We will explore the use of high-resolution rainfall measurements and modeling for the design and pricing of short-period rainfall options.

Stochastic Analysis of Life Insurance Surplus

Natalia Lysenko
Simon Fraser University, Canada

Abstract: The behaviour of insurance surplus over time for a portfolio of homogeneous life policies in an environment of stochastic mortality and rates of return is examined. We distinguish between stochastic and accounting surplus and derive their first two moments. A recursive formula is proposed for calculating the distribution function of the accounting surplus. We then examine the probability that the surplus remains positive in every insurance year. Numerical examples illustrate the results for portfolios of temporary and endowment life policies assuming an AR(1) process for the rates of return.

Broken-Heart Mortality

Lily Yunsui Li (with Mary Hardy)
University of Waterloo, Canada

Abstract: Although actuaries often assume independence in joint life and last

survivor insurance, it is generally accepted that mortality of married lives worsens after the death of their spouse. Earlier work proposed a Frank's copula to manage this dependence. In this work we propose a multiple state approach, which will provide a more direct measure of dependency. We will explore data on deaths by marital status and look at the impact of this model on annuity and insurance pricing.

Entropy, Longevity and Annuities

M. Khalaf-Allah, S. Haberman and R. Verrall
City University, UK

Abstract: In this paper the entropy measure applied in population biology by Demetrious (1976) is extended to measure the effect of any changes in the force of mortality on the cost of life annuity for different interest rate scenarios and levels of mortality improvements. This allows different sources of risk in a life annuity contract to be summarized in a one figure index. Numerical values for the entropy measure are derived using an approach which extends that of Keyfitz (1977). Results are illustrated using English life tables over the period from 1851 till 1991 and also by applying different mathematical models for mortality projections such as the Gompertz and the Sithole et al (2000) mortality projection models for both males and females aged 60 as an attempt to get a better understanding regarding the properties of the entropy measure. This is followed by testing the sensitivity of the results obtained with regard to the different factors that are likely to affect the value of the entropy measure. In this study the effect of gender, age, assumed interest rate and the level of mortality improvement are investigated.

Method to Develop a Provision for Adverse Deviation (PAD) for the Longevity Risk for Impaired Lives

Sudath Ranasinghe
University of Connecticut

Abstract: This paper examines longevity risk and its implications for impaired lives. Life annuities and other long-term living benefits are the most important insurance products concerned with longevity risk. There are no standard mortality tables for impaired annuitants, especially for those with chronic medical and acute conditions. Therefore, it is difficult to capture longevity risk using traditional mortality projection methods.

This study recognizes the main components of longevity risk and treats them separately to develop an analytical method to calculate the provision for adverse deviation (PAD). The study also considers implications of this PAD model to the other insurance products such as Life Settlement and Structured Settlements.

Mortality Improvement Scales for the Canadian Insured Lives

Johnny Siu-Hang Li, Mary Hardy and Ken Seng Tan
University of Waterloo, Canada

Abstract: In the valuation of products involving life contingencies, actuaries often rely on life tables with a forecast of future trends, typically summarized by

multiplicative mortality improvement scales. In this study we derive mortality improvement scales for the Canadian insured lives by means of a joint model that consists of multiple fitting stages. In the first stage we project the mortality experience of the whole Canadian population using an extended version of the Lee-Carter model in which special attention is paid to the measure of uncertainty and to the possible recurrence of outliers. In the second stage we relate the experience of the insured lives and the whole population by the so-called Brass-type approach. We then summarize the projection by a small number of parameters to obtain the improvement scales. Finally, we discuss the effects of selection and smoker-status.

Pandemic Influenza Claims Risk in the U.S.

Tom Edwalds
Munich American Reassurance Company, Chicago, IL, USA

Abstract: The Society of Actuaries (SoA) is sponsoring a study of the potential effect on U.S. life and health insurers of an influenza pandemic. Will insurers be able to pay all the claims if a new deadly and transmissible strain of the influenza virus appears? We will explain the model developed for the SoA by Jim Toole, FSA, of MBA Actuaries, and discuss the results of this model for a severe pandemic scenario.

