1985 VALUATION ACTUARY
SYMPOSIUM PROCEEDINGS

SESSION 6

NEW METHODS OF ANALYZING EXISTING ASSETS AND THEIR ABILITY TO MEET FUTURE LIABILITY DEMANDS

MR. JAMES TILLEY: The number of problem-solving methods available to the valuation actuary is larger than anyone might have liked, and is still growing. I'm not sure what kind of curve it's growing on, but it is indeed growing. Unfortunately, in this discussion I am not going to resolve any confusions about methodology, rather I am going to add to that by presenting yet another method, one involving concepts of duration and market values. My methodology will appear, on the surface, to differ considerably from extrapolation and simulation. In fact it does not, but I believe it may sound very different. I will do my best to establish that this new methodology is just a different way of summarizing information leading to the same sound conclusions one would obtain from following other methods.

My first goal is to provide a demonstration of the usefulness of the concepts of duration, interest-rate sensitivity and, in particular, market values. When I talk about market value surplus, I am referring to both sides of the balance sheet. Most actuaries feel they know how to value assets to the market, certainly tradeable assets. However, nobody engages in much trading of liabilities. One could argue that some reinsurance arrangements amount to trading liabilities, but those are never really valued at market rates in transactions. Techniques described by some of the other speakers at this symposium amount to other ways of putting a market value on liabilities. I am going to compare the market value of liabilities with the market value of assets and then look at the resulting market value of surplus. Following this, I will examine how the market value of surplus is sensitive to changes in interest rates.

I have deliberately tailored a GIC example to be quite simple. It is a somewhat artificial example, with no expense or tax assumptions. Those items are clearly important, and they can both be handled in the methodology. What I really want to do here is show what the traditional kind of C-3 risk methodology has to say about my GIC example shown in Exhibits 6-1 through 6-3. In later exhibits, illustrations of further concepts are presented, such as real market value surplus and convexities of assets and liabilities. These are very useful, as the main reason for looking at market values and interest-rate sensitivity is that those allow one to understand a complex situation in very simple terms. One does not have to carry around computer output, nor does one have to employ trial and error methods to discover whether assets are too short or too long for liabilities. These concepts are so powerful that the essential information about a portfolio's exposure can be carried around on one little card. Once the new concepts are understood, they allow one to speak concisely with senior management and fashion strategies that properly control interest-rate risk.

I will discuss the GIC example briefly. There are four bonds in the portfolio and six bullet GIC liabilities. This demonstration is arranged so that it starts off with $100 million in book value for both assets and liabilities. Comparing the weighted average cost on funds on the liabilities to the weighted average book yield on the assets, the spread is 72.5 basis points. That is an unrealistically large number for a GIC portfolio, and is used just as an example.

At the bottom of Exhibit 6-1, the cash flow runout of the assets and liabilities in the portfolio is shown. The difference between asset and liability cash flow is called, not surprisingly, surplus cash flow. The surplus cash flow stream lies at the heart of C-3 risk techniques.

EXHIBIT 6-1

ASSET PORTFOLIO

SECURITY	MATURITY	COUPON*	PAR VALUE	BOOK VALUE
BOND AA	3 YEARS	13.250	5 MM	5 MM
BOND BB	4 YEARS	13.000	15 MM	15 MM
BOND CC	5 YEARS	14.000	45 MM	45 MM
BOND DD	6 YEARS	13.250	35 MM	35 MM
AVERAGES/TOTALS		**13.550**	**100 MM**	**100 MM**

***ANNUAL COUPONS**

LIABILITY PORTFOLIO

CONTRACT/CLASS	MATURITY	GUARANTEE RATE	FUND VALUE
GIC BULLET-PF1	2 YEARS	12.000	10 MM
GIC BULLET-PF2	3 YEARS	12.750	15 MM
GIC BULLET-PF3	4 YEARS	12.500	20 MM
GIC BULLET-PF4	5 YEARS	13.500	30 MM
GIC BULLET-PF5	6 YEARS	12.750	20 MM
GIC BULLET-PF6	7 YEARS	12.250	5 MM
AVERAGES/TOTALS		**12.825**	**100 MM**

CASH FLOW ANALYSIS FOR EXISTING PORTFOLIO

TIME	ASSET CASH FLOW	LIABILITY CASH FLOW	SURPLUS CASH FLOW
1 YEAR	13550000	0	13550000
2 YEARS	13550000	12544000	1006000
3 YEARS	18550000	21500121	-2950121
4 YEARS	27887500	32036132	-4148632
5 YEARS	55937500	56506780	-569280
6 YEARS	39637500	41089352	-1451852
7 YEARS	0	11227277	-11227277

The surplus cash flow column data indicates a quite good match between asset and liability cash flows. The example was constructed that way. The biggest mismatches are at the end of the first and seventh years. Because the liabilities run out at the end of seven years, a seven-year projection period will be used. The reason I've constructed this type of cash flow pattern for surplus is that I can examine the benefits of a duration matching strategy by using a straight forward reinvestment strategy. At the end of the first year, one invests the big excess asset cash flow over liability cash flow in a bond that matures at the end of the analysis horizon — in other words, in a six-year bond. The relatively minor deficiencies occurring thereafter are also invested in bonds maturing at the horizon; that is, at the end of the second year the surplus cash flow is invested in a five-year bond, those at the end of the third year in a four-year bond, and so on.

Exhibit 6-2 shows a summary of some fairly naive reinvestment strategies. Those are constant strategies in which one uses five-year bonds or ten-year mortgages for all reinvestments. Three such strategies are examined. Exhibit 6-3 contains the yield curve scenarios. I will not discuss these in detail but want to note that I've used yield curves with nonzero shapes.

Let's study the table titled "Market Value Surplus (in millions) at End of Liability Runout," Exhibit 6-2. It is the same type of table used in presentations where one carries forward asset and liability cash flows according to an assumed reinvestment strategy against the different scenarios, and computes at the end of the projection period (seven years in this case) the terminal market value of surplus. Since all liabilities are fully paid off after seven years in this example, the surplus figure is just the market value of the terminal asset portfolio.

EXHIBIT 6-2

MARKET VALUE SURPLUS (IN MILLIONS)
AT END OF LIABILITY RUNOUT

SCENARIO	REINVESTMENT STRATEGY CONSTANT 1-YEAR	CONSTANT 5-YEAR	CONSTANT 10-YEAR	6,5,4,3,2 1-YEAR
1 - UP, THEN LEVEL	6.075	4.305	3.868	4.220
2 - DOWN, THEN LEVEL	-2.725	1.328	4.671	2.037
3 - RAPID UP/DOWN	1.627	8.657	10.847	6.485
4 - RAPID DOWN/UP	-0.040	-2.437	-3.664	-1.658
5 - MODERATE UP/DOWN	2.178	3.148	7.364	5.124
6 - MODERATE DOWN/UP	-0.486	1.632	-0.339	0.561
7 - SLOW UP/DOWN	4.008	4.617	6.213	4.258
8 - SLOW DOWN/UP	-1.888	1.582	2.127	1.934
9 - LEVEL THROUGHOUT	0.281	3.047	4.476	3.352

EXHIBIT 6-3

YIELD CURVE SCENARIO 1

TERM TO MATURITY	TIME 0 YR	1 YR	2 YR	3 YR	4 YR	5 YR	6 YR	7 YR
1 YEAR	8.00	10.25	12.50	14.75	14.75	14.75	14.75	14.75
2 YEARS	9.00	10.80	12.60	14.40	14.40	14.40	14.40	14.40
3 YEARS	9.75	11.23	12.70	14.18	14.18	14.18	14.18	14.18
4 YEARS	10.25	11.50	12.75	14.00	14.00	14.00	14.00	14.00
5 YEARS	10.65	11.70	12.75	13.80	13.80	13.80	13.80	13.80
6 YEARS	10.95	11.85	12.75	13.65	13.65	13.65	13.65	13.65
7 YEARS	11.20	12.00	12.80	13.60	13.60	13.60	13.60	13.60
8 YEARS	11.35	12.10	12.85	13.60	13.60	13.60	13.60	13.60
9 YEARS	11.45	12.15	12.85	13.55	13.55	13.55	13.55	13.55
10 YEARS	11.50	12.18	12.85	13.53	13.53	13.53	13.53	13.53
11 YEARS	11.50	12.18	12.85	13.53	13.53	13.53	13.53	13,53
12 YEARS	11.50	12.18	12.85	13.53	13.53	13.53	13.53	13.53
13 YEARS	11.50	12.18	12.85	13.53	13.53	13.53	13.53	13.53
14 YEARS	11.50	12.18	12.85	13.53	13.53	13.53	13.53	13.53
15 YEARS	11.50	12.18	12.85	13.53	13.53	13.53	13.53	13.53

EXHIBIT 6-3
(Continued)

YIELD CURVE SCENARIO 2

TERM TO MATURITY	TIME							
	0 YR	1 YR	2 YR	3 YR	4 YR	5 YR	6 YR	7 YR
1 YEAR	8.00	6.50	5.00	3.50	3.50	3.50	3.50	3.50
2 YEARS	9.00	7.50	6.00	4.50	4.50	4.50	4.50	4.50
3 YEARS	9.75	8.25	6.75	5.25	5.25	5.25	5.25	5.25
4 YEARS	10.25	8.75	7.25	5.75	5.75	5.75	5.75	5.75
5 YEARS	10.65	9.15	7.65	6.15	6.15	6.15	6.15	6.15
6 YEARS	10.95	9.45	7.95	6.45	6.45	6.45	6.45	6.45
7 YEARS	11.20	9.70	8.20	6.70	6.70	6.70	6.70	6.70
8 YEARS	11.35	9.85	8.35	6.85	6.85	6.85	6.85	6.85
9 YEARS	11.45	9.95	8.45	6.95	6.95	6.95	6.95	6.95
10 YEARS	11.50	10.00	8.50	7.00	7.00	7.00	7.00	7.00
11 YEARS	11.50	10.00	8.50	7.00	7.00	7.00	7.00	7.00
12 YEARS	11.50	10.00	8.50	7.00	7.00	7.00	7.00	7.00
13 YEARS	11.50	10.00	8.50	7.00	7.00	7.00	7.00	7.00
14 YEARS	11.50	10.00	8.50	7.00	7.00	7.00	7.00	7.00
15 YEARS	11.50	10.00	8.50	7.00	7.00	7.00	7.00	7.00

YIELD CURVE SCENARIO 3

TERM TO MATURITY	TIME							
	0 YR	1 YR	2 YR	3 YR	4 YR	5 YR	6 YR	7 YR
1 YEAR	8.00	14.75	8.00	3.50	8.00	14.75	8.00	3.50
2 YEARS	9.00	14.40	9.00	4.50	9.00	14.40	9.00	4.50
3 YEARS	9.75	14.18	9.75	5.25	9.75	14.18	9.75	5.25
4 YEARS	10.25	14.00	10.25	5.75	10.25	14.00	10.25	5.75
5 YEARS	10.65	13.80	10.65	6.15	10.65	13.80	10.65	6.15
6 YEARS	10.95	13.65	10.95	6.45	10.95	13.65	10.95	6.45
7 YEARS	11.20	13.60	11.20	6.70	11.20	13.60	11.20	6.70
8 YEARS	11.35	13.60	11.35	6.85	11.35	13.60	11.35	6.85
9 YEARS	11.45	13.55	11.45	6.95	11.45	13.55	11.45	6.95
10 YEARS	11.50	13.53	11.50	7.00	11.50	13.53	11.50	7.00
11 YEARS	11.50	13.53	11.50	7.00	11.50	13.53	11.50	7.00
12 YEARS	11.50	13.53	11.50	7.00	11.50	13.53	11.50	7.00
13 YEARS	11.50	13.53	11.50	7.00	11.50	13.53	11.50	7.00
14 YEARS	11.50	13.53	11.50	7.00	11.50	13.53	11.50	7.00
15 YEARS	11.50	13.53	11.50	7.00	11.50	13.53	11.50	7.00

EXHIBIT 6-3
(Continued)

YIELD CURVE SCENARIO 4

TERM TO MATURITY	TIME 0 YR	1 YR	2 YR	3 YR	4 YR	5 YR	6 YR	7 YR
1 YEAR	8.00	3.50	8.00	14.75	8.00	3.50	8.00	14.75
2 YEARS	9.00	4.50	9.00	14.40	9.00	4.50	9.00	14.40
3 YEARS	9.75	5.25	9.75	14.18	9.75	5.25	9.75	14.18
4 YEARS	10.25	5.75	10.25	14.00	10.25	5.75	10.25	14.00
5 YEARS	10.65	6.15	10.65	13.80	10.65	6.15	10.65	13.80
6 YEARS	10.95	6.45	10.95	13.65	10.95	6.45	10.95	13.65
7 YEARS	11.20	6.70	11.20	13.60	11.20	6.70	11.20	13.60
8 YEARS	11.35	6.85	11.35	13.60	11.35	6.85	11.35	13.60
9 YEARS	11.45	6.95	11.45	13.55	11.45	6.95	11.45	13.55
10 YEARS	11.50	7.00	11.50	13.53	11.50	7.00	11.50	13.53
11 YEARS	11.50	7.00	11.50	13.53	11.50	7.00	11.50	13.53
12 YEARS	11.50	7.00	11.50	13.53	11.50	7.00	11.50	13.53
13 YEARS	11.50	7.00	11.50	13.53	11.50	7.00	11.50	13.53
14 YEARS	11.50	7.00	11.50	13.53	11.50	7.00	11.50	13.53
15 YEARS	11.50	7.00	11.50	13.53	11.50	7.00	11.50	13.53

