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## Session 35L
## What Is the True Value of Tax Deferral?

**Track:** Education & Research

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**Panelists:** JAMES M. CARSON†
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*Summary: It is a truism that tax deferral is a great benefit enjoyed by qualified plans, individual retirement accounts and deferred annuity products. The actual quantification of this benefit, however, is less obvious. Furthermore, tax deferral does not come free—access to funds in tax-deferred accounts is restricted. In this session, the panelists show how long it takes for tax-deferral benefits to overcome typical penalties levied on early distributions. They also discuss the effect of tax deferral on the risk profile of investments. Attendees gain a better understanding of the value of tax deferral in various accumulation products and its role in product development.*

**DR. KRZYSZTOF M. OSTASZEWSKI:** I am the actuarial program director at Illinois State University. I am the moderator and one of the two speakers in this session. The other speaker is Dr. James Carson, who is a professor and Midyette Eminent Scholar in risk management and insurance at Florida State University. We are presenting research done jointly with Dr. Yu-Luen Ma, who is assistant professor in the department of finance, insurance and law at Illinois State University.

During this session, we will discuss the issue of the value of tax deferral. This is


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**Note:** The chart(s) referred to in the text can be found at the end of the manuscript.

something that matters for us, because we offer a lot of tax-deferred products to our clients, and the accumulation products have been the main area of growth for life insurance companies. We will try to challenge some traditional ideas about tax deferral always being the perfect thing, and then maybe you can tell us whether we are right or wrong. I will let Jim Carson speak now.

**DR. JAMES M. CARSON:** Good afternoon. Krzys' mentioned that we're going to talk about the true value of tax deferral, and one question that you might have right off the bat is: from what perspective are we looking at this? We are going to approach this from the perspective of the individual, the investor purchasing a tax-deferred investment vehicle.

If you are not interested in tax deferral from the perspective of the individual, you can go now, and we will not be upset. If your 401(k)s and 403(b)s are anything like mine, you are having these issues and questions about the true value of tax deferral and what some of the limitations are that you can think about. That is what we will try to do here today. We will share some of our knowledge on how to accumulate and we will discuss some of the various aspects of accumulation. We are both university professors, and knowledge accumulates at universities because freshmen bring some with them, while graduating students do not take away any—this is a process analogous to accumulation of salt in the oceans.

You all know the merits of tax deferral, you have thought about them and are familiar with them, but I want to cover some of the main points here. There are many investment products and insurance products that offer tax deferral features. IRAs, Roth IRAs, 401(k)s and 403(b)s are among them, but the one that would certainly come to your minds immediately would be a tax-deferred annuity. In addition, we should mention something about qualified pension plans. To briefly review, our current income would be deferred, not taxed upfront, and we'd enjoy tax-free accumulation for a number of years, but then we'd be taxed at distribution as regular income.

With tax-deferred annuities, in contrast, we'd have no current deduction, but we'd enjoy tax-free accumulation for a number of years. Some examples of these are single premium, fixed premium, fixed annuity and variable annuities. Regarding IRAs and Roth IRAs, a couple of variations of these will be introduced in the near future, around 2005: the Roth 401(k) and the Roth 403(b).

At this point, there's going to be some tradeoff between the capital gains tax rate and current income tax rate, and we'll come back to that as we move on with our discussion here.

I have a couple of other points concerning the significance of tax deferrals. When it comes to qualified plans and deferred annuities, tax deferral is one of—if not the key marketing point—that is often made. However, tax deferral comes with some limitations. There are some dark sides to tax deferral. One is that access to funds,

prior to some point in the future, is restricted, and penalties are associated with trying to pull funds out before certain times.

Regarding penalties for early withdrawals, we can take loans under some circumstances, such as having a disability or being a first-time home-buyer, but there are some restrictions to that.

In addition, our investment choices are limited. With a qualified plan, the employer would be making the choice of where the funds are invested; and with a deferred annuity, we would pick the vendor or the insurer, but then we would be restricted to those funds or those investments made available by that insurer. In addition, we'd have to use legal vehicles prescribed by the tax code. For example, if we wanted to invest directly in real estate, we'd be limited there. If we wanted to play the gold or silver market, we couldn't do that in these vehicles.

We can't write options, even if we wanted to, and we can do nothing on margin. There are some investment restrictions, as well. Finally, we should note the amount of funds contributed is often restricted in qualified plans, but that is not the case with deferred annuities.

The primary issue that we want to look at is whether tax deferral is worth it. By that we mean, given the potential penalties, restrictions, and possible fees, do the advantages of tax deferral over time make it worth those potential costs?

The conventional wisdom is that, in the long run, the effects of tax deferral outweigh any potential penalties or fees. By that, we refer to taxes, fees, and penalties that might be imposed. I should mention as a side note the recent controversy in pension funding. It's the idea that defined-benefit pension plans perhaps should be funded solely with bonds since any excess investment returns, over the promised pension benefit, would not accrue to the pension holders but ultimately probably back to the firm or someone other than the pension holders. We can come back to that issue later if we have time.

To put some quantification on what the true value of tax deferral is, let me introduce a little bit of notation in Chart 1. We'll use i for the effective annual rate of return. We're going to start out as simply as possible and give a base case using simplifying assumptions. As we move forward, we'll try to complicate it and make it more realistic.

We'll use tau to denote the tax rate paid on investment income if it's held in a non-tax-deferred account, for example putting some money in a bank account. We will use tau for the rate of taxation in such an account as well, which we'll also assume is equal to the rate of taxation on the contribution if that's taxed.

We'll write T for the tax rate paid upon withdrawal of the funds from the tax-deferred account, and that T could include a penalty tax or a fee. T is going to be

larger than tau. We'll use t for the number of years—and this is what we're going to solve for—needed to wait until the taxes on withdrawal from a tax-deferred account will at least make up or make the investor no worse off than regular annual taxation of investment income.

That's about all of the notation that we're going to need. We basically set up the situation on the left-hand side, and this is taking a look at a single point in time. Think about it as a snapshot at retirement. We're looking at what if we had made a single tax-deductible contribution at a point in time before, and we let this accumulate. On the left-hand side, the first term one plus I to the T, which is just the accumulation for a number of years, multiplied by 1 minus T (again T is that tax rate at withdrawal).