Threshold Life Tables and their Applications

Johnny Siu-Hang Li
University of Waterloo, Canada

Abstract: The rapid emergence of super-centenarians has highlighted the importance of the tail of the survival distribution, and motivated researchers to look for alternative ways for closing off the life tables instead of the prevailing practice of using the value of one at an arbitrarily chosen age. Based on the asymptotic results in the extreme value theory, we propose a model -the threshold life table -that allows practitioners to extrapolate the survival distribution to the extreme ages and to determine the appropriate end point of the life table. The model is further extended to a dynamic version which takes account of the non-diversifiable longevity risk, which originates from the uncertainty in future trends. The theoretical results are finally applied to the stochastic valuation of a life annuity portfolio and to the prediction of highest attained age for various cohorts.

Pension Plan Evaluation using Conditional Tail Expectation (CTE)

Claude Pichet, Ren´e Delsanne and Carole Turcotte
UQAM, Canada

Abstract: We present a new method to calculate the liabilities of a defined benefit pension plan. The interest rate used will be obtained through the calculation of the conditional tail expectation of the 20th percentile of the lowest returns of the simulated portfolios.

Policyholder Behavior Study in Variable Annuity with Guaranteed Minimum Withdrawal Benefit

Yan Liu
University of Waterloo, Canada

Abstract: The Guaranteed Minimum Withdrawal Benefit (GMWB) guarantees a certain minimum monthly (quarterly) withdrawal amount to the policyholder equal to the total investment even if the account value reaches zero. Policyholder Behavior is hard to estimate and predict, but it's a critical part in pricing and hedging the Product. Factors affecting withdrawal and lapse decisions may include economic variables, in-the-moneyness of the benefit, personal situations (age, other income etc.), and product features (policy duration, benefit features). Assumptions here seem to be subjective since there is no published data. We will use a multinomial logistic regression model to estimate the effects of some of these factors. We model withdrawal and lapse behavior as discrete-time stochastic processes.

The Effects on the Funding and Contribution Variance using the Modified Spreading Model

Steven Haberman, M. Iqbal Owadally and D. Gomez
City University, London, UK

Abstract: The present work analyses the effects on the funding and the contribution rate when the only source of unpredictable experience is through volatile rates of return. The funding of a defined benefit scheme has been one of the main concerns of the sponsors of a pension plan. A volatile fund is not desirable, as it might imply volatile contributions. It is shown that the modified spreading model, developed by Owadally (2003), eliminates gains and losses, arisen by favourable or unfavourable experience, by paying a specific amount of unfunded liabilities through time. A comparison of the modified spreading model is made with the spreading model developed by Dufresne (1988). The modified spreading model is shown to be more efficient than the spreading model, as it minimises the variance of the fund and the contribution and as it leads to a smoother fund and contribution rate. Real investment rates of return of the pension fund, are assumed to be represented by two stochastic models: bootstrap sampling method by using historical data and the IID special case of the autoregressive model. The bootstrap sampling analysis considers two different assets, a high-risk asset given by UK equities and a low-risk asset given by gilts. Also, six different asset allocations, three different periods of time to project the value of the fund, and three scenarios for the actuarial assumptions on the rates of return are considered and analysed. The basis of our work is found mainly in Owadally and Haberman (2004) and Owadally (2003).

The Risk Management of a DB Underpin Pension Plan

Kai Chen
University of Waterloo, Canada

Abstract: Hybrid pension plans offer employees the best features of both defined benefit and defined contribution plans. In this work, we consider a hybrid design

offering a defined contribution benefit with a defined benefit guaranteed minimum underpin. We consider valuation and risk management for this hybrid pension plan. A hedging portfolio is constructed using inflation-linked bonds, and a stock index. The payoff of the guarantee of the hybrid pension plan is similar to the payoff of an exchange option, which was developed by Margrabe (1976). We separate the pension guarantee by month and implement delta hedging to each part. We also calculate the monthly hedging cost and test the sensitivity to the crediting rate and the contribution rate.