YIELD CURVE SCENARIO 5

TERM TO MATURITY	TIME 0 YR	1 YR	2 YR	3 YR	4 YR	5 YR	6 YR	7 YR
1 YEAR	8.00	11.38	14.75	11.38	8.00	5.75	3.50	5.75
2 YEARS	9.00	11.70	14.40	11.70	9.00	6.75	4.50	6.75
3 YEARS	9.75	11.96	14.18	11.96	9.75	7.50	5.25	7.50
4 YEARS	10.25	12.13	14.00	12.13	10.25	8.00	5.75	8.00
5 YEARS	10.65	12.23	13.80	12.23	10.65	8.40	6.15	8.40
6 YEARS	10.95	12.30	13.65	12.30	10.95	8.70	6.45	8.70
7 YEARS	11.20	12.40	13.60	12.40	11.20	8.95	6.70	8.95
8 YEARS	11.35	12.48	13.60	12.48	11.35	9.10	6.85	9.10
9 YEARS	11.45	12.50	13.55	12.50	11.45	9.20	6.95	9.20
10 YEARS	11.50	12.51	13.53	12.51	11.50	9.25	7.00	9.25
11 YEARS	11.50	12.51	13.53	12.51	11.50	9.25	7.00	9.25
12 YEARS	11.50	12.51	13.53	12.51	11.50	9.25	7.00	9.25
13 YEARS	11.50	12.51	13.53	12.51	11.50	9.25	7.00	9.25
14 YEARS	11.50	12.51	13.53	12.51	11.50	9.25	7.00	9.25
15 YEARS	11.50	12.51	13.53	12.51	11.50	9.25	7.00	9.25

EXHIBIT 6-3
(Continued)

YIELD CURVE SCENARIO 6

TERM TO MATURITY	TIME							
	0 YR	1 YR	2 YR	3 YR	4 YR	5 YR	6 YR	7 YR
1 YEAR	8.00	5.75	3.50	5.75	8.00	11.38	14.75	11.38
2 YEARS	9.00	6.75	4.50	6.75	9.00	11.70	14.40	11.70
3 YEARS	9.75	7.50	5.25	7.50	9.75	11.96	14.18	11.96
4 YEARS	10.25	8.00	5.75	8.00	10.25	12.13	14.00	12.13
5 YEARS	10.65	8.40	6.15	8.40	10.65	12.23	13.80	12.23
6 YEARS	10.95	8.70	6.45	8.70	10.95	12.30	13.65	12.30
7 YEARS	11.20	8.95	6.70	8.95	11.20	12.40	13.60	12.40
8 YEARS	11.35	9.10	6.85	9.10	11.35	12.48	13.60	12.48
9 YEARS	11.45	9.20	6.95	9.20	11.45	12.50	13.55	12.50
10 YEARS	11.50	9.25	7.00	9.25	11.50	12.51	13.53	12.51
11 YEARS	11.50	9.25	7.00	9.25	11.50	12.51	13.53	12.51
12 YEARS	11.50	9.25	7.00	9.25	11.50	12.51	13.53	12.51
13 YEARS	11.50	9.25	7.00	9.25	11.50	12.51	13.53	12.51
14 YEARS	11.50	9.25	7.00	9.25	11.50	12.51	13.53	12.51
15 YEARS	11.50	9.25	7.00	9.25	11.50	12.51	13.53	12.51

YIELD CURVE SCENARIO 7

TERM TO MATURITY	TIME							
	0 YR	1 YR	2 YR	3 YR	4 YR	5 YR	6 YR	7 YR
1 YEAR	8.00	10.25	12.50	14.75	12.50	10.25	8.00	6.50
2 YEARS	9.00	10.80	12.60	14.40	12.60	10.80	9.00	7.50
3 YEARS	9.75	11.23	12.70	14.18	12.70	11.23	9.75	8.25
4 YEARS	10.25	11.50	12.75	14.00	12.75	11.50	10.25	8.75
5 YEARS	10.65	11.70	12.75	13.80	12.75	11.70	10.65	9.15
6 YEARS	10.95	11.85	12.75	13.65	12.75	11.85	10.95	9.45
7 YEARS	11.20	12.00	12.80	13.60	12.80	12.00	11.20	9.70
8 YEARS	11.35	12.10	12.85	13.60	12.85	12.10	11.35	9.85
9 YEARS	11.45	12.15	12.85	13.55	12.85	12.15	11.45	9.95
10 YEARS	11.50	12.18	12.85	13.53	12.85	12.18	11.50	10.00
11 YEARS	11.50	12.18	12.85	13.53	12.85	12.18	11.50	10.00
12 YEARS	11.50	12.18	12.85	13.53	12.85	12.18	11.50	10.00
13 YEARS	11.50	12.18	12.85	13.53	12.85	12.18	11.50	10.00
14 YEARS	11.50	12.18	12.85	13.53	12.85	12.18	11.50	10.00
15 YEARS	11.50	12.18	12.85	13.53	12.85	12.18	11.50	10.00

EXHIBIT 6-3
(Continued)

YIELD CURVE SCENARIO 8

TERM TO MATURITY	TIME 0 YR	1 YR	2 YR	3 YR	4 YR	5 YR	6 YR	7 YR
1 YEAR	8.00	6.50	5.00	3.50	5.00	6.50	8.00	10.25
2 YEARS	9.00	7.50	6.00	4.50	6.00	7.50	9.00	10.80
3 YEARS	9.75	8.25	6.75	5.25	6.75	8.25	9.75	11.23
4 YEARS	10.25	8.75	7.25	5.75	7.25	8.75	10.25	11.50
5 YEARS	10.65	9.15	7.65	6.15	7.65	9.15	10.65	11.70
6 YEARS	10.95	9.45	7.95	6.45	7.95	9.45	10.95	11.85
7 YEARS	11.20	9.70	8.20	6.70	8.20	9.70	11.20	12.00
8 YEARS	11.35	9.85	8.35	6.85	8.35	9.85	11.35	12.10
9 YEARS	11.45	9.95	8.45	6.95	8.45	9.95	11.45	12.15
10 YEARS	11.50	10.00	8.50	7.00	8.50	10.00	11.50	12.18
11 YEARS	11.50	10.00	8.50	7.00	8.50	10.00	11.50	12.18
12 YEARS	11.50	10.00	8.50	7.00	8.50	10.00	11.50	12.18
13 YEARS	11.50	10.00	8.50	7.00	8.50	10.00	11.50	12.18
14 YEARS	11.50	10.00	8.50	7.00	8.50	10.00	11.50	12.18
15 YEARS	11.50	10.00	8.50	7.00	8.50	10.00	11.50	12.18

YIELD CURVE SCENARIO 9

TERM TO MATURITY	TIME 0 YR	1 YR	2 YR	3 YR	4 YR	5 YR	6 YR	7 YR
1 YEAR	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00
2 YEARS	9.00	9.00	9.00	9.00	9.00	9.00	9.00	9.00
3 YEARS	9.75	9.75	9.75	9.75	9.75	9.75	9.75	9.75
4 YEARS	10.25	10.25	10.25	10.25	10.25	10.25	10.25	10.25
5 YEARS	10.65	10.65	10.65	10.65	10.65	10.65	10.65	10.65
6 YEARS	10.95	10.95	10.95	10.95	10.95	10.95	10.95	10.95
7 YEARS	11.20	11.20	11.20	11.20	11.20	11.20	11.20	11.20
8 YEARS	11.35	11.35	11.35	11.35	11.35	11.35	11.35	11.35
9 YEARS	11.45	11.45	11.45	11.45	11.45	11.45	11.45	11.45
10 YEARS	11.50	11.50	11.50	11.50	11.50	11.50	11.50	11.50
11 YEARS	11.50	11.50	11.50	11.50	11.50	11.50	11.50	11.50
12 YEARS	11.50	11.50	11.50	11.50	11.50	11.50	11.50	11.50
13 YEARS	11.50	11.50	11.50	11.50	11.50	11.50	11.50	11.50
14 YEARS	11.50	11.50	11.50	11.50	11.50	11.50	11.50	11.50
15 YEARS	11.50	11.50	11.50	11.50	11.50	11.50	11.50	11.50

The descriptions for the interest-rate scenarios in Exhibit 6-4 through 6-7 come from examining their behavior over time. Rapid, moderate, and slow refer to the length of the interest-rate cycle. Up/down and down/up differ in the direction of the initial interest-rate movement. Notice that I have used paired scenarios. In doing this kind of analysis, especially when choosing to do any investment strategy optimization, it is a good idea to pair scenarios. If one looked at a single scenario, strictly upward for example, one would conclude with perfect foresight that it is beneficial to be invested very short. Looking at a scenario that is strictly down, one would again see with perfect foresight that it would be good to be invested very long. Bringing both those scenarios into the analysis, one tends to get tradeoffs between using short and long assets. If modeling liabilities as well, so that their duration properties emerge from the analysis, using paired scenarios induces the kinds of tradeoffs that lead to immunized strategies. So one wants to be unbiased in the choice of scenarios, and it generally makes sense to pair them much as I have done here.

I am not going to try to divide these scenarios or paths into "reasonable" and "plausible." I don't know whether I hold the majority or minority view, but I really do not think anybody can do that. To give an example, probably most of us here today might agree that over the next five years, perhaps over the next ten years, interest rates are not likely to be tame. They're likely to be quite volatile. We'd probably have wide differences of views as to whether there is going to be a trend up or trend down or no trend at all. But if we could agree that there would be at least modest volatility, then I suggest that any scenario in which interest rates are relatively tame throughout is not a "reasonable" scenario, although maybe it's a "plausible" scenario. I do not believe any of us has even thought about doing valuation actuary work with anything other than a "level throughout" scenario in the "reasonable" group. My current thinking is that a level scenario is probably one of the most "unreasonable" scenarios that one can really look at. But, because it is a bench mark in some sense, it is useful to include in the analysis, but it is certainly not "reasonable."

Again in Exhibits 6-4 to 6-7, I have used four reinvestment strategies: they're labeled "constant 1-year," "constant 5-year," "constant 10-year" and "6,5,4,3,2 1-year." A constant strategy means that anytime there is investable cash flow, whether it's positive or negative, it will be placed into one-year bonds for the constant 1-year strategy, into five-year bonds for the constant 5-year strategy, and into ten-year bonds for the constant 10-year strategy.

Let's examine the exhibits to try to decide what the "worst" paths are. We might then also decide whether to kick the worst paths out of the analysis as being unreasonable and/or implausible, but first let's suppose we try only to answer what is the worst path. A lot of traditional C-3 risk methodology is based on worst-path analysis. Let's look at the constant 1-year strategy in Exhibit 6-4. The worst path is clearly Scenario 2. The terminal surplus there is $-2.7 million. Looking down at the constant 5-year strategies in Exhibit 6-5, we see that the worst path is Scenario 4. Now if we look at the constant 10-year strategy in Exhibit 6-6, we see that the worst scenario is also Scenario 4, with the same result for the 6,5,4,3,2, 1-year reinvestment strategy shown in Exhibit 6-7. Already there is an ambiguity.

EXHIBIT 6-4

MARKET VALUES, DURATIONS, AND CONVEXITIES
CONSTANT 1-YEAR BOND REINVESTMENT STRATEGY

INTEREST RATE SCENARIO 1

TIME	MARKET VALUE ASSETS	MARKET VALUE LIAB'S	MARKET VALUE SURPLUS	DURATION ASSETS	DURATION LIAB'S	CONVEXITY ASSETS	CONVEXITY LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	119.9	117.7	2.2	3.12	3.51	11.8	14.1
2	118.0	115.4	2.6	2.47	2.78	7.3	9.0
3	108.7	105.4	3.3	1.88	2.12	4.1	5.3
4	92.1	88.2	3.9	1.33	1.53	2.0	2.8
5	48.9	44.4	4.6	1.00	1.19	1.0	1.6
6	15.1	9.8	5.3	1.00	1.00	1.0	1.0
7	6.1	0.0	6.1				

INTEREST RATE SCENARIO 2

TIME	MARKET VALUE ASSETS	MARKET VALUE LIAB'S	MARKET VALUE SURPLUS	DURATION ASSETS	DURATION LIAB'S	CONVEXITY ASSETS	CONVEXITY LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	129.4	128.5	0.9	3.17	3.54	12.0	14.4
2	133.6	133.9	-0.3	2.54	2.83	7.7	9.2
3	125.3	126.8	-1.5	1.95	2.17	4.4	5.5
4	99.9	101.8	-1.9	1.36	1.56	2.1	2.9
5	47.6	50.0	-2.3	1.00	1.21	1.0	1.6
6	8.2	10.8	-2.6	1.00	1.00	1.0	1.0
7	-2.7	0.0	-2.7				

INTEREST RATE SCENARIO 3

TIME	MARKET VALUE ASSETS	MARKET VALUE LIAB'S	MARKET VALUE SURPLUS	DURATION ASSETS	DURATION LIAB'S	CONVEXITY ASSETS	CONVEXITY LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	112.4	109.4	3.0	3.09	3.50	11.6	14.1
2	126.0	123.8	2.2	2.49	2.79	7.4	9.1
3	127.0	126.8	0.2	1.94	2.17	4.3	5.5
4	96.4	95.3	1.1	1.35	1.54	2.0	2.8
5	46.2	44.4	1.8	1.00	1.19	1.0	1.6
6	11.9	10.4	1.5	1.00	1.00	1.0	1.0
7	1.6	0.0	1.6				

EXHIBIT 6-4
(Continued)

MARKET VALUES, DURATIONS, AND CONVEXITIES CONSTANT 1-YEAR BOND REINVESTMENT STRATEGY

INTEREST RATE SCENARIO 4

	MARKET VALUE			DURATION		CONVEXITY	
TIME	ASSETS	LIAB'S	SURPLUS	ASSETS	LIAB'S	ASSETS	LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	141.3	142.2	-0.9	3.23	3.60	12.4	14.7
2	124.5	123.8	0.7	2.51	2.79	7.5	9.1
3	107.0	105.4	1.6	1.90	2.12	4.2	5.3
4	96.2	95.3	0.9	1.35	1.54	2.0	2.8
5	49.7	50.0	-0.3	1.00	1.21	1.0	1.6
6	10.4	10.4	0.0	1.00	1.00	1.0	1.0
7	0.0	0.0	0.0				

