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LETTERS

Tax-Exempt Investments

Sir:

In the November 1977 issue, Mr. Clayton A. Cardinal expressed worry about tax-exempt bonds in insurance company investment portfolios. He also expressed worry about trends in the use of credit, about inflation and federal deficits, concerns which we share. It is suggested his comment on investments does not reflect historical fact nor appreciation of inherent differences between corporate and tax-exempt issues.

(1) As a class of securities, state and local bonds have a record far superior to that of corporate bonds, both in terms of safety of principal and continuity of interest payments. This stems mainly from the fact that states and cities do not disappear into thin air when financial problems develop, as frequently occurs in a corporate bankruptcy where the entity may be liquidated. During the great Depression of the thirties, loss of principal by tax-exempt bond holders was indeed a rarity, but principal losses, occasionally 100%, were the rule in corporate bankruptcies.

(2) As inflation has placed more and more taxpayers in higher income tax brackets, the investment appeal of tax-exempt bonds has broadened, reducing the appeal of corporate bonds for most individuals. The secondary market for tax-exempt bonds has far more depth and breadth than that of corporate bonds, and is second in this respect only to U.S. Government obligations.

(3) Using New York City as a basis for condemnation of tax-exempts as a class shows a lack of appreciation of the range in quality available to the investor. New York City is at one end of the quality range and cities such as Omaha, Nebraska, rated Aaa by Moody's, are at the other end, with all shades of quality in between. It is up to the investor to choose his risk level. The same logic, of course, applies to corporate bonds. I would not suggest that because of Penn Central corporate bonds are unsuitable for investment purposes. High-quality corporate bonds are obviously suitable for investment purposes.

(4) Mr. Cardinal's comment about conventional mortgages is also subject to question in a society which seems to prefer a full employment policy by government despite possible inflationary

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SECOND TO DIE JOINT LIFE CASH VALUES AND RESERVES

by William M. Frasier

An increasingly popular tool in estate planning is the Joint Life policy payable upon the second death. The design and pricing of this policy is influenced significantly by cash value and reserve requirements. This article discusses two methods for calculating these values.

Under Method I the cash values and reserves for the Joint Survivorship policy are calculated using joint last-survivor functions while both insureds are alive and single life functions after the first death. The calculation of the statutory terminal reserves and cash values while both insureds are alive is

$A_{\overline{x+t:y+t}} - P_{\overline{xy}} \cdot \ddot{a}_{\overline{x+t:y+t}}$ where $P_{\overline{xy}}$ represents either the adjusted premium or the net valuation premium. The terminal reserves or cash values after the first death are calculated using the formula $A_{x+t} - P_{\overline{xy}} \cdot \ddot{a}_{x+t}$ where x is the survivor and $P_{\overline{xy}}$ is the same as above. Method I produces a large discontinuity in cash values and reserves upon the first death, as shown below:

Joint Equivalent Issue Age 45

End of Policy Year	*Cash Value Per $1,000*			*CRVM Term. Res. per $1,000*		
	Method II	*Method I*		*Method II*	*Method I*	
		Both Alive	*One Alive*		*Both Alive*	*One Alive*
1	−14.49	−15.99	124.25	0	0	0
10	158.95	137.00	286.33	170.96	150.58	297.56
20	389.31	332.80	474.76	398.03	343.29	483.03
30	609.61	528.85	634.51			

Method II determines the cash values and reserves based upon the survivors of the joint last-survivor status without distinguishing between the case where both insureds are alive and the case where only one survives. This results in a single cash value and reserve scale. Method II produces values that are greater than the values calculated by Method I when both insureds are alive and less than the Method I values after the first death. This approach produces precisely the same net single premium and annuity value at issue as Method I.

The surviving joint last-survivor status (call it $l_{\overline{\overline{xx}}}$) is the sum of the surviving joint life status and the surviving single lives. If we assume that both insureds are the same age or that we can calculate a joint equivalent issue age (x), the $l_{\overline{\overline{xx}}}$ values can be easily determined. The surviving joint life status, $l_x^{(T)}$, is decremented by the probability that both insureds will die, $(q_x)^2$, and the probability that one will die and the other survive, $2p_xq_x$. The number of single lives, $(hl)_x$, is incremented by the survivors after a single death and decremented by the single life deaths. This method generates a unique set of values for each equivalent issue age with the initial surviving joint life status, $l_x^{(T)}$, being the radix that we choose for our table and the initial number of single lives, $(hl)_x$, equal to zero. In general then:

$$l_{\overline{\overline{xx}}} = l_x^{(T)} + (hl)_x; \quad l_{x+1}^{(T)} = l_x^{(T)} - d_x^{(d)} - d_x^{(h)}$$

$$d_x^{(d)} = l_x^{(T)} (q_x)^2; \quad d_x^{(h)} = l_x^{(T)}\, 2 p_x q_x$$

$$(hl)_{x+1} = (hl)_x + d_x^{(h)} - (hd)_x; \quad (hd)_x = (hl)_x q_x$$

$$d_{\overline{\overline{xx}}} = d_x^{(d)} + (hd)_x; \quad q_{\overline{\overline{xx}}} = d_{\overline{\overline{xx}}} \div l_{\overline{\overline{xx}}}$$