The left-hand side is our tax-deferred account and what we get from a single withdrawal at some point in time. We want to set that equal to the non-tax-deferred account, so it's 1 minus tau—that initial tax effect of how much tax we pay on the contribution—multiplied by the income every year after tax, and so in a taxable situation. By setting those equal to each other and solving for T, we want to see how many years it takes to overcome any potential effect of the penalties or fees. Before jumping to a graph, let me mention a couple of assumptions that we'll make. As we move forward, we'll relax some of these assumptions and get a little more complex.

Here we are using a tax-deferred account that does not affect the rate of return. That's a simplifying assumption to start out. Second, we'll assume to start that the taxable and nontaxable have the same degrees of risk, which of course may or may not be the case. Finally, the investment in both sides is the same.

With respect to the tax rates, we'll start out assuming that the tau is 35% and that T, which could include some taxes and fees, is 45%. Based on those assumptions, how long does it take to overcome any potential taxes or fees that we might incur with tax deferral?

On the X-axis of Chart 2, we put the interest rate in and let it vary. On the Y-axis, which is how many years it then takes to make up, you can see that at low rates of return, given the assumptions that we've laid out so far, it takes a fair number of years to make up for the potential taxes and fees that might be incurred in a tax-deferred account.

On the other hand, as we can increase that rate of return, the number of years needed to make up for any potential taxes and fees goes down dramatically. You might say that at a six-percent rate of return, it looks like an individual better stick with that and not pull money out early around eight or 10 years to at least break even in this situation.

Admittedly, we're fairly simplified, and we have some initial assumptions. I'm sure

there are a lot of questions about reality and I'm going to turn it over to Krzys' who will get you a little farther along.

**DR. OSTASZEWSKI:** We will not complicate the picture completely, as it could be in reality, but we will ask questions of what happens when we start complicating things. By the way, it used to be that tax-deferred accounts were great in people's minds because about 20% used to be the range that people were assuming for their investment returns, and I'm wondering what people are assuming right now, but probably somewhere in the area of 3%, or maybe even negative.

There are a lot of things to be complicated in reality because the deposits do not have to be just made at one time. If you include the effect of inflation, you probably should include it as annual tax so both accounts would have some annual taxation. That's a possible complication to the model. You may have taxation of investment income that's different from the taxation of earned income, so a lot of things can be complicated.

We'll try to start complicating them, but we didn't really do all the possible cases. We just want to point out to you that this can be done. What we first showed you was a simple equation, and as whenever I teach for the first exam, I always tell my students that when there's an unknown in the exponent, just take the logarithm at both sides. This makes solving the equation easy, and we have an explicit formula for the solution.

Most of the time, it will not be possible to write down an explicit solution for that waiting time, but it's possible to solve it, and we'll get to this in just a moment when I comment about the next case.

By the way, if the tax rate of the withdrawal is the same as the beginning, the tax deferral always pays. The question is, "Do I want to pay my taxes now, or do I want to pay them later?" If the rates of return are negative, that doesn't hold anymore, so if we assume that you're losing money while you're waiting, you clearly should not wait.

Most of the time, in reality, in the long run, we have positive rates of return, and the limit of this expression that we get for the waiting time is shown in Chart 3. You can see that it mostly depends on the tax rates. We pointed out that the realistic range of returns is three to 12%, and we study it with roughly the U.S. tax rates, including federal, state and possibly local taxes. The waiting time is between 15 years and five years.

Of course, there is an inverse relationship between the rate of return and the waiting time, so that 15 years waiting time corresponds to a lower rate of return, and five years corresponds to a higher rate.

If the contribution is not tax-deductible, the tax on the tax-deferred account is paid

only at the end (see Chart 4). This tax is paid only on the income earned on the account—the excess of the dollar that stayed in the account over one dollar plus we get that dollar back at the end. That's what's included here.

Both accounts are funded with money that was already taxed, so we don't have to take into consideration the initial tax effect. In this case, we don't have an easy, closed-form solution. That doesn't stop us. It used to be harder to do things like that, but now we have a simple way to do this. We just tell Excel Solver to find these points, and some of your students can write macros for you to do it fast. You can even do it by hand. If you can write macros, I admire you. I can't, so I did this by hand and calculated the waiting time. We have a picture of how it compares in Chart 5. Naturally, it is to be expected that the waiting time is longer. It is this upper curve versus the deductible contributions of the lower curve. For this realistic range of returns of somewhere from three to about 12%, we are between 10 and about 30 years. That speaks to us about the tax-deferred annuities being long-term investments if we want to recoup the effect of the taxes unless the tax at the end is the same as at the beginning, in which case it always pays, as before.

This is a situation that applies to tax-deferred products that are not qualified plans and may be of importance for us in understanding our market for deferred annuities.

Here is the situation with a deductible contribution, tax rate tau, and tax-deferred interest becomes the equation in Chart 6. If you plug it in, and those two are equal, these things cancel and become this equation, but this is the same result as a nondeductible contribution with no ultimate taxation (if you plugged in T equal to 0 into this formula). The point I'm making is that if you take this second formula and plug in T equal to 0, you get this. If in this you plug in T equal to tau, you get this. What's the point in this?

This is something that for many people, especially actuaries, makes intuitive sense if you are absolutely certain of the fact that your Roth IRA will not be taxed anymore, which some people doubt. It's true that you can't be certain of anything, except that there will be taxes in the future. You just hope that Roth will not be taxed.

The point is that a fully deductible tax-deferred account produces the same effect as a Roth that is taxed at the beginning and no longer is taxed. In both cases, a certain fraction of your money is left to you after taxes, and if those tax rates are identical, it's the same fraction of your money that is left to you. It doesn't make any difference whether it's taken away from you at the beginning or at the end. It's true that with a Roth that you may have doubts whether it will remain untaxed forever, or it may be taxed with wealth tax or something of that nature.

