The Actuary

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Actuarial Meetings

Jan. 12, Baltimore Actuaries Club
Jan. 17, Chicago Actuarial Club
Jan. 18, Seattle Actuarial Club
Feb. 9, Baltimore Actuaries Club
Feb. 15, Seattle Actuarial Club
Feb. 21, Chicago Actuarial Club

HEALTH INSURANCE SPECIALTY MEETING REQUEST FOR PAPERS

by Robert E. Hunstad

You are invited to write a paper for publication in the *Transactions*. Papers on health insurance are specifically sought to enhance the program for a 1979 special topic meeting of the Society of Actuaries.

Past specialty meetings, co-sponsored by the Program Committee and the Committee on Continuing Education and Research, have provided educational opportunities for our members. Another purpose of the Society's continuing education effort is to encourage the development of actuarial literature. This special call, made by the Committee on Health and Group, is to encourage your individual contribution to our literature.

The final program for the 1979 special meeting on health insurance will be determined, in large part, by the subjects covered in papers submitted in response to this call. Topics are limited to health insurance, but could cover any specific subject within that general category.

Procedures for submitting papers are outlined on pages 13 and 14 of the *Year Book*. To assure that papers are available for the meeting, deadlines have been established. Potential authors are asked to submit an outline of their proposed paper to the Executive Director by July 1, 1978. Information received by this date will be used in the initial program planning. Completed papers must be submitted no later than September 15, 1978, to permit adequate time for review, editing, printing and distribution prior to the meeting. Submission of manuscripts and outlines in advance of these deadlines would help the review process of the Committee on Papers.

Additional information may be obtained from Stephen T. Carter, Chairman, Committee on Health and Group.

JOINT LIFE ANNUITY FORMULATIONS

by Samuel H. Cox

An appendix to Harold Cherry's article, "The 1971 Individual Mortality Table" (*TXA XXIII*, 1972, p. 475), contains a FORTRAN program which produces annual payment, joint life immediate annuity rates. The program has been modified by the author to compute other types of two life annuities including those designated "qualified joint and survivor annuities" in the Employee Retirement Income Security Act of 1974. The modified program is also capable of determining rates based on modes of payment other than annual. Copies of the modified program are available from the author.

The major modifications allow for frequency of payment other than annual, computation of single life in addition to joint rates and for other than straight joint life annuity forms. In allowing for frequency of payment other than annual, the problems reported in *The Actuary* by Hermann Edelstein (January 1977) and Dave Becker, Imen Bojrab and Lee Buchele (April 1977) were avoided by using the type of approximation suggested by John A. Mereu (also in *The Actuary*, April 1977). Mr. Mereu suggested using the uniform distribution of deaths (UDD) hypothesis for evaluating $\ddot{a}^{(m)}_{x:\overline{n}|}$; applied to immediate and continuous annuities this gives, respectively,

$$a^{(m)}_{x:\overline{n}|} = \frac{id}{i^{(m)}d^{(m)}}\, a_{x:\overline{n}|} + \frac{d^{(m)}-d}{i^{(m)}d^{(m)}}\,(1-{}_nE_x)$$

and

$$\bar{a}_{x:\overline{n}|} = \frac{id}{\delta^2}\, a_{x:\overline{n}|} + \frac{\delta-d}{\delta^2}\,(1-{}_nE_x).$$

This approximation amounts to assuming that ${}_{k+t}p_x$ is a linear function of $t\ (0 \le t \le 1)$ for integral values of k and x. The algorithm uses the same method to approximate $a^{(m)}_{xy:\overline{n}|}$. That is, ${}_{k+t}p_{xy}$ is assumed to be a linear function of t. This results in the following approximations:

$$a^{(m)}_{xy:\overline{n}|} = \frac{id}{i^{(m)}d^{(m)}}\, a_{xy:\overline{n}|} + \frac{d^{(m)}-d}{i^{(m)}d^{(m)}}\,(1-{}_nE_{xy})$$

$$\bar{a}_{xy:\overline{n}|} = \frac{id}{\delta^2}\, a_{xy:\overline{n}|} + \frac{\delta-d}{\delta^2}\,(1-{}_nE_{xy})$$