INTEREST RATE SCENARIO 5

	MARKET VALUE			DURATION		CONVEXITY	
TIME	ASSETS	LIAB'S	SURPLUS	ASSETS	LIAB'S	ASSETS	LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	117.9	115.5	2.4	3.11	3.51	11.7	14.1
2	114.7	111.5	3.2	2.46	2.77	7.3	8.9
3	113.6	110.4	3.2	1.89	2.13	4.2	5.3
4	98.3	95.3	2.9	1.34	1.54	2.0	2.8
5	51.1	48.7	2.4	1.00	1.20	1.0	1.6
6	13.0	10.8	2.1	1.00	1.00	1.0	1.0
7	2.2	0.0	2.2				

INTEREST RATE SCENARIO 6

	MARKET VALUE			DURATION		CONVEXITY	
TIME	ASSETS	LIAB'S	SURPLUS	ASSETS	LIAB'S	ASSETS	LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	132.2	131.7	0.5	3.18	3.56	12.1	14.4
2	138.3	139.5	-1.2	2.56	2.85	7.8	9.3
3	120.2	121.1	-0.9	1.94	2.16	4.3	5.4
4	94.6	95.3	-0.7	1.35	1.54	2.1	2.8
5	45.3	45.9	-0.6	1.00	1.20	1.0	1.6
6	9.4	9.8	-0.4	1.00	1.00	1.0	1.0
7	-0.5	0.0	-0.5				

EXHIBIT 6-4
(Continued)

MARKET VALUES, DURATIONS, AND CONVEXITIES
CONSTANT 1-YEAR BOND REINVESTMENT STRATEGY

INTEREST RATE SCENARIO 7

TIME	MARKET VALUE ASSETS	MARKET VALUE LIAB'S	MARKET VALUE SURPLUS	DURATION ASSETS	DURATION LIAB'S	CONVEXITY ASSETS	CONVEXITY LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	119.9	117.7	2.2	3.12	3.51	11.8	14.1
2	118.0	115.4	2.6	2.47	2.78	7.3	9.0
3	108.7	105.4	3.3	1.88	2.12	4.1	5.3
4	94.0	90.5	3.6	1.33	1.53	2.0	2.8
5	50.1	46.4	3.7	1.00	1.20	1.0	1.6
6	14.1	10.4	3.7	1.00	1.00	1.0	1.0
7	4.0	0.0	4.0				

INTEREST RATE SCENARIO 8

TIME	MARKET VALUE ASSETS	MARKET VALUE LIAB'S	MARKET VALUE SURPLUS	DURATION ASSETS	DURATION LIAB'S	CONVEXITY ASSETS	CONVEXITY LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	129.4	128.5	0.9	3.17	3.54	12.0	14.4
2	133.6	133.9	-0.3	2.54	2.83	7.7	9.2
3	125.3	126.8	-1.5	1.95	2.17	4.4	5.5
4	98.1	99.6	-1.5	1.36	1.55	2.1	2.8
5	46.7	48.3	-1.6	1.00	1.20	1.0	1.6
6	8.6	10.4	-1.7	1.00	1.00	1.0	1.0
7	-1.9	0.0	-1.9				

INTEREST RATE SCENARIO 9

TIME	MARKET VALUE ASSETS	MARKET VALUE LIAB'S	MARKET VALUE SURPLUS	DURATION ASSETS	DURATION LIAB'S	CONVEXITY ASSETS	CONVEXITY LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	124.0	122.3	1.7	3.14	3.52	11.9	14.2
2	125.1	123.8	1.3	2.50	2.79	7.5	9.1
3	116.7	115.7	1.0	1.92	2.14	4.3	5.4
4	96.0	95.3	0.7	1.35	1.54	2.0	2.8
5	47.9	47.5	0.4	1.00	1.20	1.0	1.6
6	10.7	10.4	0.3	1.00	1.00	1.0	1.0
7	0.3	0.0	0.3				

EXHIBIT 6-5

MARKET VALUES, DURATIONS, AND CONVEXITIES
CONSTANT 5-YEAR BOND REINVESTMENT STRATEGY

INTEREST RATE SCENARIO 1

	MARKET VALUE			DURATION		CONVEXITY	
TIME	ASSETS	LIAB'S	SURPLUS	ASSETS	LIAB'S	ASSETS	LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	119.9	117.7	2.2	3.47	3.51	13.7	14.1
2	117.8	115.4	2.4	2.81	2.78	9.0	9.0
3	108.0	105.4	2.5	2.12	2.12	5.1	5.3
4	91.1	88.2	2.9	1.40	1.53	2.0	2.8
5	47.7	44.4	3.4	0.95	1.19	0.7	1.6
6	13.7	9.8	3.9	3.73	1.00	17.0	1.0
7	4.3	0.0	4.3				

INTEREST RATE SCENARIO 2

	MARKET VALUE			DURATION		CONVEXITY	
TIME	ASSETS	LIAB'S	SURPLUS	ASSETS	LIAB'S	ASSETS	LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	129.4	128.5	0.9	3.51	3.54	14.0	14.4
2	134.9	133.9	0.9	2.85	2.83	9.2	9.2
3	127.9	126.8	1.1	2.14	2.17	5.1	5.5
4	103.1	101.8	1.2	1.39	1.56	1.9	2.9
5	51.1	50.0	1.1	0.88	1.21	0.4	1.6
6	11.7	10.8	0.9	4.45	1.00	22.3	1.0
7	1.3	0.0	1.3				

INTEREST RATE SCENARIO 3

	MARKET VALUE			DURATION		CONVEXITY	
TIME	ASSETS	LIAB'S	SURPLUS	ASSETS	LIAB'S	ASSETS	LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	112.4	109.4	3.0	3.44	3.50	13.6	14.1
2	127.4	123.8	3.6	2.82	2.79	9.1	9.1
3	131.4	126.8	4.6	2.16	2.17	5.3	5.5
4	100.1	95.3	4.7	1.43	1.54	2.1	2.8
5	49.6	44.4	5.2	1.02	1.19	1.1	1.6
6	15.8	10.4	5.4	3.74	1.00	17.0	1.0
7	8.7	0.0	8.7				

EXHIBIT 6-5
(Continued)

MARKET VALUES, DURATIONS, AND CONVEXITIES
CONSTANT 5-YEAR BOND REINVESTMENT STRATEGY

INTEREST RATE SCENARIO 4

	MARKET VALUE			DURATION		CONVEXITY	
TIME	ASSETS	LIAB'S	SURPLUS	ASSETS	LIAB'S	ASSETS	LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	141.3	142.2	-0.9	3.56	3.60	14.3	14.7
2	123.1	123.8	-0.7	2.82	2.79	9.1	9.1
3	104.5	105.4	-0.9	2.10	2.12	4.9	5.3
4	94.4	95.3	-1.0	1.35	1.54	1.7	2.8
5	47.7	50.0	-2.2	0.74	1.21	-0.3	1.6
6	8.7	10.4	-1.7	4.52	1.00	23.1	1.0
7	-2.4	0.0	-2.4				

INTEREST RATE SCENARIO 5

	MARKET VALUE			DURATION		CONVEXITY	
TIME	ASSETS	LIAB'S	SURPLUS	ASSETS	LIAB'S	ASSETS	LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	117.9	115.5	2.4	3.46	3.51	13.7	14.1
2	114.2	111.5	2.6	2.80	2.77	9.0	8.9
3	113.6	110.4	3.2	2.13	2.13	5.1	5.3
4	99.2	95.3	3.8	1.42	1.54	2.1	2.8
5	52.5	48.7	3.8	0.97	1.20	0.9	1.6
6	14.4	10.8	3.5	4.16	1.00	20.0	1.0
7	3.1	0.0	3.1				

INTEREST RATE SCENARIO 6

	MARKET VALUE			DURATION		CONVEXITY	
TIME	ASSETS	LIAB'S	SURPLUS	ASSETS	LIAB'S	ASSETS	LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	132.2	131.7	0.5	3.52	3.56	14.1	14.4
2	139.9	139.5	0.4	2.87	2.85	9.3	9.3
3	121.5	121.1	0.4	2.13	2.16	5.1	5.4
4	95.9	95.3	0.5	1.37	1.54	1.8	2.8
5	46.5	45.9	0.6	0.85	1.20	0.2	1.6
6	10.9	9.8	1.1	3.98	1.00	19.0	1.0
7	1.6	0.0	1.6				

EXHIBIT 6-5
(Continued)

MARKET VALUES, DURATIONS, AND CONVEXITIES
CONSTANT 5-YEAR BOND REINVESTMENT STRATEGY

INTEREST RATE SCENARIO 7

TIME	MARKET VALUE ASSETS	MARKET VALUE LIAB'S	MARKET VALUE SURPLUS	DURATION ASSETS	DURATION LIAB'S	CONVEXITY ASSETS	CONVEXITY LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	119.9	117.7	2.2	3.47	3.51	13.7	14.1
2	117.8	115.4	2.4	2.81	2.78	9.0	9.0
3	108.0	105.4	2.5	2.12	2.12	5.1	5.3
4	93.5	90.5	3.0	1.41	1.53	2.0	2.8
5	49.7	46.4	3.3	0.95	1.20	0.7	1.6
6	13.6	10.4	3.2	3.89	1.00	18.2	1.0
7	4.6	0.0	4.6				

INTEREST RATE SCENARIO 8

TIME	MARKET VALUE ASSETS	MARKET VALUE LIAB'S	MARKET VALUE SURPLUS	DURATION ASSETS	DURATION LIAB'S	CONVEXITY ASSETS	CONVEXITY LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	129.4	128.5	0.9	3.51	3.54	14.0	14.4
2	134.9	133.9	0.9	2.85	2.83	9.2	9.2
3	127.9	126.8	1.1	2.14	2.17	5.1	5.5
4	100.9	99.6	1.3	1.39	1.55	1.9	2.8
5	49.7	48.3	1.4	0.88	1.20	0.4	1.6
6	11.9	10.4	1.5	4.06	1.00	19.5	1.0
7	1.6	0.0	1.6				

INTEREST RATE SCENARIO 9

TIME	MARKET VALUE ASSETS	MARKET VALUE LIAB'S	MARKET VALUE SURPLUS	DURATION ASSETS	DURATION LIAB'S	CONVEXITY ASSETS	CONVEXITY LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	124.0	122.3	1.7	3.48	3.52	13.8	14.2
2	125.7	123.8	1.9	2.82	2.79	9.1	9.1
3	117.8	115.7	2.1	2.13	2.14	5.1	5.4
4	97.7	95.3	2.4	1.40	1.54	2.0	2.8
5	49.9	47.5	2.5	0.92	1.20	0.6	1.6
6	12.8	10.4	2.4	3.96	1.00	18.8	1.0
7	3.0	0.0	3.0				

EXHIBIT 6-6

MARKET VALUES, DURATIONS, AND CONVEXITIES
CONSTANT 10-YEAR BOND REINVESTMENT STRATEGY

INTEREST RATE SCENARIO 1

	MARKET VALUE			DURATION		CONVEXITY	
TIME	ASSETS	LIAB'S	SURPLUS	ASSETS	LIAB'S	ASSETS	LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	119.9	117.7	2.2	3.71	3.51	17.4	14.1
2	117.8	115.4	2.4	3.13	2.78	13.2	9.0
3	108.0	105.4	2.6	2.48	2.12	9.0	5.3
4	91.1	88.2	2.9	1.85	1.53	5.5	2.8
5	47.6	44.4	3.2	1.96	1.19	7.1	1.6
6	13.3	9.8	3.6	3.97	1.00	17.7	1.0
7	3.9	0.0	3.9				

INTEREST RATE SCENARIO 2

	MARKET VALUE			DURATION		CONVEXITY	
TIME	ASSETS	LIAB'S	SURPLUS	ASSETS	LIAB'S	ASSETS	LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	129.4	128.5	0.9	3.76	3.54	17.8	14.4
2	135.4	133.9	1.5	3.22	2.83	13.9	9.2
3	129.5	126.8	2.7	2.59	2.17	9.8	5.5
4	104.9	101.8	3.1	1.91	1.56	5.8	2.9
5	53.4	50.0	3.4	2.02	1.21	7.2	1.6
6	14.8	10.8	4.0	3.91	1.00	15.7	1.0
7	4.7	0.0	4.7				

INTEREST RATE SCENARIO 3

	MARKET VALUE			DURATION		CONVEXITY	
TIME	ASSETS	LIAB'S	SURPLUS	ASSETS	LIAB'S	ASSETS	LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	112.4	109.4	3.0	3.70	3.50	17.3	14.1
2	127.4	123.8	3.6	3.16	2.79	13.4	9.1
3	134.1	126.8	7.3	2.67	2.17	10.6	5.5
4	100.4	95.3	5.1	1.92	1.54	6.1	2.8
5	49.5	44.4	5.1	2.06	1.19	7.9	1.6
6	17.3	10.4	6.9	4.07	1.00	19.3	1.0
7	10.8	0.0	10.8				

EXHIBIT 6-6
(Continued)

MARKET VALUES, DURATIONS, AND CONVEXITIES
CONSTANT 10-YEAR BOND REINVESTMENT STRATEGY

INTEREST RATE SCENARIO 4

TIME	MARKET VALUE ASSETS	MARKET VALUE LIAB'S	MARKET VALUE SURPLUS	DURATION ASSETS	DURATION LIAB'S	CONVEXITY ASSETS	CONVEXITY LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	141.3	142.2	-0.9	3.85	3.60	18.6	14.7
2	121.6	123.8	-2.2	3.09	2.79	12.7	9.1
3	103.1	105.4	-2.3	2.39	2.12	8.1	5.3
4	92.7	95.3	-2.6	1.70	1.54	4.2	2.8
5	48.0	50.0	-2.0	1.70	1.21	4.4	1.6
6	7.6	10.4	-2.8	3.45	1.00	7.6	1.0
7	-3.7	0.0	-3.7				