However, you can have the same worries that your regular account. For example, a qualified account that is accruing tax-free and eventually is to be taxed, might be

taxed more than you expect. Why would Roth become taxable at some point? It might become taxable because the government needs the revenue. You may be able to think of some other reason, but that seems to be the natural one.

That would be the same reason why your qualified plan may become taxed: The government needs the revenue. The same fears that apply to Roth apply to any other account. It's good to remember this. By the way, the Roth IRA is designed to be taxed, at least from a perspective of the employee contributions, the same way as the original design of Social Security was.

Just as there are plans out there to privatize Social Security, maybe this is the stealth way to propose privatization, the one that would be neutral in the sense of a design. It would privatize for the individual accounts and privatize Social Security to be in Roth IRAs, not to be in some copies of qualified plans, but rather in Roth accounts because for the employee, the taxation would become identical in nature. That is, the money going in is taxed money, but when it's received, it's not taxed, except that since 1991 we do tax some of the Social Security benefits, but the original design was basically for the employees, just like a Roth IRA.

All other things being equal, the Roth IRA produces the same effect as a fully deductible IRA or a qualified plan, as long as the ultimate tax level is the same as the original tax level. That applies also to a series of contributions because if it works for every one, it works for the whole thing. As we mentioned, Roth IRA has a greater uncertainty of future taxes. It all depends on how uncertain you are about those taxes.

What is the situation for a series of level contributions? We can write out the equation (see Chart 7). Time n when a tax-deferred account with deductible contributions produces the same effect as a taxable account would now be a solution of this kind of an equation, where we have a different interest rate affected by taxes. This one is not taxed. There is a tax at the end of T and again, we assume that the initial money going into a regular taxable account has the same tax rate as the investment income tax.

You may object to this, but you can see that that's not such a bad thing in terms of a design of the model. If you want the initial money to be taxed differently from the investment income tax every year, all you need to do is just change this tau to some other number, and we will have a third variable that can change, and that's all there is to it. We would still have this equation in which the variable that we would most likely be solving for is n. I don't have a simple solution.

Unfortunately, it has an exponential expression and we would have to deal with it and try to solve it, but you don't have to worry about it because all you need to do is tell Solver to find it for you, and it can be done. These calculations can be done, and it may be a good idea to tell people more about this because I think that most people are not fully aware of the time length, and it is dependent on the interest

rate under the assumed rate of return.

In this case, one significant difference is that there are a lot of contributions that come close to n because it's a series of payments. If they're level, there would be more than in the previous design that come close to n, and remember that with most typical tax ranges, if there is a 10% penalty or anything like that, it's not possible to overcome it over a short period of time.

If this is a series of contributions, it's unlikely that the effect of penalty at the end will be easy to overcome. With some example calculations that I did, it was always a bad idea to take your money out unless you're taking only that portion from the beginning that had a long time to wait there, and then it's the same problem as before.

It's like the story of the mathematician and physicist who were given a problem. The physicist was given a problem of moving a glass of water from the table to the top of the podium, so that's how he solved it. The mathematician had the glass on the floor and was asked to put it on the podium. He moved it to the table and said that this reduces it to the previously solved problem, so we don't have to worry about it any more. If you really are taking the money only from the early contributions, this reduces it to the previously solved problem, so we have nothing more to worry about.

We made some unrealistic assumptions in this. As I already said, we should be looking at varying tax rates, possibly even varying throughout the period of existence of the account. After all, we just had a change in the tax rate on investments, although I think there is a sunset provision in this new law, which amazes me. I find those sunset provisions fascinating.

As I understand, the death tax will expire in 2011. That means that if you're smart, you should plan your departure appropriately because 2010 is when you save a lot of money. You can't take it with you, but still, the temptation is there. I remember that in the late 1990s, I gave my students a life contingencies problem. Assuming the rate of return that Microsoft had throughout the late '90s, they had to figure out how much death tax Bill Gates would pay given the mortality table that they had at the time and how old he was.

We also made an assumption about the growth rate of the federal government budget because I wanted them to compare the death tax on Bill Gates with that growth rate versus the federal government budget, and with the data from that period, Bill Gates would have paid for the government budget for three years. That's of course no longer the case.

However, we want to bring to your attention that, on top of the fact that the tax rates are uncertain, and we might want to change the tax rates for a period and that the investment income tax may be different from the earned income tax, all of

these things that most people probably are aware of that make this calculation more complicated, there is more that gets in the way, and that's what we mentioned at the beginning: In reality, most investment vehicles are risky, and what we were assuming was that there was a risk-free accumulation at a certain interest rate.

Ultimately there will be some realized rate of return, but there's an important thing to think about here. Imagine that you just went through this process of having your 401(k) becoming a 201(k), and you are the one who among the choices provided by the employer decides how to allocate the money. It is established by various studies that people who have higher wealth have more risk tolerance. Those with lower wealth have less risk tolerance.

It is likely that people will adjust their investment portfolio in response to the losses that they experience. In fact, almost all of the investment houses and mutual fund companies say, as you lost half of your money, you should take less risk. That's what the ads on television they tell you, which I think is absolutely false, but that's what we are told. They will provide you for a fee with a consultation about how you can take less risk now as they lost half of your money. While that's nice, I'm not sure if it's the best investment advice.

The fact is that it is not a smooth process, but unfortunately people look at their investment portfolios in the meantime and change asset allocation and that risk matters. It may be a good idea not to look and have a plan and at the beginning, sit down and decide what you're going to do at 25 and then at 45 and then stick to it until you retire. You follow the plan. That's probably not going to be likely in many cases. People will be affected by risk. If they have big losses in their investment accounts, they're likely to change asset allocation, which means that this rate of return doesn't exist in a vacuum. It exists with the riskiness of the investment.

This is basically what started modern finance, what Markowitz started when he introduced the concept of the efficient frontier and how we should invest considering both risk and return. The simplest thing to immediately observe is that if you have a regular taxable account, some of the losses will become deductible and if you have the same losses in the tax-deferred account, tax-deferred means tax-deferred. You will defer your losses.