A Bias Reduction Technique for Monte Carlo Pricing of Early Exercise Options

Tyson L. Whitehead, Matt Davison and R. Mark Reesor
The University of Western Ontario, London, Canada

Abstract: We present a new method for reducing the bias in Monte Carlo estimators of the price of American-style contingent claims. At each exercise opportunity (in a time discretization), we assume there is an unbiased estimator of the claim value at the next exercise opportunity. We approximate the distribution of this statistic using the Central Limit Theorem and use this to derive an expression for the bias. This expression is easily estimated in the context of a simulation, which allows for the straightforward computation of bias-reduced estimators of the claim value. We conclude by presenting a well-studied multivariate pricing example to show that this method offers significant improvements over the vanilla stochastic mesh technique and that it is a much more computationally effcient approach to reducing bias than nonparametric bootstrapping.

Decomposing Loan Portfolio Value-at-Risk and Expected Shortfall

Tingting Fan and Zhongfei Li
University of Waterloo, Canada
and Lingnan College (University), Sun Yat-Sen University, Guangzhou, PRC

Abstract: A variety of models have been proposed with the objective of calculating the loss distribution of a credit portfolio. From the loss distribution, one can obtain useful risk measures such as the Values-at-Risk (VaR) and Expected Shortfall (ES). While these risk measures are important, it is also of significant interest of obtaining information about attributing fair VaR contributions and ES contributions to loans underlying the portfolio. This paper presents a new and efficient tool of decomposing loan portfolio's VaR and ES using the probability generating function corresponding to the portfolio loss distribution. Our objective is to assign appropriate risk contributions to the parts of a loan portfolio with dependent default risks. We consider a factor copula approach which has the capability of modeling the default dependent structure while maintaining a balance between tractability and parsimony. Using our proposed approach, we derive semi-explicit expressions of credit risk contributions to VaR and ES, which in turn allow us to calculate the risk contributions quickly and exactly. We illustrate our proposed technique via some numerical examples.

Econometric Models for Interest Rates

R. Keith Freeland and Mary R. Hardy
University of Waterloo, Canada

Abstract: In this paper, we fit numerous econometric models to short-term and long-term UK interest rates. Most of the models considered are regime switching models, where the distribution depends on a latent regime. These models are routinely used to capture the dynamics of financial returns. We compare both regime switching models and single state models. In particular we consider the CIR and GARCH models. In addition to considering Gaussian returns we also consider the more complex non-central chi-square distribution. The models are all estimated via maximum likelihood estimation. The models are evaluated based on their complexity, likelihood value and residuals.

Fuzzy Volatility Forecasts and Fuzzy Option Values

K. Ranee Thiagarajah
Illinois State University, Normal, USA

Abstract: Many financial time series, such as returns on stocks and foreign exchange rates, exhibit leptokurtosis and volatility varying in time. Decision-making problems, in general, are not well defined as their model parameters are not precisely known. As a result there has been growing interest in using fuzzy models in such cases. Fuzzy probability theory is a simple and potentially a useful way to propagate impreciseness through a cascade of calculations. The use of fuzzy probability theory, introduced by Buckley (2004) as a methodology for modeling and analyzing certain financial problems, is of particular interest to a number of researchers due to fuzzy probability theory's ability to quantitatively and qualitatively model those problems which involve vagueness and imprecision. In this paper, we summarize the basics of fuzzy random variables and introduce a class of fuzzy random coefficient (FRC) volatility models. Fuzzy option values and fuzzy forecasts are also discussed in some details.