INTEREST RATE SCENARIO 5

TIME	MARKET VALUE ASSETS	MARKET VALUE LIAB'S	MARKET VALUE SURPLUS	DURATION ASSETS	DURATION LIAB'S	CONVEXITY ASSETS	CONVEXITY LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	117.9	115.5	2.4	3.71	3.51	17.4	14.1
2	114.2	111.5	2.6	3.13	2.77	13.2	8.9
3	113.6	110.4	3.2	2.50	2.13	9.1	5.3
4	99.4	95.3	4.1	1.89	1.54	5.8	2.8
5	54.3	48.7	5.6	2.11	1.20	8.2	1.6
6	18.5	10.8	7.7	4.11	1.00	19.0	1.0
7	7.4	0.0	7.4				

INTEREST RATE SCENARIO 6

TIME	MARKET VALUE ASSETS	MARKET VALUE LIAB'S	MARKET VALUE SURPLUS	DURATION ASSETS	DURATION LIAB'S	CONVEXITY ASSETS	CONVEXITY LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	132.2	131.7	0.5	3.78	3.56	18.0	14.4
2	140.8	139.5	1.3	3.26	2.85	14.3	9.3
3	121.3	121.1	0.3	2.50	2.16	9.0	5.4
4	94.9	95.3	-0.4	1.77	1.54	4.8	2.8
5	45.5	45.9	-0.4	1.76	1.20	5.5	1.6
6	9.1	9.8	-0.6	3.72	1.00	13.0	1.0
7	-0.3	0.0	-0.3				

EXHIBIT 6-6
(Continued)

MARKET VALUES, DURATIONS, AND CONVEXITIES
CONSTANT 10-YEAR BOND REINVESTMENT STRATEGY

INTEREST RATE SCENARIO 7

	MARKET VALUE			DURATION		CONVEXITY	
TIME	ASSETS	LIAB'S	SURPLUS	ASSETS	LIAB'S	ASSETS	LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	119.9	117.7	2.2	3.71	3.51	17.4	14.1
2	117.8	115.4	2.4	3.13	2.78	13.2	9.0
3	108.0	105.4	2.6	2.48	2.12	9.0	5.3
4	93.5	90.5	3.1	1.86	1.53	5.6	2.8
5	50.0	46.4	3.6	1.99	1.20	7.3	1.6
6	15.0	10.4	4.6	3.99	1.00	17.9	1.0
7	6.2	0.0	6.2				

INTEREST RATE SCENARIO 8

	MARKET VALUE			DURATION		CONVEXITY	
TIME	ASSETS	LIAB'S	SURPLUS	ASSETS	LIAB'S	ASSETS	LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	129.4	128.5	0.9	3.76	3.54	17.8	14.4
2	135.4	133.9	1.5	3.22	2.83	13.9	9.2
3	129.5	126.8	2.7	2.59	2.17	9.8	5.5
4	101.8	99.6	2.2	1.87	1.55	5.6	2.8
5	50.3	48.3	2.1	1.92	1.20	6.6	1.6
6	12.4	10.4	2.0	3.88	1.00	15.7	1.0
7	2.1	0.0	2.1				

INTEREST RATE SCENARIO 9

	MARKET VALUE			DURATION		CONVEXITY	
TIME	ASSETS	LIAB'S	SURPLUS	ASSETS	LIAB'S	ASSETS	LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	124.0	122.3	1.7	3.72	3.52	17.4	14.2
2	125.6	123.8	1.8	3.14	2.79	13.2	9.1
3	117.9	115.7	2.2	2.49	2.14	9.0	5.4
4	97.9	95.3	2.6	1.85	1.54	5.5	2.8
5	50.5	47.5	3.0	1.96	1.20	7.1	1.6
6	14.0	10.4	3.6	3.96	1.00	17.2	1.0
7	4.5	0.0	4.5				

EXHIBIT 6-7

MARKET VALUES, DURATIONS, AND CONVEXITIES
6,5,4,3,2 1-YEAR REINVESTMENT STRATEGY

INTEREST RATE SCENARIO 1

	MARKET VALUE			DURATION		CONVEXITY	
TIME	ASSETS	LIAB'S	SURPLUS	ASSETS	LIAB'S	ASSETS	LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	119.9	117.7	2.2	3.53	3.51	14.4	14.1
2	117.8	115.4	2.4	2.88	2.78	9.6	9.0
3	107.9	105.4	2.5	2.21	2.12	5.7	5.3
4	91.0	88.2	2.8	1.56	1.53	2.9	2.8
5	47.6	44.4	3.2	1.25	1.19	1.7	1.6
6	13.5	9.8	3.7	1.00	1.00	1.0	1.0
7	4.2	0.0	4.2				

INTEREST RATE SCENARIO 2

	MARKET VALUE			DURATION		CONVEXITY	
TIME	ASSETS	LIAB'S	SURPLUS	ASSETS	LIAB'S	ASSETS	LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	129.4	128.5	0.9	3.57	3.54	14.7	14.4
2	135.0	133.9	1.1	2.93	2.83	9.9	9.2
3	128.4	126.8	1.5	2.25	2.17	5.9	5.5
4	103.6	101.8	1.8	1.57	1.56	2.9	2.9
5	51.8	50.0	1.9	1.24	1.21	1.7	1.6
6	12.8	10.8	2.0	1.00	1.00	1.0	1.0
7	2.0	0.0	2.0				

INTEREST RATE SCENARIO 3

	MARKET VALUE			DURATION		CONVEXITY	
TIME	ASSETS	LIAB'S	SURPLUS	ASSETS	LIAB'S	ASSETS	LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	112.4	109.4	3.0	3.51	3.50	14.3	14.1
2	127.4	123.8	3.6	2.90	2.79	9.7	9.1
3	132.0	126.8	5.2	2.27	2.17	6.0	5.5
4	100.2	95.3	4.8	1.58	1.54	3.0	2.8
5	49.4	44.4	5.0	1.26	1.19	1.8	1.6
6	16.4	10.4	6.0	1.00	1.00	1.0	1.0
7	6.5	0.0	6.5				

EXHIBIT 6-7
(Continued)

MARKET VALUES, DURATIONS, AND CONVEXITIES
6,5,4,3,2 1-YEAR REINVESTMENT STRATEGY

INTEREST RATE SCENARIO 4

	MARKET VALUE			DURATION		CONVEXITY	
TIME	ASSETS	LIAB'S	SURPLUS	ASSETS	LIAB'S	ASSETS	LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	141.3	142.2	-0.9	3.63	3.60	15.1	14.7
2	122.8	123.8	-1.0	2.89	2.79	9.7	9.1
3	104.1	105.4	-1.3	2.18	2.12	5.6	5.3
4	94.1	95.3	-1.3	1.53	1.54	2.8	2.8
5	18.5	50.0	-1.5	1.20	1.21	1.6	1.6
6	8.9	10.4	-1.5	1.00	1.00	1.0	1.0
7	-1.7	0.0	-1.7				

INTEREST RATE SCENARIO 5

	MARKET VALUE			DURATION		CONVEXITY	
TIME	ASSETS	LIAB'S	SURPLUS	ASSETS	LIAB'S	ASSETS	LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	117.9	115.5	2.4	3.52	3.51	14.4	14.1
2	114.2	111.5	2.6	2.87	2.77	9.6	8.9
3	113.6	110.4	3.2	2.22	2.13	5.7	5.3
4	99.3	95.3	4.0	1.57	1.54	2.9	2.8
5	53.2	48.7	4.5	1.26	1.20	1.8	1.6
6	15.8	10.8	5.0	1.00	1.00	1.0	1.0
7	5.1	0.0	5.1				

INTEREST RATE SCENARIO 6

	MARKET VALUE			DURATION		CONVEXITY	
TIME	ASSETS	LIAB'S	SURPLUS	ASSETS	LIAB'S	ASSETS	LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	132.2	131.7	0.5	3.58	3.56	14.8	14.4
2	140.1	139.5	0.6	2.95	2.85	10.0	9.3
3	121.5	121.1	0.5	2.23	2.16	5.8	5.4
4	95.8	95.3	0.4	1.55	1.54	2.8	2.8
5	46.3	45.9	0.5	1.21	1.20	1.6	1.6
6	10.3	9.8	0.5	1.00	1.00	1.0	1.0
7	0.6	0.0	0.6				

EXHIBIT 6-7
(Continued)

MARKET VALUES, DURATIONS, AND CONVEXITIES
6,5,4,3,2 1-YEAR REINVESTMENT STRATEGY

INTEREST RATE SCENARIO 7

	MARKET VALUE			DURATION		CONVEXITY	
TIME	ASSETS	LIAB'S	SURPLUS	ASSETS	LIAB'S	ASSETS	LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	119.9	117.7	2.2	3.53	3.51	14.4	14.1
2	117.8	115.4	2.4	2.88	2.78	9.6	9.0
3	107.9	105.4	2.5	2.21	2.12	5.7	5.3
4	93.5	90.5	3.0	1.56	1.53	2.9	2.8
5	49.9	46.4	3.5	1.25	1.20	1.8	1.6
6	14.3	10.4	3.9	1.00	1.00	1.0	1.0
7	4.3	0.0	4.3				

INTEREST RATE SCENARIO 8

	MARKET VALUE			DURATION		CONVEXITY	
TIME	ASSETS	LIAB'S	SURPLUS	ASSETS	LIAB'S	ASSETS	LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	129.4	128.5	0.9	3.57	3.54	14.7	14.4
2	135.0	133.9	1.1	2.93	2.83	9.9	9.2
3	128.4	126.8	1.5	2.25	2.17	5.9	5.5
4	101.2	99.6	1.6	1.57	1.55	2.9	2.8
5	50.0	48.3	1.7	1.23	1.20	1.7	1.6
6	12.2	10.4	1.8	1.00	1.00	1.0	1.0
7	1.9	0.0	1.9				

INTEREST RATE SCENARIO 9

	MARKET VALUE			DURATION		CONVEXITY	
TIME	ASSETS	LIAB'S	SURPLUS	ASSETS	LIAB'S	ASSETS	LIAB'S
0	110.9	108.9	2.1	4.01	4.51	18.7	22.1
1	124.0	122.3	1.7	3.54	3.52	14.5	14.2
2	125.7	123.8	1.9	2.89	2.79	9.7	9.1
3	117.9	115.7	2.2	2.22	2.14	5.8	5.4
4	97.9	95.3	2.6	1.56	1.54	2.9	2.8
5	50.3	47.5	2.8	1.24	1.20	1.7	1.6
6	13.5	10.4	3.1	1.00	1.00	1.0	1.0
7	3.4	0.0	3.4				

Let's suppose we all agree that rapid up/down and rapid down/up scenarios are probably implausible, and should not be included in the analysis. Suppose then that we cross those scenarios out, so that the new universe of scenarios includes Scenarios 1, 2, 5, 6, 7, 8 and 9 in each of the exhibited investment strategies. With this new universe, the worst case is still Scenario 2 under the constant 1-year reinvestment strategy, and now is Scenario 2 under the 5-year strategy as well, but is it Scenario 6 under both the 10-year and the 6,5,4,3,2 1-year reinvestment strategies?

I don't want to make too much of the worst path ambiguity in this example. But, one thing we can conclude unambiguously from this example is that the assets are too short for the liabilities. The way to see that is to look at every pair of scenarios. Look at 1 and 2 as a pair. Or look at 3 and 4 as a pair, and 5 and 6, and so on. You'll see that with the exception of the pair of Scenarios 1 and 2 for the 10-year strategy, you get a higher terminal surplus on a scenario that starts up than on one that starts down. That's a strong indication that the assets are too short for the liabilities. Nevertheless, it's unfortunate to have to look at a tabular array in order to make that conclusion. I've seen a lot of analyses of actual insurance company portfolios where this type of analysis does not yield unambiguous results.

Looking at some reinvestment strategies and some pairs of scenarios, one can conclude that assets are too short. Looking then at other strategies and other pairs of scenarios, one can conclude that assets are too long. Well, they can't be both too short and too long! They are one or the other. Maybe the ambiguity is resolved by saying that assets and liabilities are actually pretty well balanced. If indeed that's the case (and it need not be), it's a rather strange way to arrive at that conclusion. One would ideally like much cleaner methods for understanding what is going on.

Another element I'd like to examine in Exhibits 6-4 through 6-7 is the range of variation of the results over the scenarios for any given reinvestment strategy. For example, if you look at the constant 1-year strategy, you'll see that the range is from minus $2.7 million under Scenario 2 to plus $6.1 million under Scenario 1. Now do this similarly for the other reinvestment strategies. You'll find that the tightest range of terminal surplus comes from the 6,5,4,3,2, 1-year strategy. We'll see why in a moment. In fact, if you put equal probability weights on all scenarios (there's no very good reason for doing that but if you do it anyway), the average terminal surplus lies between $3 million and $4 million. Remember that number as we continue this analysis.

What I'd like to do now is to start a new topic. Exhibits 6-4 through 6-7 also provide information of a kind not usually looked at by actuaries in asset/liability studies. For each reinvestment strategy, each scenario shows the market values of assets, liabilities, surplus, and numbers called "durations." I have used the Macaulay duration measure, the definition is in Exhibit 6-8. The convexity number shown is exactly the same as D_z, the second moment used by Redington and Vanderhoof. Its definition also appears in Exhibit 6-8.