It's not necessarily perceived as such by most people. It's tax-deferred income. The advertisements don't say, "Put your money in our tax-deferred loss account." That's not necessarily what we hear in the ads, but that's how it is when markets decline. We had a fascinating phenomenon of many people switching to a Roth IRAs in late 1990s. That one is particularly fascinating because that means deferral forever. We'll no longer be taxed. That's it. Therefore, any losses in a Roth are just losses. Tax-deferred forever! Yet we have large amounts of money transferred from regular IRAs, where at least when you take the money out, you have less income to

be taxed to a degree. But in Roths, you lost and the loss is forever.

The situation where at least some of the losses are deductible is dramatically different from the perspective of risk versus the situation where none of the losses are. I'm not a creative accountant, and I'm not a tax lawyer and don't even pretend to be one, so there may be some ways to make some of the losses deductible, and if anybody wants to comment on that, I welcome such comments because we're checking on this.

It still is much harder to realize some of these losses on your tax return. They are losses most of the time, so we are proposing the old Markowitz idea that got him a Nobel Prize in economics that we should compare only investments that had the same degree of risk and that we should make some adjustment for the fact that these are not investments of the same level of risk.

We looked at two possible measures of risk. Not being devoted followers, but being accepting followers of capital asset pricing models, we believe that beta is okay as a risk measure for equity portfolios (see Chart 8). Of course, you may want to debate that. That's something that may not be ideal, but then the question is whether there's any research on what is the effect of taxes on beta. The answer is yes: because it matters for property/casualty companies. Richard Derrig wrote a paper in 1994, where he studied the effect of federal income tax. He was studying this from a perspective of P&C companies, which invest a lot in tax-free bonds because they think that they get great benefit from it and because their investment income for practical purposes is taxable. He tries to convince them that's not the case.

They also invest large portions of their surplus funds in equities, and so he was interested in the question of what the effect of taxes on beta is. He showed that after tax, beta for risky assets whose losses are deductible is given by this formula (Chart 8). There was a paper in *Journal of Risk and Insurance* in '94 about this. It's intuitively reasonable to understand that the government becomes your partner in taking the risk. If you get to deduct the losses in some way against something, that's great. You don't suffer as much, although you have to pay the tax on the upside.

Having this government partner in this process reduces your beta. The after-tax beta is less. It's true that the pesky reality is more complicated than this because, for most individuals, only $3,000 worth of losses are deductible in the same year, and then you have to carry it forward. The question is more complicated, and estimating beta for a multi-period amortization of these losses has not been done.

That's something that may be interesting, but the point remains that a risky investment is a little bit less risky if you get to deduct some of your losses on that investment. After all, if it's risky, you may have losses on it. That's risk. A taxable investment is less risky than a tax-free one for the same asset held in those two accounts. This is exactly the same asset. If it's equities, once you place it in an

account that enjoys tax privileges, it becomes riskier because your losses are losses.

This is especially pronounced for a Roth, and it's fascinating that we are expanding Roth IRAs into 401 and 403(b) Roths at the time when the market is dropping. Maybe after three years of losses, it's possible the market will go up for a while. In fact, it is true that for the entire 20th century, the U.S. market had an average real rate of return of slightly over seven percent, so you can make money in stocks. It's been possible for the last couple of months, so that's not bad.

What about fixed income instruments? In a paper by Jerome Baesel that was published in 1977, he pointed out something that is reasonable if you get some intuition to it. For fixed-income instruments, their after-tax duration generally increases with the tax rate of the investor, and why is that? When do you have a bigger duration? Suppose you have a bond. The bond will return your principal to you as return of principal. That's not taxable, so it's somewhere there at the end. It's hanging out there and is quite heavy.

Duration is this weighted average. Let's think first of Macaulay duration. It's a weighted average of times, and the weights are by the amount of money that you get in terms of the present value. If the coupons on the way are clipped by taxes, they have lower rates, so they contribute less to duration. Having all the cash flows clipped by taxes on the way increases duration. All other things being equal, assuming that the bond is at par, and assuming everything else works reasonably well, if you place a bond in a taxable account and compare it to the same bond in a tax-deferred account, the bond will have a longer duration in the taxable account. That's unfamiliar. It's the opposite effect of what we just discussed with stocks.

The risk if measured by duration is reduced if the investment is placed in a tax-deferred account for bonds—for something that produces regular income. In fact, the effect would be most pronounced if the investment never paid you any principal and paid the income over and over, and if instead of having it taxed, you could not have it taxed. That would be the nicest situation, which I think should give you something to think about. I'm trying to hint at something.

This effect dominates. There also is the second effect, which is that the bond itself may be bought not at par but at a discount or premium. Capital gains taxes cause the after-tax duration to be shorter than before-tax stream. This effect concerns the principal repayments, so the principal repayment may complicate the picture.

Let's take a look at the next one. This is something to think about now. What we basically observed is that in the purest situation, where you're looking at being taxed versus not being taxed, stocks generally become a riskier asset class when placed in a tax-deferred account. We are told most of the time that you have a long time, you're young and you should just buy stocks. Your tax-deferred account should be in stocks. I am somewhat in agreement with that, but from a different

perspective. If you have a very stable and reliable income, as tenured professors and actuaries, you have a fixed income security—your human capital. You are a bond. For professors, that stream of income, is not very high, but stability is there. For actuaries, the income is high, and there's almost no unemployment of actuaries.

The actuaries have job security and provide to themselves a relatively high stream of income until retirement, so they act as bonds for their own personal portfolio. Their human capital acts like a bond. It provides a stream of income, a regular income. You are a bond. You can even introduce yourself that way if you want to. That creates a situation when it's more reasonable to have higher equity exposure because the risk of your equity portfolio is somewhat balanced by your human capital-producing regular income.

Of course, we know the flip side of this. If your income is uncertain, especially if it's correlated with your stock portfolio, and if you work for a company that is risky and all of your investments are in the company stock, you're asking for trouble. There's no balance in your portfolio.

Those risky stocks in your retirement pension portfolio are even riskier, so you're going wild and it's time to become a bond. If stocks are held for a long time in a taxable account, there is a problem in that the effect is somewhat muted by the fact that capital gains are deferred. The effective rate of taxation in the long run on stocks is less because of the lower rate of capital gains taxation. What we're trying to say here may not be as pronounced, on the other hand, if we have this reduction in the dividends tax rate. We'll end up with stocks being taxed much less, and the entire consideration will change.