Sample values, shown below, are calculated using the 1958 CSO Mortality Table and the Security Life of Denver graduation of the 55-60 Basic Select and Ultimate Table:

1958 CSO Mortality

Att. Age	$l_{\overline{\overline{xx}}}$	$l_x^{(T)}$	$d_x^{(d)}$	$d_x^{(h)}$	$(hl)_x$	$(hd)_x$
45	1.000.00	1,000.00	.03	10.64	.00	.00
46	999.97	989.33	.03	11.47	10.64	.06
47	999.88	977.83	.04	12.36	22.05	.14
48	999.70	965.43	.05	13.32	34.27	.23
49	999.41	952.06	.06	14.36	47.36	.36
50	999.00	937.64	.06	15.47	61.36	.51

55-60 Basic Select and Ultimate Table

Att. Age	$l_{\overline{\overline{xx}}}$	$l_x^{(T)}$	$d_x^{(d)}$	$d_x^{(h)}$	$(hl)_x$	$(hd)_x$
45	1.000.00	1,000.00	.01	3.77	.00	.00
46	1,000.00	996.22	.01	5.06	3.77	.00
47	999.94	991.15	.01	6.28	8.83	.02
48	999.94	984.86	.02	7.44	15.09	.06
49	999.87	977.40	.02	8.56	22.47	.10
50	999.75	968.82	.02	9.66	30.93	.15

Having calculated the $l_{\overline{\overline{xx}}}$'s for an equivalent issue age the calculation of the commutation functions, net single premiums and annuity values proceeds as if we had a single decrement table. Thus,

$$D_{\overline{\overline{xx}}} = v^x l_{\overline{\overline{xx}}}, \quad C_{\overline{\overline{xx}}} = v^{x+1}\left(l_{\overline{\overline{xx}}} - l_{\overline{\overline{x+1:x+1}}}\right),$$

$$N_{\overline{\overline{xx}}} = \sum_{t=0}^{\infty} D_{\overline{\overline{x+t:x+t}}}, \quad M_{\overline{\overline{xx}}} = \sum_{t=0}^{\infty} C_{\overline{\overline{x+t:x+t}}},$$

$$A_{\overline{\overline{xx}}} = M_{\overline{\overline{xx}}} / D_{\overline{\overline{xx}}} \text{ and } \ddot{a}_{\overline{\overline{xx}}} = N_{\overline{\overline{xx}}} / D_{\overline{\overline{xx}}}.$$

The cash values and reserves can then be determined in the usual manner.

If one compares the net level reserves or the minimum cash values, calculated by the two different methods using a group of lives, where the mortality of the group of lives involved equals 1958 CSO Mortality, the total terminal reserve or cash value for the block of business will be equal.

The illustration below shows the above equality for minimum cash values with the small differences due to the rounding of the cash values per $1,000 to near cent. It also compares the CRVM reserves calculated by the two different methods. Starting with a block of 1,000 joint lives and assuming 1958 CSO Mortality, the total reserve on the block of business is slightly smaller using Method II in the early durations. This difference decreases as the business matures. If the mortality experienced is equal to 55-60 Basic Select and Ultimate Table, the total reserve using Method II is larger at all durations than the reserve calculated using Method I:

Joint Equivalent Issue Age 45

End of Policy Year	*Total Cash Value*		*Total Reserve*			
			58 CSO		*55-60 Basic*	
	Method II	*Method I*	*Method II*	*Method I*	*Method II*	*Method I*
10	157,950.17	157,946.53	169,884.63	171,097.93	170,589.36	163,303.54
20	365,273.68	365,270.78	373,455.31	374,284.02	383,488.56	373,957.11
30	429,465.49	429,464.62				

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effects. A portfolio of conventional mortgages, carefully selected and diversified geographically, will weather business storms as well as a portfolio of corporates or tax-exempts. Mortgage payments have a high priority in family budgets, especially in conventional mortgages where there is an equity to preserve.

Finally, if Mr. Cardinal wants to worry about something, he should consider the private placement corporate loans that dominate the investment portfolios of most life insurance companies. These issues are not readily marketable and you can bet your life that many which looked good when acquired have marginal quality now.

William W. Hill

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Par *vs.* Non-Par

Sir:

I found Robin Leckie's suggestions about non-participating permanent life insurance (November issue of *The Actuary*) provocative, to say the least.