EXHIBIT 6-8

INTEREST SENSITIVITY INDEXES FOR FIXED AND CERTAIN CASH FLOW STREAMS

CF(T) = CASH FLOW OCCURRING AT TIME T

B(T) = PRICE OF ZERO-COUPON BOND MATURING AT TIME T

MARKET VALUE = SUM_{T} CF(T)x B(T)

DURATION = (SUM_{T} T x CF(T) x B (T1) ÷ MARKET VALUE

CONVEXITY = (SUM_{T} T-SQUARED x CF(T) x B(T1) ÷ MARKET VALUE

NOTE:

THE DEFINITIONS OF DURATION AND CONVEXITY GIVEN ABOVE ARE BASED ON RIGID (SHAPE-PRESERVING) SHIFTS OF THE TERM STRUCTURE OF INTEREST RATES. SUCH A MODEL OF INTEREST RATE MOVEMENTS CANNOT REPRESENT REALITY BECAUSE IT LEADS TO THE POSSIBILITY OF RISKLESS ARBITRAGE. NEVERTHELESS, THESE MACAULAY/REDINGTON FIRST- AND SECOND-ORDER INTEREST SENSITIVITY MEASURES ARE VERY USEFUL TO UNDERSTANDING INTEREST RATE EXPOSURE AND TO EXECUTING ASSET/LIABILITY MANAGEMENT STRATEGIES. THEY ALLOW IMMUNIZATION AGAINST ANY COMBINATION OF CONSTANT AND PROPORTIONAL SHOCKS (IN OTHER WORDS, ANY LINEAR SHOCKS) ACROSS THE INTEREST RATE TERM STRUCTURE, AND THUS PROTECT AGAINST CHANGES IN THE LEVEL AND SLOPE OF THE TERM STRUCTURE. EXPOSURE TO CHANGES IN THE TERM STRUCTURE'S CURVATURE AND OTHER HIGHER-ORDER SHAPE CHARACTERISTICS CANNOT BE IMMUNIZED BY STRATEGIES UTILIZING DURATION AND CONVEXITY ALONE.

EXHIBIT 6-8
(Continued)

OBSERVATIONS

1. THE CHOICE OF REINVESTMENT STRATEGY DOES NOT AFFECT THE MARKET VALUE SURPLUS POSITION AT THE END OF THE FIRST YEAR.

2. FOR ANY GIVEN SCENARIO, THE MARKET VALUE SURPLUS POSITION AT THE END OF THE FIRST YEAR CAN BE COMPUTED QUITE ACCURATELY FROM THE ASSET AND LIABILITY MARKET VALUES, DURATIONS, AND CONVEXITIES AT THE START OF THE PROJECTION PERIOD.

3. THE CHOICE OF REINVESTMENT STRATEGY DOES AFFECT THE ULTIMATE MARKET VALUE SURPLUS POSITION BECAUSE IT AFFECTS THE SUCCESSIVE ASSET/LIABILITY DURATION AND CONVEXITY MISMATCHES.

4. THERE IS AN ORDERLY PATTERN OF MARKET VALUE SURPLUS ACCUMULATION WHEN IMMUNIZING CONDITIONS ARE MET (OR NEARLY MET). HOWEVER, THE PATTERN OF SURPLUS ACCUMULATION SHOWS CONSIDERABLE VARIABILITY WHEN THERE ARE SIGNIFICANT MISMATCHES IN DURATIONS AND CONVEXITIES BETWEEN ASSETS AND LIABILITIES.

CONCLUSION

THE OBSERVATIONS SUGGEST THAT MARKET VALUE SURPLUS AND ITS DURATION AND CONVEXITY CONTAIN THE ESSENTIAL INFORMATION ABOUT THE ECONOMIC VALUE OF A PORTFOLIO, AND THAT DETAILED PROJECTIONS ALONG SEVERAL DIFFERENT YIELD CURVE SCENARIOS ARE NOT NECESSARY TO UNDERSTANDING THE NATURE OF C-3 RISK WHEN ALL THE RELEVANT CASH FLOWS ARE FIXED AND CERTAIN. NAIVE 'STATIC' REINVESTMENT STRATEGIES CAN LEAD TO AN INCOMPLETE (AND SOMETIMES AMBIGUOUS) CHARACTERIZATION OF C-3 RISK, WHILE THE MODELING OF 'DYNAMIC' MATCHING STRATEGIES ALONG DIFFERENT SCENARIOS MERELY PROVES THAT MODERN IMMUNIZATION THEORY WORKS.

Notice that the market value of surplus, in Exhibits 6-4 through 6-7, at times is zero. In other words, today's valuation is $2.1 million in each and every case. This is independent of investment strategy, it is independent of scenario. It's independent of everything else. It is the answer.

Where does the $2.1 million come from? It comes from taking the difference between the market value of assets and the market value of liabilities. Here we have a very simple example of fixed and certain cash-flow streams without options. There are no interest-rate fluctuations in the cash flow streams, there are no laspe rates, there are no reoccurring deposits, these are just bullet GICs. This is the simplest of all situations. In this case (and I'm assuming no credit problems on the assets), the asset cash flows are fixed and certain. To get the market value of assets, one can merely use the interest rates implicit in today's yield curve to discount the cash flows back to the present. One can do the same calculation for the liabilities. Now obviously the asset and liability market values derived from such a calculation are not a function of how one reinvests in the future. The market values come from a straight forward discounting calculation. They are not a function of where interest rates are going to go in the future. The discounting calculation uses only today's interest rates. I need only today's yield curve because I have fixed and certain cash flows here.

So one thing I would say about Exhibits 6-4 through 6-7 is that they give me three very useful numbers: the $110.9 million market value of assets, the $108.9 million market value of liabilities and the $2.1 million market value of surplus.

Should I feel comfortable that this is the true economic surplus position of the company? Well yes I should, but unfortunately it is subject to changes in interest rates.

The duration for the assets is 4.1 years, and for the liabilities the duration is 4.5 years. There's nearly a half-year mismatch. This tells us unambiguously that assets are shorter than liabilities and that the portfolio is vulnerable to a drop in interest rates. It does not have to be a sudden or a big drop, it can develop slowly over the period of a year, but the portfolio is exposed to a drop in interest rates.

One can use fairly simple formulas based on a Taylor series expansion of the market value surplus function, up to second order, to determine how big a shock the portfolio can withstand given this duration mismatch, before the surplus is driven down to zero. This is a very simple calculation. A complicated computer program is not needed to obtain the answer. That is one of the advantages of focusing on these six numbers: the market values of assets and liabilities, the durations of assets and liabilities, the second moments or convexities.

I'm not going to spend too much more time on this. What I would like to do is go back to Exhibit 6-7, the market value, duration and convexity detail by scenario for the investment strategy 6,5,4,3,2 1-year. I constructed this as a teaching example. I wanted to be able to model a duration and convexity matching strategy without working very hard. Because there is an initial half-year mismatch between assets and liabilities, and because interest rates can change under the nine scenarios during the first year, we should expect a shock to occur to market value surplus. We should not expect an immunized surplus position. And, in fact, as you look through the various scenarios you generally see a shock to surplus over the first year. In Scenario 1, the shock is mild. It goes from 2.1 to 2.2. None of us would really view that as much of a shock. Under Scenario 2, it goes from 2.1 down to 0.9. That's a fair shock. The move from 2.1 to 3.0 in Scenario 3 is a favorable shock, but still a shock. Then the move from 2.1 to -.9 in Scenario 4 is really quite a shock. There will be shocks to surplus over the first year, which is not surprising because the portfolio is mismatched and thus exposed to changes in interest rates. Many of the scenarios exhibit a fairly dramatic change in interest rates over the first year.

Under this reinvestment strategy, all excess cash flow at the end of the first year is invested in a six-year bond that matures at the investment horizon. For this particular example, the portfolio is then essentially cash flow matched. If you look at the duration and convexity columns starting from year two and running through the end of seven years, you'll notice that asset and liability numbers are very close. They are almost matched. Asset and liability durations are almost equal. Asset and liability convexities are almost equal. This follows from the near cash flow match.

If immunization works, you should notice that, after the first year, the market value surplus grows in a stable fashion throughout the balance of the projection period. If the surplus position at the end of the first year is negative, the growth is really a smooth decline.

Under this reinvestment strategy, asset and liability characteristics — duration, convexities, and so on — are matched after the first year. This immunization leads to development of stable market value surplus. The reason one didn't have stable development from the beginning is that the portfolio carried an initial mismatch through a shock in interest rates, before it was rebalanced to eliminate the mismatch.

The one thing I would like to comment on before I pass along to the second part of my presentation is the note in the middle of Exhibit 6-8. There has been a lot of talk recently, and there have been a number of papers lately, casting aspersions on the value of Macaulay duration, particularly on convexity because calculation assumes parallel shifts in the term structure of interest rates. I think that too much is made of that point. Let me try to explain why.

One of the statements made is that because these calculations of first and second moments are based on a rigid (shape-preserving) shock to the yield curve or term structure, their use must imply that this is the way the world behaves. Well, in fact we know (as correctly pointed out in one TSA paper) that the world does not offer merely parallel shifts in all interest rates. If it worked that way, there would be riskless arbitrage opportunities all over the marketplace. We know the world doesn't work that way — at least not for very long!

So we know that the world doesn't move in rigid, parallel shock fashion. One can simply examine yield curve movements over the last five, ten or fifteen years to ascertain that fact. However, because the world behaves differently does not mean that you can't calculate sensitivity indices as if only parallel shifts took place. However, because it doesn't actually work that way a single sensitivity index — in other words just the duration index — isn't going to capture the whole story. In fact, duration and convexity aren't going to capture the whole story for big shocks or wild changes in the shape of the yield curve, but they capture important parts of the story.

An analogy from probability theory might be to look at moments of a probability distribution and notice that if you define things properly, you don't need more than two parameters to specify the normal distribution. But if you chose a different way of calculating moments, a different basis if you will, you might need an infinite number of parameters to characterize the normal distribution. (They'd be highly independent, of course.) The problem in portfolio management is tantamount to choosing a set of measuring sticks. Macaulay duration and Macaulay convexity, the D1 and D2 measures, are very useful measuring sticks. They do not imply that the world must work in a parallel, rigid-shock fashion, and they really are quite useful. You can work through some of these ideas yourself. I wanted to stand up in public and state for the record that a lot of things being said about these measures are vast overstatements of their disadvantages.

Moving on to the next part of the presentation, the substance of the life insurance industry is cash flows that are interest-rate sensitive, not ones that are fixed and certain. Looking back in history, I believe if it had been possible at an earlier point in time to properly and fairly discount a stream of interest-rate sensitive cash flows, I suspect that much of what the Society's C-3 Risk Task Force has done over the last three or four years would not have been necessary.

I do not mean to imply that the work on that task force is useless or that one does not gain a lot of insight by doing simulation studies. One surely does. In fact, I know of no way to gain more insight than by playing around with simulation models. But the point is, if we did know how to discount in a very simple fashion, we probably would have realized that we needed to make assumptions only about the nature and extent of interest-rate fluctuations, and then apply this grand methodology to come up with an answer. And we could have worked with just that answer. In other words, we could have market-valued liabilities. Instead we have a crude trial and error approach using different reinvestment strategies and modeling to try to guess at what the market value of liabilities really is.

What I'm going to talk about now is a theory that can be properly viewed as an extension of the theory of interest. It is intended to discount, fairly and properly, a stream of interest-rate sensitive cash flows. You may wonder why I'm going to talk about new and difficult things when the conventional C-3 risk theory is already quite complicated. The reason is to attempt to make some sense out of a complicated situation. The task force members are trying to eliminate the paper blizzard that the conventional approaches tend to bring. We are trying to make sensitivity analyses more straightforward. We are trying to bring actuaries up to speed on concepts and techniques that are highly useful, and are used everywhere else in the financial marketplace. These approaches apply with equal force to the life insurance business, even though it is much more complicated than any other financial service business.

The whole purpose of this part of my presentation is to extend the theory of interest to interest-rate sensitive cash flow streams; to show that given a set of assumptions, which includes an assumption about the volatility of interest rates, one can discount the cash flow stream to a unique market value. One can actually do that. Then one can calculate what the market value becomes after a shock to interest rates, and thus calculate the interest sensitivity of the cash flow stream. That's what durations are all about. If one calculates the market value in three different interest rate environments — today's rates, today's rates with a shock up, today's rates with a shock down — one can put the results together and calculate a second moment or convexity. And by repeating those calculations in different starting interest-rate environments, one can then calculate durations and convexities.

This method of discounting cash flow streams must be done in a manner consistent with concepts in the modern theory of finance, namely, term structure theory and option pricing theory.

My material is practical, and not just theoretical. I will end my presentation with results of a SPDA analysis of a type carried out by my firm for several clients. I use these techniques every day. This is my approach to the C-3 risk problem for single premium products. It is actually my approach for all types of business, but I've only written what I consider fairly complete software for the SPDA problem.

Remember that I defined the surplus cash flow stream to be the difference between asset and liability cash flow streams for the in-force business. This net cash flow stream is interest-rate sensitive in actual portfolios. So when interest rates rise, there tends to be disintermediation, and there tends to be an acceleration of liquidity problems to current time or to the early part of the projection period. The opposite happens when interest rates fall. Then there are prepayments and calls on the assets.

The dilemma, then, is to distinguish cash flow behavior as either fixed and certain or interest-rate sensitive. The risk control strategy for fixed and certain cash flows is obviously a matching strategy. Although it probably is too cost prohibitive for anyone to do that and sell business, for example in a GIC portfolio, a structured settlement portfolio or a pension plan close-out portfolio, it is, nevertheless, the risk control bench mark. There's no question that cash flow matching provides full immunization. However, because of the interest sensitive behavior of the surplus cash flow stream that was discussed earlier, matching cash flows clearly does not work when the cash flows are interest-rate sensitive. So we are in search of a way of solving the problem of interest-rate sensitive cash flows that will be highly effective, whether the cash flow stream is fixed and certain or interest-rate sensitive.

The solution is to focus on the present value of the surplus cash flow stream, and, as I said a few moments ago, the question is what technique to use to discount that cash flow stream to a unique present value independently of where interest rates may wander. Once that can be done, it is possible to compute the sensitivity of the present value to changes in interest rates. If the calculation is done for today's interest rate environment, one simply shocks today's interest rate environment a little, and then continues the calculation to determine what the sensitivity of the present value is to the change in interest rates. Then through duration and convexity matching, in other words, through immunization extended to this more complicated situation, one can control the sensitivity characteristics of the cash flow stream. This was done in the 6,5,4,3,2 1-year reinvestment strategy in the GIC example where there was good control over the evolution of market value surplus after the first year.