Nevertheless, we do have this effect. Bonds may become riskier or less risky depending on whether they are held for capital appreciation or for income. If they are mainly for the purpose of income, not having this income clipped makes it nice. There is this controversy already, and there's going to be a special meeting in Vancouver discussing the pension controversy that we mentioned earlier. The controversy is more from the finance perspective.

Why is it that you would be investing in stocks in a defined-benefit pension plan, when if there are any great gains from this, they will not in any way accrue to plan participants? They will accrue to the stockholders of the company or possibly even the managers of the company if the compensation is based on the results of the company, while if things go bad, the government will pick up the tab. The sentiment is, "Let's take all the bets we can because we can take the risk. The government is paying for this."

On the other hand, bonds will provide greater security in the long run. The standard argument is that bonds have greater inflation risk, but there are inflation-protected bonds out there, and it is possible to structure a bond portfolio to have some

degree of protection from inflation. After all, insurance companies offer the deferred annuities that generally beat inflation in the long run and provide something that provides a regular stream of income without getting into capital appreciation on the principal. It's just a stream of income all the time, which is something that gives you the greatest possible benefit of risk control or reduction in a tax-deferred account.

We may be better for our customers than they admit. This basically implies that from the perspective of taking advantage of the tax deferral status and controlling risk, of not increasing risk at all, fixed-income annuities or stable-value funds, which have become somewhat popular, have outstanding and under-appreciated risk characteristics in tax-deferred accounts. In practice, such low-risk or no-risk accounts are quite popular in defined-contribution plans.

There is a complaint about many people having their defined-contribution plans in guaranteed types of accounts. The complaint is that people don't know what they're doing. Well, they sure have looked smart over the past three years. But for somebody who wants to minimize risk, who is conscious about risk or who has low risk tolerance, these may be ideal investments for their tax-deferred accounts.

In any case, there is a greater benefit to using fixed annuities and stable-value funds in tax-deferred accounts than is commonly acknowledged. The most standard argument about the use of these funds is that they provide a good balance in the portfolio. In the portfolio context, they help, and you can enhance the efficient frontier. That is a good argument. It is used in marketing the products, but these are also good products just from the point of view of risk management for people who have low risk tolerance. They are not as bad as they often are presented, and maybe we should be bolder in telling the world, "We're giving these baby boomers something worth considering for their retirement portfolios."

We want to present to you methodologies for comparing investment choices on the risk-adjusted basis. We'll present them to you and discuss them, and one of them seems especially appropriate for the context of tax-deferred accounts, although we'll work on that discussion. We should always adjust the rates of return for risk. That's the basic premise: When you compare two investments, you should not compare them without consideration for how much risk you're taking when you invest in them.

There have been various measures of performance that are proposed for that. We already know that when we compare investment managers, we should not just compare a bond manager to a stock manager and then after a good year in the stock market, fire the bond manager because this person did a terrible job. It's now an accepted premise that we should compare people within their risk class. Investment managers will remind you if you ever forget that because they are sensitive about it. A value-fund manager will not want to be compared to a growth-fund manager. The bond managers will not want to be compared to stock managers

except for the past three years (in this case they will gladly take it).

Chart 9 shows some measures of performance that are studied in theoretical finance when comparing investment performance adjusted for risk. One is the Sharpe measure. That bar stands for average rate of return. If you look for a series of data with the rates of return for your portfolio (P stands for portfolio), Sharpe's ratio in general is the excess of the rate of return over the risk-free rate, divided by standard deviation of the portfolio. That is a measure of return over the risk-free rate that you get for the risk you're taking if sigma, the standard deviation, is the measure of risk. That's where this comes from, and the bars are there because this refers to a practical calculation. You have a series of data of rates of return of a portfolio. You take the average of those rates of return, you take the average of the risk-free rates over the periods that you're looking at and then take those differences and divide it by the standard deviation that you have that you observe. That's one measure that is commonly used in adjusting the rate of return for risk.

The second one is the same thing but divides by beta of the portfolio. It's called the Treynor measure, and the reason why some people prefer it is because standard deviation may include what is called the firm-specific risk or portfolio-specific risk, which is the risk that is taken outside of the general macroeconomic market risk that everybody has to take. When I teach this, I like to say that if you're a policeman, a fireman or a soldier, it's generally an accepted idea that you should be paid for the risky nature of your work. It's a part of your job. But if you want to go bungee jumping, it is an effort to convince people that they should pay you for this because you're basically creating a risk that didn't exist. However, if you're a fireman, the risk that you assume is the risk that as a result of your action is taken away from somebody else. It's basically the idea that the market risk is the risk that affects everybody in the society, and the markets serve the purpose of distributing this among people.

Some willing participants assume more of it, and some people don't want to assume it, but because the people assume it with the benefit for the entire society and the entire economy, it makes sense that there should be compensation for it. That's basically the reasoning. But the reasoning says that if you assume the risk that you want to take and that doesn't take away from anybody else's risks, there's not much societal benefit from this. For example, if you want to buy 10,000 shares of a single penny stock, go for it and hope that you'll be rich tomorrow. If you lose your money, that's too bad, but why should you benefit in the long run? If you diversify your portfolio, those firm-specific risks tend to cancel out, and that's why diversified portfolios of stocks start behaving much like the market. The reasoning is that you should be compensated only for the market risk that you take, but you can leverage the market risk by buying higher beta stocks. You should make an adjustment for the beta that you have because if you leverage up your bet and in a good year make twice as much money, you're making it because you took the risk. You're not making it because it's your skill. That's the source of this adjustment.

Another measure is the Jensen measure, or alpha, as we call it, which is the excess of the rate of return versus what is predicted by a capital-asset-pricing model. That is something that investment managers often like to say is a good measure. When they have this great alpha, they talk about it, and when they don't have it, they typically talk about something else. It's an accepted measure, and anybody who has a positive alpha is proud of it.