At first glimpse, it appears a contradiction in terms to suggest that stock life companies should sell only participating insurance and that non-participating products should only be sold by mutual companies. Any company operating under New York Law would certainly have trouble with this concept.

Mr. Leckie feels there is an additional risk on non-par insurance from adverse experience, and that the "owners" of a mutual company are in a better position to accept that risk (by borrowing from par-fund surplus and premium margins) than the professional businessman and risk-taker whose capital supports a stock company. I wonder if the policyholder/owner of a mutual company would agree. Par dividends would go down, not only because of the adverse experience within the policyholder's own class of business, but also to recoup simultaneous losses in the non-par fund (a double-whammy effect which few would appreciate).

The owners of a stock company, on the other hand, are in the same risk-taking business as any capitalist, and should be in a better position to assess and accept that risk, given adequate information from the management.

I would go so far as to suggest that no mutual company should write a sig-

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nificant portion of its business on a non-participating basis without full disclosure to its policyholder/owners of the expected profits and risks inherent in this investment. If Mr. Leckie had suggested that no company, stock or mutual, should write non-par business without a margin of capital and surplus sufficient to cover adverse experience within a conservative confidence level, I would agree whole-heartedly. *D. C. Townsend*

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Replacement Ratios

Sir:

Ray Peterson's letter in the December issue makes a good point about Social Security replacement ratios. However, the ratios for *net* income are even higher when FICA taxes are also deducted from the pre-retirement income.

Lower marginal tax rates after retirement will also result in a better net return on any investment income.

Participation limits for disability income benefits should be subjected to a similar analysis of spendable income after taxes. In this case, I recommend a further adjustment for waiver of premium benefits. Premiums could be deducted from net income before disability or added to benefits received.

Don't overlook *life* insurance in this one; the buyer of substantial disability income should be assumed to have significant life insurance premiums subject to waiver of premium. *Curt Greene*

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Adjusted Premiums

Sir:

In response to the letter in the February, 1978 issue by Amy Hicks and Jeff Sonheim adjusted premium for cash values, I believe reference should be made to equation (6.9) in Jordan which defines the adjusted premium as a function of E′. If E′ is minimized, then the adjusted premium is necessarily minimized.

It is possible to develop expressions for the adjusted premium for each possible relationship between the items in the brackets of equation (6.11). After simplifying the algebra gives three different equations for the adjusted premium. To use the equations just compute the values of all three and take the smallest value. This approach is especially handy in computer applications.

Larry Cohen

Letters

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Treasury Bill Yields

Sir:

Harry Ploss in the December issue notes that in the financial world, interest formulas are often less logical than those encountered in Part 3. He cites the rates used by Savings Banks for Certificates of Deposit as an example. Unfortunately, he compounds daily instead of continuously, leading to some confusion.

Since the banks advertise continuous compounding, one might expect the accumulation of \$1 in one year at nominal i to be e^i. But the nominal rates are given relative to a 360 day "bank year"; a calendar year accumulation is actually $(e^i)^{365/360} = e^{\frac{365}{360}i}$. With this formula it is easy to verify the yields quoted by Mr. Ploss.

A more interesting question is the one asked from time to time by an adventurous lawyer or accountant: How significant is the difference between daily and continuous compounding? Using the binomial theorem we can see

$$\left(1+\frac{i}{n}\right)^n = 1 + i + \frac{i^2}{2}\left(1-\frac{1}{n}\right) + \frac{i^3}{3!}\left(1-\frac{3}{n}+\frac{2}{n^2}\right) + \ldots$$

so that $e^i - \left(1+\frac{i}{n}\right)^n \approx \frac{i^2+i^3}{2n}$. Over the course of a year, with $i = 8\%$ and n = 365 the error is about .00095% or roughly one ten-thousandth of the yield. Should the compounding be weekly (n = 52), the error is about a thousandth of the yield which, considering the large principal often encountered in Certificates of Deposit, might be significant.

Ken Avner

Second To Die

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Method I is currently being used. Method II would seem to be an acceptable method for calculating reserves since in aggregate the reserves calculated by Method II are close to or greater than those calculated by Method I. If one takes a prospective view of cash value equity, the Method II cash values are inadequate. This may cause regulatory problems. However, if one looks at the cash value calculation retrospectively, the Method II cash values are adequate.

Method II has several advantages from both the company's and the insured's point of view:

(1) The single cash value and reserve scale simplifies many steps in product development and administration,

(2) The asset share model will only be concerned with a single cash value and reserve scale and the Method I discontinuity in reserve and cash value upon the first death need not be considered,

(3) Method II eliminates the adverse effect upon earnings and surplus resulting from the first death under Method I,

(4) The number of cash value and reserve files in the policy administration systems will be greatly reduced,

(5) The policy form language can be simpler under Method II,

(6) The single cash value scale is easier for the policyholder to understand, and provides them with more cash value while they are both alive. □