Why is market value surplus important? After all, it is yet another set of books, and no one likes more work than necessary. It is important because it portrays the true financial condition. As a matter of fact, traditional C-3 risk methodology is a somewhat crude way of measuring market value surplus. And we know that the whole reason for doing C-3 risk calculations is to get a handle on the true economic picture. Market value surplus also turns out to be a leading indicator of book earnings. At the end of the projection period, whether one cashes out the remaining portfolio at market value or just runs out remaining cash flows on a level path of interest rates, one ends up with results that are, essentially, financially equivalent. This is the case regardless of whether one borrows needed funds or liquidates assets, even though in one case there is an immediate realization of capital effects, and in the other the impact emerges over time as changes in book earnings. Market value surplus is, in an important sense, a leading indicator of book earnings. Therefore, it is obviously a key to understanding product economics. As we will see soon, it also responds to risk-control techniques. In other words, duration and convexity matching control the volatility of market value surplus.

The basic methodology involves four steps. In Step #1, create the yield curve lattice, I'll expand on that in a minute, but for now you can think of this step as equivalent to choosing a universe of interest rate scenarios over which to do the analysis. In Step #2, project cash flows on the lattice. This is nothing more than projecting cash flows over the universe of interest scenarios. So, there's nothing profound in these two steps. These two steps coincide exactly with what you must do when using all the other techniques presented at this symposium. And the same kinds of assumptions and models have to be built to accomplish these first two steps. The last two steps are what the new methodology is all about. These steps provide a way to summarize the information resulting from the projections over the universe of interest-rate scenarios, a way to condense everything back to market value numbers, and a way to compute sensitivities of the market values, that is, durations and convexities.

I will expand a little on each of the four steps, and then spend the remainder of my discussion time on the nature of the discounting calculations.

Creating the yield curve lattice begins with the current Treasury curve, adding to it a quality spread assumption for the mix of investments, and adjusting that quality spread assumption for asset defaults (if not modeled explicitly elsewhere). Also, one should adjust for investment expenses if those are not modeled explicitly as cash flows. Finally, one must make an interest-rate volatility assumption. Remember, for a fixed and certain cash flow stream, an interest volatility assumption does not have to be made. One can use just the current yield curve to discount the stream of fixed and certain cash flows. But when the cash flows themselves depend on where interest rates go in the future, it is necessary to make an assumption about how volatile interest rates will be. That is one of the advantages of creating this kind of lattice, because for the lattice, unlike the situation for hand-fashioned scenarios, one actually has a controllable volatility parameter that can be varied. There is a number that characterizes the volatility of interest rates. Looking at any of the collections of interest-rate scenarios that have been presented during this symposium, I rather doubt that either you or I could characterize them by a single volatility parameter. In fact, I rather doubt that the scenarios were drawn from a universe characterized by a fixed volatility assumption. In sensitivity analysis, it is very nice to be able to find out how the values of the cash-flow streams change as adjustments are made in a volatility parameter.

Projecting the cash flows on the lattice involves making usual pricing assumptions about expenses, mortality and so on, and then, making assumption about interest rates, incremental lapse rates, interest-rate crediting strategy, bond calls, mortgage prepayments and so on. I concur with the Tillinghast observations about the importance of very carefully modeling the interest crediting strategy. It is a much overlooked factor.

Discounting the cash flows to obtain market values must be done in a manner consistent with term structure and option pricing theory. I will start off with a notion that one uses when discounting fixed and certain cash flows because it is also needed for interest-rate sensitive cash flows. I will presume that most of you in the audience do not know what "spot rates" of interest are. These are rates of interest, or yields to maturity, applying to zero-coupon bonds, bonds having no cash flow until their maturity dates. I define a term to be the price of an N-period zero-coupon bond. The period can be months, quarters or half-years, not necessarily years as the fundamental revaluation cycle. The yield corresponding to the price $B^{(N)}$ is $r^{(N)}$ called the N-period spot rate. It is the interest rate used to discount cash flow occurring N periods out, back to today. The relationship between the price of zero-coupon bond and the spot rate is given as follows: A bond that matures for $1 at the end of N periods has a price of 1 divided by the quantity $\left[1 + r^{(N)}\right]$. The interest rate is convertible every period. Therefore, $r^{(N)}$ would be one-half the bond equivalent rate if I were using half-year periods. It would be the annual effective rate if I were using annual periods. It would be one-twelfth of a typical mortgage equivalent rate if I were using monthly periods. I don't like to carry around a lot of divisors of N, so I define these to be per-period rates.

How does one use spot rates, then, to discount a stream of fixed and certain cash flows? Let's look at Exhibit 6-9. The five-year schedule of end-of-year cash flows is shown. Notice that they are cash flows from a bond paying annual coupons of $10.65 per $100 par amount; in other words, a 10.65 percent effective annual rate. The spot rates correspond to the starting yield curve in the earlier GIC example. To compute the discount factors, or the prices of the zero-coupon bonds, divide the spot rate, as a percent, by 100 to express it as a fraction. Add 1 to the fraction and take the reciprocal of the result. Then raise that result to the Nth power again, where N is the number of periods. These are the discount factors. Those are applied to the cash flows in discounting. In this particular example, the end result is $100 exactly. In other words, this bond is priced at par. Thus, for this carefully contrived example, one knows that the five-year yield of a par instrument is 10.65 percent. But the spot rate, the yield

EXHIBIT 6-9

Discounting a Stream of Certain Cash Flows

Time	Cash Flow	Spot Rate	Discount Factor	Discounted Cash Flow
1	$10.65	8.00%	.92593	$9.86
2	10.65	9.05	.84098	8.96
3	10.65	9.86	.75419	8.03
4	10.65	10.42	.67264	7.16
5	110.65	10.89	.59635	65.99
				$100.00

Term Structure

Present Value

to maturity, of a five-year zero-coupon bond is 10.89 percent. Those are two different numbers. Only the five-year spot rate tells one how to discount cash flow occurring at the end of the fifth year.

Let's move on to the more interesting situation of interest-rate sensitive cash flows. Exhibit 6-10 shows a binomial lattice. That is a fancy way of displaying how a yield curve today can move to one of two yield curves in the next period. Start at the left-most point on the vertical interest rate axis. That point represents today's yield curve, or today's spot rate curve. One assumes that over the next period, interest rates can go up or down. Every point, or node, in the lattice leads to two nodes at the start of the next period. But this lattice is connected, the points converge. So there are only three possible states of the world, three different term structure curves at the end of the second period, four at the end of the third period, five at the end of the fourth period and so on. In this lattice, volatility is represented by the spacing of the nodes. If volatility is high, today's yield curve will change substantially to become tomorrow's yield curve. If volatility is low, it won't change very much. The arrows on the lattice indicate two possible interest-rate paths through three periods: up, up, up; up, up, down.

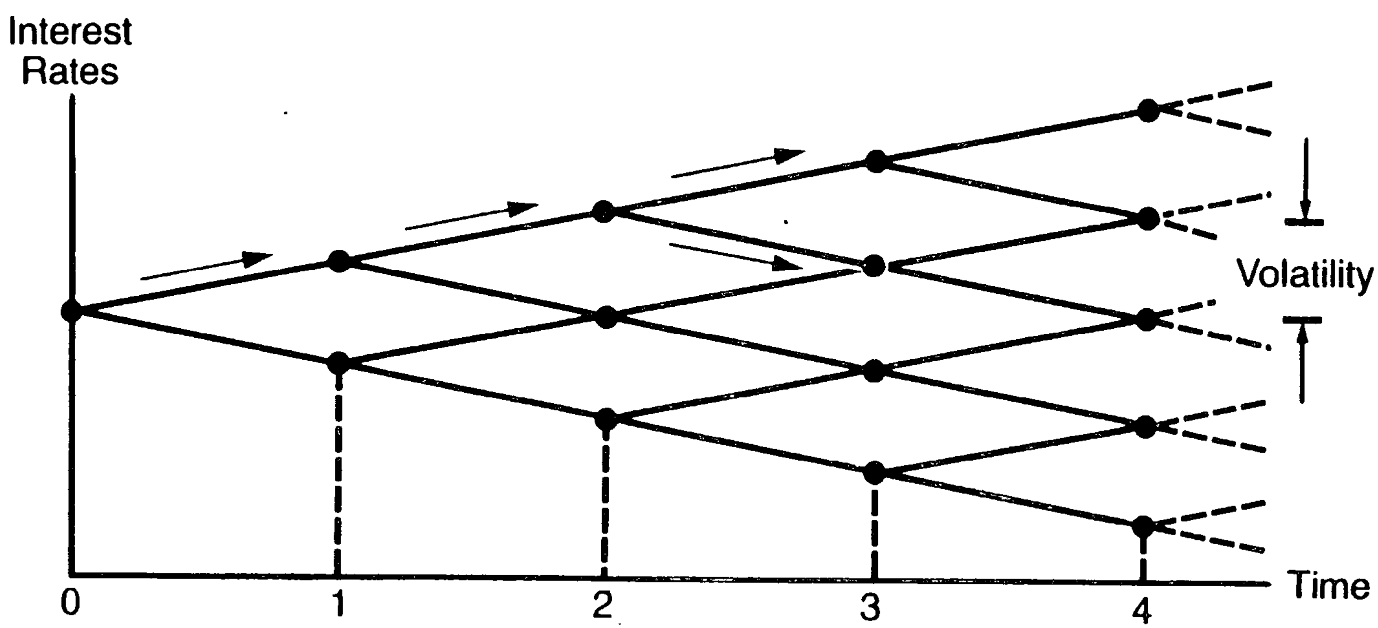
Let's examine the fundamental vertex in the lattice to see if we can understand how to discount cash flows that might occur at the ends of the fork.

For this I will base my comments on the idea of two fundamental discounting units. It is not necessary, but quite useful, to have a name for these units. I have chosen to call them "quarks," a term borrowed from particle physics. In

EXHIBIT 6-10

Pricing/Valuation Methodology

YIELD CURVE LATTICE



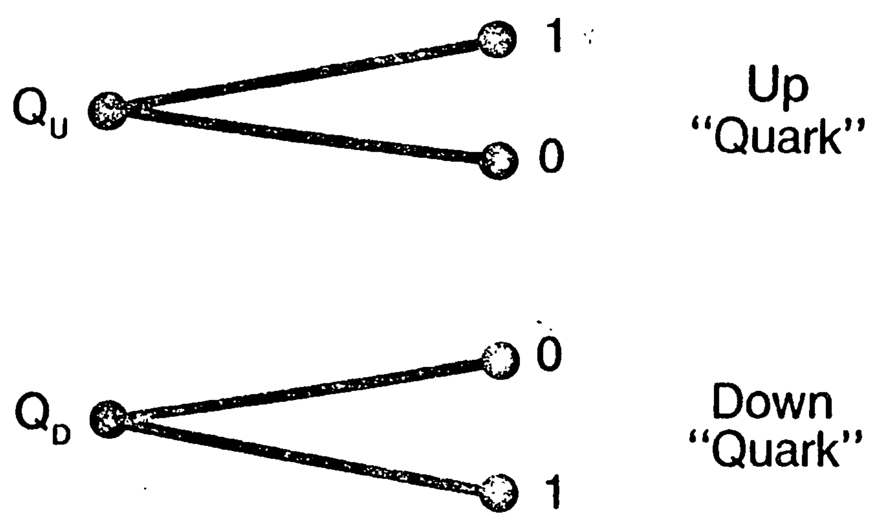
physics, quarks are currently thought to be the most fundamental building blocks of matter. Molecules are composed of atoms, atoms of a nucleus and electrons, the nucleus of neutrons and protons, and neutrons and protons of quarks. It is the building block analogy I want to carry over to the interest-rate lattice and the discounting calculations to be performed on the lattice. The key point is that if one knows how to perform the cash flow discounting calculation for an elemental fork, then one knows how to do it for a larger lattice composed of connected forks. In turn, an elemental fork can be considered to be built up from the fundamental discounting units or quarks. Two fundamental quarks for a binomial interest-rate lattice — an up quark and a down quark — are illustrated in Exhibit 6-11.

The up quark has a 1 at the end of the up node and a 0 at the end of the down node. It can be thought of as a financial instrument that promises the following: At the end of one period, the holder of this instrument is paid $1 if the up state occurs, $0 if the down state occurs. What is the fair price for this instrument? Well, I'm going to say it is Q_U. I will show how to derive Q_U in a minute, but for now it's just Q_U. That's the fair price for this instrument. It is what one would pay for it in a fair market. Those of you who know something about options will realize that it is a very basic put option. If the up state represents interest rates rising, the option expires in the money. If interest rates fell, it expires out of the money. The down quark pays off $1 if the down state occurs, and $0 if the up state occurs. The fair price, in other words the present value, one period from now is Q_D.

EXHIBIT 6-11

Pricing/Valuation Methodology

FUNDAMENTAL "DISCOUNTING" BUILDING BLOCKS



Knowing Q_U and Q_D tells us how to discount an interest-rate sensitive cash flow stream. This is described in Exhibit 6-12. Suppose that in the state of the world represented by the up node (remember this is just a yield curve at some point on some path through the lattice) the cash flow is CF_U, which might include deaths, expenses, lapses (if it represents a liability stream), or principal and interest payments (if it represents an asset stream). The principal would include both scheduled and unscheduled principal prepayments.

Let's suppose we know the present value, MV_U of all the future cash flows on the lattice emanating from the up node, and also know MV_D for the down node. How do we find out what the present value is back at node 0, the origin of the fork? The answer is pretty simple. Decompose the cash flows on the fork into two pieces: the up quark and the down quark each scaled by the appropriate cash flows. The up quark is an instrument that pays \$1 in the up state and \$0 in the down state. So scale the quark value by CF_U plus MV_U, because that is the fund value one must have in the up state to pay off all obligations. Similarly, scale the down quark by CF_D plus MV_D.