Finally, this is the notation for the firm-specific or portfolio-specific risk. I don't have a way to write it, but we decompose the standard deviation squared, so the variants of the portfolio get decomposed into beta squared multiplied by the variance of the market plus the square of this. This is the stupid risk that you took. You produced an alpha, but was it produced because you took a stupid risk? If so, it's not worth it. That's basically the thinking behind the appraisal ratio.

These are four widely accepted measures, although most of the time the investment managers will talk about alpha as the standard measure of performance as adjusted for risk.

One more measure that is appropriate for comparison of the same investment in a taxable versus tax-deferred status is commonly called $M^2$. Because there is an $M^2$ measure in duration convexity analysis, I need to stress that it's a totally different thing.

This one is called $M^2$ because it was introduced by Leah Modigliani and Franco Modigliani. I believe Leah is the granddaughter of Franco. This measure is an ingenious way to compare investments, and the procedure for calculation of this measure is that you mix the managed portfolio, the one that you're evaluating, with Treasury bills, so that the standard deviation matches the standard deviation of the index or another portfolio it is being compared against. We call such an adjusted portfolio P*. The $M^2$ is then defined as the difference in performance of the P* portfolio and the index it is being compared against.

In other words, bring your portfolio to the level of risk of the thing that you're comparing it with. This means that if you want to use this for comparing taxable and nontaxable investments, for equity portfolios, adjust the standard deviation of the nontaxable investment to bring it to the level of the taxable investment.

What this means is that if you place a stock-equity portfolio in a tax-deferred account, its return will become worth less than if it were outside of that account, adjusted for that beta after tax. You should have clipped that return a bit because it's obtained with higher risk. That should be an appropriate way to compare returns.

For fixed-income portfolios, adjust duration of the nontaxable investment to bring it to the level of the taxable investment if we assume that the duration is entirely responsible for the variance of the portfolio. You may want to look for other

adjustments if you don't believe so, but duration is a reasonable measure to use for this. When, for example, you look at estimation of value at risk for fixed-income portfolios, it typically is done with the use of duration. That would be a similar procedure. The adjustment here typically would be that the nontaxable account would benefit from the fact that the coupon income is not taxed and that it has shorter duration as a result.

This is because most of the time, the yield curve slopes up. What we would have to do is look up how much higher the rate of return would be if we extended the duration of the tax-deferred account to the level of the taxable account. There would be some benefit from it, although not huge, but the yield curve does slope up usually. Right now, though, it's quite a slope because a three-year Treasury currently is at a little bit over one percent, and a 30-year bond is over four percent. That's quite a spread.

If you have to make an adjustment by adding almost three percent to the return to adjust for the level of risk, that's a sizable adjustment. If you're offering a tax-deferred annuity that pays current income of three percent, you may want to tell the customers that, adjusting for risk, this is worth six percent. Do you want to use it or not? I think it's worth considering. I'm maybe exaggerating this marketing point here, but you should consider it.

What does this mean for a situation of a Roth IRA and these new Roth vehicles that will be introduced soon? While a Roth IRA is a tax-efficient investment vehicle, there are several risks rarely accounted for: future taxes are uncertain, beta of equity held in a Roth IRA is dramatically increased and the people who hold these accounts may not be fully aware that they're holding risky accounts. I have a Roth IRA, so I'm not saying that it's a bad thing. It's just that there is a great uncertainty, and you don't have the government as a partner in your losses, which makes it a risky investment.

*Stocks for the Long Run* is Jeremy Siegel's book title and a common premise for retirement accounts. Now it is a statement of what in finance is referred to as equity premium paradox, and that is the idea that stocks return too much, that it's unbelievable how much money people make in stocks. I know it doesn't feel like it after the past three years, but if you look at the entire 20$^{th}$ century, Ibbotson produces a book that gives you those numbers, and for the excess of stocks over a diversified portfolio of bonds, the excess return is in the range of five percent. That's the premise. It seems to be way too much for American data.

Maybe this is an expression of what is commonly called a survivorship bias. My favorite comment about the survivorship bias is just to quote Steven Ross about how he explained it. He explained it in one of the SOA meetings. We really don't know the long-term history of many places in terms of data. We have, for example, some data over the past 500 years of interest rates in Europe, but we have much less data about interest rates in Latin America or what used to be Latin America

500 years ago.

There is one river for which we have data for close to 4,000 years. That's the Nile River in Egypt. It's not just in Egypt but most of the data we have is for Egypt, and what is observed is that there are fascinating cosmic cycles in the level of water in the Nile River. It has gone up and down for 4,000 years, although not with the same size cycles. Isn't this a proof of some great mystical force controlling a situation? What distinguishes the Nile River is that it's still around. It survived for 4,000 years. What happened to the rivers that didn't survive for 4,000 years? As they were going up and up, they became lakes, so if they didn't come down, they stayed there. Or they were going down and eventually dried up. If the river didn't become a lake and didn't dry up, if it survived, it had a choice of either staying flat forever or having some mystical cycles. It just took its time to change the level. The point that he is making is that the reason why we observe those mysterious mystical cycles of the Nile River is because it survived 4,000 years. It's just an expression of its survival.

Let me try to adjust it to stocks. The three largest equity markets in 1900 were in New York, London and St. Petersburg in Russia. You did okay with the first two if you held a diversified portfolio, but your rate of return on the diversified portfolio in St. Petersburg if you purchased a basket of stock was negative 100% by 1918. You lost all of your money because it was nationalized by the Bolshevik government. You didn't do so well.

If you combined the three baskets, your equity premium probably didn't work out to be as much as five percent. It was probably lower, meaning that a possible reason why we observed such high five-percent equity risk premium in the United States, although I'm not making a definite pronouncement here because this is an issue debated among finance researchers, is that the United States has successfully survived for 100 years. Had we not survived, some people invested in the U.S. stock market wouldn't make that much money.