Multivariate Modeling of Asset Returns for Investment Guarantees Valuation

Mathieu Boudreault, HEC Montreal, Canada
Christian-Marc Panneton, Industrial Alliance

Abstract: Stochastic models have been used for valuing the CTE provision of segregated fund guarantees in Canada since 2002. Most publications on this topic have used a univariate estimation technique and applied it to a multivariate context. In this talk, we will analyze some of these approaches and compare them with a true multivariate estimation using the Canadian, U.S., U.K. and Japanese markets. Such a multivariate estimation requires the modeling of both the volatility and correlation structures. We therefore discuss the use of the regime-switching model in a multivariate setting. Many issues need to be considered when using a Markovian environment for multiple indices such as how the regimes and transitions are defined. Numerous multivariate GARCH models are also considered (VECH, BEKK, CCC, FARCH), notably, Engle's DCC class of GARCH models, which provides for a 2-step consistent estimation of volatilities and correlations. This approach is very useful to represent the dynamics of large correlation structures. Copula techniques are also explored to

build multivariate distributions. As a final exercise, we have conducted a Monte Carlo experiment in which we have supposed the data come from a multivariate stochastic volatility model having leverage and/or fat tails with known parameters. This helps determine how the CTE computed with either the multivariate GARCH or regime-switching models compare with the true CTE. Our analysis shows that the model selection process is not the same in a multivariate setting than in a univariate framework. With the four markets used in our analysis, we have come up with the following conclusions: (1) the constrained univariate regime-switching model used by many practitioners provides the worst fit to historical data; (2) the multivariate GARCH models provide a better fit than multivariate regime-switching models; (3) there is significant variability in the 10-year CTE provisions computed with the multivariate models. As a result, the most appropriate model will depend on the specific portfolio that needs to be modeled.

Optimal Mean-Variance Investment for an Insurer

Wenjing Guo
Nanjing University of Finance and Economics, PRC
and University of Waterloo, Canada

Abstract: How to select the optimal investment strategy turns into a difficult problem faced by insurance corporations now. The traditional investment model considers that insurer invests all its surplus in risky assets. Anna finds that the ruin probability of insurance business with a risky investment equals 1. To reduce the ruin probability insurer should invests in both risky and non-risky asset and set part of its wealth aside to cope with random claims. In this paper, a finite interval investment of insurer is studied. The insurer is permitted to invest in risky assets and non-risky assets. The wealth of the insurer is divided into two parts according to a fixed proportion, one is retained to meet the random claims and the other is used to invest. Variance is applied to measure insurer's risk. The relationship between underwriting risk and investment risk is considered in the model. The optimal investment strategy is defined to minimize the investment risk and the insurance risk for a given expected eventual wealth. By solving the model, the explicit expressions of the optimal strategy and the efficient frontier are derived. In addition, the dynamical properties of the optimal strategy and the efficient frontier are discussed. We find that the amount invested in risky asset is positively relative to the insurer's expected eventual wealth and negatively relative to its safe load.

Quality Control of Risk Measures: Backtesting Var Models

Victor H. de la Pena, Columbia University
Ricardo Rivera, State of New York Banking Department and NYU
Jesus Ruiz-Mata, Lehmann Brothers

Abstract: This paper introduces a statistical approach to assess the quality of risk measures (QCRM). The approach is applied to the problem of evaluating the accuracy of Value-at-Risk (VaR) models used to predict the maximum losses in a bank's portfolio with a given confidence level. The current value VaR back-testing method developed by the Basel Committee for Banking Supervision controls the probability of rejecting the VaR when it is correct. However, the test has limited power to distinguish an accurate model from an inaccurate one and its power is

not an appropriate measure of the model's validity (the probability of accepting the model when it is correct). Using the same information and changing the role of the null and alternative hypotheses, QCRM exploits the binomial structure of the testing problem to: a) provide a uniform reduction (over Bassell's) of the probability of accepting the model when it is incorrect and b) provides a uniformly most powerful test which measures the probability of accepting the model when it is correct. These improvements are partially offset by a small increase in the probability of rejecting the model when it is correct. The test is based on correct coverage one-sided confidence intervals for the probability of an exception using the technique of pivoting the cumulative distribution function. The test results in new acceptance and rejection regions that are a complementary standard to the current regulatory regions. We compare the proposed confidence intervals to the ones used in financial literature and show their comparative strength.