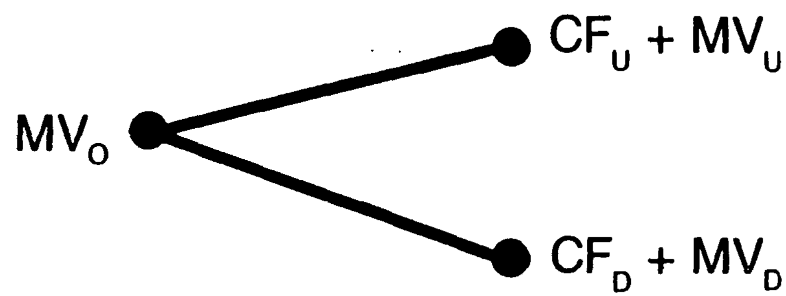
I still haven't told you how to calculate Q_U and Q_D, and you need to know those to calculate the expression for MV_0.

In terms of the lattice and the allowable states of the world, consider a cash flow structure represented by a one-period bond, similar to that shown in Exhibit 6-13. No matter which of the two states of the world occur, it pays exactly \$1. It matures. It pays \$1 whether the up state or down state occurs. What's the value of that? Well, we can use the equation we just derived to find out. There

EXHIBIT 6-12

Pricing/Valuation Methodology

PRESENT VALUE CALCULATION

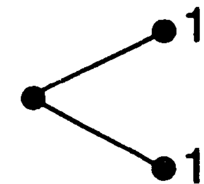


$$MV_O = [CF_U + MV_U] \bullet Q_U + [CF_D + MV_D] \bullet Q_D$$

EXHIBIT 6-13
PART 1

Determining the "Quark" Values

One-Period
Zero-Coupon Bond

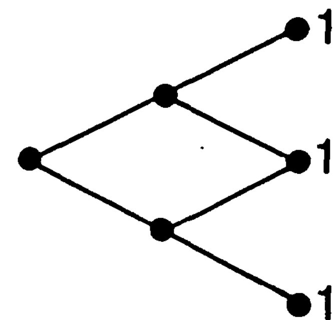


$$B_O^{(1)} = 1 \bullet Q_U + 1 \bullet Q_D$$

EXHIBIT 6-13
PART 2

Determining the "Quark" Values

Two-Period
Zero-Coupon Bond

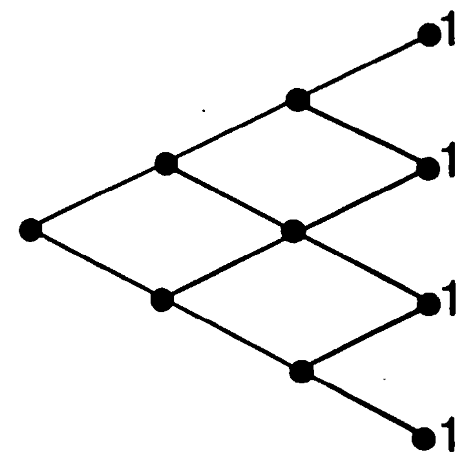


$$B_O^{(2)} = B_U^{(1)} \bullet Q_U + B_D^{(1)} \bullet Q_D$$

EXHIBIT 6–13
PART 3

Determining the "Quark" Values

Three-Period
Zero-Coupon Bond



$$B_O^{(3)} = B_U^{(2)} \bullet Q_U + B_D^{(2)} \bullet Q_D$$

are no cash flows beyond the first period because this is a one-period bond, so the present value is simply 1 times Q_U plus 1 times Q_D. But the price of a one-period zero-coupon bond is also $B_0^{(1)}$. And we know that price today, because we know today's yield curve. In this equation, we know $B_0^{(1)}$ value, not the Q_U or Q_D.

You may not think of a two-period zero-coupon bond in terms of the picture shown in Exhibit 6-13, but that is exactly what the cash flow structure represents. For the sake of completeness I probably should have marked the zero cash flows, which occur everywhere except at the terminal nodes of the two-period lattice, where all the cash flows are $1. So all possible states in this particular lattice would pay $1 at the end of two periods, with no cash flows before then.

To calculate the value of this cash flow pattern, when the bond matures, we apply the technique recursively. Let's discount the upper two end nodes back one period. We determine that to be the price of a one-period bond at the up node. In other words, that is $B_U^{(1)}$. Similarly, $B_D^{(1)}$ is the price of the future cash flow associated with the down node. Now we discount $B_U^{(1)}$ and $B_D^{(1)}$ back one period. That involves the Q_U and Q_D. This leads us to consider the equation shown in Exhibit 6-13 for the two-period zero-coupon bond. What do we know from the equation shown? We know the left-hand side from today's yield curve, and by construction of the lattice (although there are some constraints on the construction). We can assume that we know what the one-period yields are in the up and down states.

We have derived two equations in the two unknowns, Q_U and Q_D. One may think that one can just solve the equations to get the answers. Actually that is right. The whole story is that simple.

The bottom of Exhibit 6-13 contains an equation for the price of a three-period zero-coupon bond, derived by applying the same procedures as before. It involves the same Q_U and Q_D as in the equation for the two-period zero-coupon bond. And if we went out four periods, we would get yet another equation involving the same quantities. In other words, if I replace the superscript 3 on the left-hand side with an N, and the superscripts 2 on the right-hand side with N-1, the equation will hold for all N on the yield curve. So there are a lot of equations and only two unknowns. That's not as bad as it looks. What it really means is that for the lattice to be arbitrage free, in other words, for all bonds to be fairly priced, one can't allow any arbitrary yield-curve evolution through the lattice. These equations are used not only to find Q_U and Q_D, but also to specify how the yield curves at the up and the down nodes must be related to the starting yield curve.

Now let me define some new notation, a probability P_U in terms of Q_U and corresponding P_D in terms of Q_D, as shown in Exhibit 6-14. Notice the following properties:

(1) P_U plus P_D equals 1. Why is that? Look back at the one-period bond example of Exhibit 6-13. Look at the equation for the price of the one-period bond. It implies that P_U plus P_D equals 1, because $B_0^{(1)}$ is the reciprocal of $1+r_0^{(1)}$.

(2) P_U and P_D are positive. This is because Q_U and Q_D are positive (no one would sell an instrument that pays \$1 in one state and \$0 in the other state for a nonpositive amount).

EXHIBIT 6-14
PART 1

Simulation Approach to Determining Market Values

- DEFINE $P_U \equiv Q_U \bullet [1 + r_O^{(1)}]$

  $P_D \equiv Q_D \bullet [1 + r_O^{(1)}]$

- THEN $P_U + P_D = 1$ AND $P_U, P_D \geq 0$

- INTERPRET P_U AND P_D AS "PROBABILITIES"

EXHIBIT 6-14
PART 2

Simulation Approach to Determining Market Values

$$MV_O = [CF_U + MV_U] \bullet Q_U + [CF_D + MV_D] \bullet Q_D$$

$$MV_O = P_U \bullet \frac{[CF_U + MV_U]}{[1 + r_O^{(1)}]} + P_D \bullet \frac{[CF_D + MV_D]}{[1 + r_O^{(1)}]}$$

So, at every node in the lattice there are two (different) numbers, P_U and P_D, which sum to one, and are both positive. I can therefore choose to interpret them as probabilities. In fact, these can be interpreted as "arbitrage probabilities" in pricing options.

Why do I want to interpret P_U and P_D as probabilities? The reason is very simple. In the real world, there are path-dependent cash flows. Think of a GNMA or think of a SPDA. I'll briefly describe how it works with each one, first the SPDA. Whether interest rates go up, then down, or down and then up, one might think one has arrived at the same state of the world. After all, the binomial lattice is connected. But, even though one has arrived at the same interest-rate state of the world, one surely arrives with a different book of business than one began with. The in-force business is quite different. If rates go up first and then down, business is lost through cash surrenders. Maybe a lot of it. If rates go down and then up, a different amount of business is lost. Thus, the cash flows are path dependent. The same is true for GNMAs. If interest rates go down and then up, one gets a lot of prepayments. If interest rates go up and then down, prepayments drop, and then may come back to the previous level. In general with path-dependent cash flows, one can't "solve" the lattice the way I did for the zero-coupon bonds — by moving backwards recursively from the end to the beginning. Although the lattice appears connected, the cash flows depend on the particular path through the lattice. So, the probabilities turn out to be very useful. Those allow us to rewrite the fundamental discounting equation in a way to process the lattice forward from the beginning to the end.

The equation at the bottom of Exhibit 6-14 gives the market value at the starting node in terms of the market values and the cash flows at the up and down nodes. Substituting P_U and P_D for Q_U and Q_D respectively, an equivalent equation for MV_0 results. One can attach a nice set of words to this equation: The market value at node 0, the origin of the lattice, is equal to the probability of going to the up node multiplied by the cash flow plus market value at the up node, plus the probability of going to the down node multiplied by the cash flow plus the market value at the down node, with the whole result discounted back one period at the one-period rate of interest. These probabilities are not probabilities in the sense that they correspond to either your or my subjective views as to the chances that the up and down states will occur. They are probabilities in the sense of path weights needed to prevent riskless arbitrage opportunities from arising.

The revised equation is consistent with other discounting calculation techniques. One attaches probability weights and then looks at the resulting cash flows. Those are discounted back one period at the proper interest rate for that period. And, lo and behold, the right answer emerges. Armed with this intuitive interpretation, we now see how the process the whole lattice goes forward. By construction of the lattice, we know the (different) values of Q_U and Q_D at every node. Therefore, we know P_U and P_D at every node. For any path through the lattice, the cash flows can be projected just as in any of the other models you heard about at this symposium. The probabilities for all the various path segments can be multiplied to get an overall path probability. The cash flows can be discounted back to the beginning of the lattice, one period at a time, using successive one-period spot rates. This results in the contribution of that path to the overall market value result. If this calculation is done for every path in the lattice, the results can be summed to get the correct answer.

Unfortunately, one cannot solve a real-world lattice by enumerating all paths. In a binomial model where each state leads to two other states, let us suppose we are modeling quarterly cash flows over a twenty-year projection period, there would be eighty periods and 280 paths. That is an awful lot of paths! It's more than all the Cray computers in the world all working together could solve in our lifetimes. So what one has to do is draw a sample of paths — do a Monte Carlo calculation. This technique is starting to sound like the C-3 risk methodology. One chooses interest-rate scenarios, and estimates the market value of surplus using that sample of scenarios. If appropriate variance reduction techniques are used, in other words, the sample of paths is chosen carefully, answers for reasonably interest-rate sensitive cash flow streams are obtainable, accurate to about 1 percent.

What I want to do now is give you a refresher course on duration. Most of you know something about duration, but you probably know it in terms of a typical Macaulay definition, as the time-weighted present value of a stream of cash flows. And you probably think convexity, or the second moment measures the time-squared-weighted present value. Those are valid definitions for a stream of fixed and certain cash flows, but we need to generalize the definitions so we can use them for situations involving interest-rate sensitive cash flows. The old definitions make no sense here, because a single time line cannot be drawn with uniquely marked cash flows for discounting. You might think that I could use the arbitrage probabilities to weight the Macaulay durations for each path, but, believe me, you get the wrong answer. So, we need a better definition of duration. If we start with a fixed and certain stream of cash flows and the Macaulay definition of duration, we can derive a useful property of duration. Then we can turn around and adopt that property as a fundamental definition of duration, viewing the original definition as a special property, one that makes sense only for fixed and certain cash flows. This is the approach I'm going to use.

The new definition is: Duration is a measure of the sensitivity of the present value of the cash flow stream to changes in interest rates. It measures how the present value of the cash flow stream changes for a given shock in interest rates. In Exhibit 6-15, this definition is expressed in an equation. We characterize the current interest-rate environment by some bench mark interest rate I, and the present value of the cash flow stream (fixed and certain or interest sensitive, a GNMA with all its implicit options or a noncallable bond without any) by PV. Then we shock the interest rate environment. In other words, we make a parallel shock to the term structure of I and recalculate the present value. We will see that it becomes PV plus PV. (One calculates PV and PV using the methodology I described earlier.) Duration, expressed as a price sensitivity is given by the ratio of (PV/PV) to I. The minus sign appears in the formula by convention so that bonds have positive duration namely, bonds depreciate in value as interest rates go up, and appreciate in value as interest rates go down. So a minus sign is needed. We would obtain a negative duration for a bond otherwise. The (1+I) factor recovers the Macaulay duration for a stream of fixed and certain cash flows. If a continuously compounding rate is used, that is, all rates are expressed as forces of interest, then the (1+I) factor is not used.

EXHIBIT 6–15
PART 1

What is Duration?

	Interest Rate	Present Value
Current Environment	I	PV
"Shocked" Environment	$I + \Delta I$	$PV + \Delta PV$

$$\text{Duration} = -(1 + I) \times \frac{(\Delta PV)/PV}{\Delta I}$$

EXHIBIT 6-15
PART 2

What Is Convexity?