The market had a significant drop after September 11, and some people had doubts about the survival of certain institutions. The market is hopefully recovering, but bad things happen when there is doubt about survival. The rates of return can become significantly negative. The premium may be an expression of the fact that we don't account for those entities that did not survive. If you were purchasing a diversified portfolio a long time ago, it would have included some unsuccessful investments. This happened not just in St. Petersburg but for a company that I think was called American Ice Corporation. It used to be a company in 1900 that would make ice because people didn't have refrigerators. It used to be one of the larger companies in the United States. It's not in business anymore. Danny DeVito (well, the person played by him) suggested in the movie "Other People's Money" that the last buggy-whip maker in the United States was probably a top-grade buggy-whip maker, but eventually the business wasn't that good to invest in and disappeared. Maybe it didn't disappear completely, but mostly.

Some companies disappeared, and we tend to not include them in the indices because at some point before disappearance, they disappear from the index as well. It's usually before disappearance. WorldCom was taken off the S&P 500 when it was still selling for dollars. I don't know what the price is right now, but WorldCom was selling for around $65 at some point and for 25 cents at some point. On the way from $65 to 25 cents, at some point it was taken out of the S&P 500. Had it stayed, the performance wouldn't have been so hot, but that's how it works. We have traditional products that over the past three years probably performed nicely for our customers and provided them a lot of good feelings if they held them: fixed annuities and stable-value products. They are not hot. They don't deliver 25% a year. That would be bad news if they did. But they deliver good risk-adjusted performance. The point we'd like to make is that, as the industry, we may be a bit too timid. I ask you to reconsider your pride level in the value of the fixed annuities and similar guaranteed products. They do offer value to their customers. A similar point could be made about annuities in the payout status. I would like to make a pitch here for the value of what we offer to consumers, and also the guarantees provided in other annuities. Equity-linked annuities with guarantees are a good product for the customers to consider. I gave an interesting point about annuities. I sometimes hang around investment people when I grade the CFA exams, and the point that fascinates me is that I was discussing some things with those investment people and said that. "Well, of course everybody has a short position in an annuity, and it's a good idea at some point to cover it." I mostly receive blank stares when I say that to investment folks. I wonder why. What am I talking about here? When you stop working, you will need to pay yourself income for the rest of your life. It doesn't matter what you think about where you're going to get it. You already have that position. You're either going to be working until you drop dead or you will have to pay yourself income from the moment you stop working to the moment you die. You have that position already. It's a short position. The only institutions out there who provide a long position to hedge it are the life insurance companies, and while this is not a hot product and doesn't sell that greatly, it is out there, and it's valuable. Amazingly, it's not valued by the consumers as much as it probably should be.

With this optimistic comment, I'll stop and we welcome your questions.

**MS. DIANE LLOYD:** I'm with Mony Life Insurance Co. In your simplified model, you have T versus tau, and you're showing the potential penalty that you might have to pay, and then you say that it's going to take 10 or 20 years for the tax-deferred to be worthwhile. I'm wondering if T would be closer to tau in the short run because you would be less likely to incur penalties if you had only a few years to go to retirement because you'd be less likely to pull your money out for something unexpected at the shorter period of time.

**DR. OSTASZEWSKI:** It's true that it's also possible that the ultimate rate of taxation on the withdrawal could be close or even identical to the initial rate of

taxation if you're close to retirement age. In that case, deferral always makes sense. There's no doubt about it, although the risk issue is still there. If you are relatively close to retirement, deferral makes sense. From that perspective, it will be beneficial to the consumers that in these new tax laws we increase the contribution limits for people who are closer to retirement. That is a significant benefit to them. We'll see how it works, but Americans are ingenious about finding ways not to save money, which still is not such a bad thing. I ask my students a question when I talk about the effect of savings in the country and the interest rate. The question is whether, if the rate of return on your investments increases, you would increase your savings rate or decrease your savings rate. It's fascinating because almost all foreign students, and we have a lot of them, say they would increase their savings rate, and almost all Americans say they'd decrease their savings rate. They are both, of course, quire right. After all, Americans tend to not leave much inheritance to their heirs, while non-Americans do leave money to their heirs. As the great Walter Williams puts it, he smokes because he wants to make sure that when he dies, his body is used up completely.

**MR. VANCE BERGER:** Along the same lines as the previous question, I have one challenge to the assertions that you made. First of all, is it not possible that, if you defer your taxes until you retire, the T is less than tau if you're taxed at a lower rate? Second, does that necessarily mean that you should defer anyway? You did mention other drawbacks of deferral that you didn't quantify in the equation (those limitations placed upon what you can invest in and the time at which you can take it out), so isn't it possible that you can say that even with a lower T than tau, you may still have a tradeoff to consider?

**DR. OSTASZEWSKI:** If the ultimate tax rate is lower, generally it does make sense to defer the income. On the other hand, you increase the risk if you use a risky investment in the tax-deferred account, and then it's an interesting question of when the two balance each other. This is another argument for using a fixed annuity. If your ultimate tax rate is going to be low, that's great. Use a tax-deferred fixed annuity.

**MR. DAVID MERKEL:** I have two comments. I think there's one other factor that might go into the equity premium puzzle that I don't often see talked about. One is that most of the time when they talk about the equity premium, they're talking about time-weighted returns. But the ways people get returns in the market are more often dollar-weighted returns, and people have a tendency to not be investing in stocks when things are near the lows, and they tend to be investing aggressively when things are near the highs. You could have a great time-weighted return on that, and your dollar-weighted return which is how GDP works, would not be all that great. Some of the small cap outperforms can be explained that way, too, because they rely heavily, to some extent, on the returns during the Great Depression of a few small cap stocks. But who had the chutzpah to put money into it at that point in time?

**DR. OSTASZEWSKI:** Keynes did. Keynes had double the rate of return of the market during the Great Depression. He also had double the rate of risk of the market.

**MR. MERKEL:** My second comment is perhaps we can create a more complex model than this for trying to predict what the tax rate will be in the future, simply because the states, the Feds and everybody as a whole extract about 25% of the GDP on a fairly regular basis.

**DR. OSTASZEWSKI:** We're now close to 35.

**MR. MERKEL:** Still, you can start with that and then say, "How many of the factors of production are going to get drawn off because they have to equal the sum total of the GDP?" which is the same way that you can do it on a production basis. You then say that because of Roth and a few other things, these are now blocked out from contributing. How much do the rest have to bear to do it? At some rate, labor begins to rebel. This may be true particularly in 2020 or 2030, when you're down to a 2:1 ratio of workers to retirees, so you can begin to try to get an estimate of those as to whether T actually might be considerably higher or whether you might breach the Roth IRA at some point.