If the interest only functions are evaluated with $i = 0$, then the more common approximation result:

$$a^{(m)}_{xy:\overline{n}|} = a_{xy:\overline{n}|} + \frac{m-1}{2m}\,(1-{}_nE_{xy})$$

$$\bar{a}_{xy:\overline{n}|} = a_{xy:\overline{n}|} + 1/2\,(1-{}_nE_{xy})$$

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Joint Life Annuity Formulations

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Mr. Edelstein and Mr. Becker, et al, report that approximations of this latter type lead to anomalies like

$$\ddot{a}^{(12)}_{x:\overline{n}|} > \ddot{a}^{(12)}_{\overline{n}|}$$

and

$$\ddot{a}^{(m)}_{x} > \ddot{a}^{(m)}_{\overline{x:\overline{n}|}}$$

at high interest rates and low issue ages. The first "inequality" is equivalent to the second because $\ddot{a}^{(m)}_{\overline{x:\overline{n}|}}$ is identical to $\ddot{a}^{(m)}_{x} + \ddot{a}^{(m)}_{\overline{n}|} - \ddot{a}^{(m)}_{x:\overline{n}|}$.

After substituting this expression in the second inequality and rearranging terms, one obtains the first. As Mr. Becker, et al, conjecture, the anomalies occur when p_x is close to one and i is high. Mr. Mereu proves this, at least for the case of continuous payments. The case of payments m times per year is similar. For example, the common approximation will lead to the absurdity $a^{(m)}_{\overline{xy}:\overline{1}|} > a^{(m)}_{\overline{1}|}$ exactly when

$$p_{\overline{xy}} > 2m(1+i)\left(d/i^{(m)} - (m-1)/2m\right)/(m+1).$$

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LETTERS

Theory of Interest

Sir:

Your readers might find the following problem entertaining:

Prove using the theory of interest techniques the following inequality:

For n an interger $e^n < \frac{(n+1)^{n+1}}{n!}$

The solution is to show that if the force of interest at time t is $\delta_t = 1/1+x$ then the present value of an *n* year continuously increasing annuity paid continuously is $n - \ln(n+1)$ and the present value of an *n* year annuity paid continuously where payments in the t'th year total $\$\,t$ is $\ln\left[\frac{(n+1)^n}{n!}\right]$

And since the former is less than the latter, the solution follows.

Ralph Garfield

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To Be Continued

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rier will. The insurance certificates must be changed to incorporate this type of wording. No reserves are required of the policyholder by the insurance carrier. It remains to be seen as to how popular this approach will become.

The Future

The demand by policyholders for some type of special financing arrangement is continuing, and we may well expect that in a few years a substantial portion of group medical business will be on this basis.

Group insurance has become a large and complex industry with a myriad of new and sometimes complicated financial terms. Undoubtedly in the future many more financial arrangements will emerge. □

International Congress

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are five general subjects on which papers by individuals are requested:

(1) Generalized models of insurance business (Life and/or Non-Life), (2) Testing hypotheses by statistical investigations (Life and/or Non-Life), (3) Statistical bases and experience under Disability, Sickness and Accident insurance, (4) Estimating the value of insurance companies and insurance portfolios, (5) Interrelationships between demographic and economic development and social security (including occupational and private insurance).

Detailed descriptions of the above topics will be sent out to members of the International Actuarial Association early in 1978.

Papers may be submitted any time up until January 31, 1979, to the appropriate National Correspondent, Lawrence Coward in Canada and John Wooddy in the United States. If the paper is to be submitted after September 30, 1978, (but by January 31, 1979) the author or authors should notify the appropriate National Correspondent of the title of the paper prior to September 30, 1978. Papers may be submitted in English, French, German, Spanish or Italian.

Attendance at the Congress will be limited to 1,000 members from outside Switzerland. □

About VLI

(*Continued from page 1*)

search Corporation, can be found in "The Life Insurance Industry's Marketing Dilemma." Most probably, only the forewarned would guess that a report so titled would be about VLI. This is another of TLIIMD's strengths, for it offers an interesting viewpoint as to how VLI may fit in context, from the standpoint of not only the industry but also the public.