Quantifying and Correcting the Bias in Estimated Risk Measures

Joseph H. T. Kim and Mary Hardy
University of Waterloo, Canada

Abstract: In this paper we explore the bias in the estimation of the Value at Risk and Conditional Tail Expectation risk measures using Monte Carlo simulation. We assess the use of bootstrap techniques to correct the bias for a number of different examples. In the case of the Conditional Tail Expectation, we show that application of the exact bootstrap can improve estimates, and we develop a practical guideline for assessing when to use the exact bootstrap.

Strategic Valuation

Steve Craighead, Towers Perrin - Tillinghast Software Solutions
Greg Slone, Nationwide Insurance

Abstract: In the process of assigning interest-crediting rates for accumulation of net premium deposits, there is competition between insurance companies. One company may set their rates based on new money rates, whereas another may set theirs based upon current portfolio rates, or any combination of the two. The different companies also use different metrics to determine their rate-setting practices. These metrics may be designed to measure competitive information, expected customer behavior, business line and/or company profitability, or other possible indicators. The determination of a competitive advantage must also account for regulatory control as well as the economic impact on the competitive and regulatory environment. The modeling of competitive rate setting must address all of these issues. In our initial research, we attempted to implement the above strategic situation within a traditional game theoric environment. However, due to the complexity of our industry, any attempt to recast the problem therein was frustrated. This has led us to develop the strategic valuation system Simian, which allows for the intricate inter-relationships required. We outline the fundamental C++ classes that allow us to model different rate-setting processes in the presence of competitors, regulators, and various states of the economy. We have found that these classes are very flexible and can be easily modified to account for an array of strategic questions.

The Theoretical Problem of Managing a fund by entering into independent bets

Matthieu Dufour
Universite du Quebec a Montreal, Canada

Abstract: Let us define a bet (of an amount M) as a random experiment where the gain is M with probability p and $-M$ with probability $1 - p$. A necessary condition for a risk-averse person to enter this bet is that $p > 0{:}50$. Suppose that a fund manager can, at the end of each period, gamble a proportion x_i of his portfolio in a bet with $p_i > 0{:}50$, for a finite choice of bets $i = 1, 2, \ldots, n$. It is easily seen that a strategy that aims to maximise the expected gain yields to ruin. However, there exists a strategy that maximises the expected return. Determining it, quickly becomes more complex as the number n of possible bets increases, especially if the outcomes are not independent. We shall study this problem, which has surprising links with Shannons's information theory.

An Integro-differential equation for a Sparre Andersen Model with investments

Corina D. Constantinescu and Enrique A. Thomann
Oregon State University, Corvallis, USA

Abstract: This talk considers one of the classical problems in the actuarial mathematics literature, the collective risk model. The claim number process $N(t)$ is assumed to be a renewal process, the resulting model being referred as the Sparre Andersen risk model. The inter-claim times form a sequence of independent identical distributed random variables with distribution Erlang(n, β). The additional non-classical feature is that the company invests in a risky asset with returns modeled by a diffusion. The analysis is focused on the probability of ruin $\psi(u)$, where u is the initial surplus. It is shown that $\psi(u)$ satisfies a certain integro-differential equation. As an example the equation for an Erlang (2,β) inter-arrival times distribution and a geometric Brownian motion for the returns from investments is considered and the asymptotic decay of the probability of ruin is investigated.