**DR. OSTASZEWSKI:** You know the case for this is probably much stronger in Europe than in the United States.

**MR. MERKEL:** The United States has immigration, and the United States has a higher birth rate.

**DR. OSTASZEWSKI:** Yes, and...

**MR. MERKEL:** So watch them and see...

**DR. OSTASZEWSKI:** The German government consumes more than 60% of its GDP. Even the most pessimistic scenarios for the future of the United States don't reach that level of taxation, and on top of this, the German government in the long run has promised its citizens to pay them more than the GDP. That would be a neat trick.

But you're absolutely right, and this has been raised by William Sharpe. The alternative way to put what you said is that you could explain it by the fact that these excess rates of return are earned by people who take more risk than we normally account for because they are willing to buy when nobody else is willing to buy. This is the same phenomenon as with small stocks. If this is a diversified societal risk, they probably should earn the higher rate of return. That point is definitely an important one.

Chart 1

How long does it take for deferral to work?
(single tax-deductible contribution)

$$(1+i)^t(1-T)=(1-t)(1+(1-t)i)^t$$

$$t=\frac{\ln\left(\frac{1-T}{1-t}\right)}{\ln\left(\frac{1+(1-t)i}{1+i}\right)}$$

Chart 2

# How long does it take?

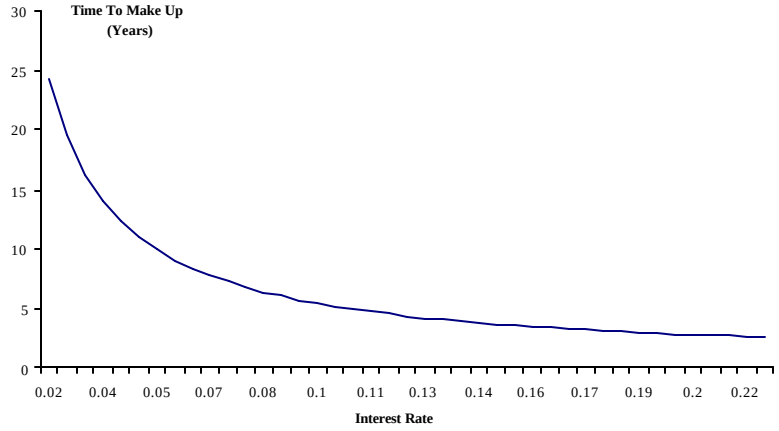

Chart 3

# Waiting time values

- If tax rate at withdrawal same as at the beginning, tax deferral *always* pays (for a positive interest rate).
- Limit $$\lim_{i\to\infty}\frac{\ln\left(\frac{1-T}{1-t}\right)}{\ln\left(\frac{1+(1-t)i}{1+i}\right)}=\frac{\ln(1-T)}{\ln(1-t)}-1$$
- For a realistic range of returns: 3% to 12%, and U.S. tax rates, the waiting time is between 15 years and 5 years.

Chart 4

What if contribution is not deductible?

$$\left((1+i)^t-1\right)(1-T)+1=\left(1+(1-t)i\right)^t$$

- There is no easy closed-form solution, but Excel's Solver can find one easily.

Chart 5

## Waiting time with non-deductible contribution (upper curve) vs. deductible contribution (lower curve)

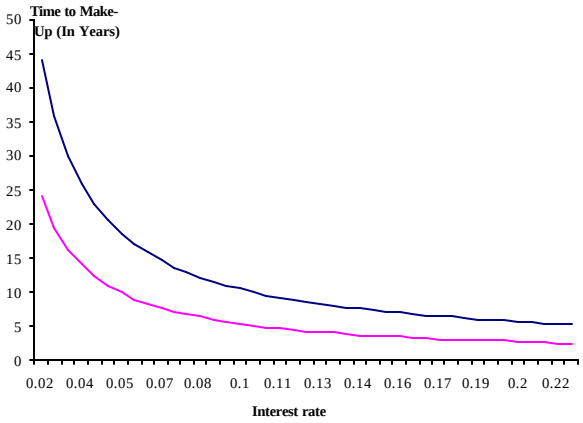


Chart 6

## Compare deductible and non-deductible

Deductible contribution with $T = t$ and tax-deferred interest:

$$(1+i)^t(1-T) = (1-t)(1+(1-t)i)^t$$

becomes $$(1+i)^t = (1+(1-t)i)^t$$

produces the same result as non-deductible contribution with no ultimate taxation ($T = 0$) and tax-deferred interest:

$$((1+i)^t - 1)(1-T) + 1 = (1+(1-t)i)^t$$

becomes $$(1+i)^t = (1+(1-t)i)^t$$

Chart 7

What is the situation for a series of level contributions?

The time *n* when tax-deferred account with deductible contribution produces the same effect as taxable account, is given by:

$$(1-T)a_{\overline{n}|i} = (1-t)a_{\overline{n}|i(1-t)}$$

and, unless the penalty tax is very small, its effect is generally not possible to overcome, because many payments have only short time to accumulate higher rate of return.

Chart 8

# After-tax beta

- Richard Derrig (1994) shows that after-tax beta for a risky asset, whose losses are deductible is:

$$b_{\text{AFTER-TAX}} = b\frac{(1-t)(1+r_F)}{1+(1-t)r_F} < b$$

- Thus, a taxable investment is *less risky* than tax-free one, for the *same asset* held in the two accounts.

Chart 9

# Measures of performance

- Sharpe's Measure $\dfrac{\bar{r}_P - \bar{r}_f}{s_P}$
- Treynor Measure $T_P = \dfrac{\bar{r}_P - \bar{r}_f}{b_P}$
- Jensen Measure $a_P = \bar{r}_P - \left(\bar{r}_f + b_P\left(\bar{r}_M - \bar{r}_f\right)\right)$
- Appraisal Ratio $\dfrac{a_P}{s\left(e_P\right)}$