The dilemma, as stated in the first chapter of the report, is that there are serious doubts as to the future of permanent life insurance in general and non-par permanent life insurance in particular, while the industry continues to lose its share of the savings dollar. At the same time, agents' earnings have flattened out and now barely keep pace with inflation. And policyowners continue to have problems of underinsurance and keeping the coverage they have current with inflation. I would agree that this is a rational summary of today's situation; the real question is whether (as this report suggests) VLI does indeed offer a solution to the industry and policyowners alike.

With this thesis established, the next two chapters of TLIIMD concisely and (for the most part) accurately summarize the regulatory scene, past, present, and future outlook. My one reservation is that the SEC's final rules are characterized as "a qualified victory" for the mutual fund industry. This is akin to saying Muhammed Ali's bloody Manila knockout of Joe Frazier was "a qualified victory" for Ali because his opponent remained alive. It is also interesting to see Mr. Johns state that "most knowledgeable observers" believe the eventual outcome of VLI tax treatment at the company level is an approach under which there would be "virtual tax neutrality between variable and fixed policies." I would agree (thus making me a knowledgeable observer) but legislation is almost certainly involved if this goal is to be truly reached. It is with sadness that I note a history like this can (properly, in context) describe what happened without ever mentioning the people who made it happen. Thus, there is no mention at all of Charlie Sternhell and John Fraser, and only an inconsequential reference later in the report to Harry Walker.

The next chapter describes Equitable's VLI product, markets and experience up to the mid-1977 date when the report was published. The material describing the product itself and how it works is generally clear. The only portion where the reader may run into trouble because of tangled language is the section describing the mechanics of how the death benefit changes. This section is quoted verbatim from the prospectus.

There are two interesting observations in this chapter: (a) that, with gross investment returns of 8% or less (as illus-

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Joint Life Annuity Formulations

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For monthly payments and i = .05, for example, we can expect distorted values of joint and last survivor annuities with 12 payments certain if the issue ages x and y are such that $p_{\overline{xy}} = p_x + p_y - p_{xy}$ exceeds 0.999628. This occurs in the 1971 Individual Annuity Mortality table for ages as high as x = y = 65. Moreover, the right hand side of the inequality is a decreasing function of i; so we can expect more distortions at higher values of i. Mr. Mereu indicates that these anomalies can be avoided by using the uniform distribution of deaths assumption. This is easy to prove. Starting with the approximation

$$a^{(m)}_{xy:\overline{n}|} = \frac{id}{i^{(m)}d^{(m)}}\, a_{xy:\overline{n}|} + \frac{d^{(m)}-d}{i^{(m)}d^{(m)}}\,(1 - {}_nE_{xy})$$

and successively using

$$a_{xy:\overline{n}|} = a_{xy:\overline{n-1}|} + {}_nE_{xy} \le a_{\overline{n-1}|} + {}_nE_{xy}$$

and

$${}_np_{xy} \le 1$$

we obtain

$$a^{(m)}_{xy:\overline{n}|} \le \frac{id}{i^{(m)}d^{(m)}}\,(a_{\overline{n-1}|} + {}_nE_{xy}) + \frac{d^{(m)}-d}{i^{(m)}d^{(m)}}\,(1 - {}_nE_{xy})$$

$$a^{(m)}_{xy:\overline{n}|} \le \frac{d - dv^{n-1} + d^{(m)} - d}{i^{(m)}d^{(m)}} - \frac{d^{(m)} - d - id}{i^{(m)}d^{(m)}}\, v^n\, {}_np_{xy}$$

$$a^{(m)}_{xy:\overline{n}|} \le \frac{1}{i^{(m)}d^{(m)}}\,(d^{(m)} - iv^n + (i - d^{(m)})v^n\, {}_np_{xy})$$

$$a^{(m)}_{xy:\overline{n}|} \le \frac{1}{i^{(m)}d^{(m)}}\,(d^{(m)} - v^n d^{(m)}) = a^{(m)}_{\overline{n}|}$$

Therefore, like Mr. Mereu's suggestion, the linearity of ${}_{k+t}p_{xy}$ produces values which never lead to the anomalies reported earlier in *The Actuary*. □