Bounds on the Ruin Probability in a Controlled Risk Model

Maikol Diasparra and Rosario Romera
Universidad Carlos III de Madrid, Spain

Abstract: We consider a discrete risk process modeled by a Markov Decision Process. The surplus could be invested in stock market assets. We adopt a realistic point of view and we let the investment return process to be statistically dependent over time. We assume that follows a Markov Chain model. To minimize the risk there is a possibility to reinsure a part or the whole reserve. We consider proportional reinsurance. Recursive and integral equations for the ruin probability are given. Generalized Lundberg inequalities for the ruin probabilities are derived. Stochastic optimal control theory is used to determine the optimal stationary policy which minimizes the ruin probability. To illustrate these results numerical examples are included considering claim distribution of the PH type.

Extreme Value Analysis for Partitioned Insurance Losses

Ping-Hung Hsieh and John B. Henry III
Oregon State University, Corvallis, USA

Abstract: Extreme value theory has become increasingly important in the actuarial literature for modeling the distribution of large claims in the situation when individual losses have been recorded. However, it is often the case that losses are available only in a partitioned form, i.e., that only the frequencies of losses residing in certain loss intervals are recorded. The objective of our project is to expand on our prior study of extreme partitioned data so as to offer a comprehensive extreme value analysis of losses in the partitioned data setting. By assuming only that the underlying survivor function of losses is regularly varying at infinity, we have derived a maximum likelihood estimator for the tail-index and investigated its performance through a simulation study. A necessary next step in this project is to propose a threshold selection procedure for determining a threshold above which the assumption of regular variation is valid. Estimators of important quantities such as expected loss above a high threshold, extreme quantiles, and tail probabilities will also be derived. We illustrate the proposed techniques in an example using actual partitioned loss data obtained from the Insurance Services Office.

Lundberg-type Approximations for Defective Renewal Equations: a Heavy-traffic Perspective

Jose Blanchet
Harvard University, USA

Abstract: Renewal equations for which the renewal time distribution, $F(\cdot)$, is defective (that is $F(\infty) < 1$) arise often in many applied probability settings, such as insurance risk theory, queueing theory, reliability and branching processes among others. The text by Willmot and Lin (2001) discusses a great variety of applications to insurance and connections between defective renewal equations (DRE's) and compound geometric sums. Our focus is on DRE's that are close to be proper, that is, $F(\infty) \backsimeq 1$. In the queueing context, this situation is often related to systems operating under heavy-traffic and it also arises naturally in risk theory applications that involve low net profit environments (for instance, low safety loading or small interest rates). Motivated by these applications, we develop asymptotic expansions for the solution of DRE's in powers of $(1 - F(\infty))$. In the presence of light-tailed claims, our expansions can be connected naturally to so-called Lundberg approximations (popular in the analysis of DRE's). However, our expansions can also be developed in the context of heavy-tailed claims; hence, we are able to develop Lundberg-type approximations in the presence heavy-tailed claims. Connections to more standard heavy-tailed asymptotics for ruin probabilities and applications to ruin models perturbed by a diffusion will also be discussed.

Multivariate Phase Variables and Recursion Principles

Karl-Theodor Eisele
Universite Louis Pasteur, Strasbourg, France

Abstract: We present discrete and continuous multivariate phase variables. We show the rationality of their generating functions, resp. their Laplace transforms and calculate the moments in the bivariate case. Finally, a recursion principle is given for compound multivariate phase variables. Examples, calculated by a VB-program, demonstrate how the results are easily applicable.

The Dividends-Penalty Identity and the Optimal Dividend Barrier

Sheldon Lin
University of Toronto, Canada

Abstract: For a general class of risk models, the dividends-penalty identity is derived by probabilistic reasoning. This identity is the key for understanding and determining the optimal dividend barrier, which maximizes the difference between the expected present value of all dividends until ruin and the expected discounted value of a penalty at ruin (which is typically a function of the deficit at ruin). As an illustration, the optimal barrier is calculated in two classical models, for different penalty functions and a variety of parameter values. Joint work with Hans Gerber and Hailiang Yang.