A MEASURE OF THE RATE OF DURATION DRIFT DUE TO CHANGES IN INTEREST RATES

$$\text{Convexity} = -(1 + I) \times \frac{(\blacktriangle[PV \times D])/PV}{\blacktriangle I}$$

EXHIBIT 6-15
PART 3

Pricing/Valuation Methodology

CALCULATE INTEREST SENSITIVITY CHARACTERISTICS

$$D = -(1 + I) \times \frac{(\Delta MV)/MV}{\Delta I}$$

$$C = -(1 + I) \times \frac{(\Delta [MV \times D])/MV}{\Delta I}$$

The essence of the duration formula is in having ΔPV divided by PV in the numerator and ΔI in the denominator. ΔPV divided by PV is the fractional change in the present value associated with the change in the interest rates ΔI. Forgetting for the moment that ΔI should perhaps be small, so that the equations involve derivatives rather than numerical approximations, what it really says is that if a 100 basis point interest-rate change causes the present value of the cash flow stream to change by 5 percent, then there is a five-year duration. There is a 5 percent change in present value divided by a 1 percent change in interest rate. That is five years. It is an index number. It indicates how sensitive the cash flow stream is to changes in interest rates. If the cash flow stream is also fixed and certain, the same answer is obtained by putting time weights on all the cash flows and doing the discounting calculation using the usual Macaulay formula. But if it's not fixed and certain, and a lattice is used to calculate the PVs, the result is still a number that makes sense. So think of duration as an index number that measures the sensitivity of present values to changes in interest rates. Duration is not all the same thing as maturity, in most cases. In Exhibit 6-16, four instruments are compared. All have ten-year maturities but otherwise have different properties, obviously their durations are different. Notice also that the durations are a function of the interest-rate environment. More interesting is the graph in Exhibit 6-17 that tracks asset and liability durations against interest rates for three different instruments: an SPDA, a putable bond, and a current GNMA. The SPDA shown here provides a seven-year guarantee of interest, not the annual reset type in which the insurer has the right to change the credited rate every year. As interest rates fall, the policyholders' cash surrender rights lose economic value and the SPDA behaves like a GIC — hence the lengthening duration. As interest rates rise, the

EXHIBIT 6-16

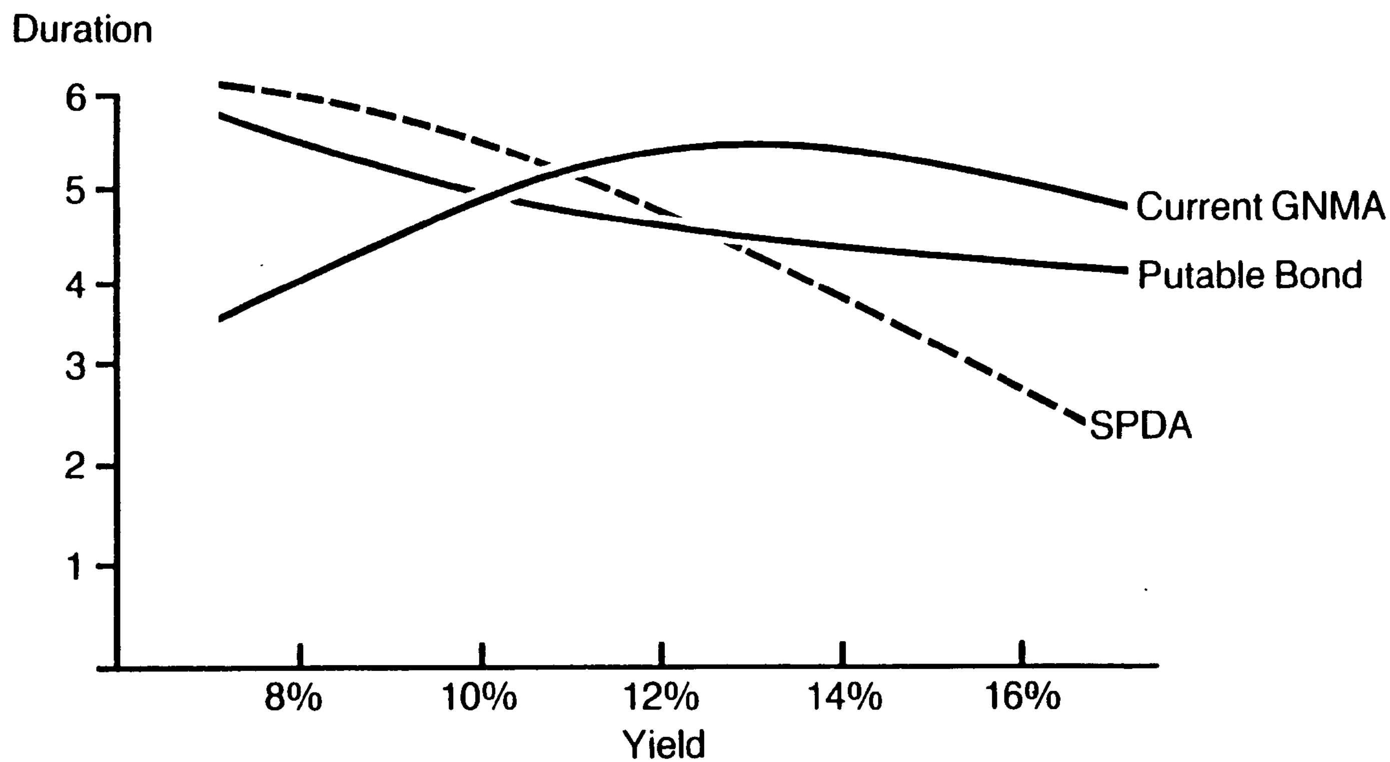
Duration Examples

10-YEAR INSTRUMENTS

	Interest Rate Environment	
	10%	15%
Zero-Coupon Bond	10.0 Years	10.0 Years
Current-Coupon Bond	6.8 Years	5.8 Years
Level-Payment Mortgage	4.7 Years	4.4 Years
Floating-Rate Note	0.5 Years	0.5 Years

EXHIBIT 6-17

Asset and Liability Durations



policyholders' cash surrender rights become very valuable and the duration of the SPDA shortens considerably. The policyholders' surrender rights can be viewed as a put option that the insurer has granted. It is not surprising then that the duration curve for this SPDA product looks very much like the duration curve for the putable bond. The putable bond illustrated has a ten-year maturity and a right to put the bond back to the issuer at par at the end of five years.

The most interesting duration curve is that for the GNMA. Duration does not merely decrease as interest rates go up as it does for most bonds. It increases, for a while, as interest rates go up. And why is that? Prepayments from the GNMA dry up as interest rates go up. It's not financially advantageous to refinance a home as interest rates are going up. It is to a homeowner's advantage to refinance as interest rates are going down. Thus, as interest rates go up the economic component of the prepayment rate, which can be referred to as an SMM or a CPR (single monthly mortality or conditional prepayment rate) decreases. It's the opposite of a lapse rate increasing as interest rates go up. This causes the GNMA to first increase as interest rates rise. The curve tops out and begins to decrease when the interest-sensitive component of the prepayment rate is pretty much zero, leaving only the now-interest-sensitive demographic component.

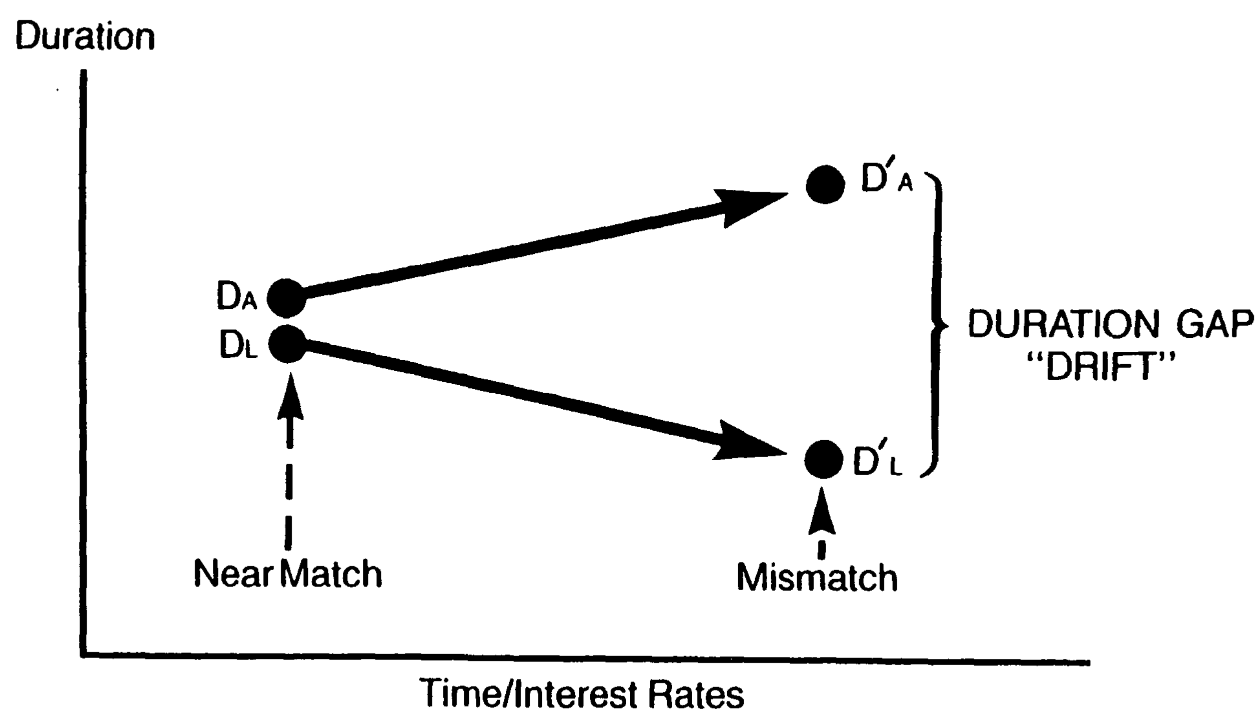
Why should one use durations? Because duration captures, in a single index, the essence of interest-rate exposure. One probably ought to expand that to say that duration captures the essence of interest-rate exposure in a few indexes, but for many situations duration gives the most important part of the story. Duration, as a risk-management tool, permits a consistent quantification of surplus requirements among various lines of business. If this methodology is used to calculate market values and durations of assets, liabilities and surplus, the market values can be weighted by the duration indexes by line of business to obtain a market-value-weighted duration for the whole company. Although it doesn't solve the combination of risks problem for C-1, C-2 and other kinds of risks, it does allow putting the C-3 risk exposures together for very different lines of business. Duration provides a useful target for asset managers. I have never met a portfolio manager who likes hundreds of pages of computer output. When he asks what to do with the portfolio, he can be told what the liability duration target is. If the company doesn't want to match asset and liability durations, but wants to time the market to take a bit of a mismatch, at least the risk control bench mark is known. Duration also allows one to standardize the comparison of various investment strategies and tools.

Once one understands what the risk-neutral duration target is, the investment strategy becomes one of putting the cheapest cost portfolio together that meets the chosen and/or convexity constraints. One does not have to build a simulation model that can handle every type of asset that Wall Street has ever created. Even if there were such a model, it would be no good tomorrow because people on Wall Street are very busy. A simple index such as duration allows comparison of investment strategies without full-blown simulations.

Unfortunately, durations tend to move as interest rates change. Duration gap, shown in Exhibit 6-18, tends to drift over time and as interest rates change, and convexity can be interpreted as a measure of how fast duration drifts for changes in interest rates.

EXHIBIT 6-18

Need for Portfolio Rebalancing



Rather than actually examining the convexity formula to illustrate this, I will draw another analogy from physics. If you toss a ball or launch a projectile, you can define the whole trajectory of its motion by position, speed and acceleration (assuming no air resistance). Knowledge of those variables at a particular point on the trajectory allows you to map out the entire trajectory. In much the same way, the market value of a stream of cash flows along with its duration and convexity allow you to map out the trajectory of the cash flow stream, the path taken by the present value of the cash flows as interest rates change. It can be said that the asset-liability matching process is one of arranging asset and liability trajectories through interest-rate space to more or less track each other. If assets and liabilities start off in a proper relationship, they should stay that way. Convexity measures how fast duration changes, much the way that acceleration measures how fast speed changes.

In Exhibit 6-19, we examine a SPDA portfolio with five homogeneous blocks of business. A cell (a block or a pool) of business is defined to be a group of SPDA policies having much the same characteristics. If you analyze your in-force portfolio, you will find several factors that distinguish one block of policies from another, for example, different policy forms, different crediting rates, different lapse-rate behavior, and so on. The idea is to model the insurance portfolio to break it into homogeneous pools. In this example there are five pools. The data in the two columns following the block number are derived from an inventory of the in-force business. There are five groups of policies at these five different credited rates on the valuation date, and their aggregate accumulation values are as shown.

EXHIBIT 6-19
PART 1

SPDA Liability Valuation Report

Block	Rate	Accumulation Value	Market Value	Duration
1	13.25%	$65.3MM	$69.7 MM	5.4 Years
2	12.75	37.1	38.3	4.3
3	12.00	59.0	59.2	4.7
4	11.50	14.6	14.4	3.9
5	10.50	92.8	89.2	3.1
Totals Averages	11.86%	$268.8MM	$270.8 MM	4.1 Years

EXHIBIT 6-19
PART 2

SPDA Asset/Liability Example

CURRENT INTEREST RATES

	Market Value	Duration
Assets	$281.7 MM	5.2 Years
Liabilities	$270.8 MM	4.1 Years
Surplus	$ 10.9 MM	32.5 Years

Given a set of assumptions as to interest-rate volatility and interest-sensitive lapse rates, the lattice methodology can be used to place a market value on each block of business. The model calculates market values for these blocks of liabilities. And by repeating the market value calculation in a changed interest-rate environment, we can calculate durations. Market values and durations are shown for each of the five blocks. Exhibit 6-19 contains the aggregate market value of the liability portfolio and its market-value-weighted duration. Once we know the durations for each of the blocks, we simply weight them by the corresponding market values to compute the overall (total averages) duration. Thus, the 4.1 years result is equal to (69.7 times 5.4) plus (38.3 times 4.3) and so on, all divided by 270.8.

Now, by itself the liability valuation report doesn't tell us very much. It may be surprising to notice that the market value of the liabilities (their real worth) is more than the accumulation value. Is the line of business in bad shape? Is there a $2 million deficit because that's the difference shown? No, not at all. We haven't even looked at the other side of the balance sheet. We might actually have to analyze the assets in modeling the liabilities, because the SPDA credited rates could be tied to the asset portfolio's earned rate. Whether or not that is the case, only the liability numbers have been displayed so far. It may happen that the asset portfolio has appreciated much more than the liabilities have. In other words, while the portfolio was mismatched, interest rates could have moved the right way, with a consequent strengthening of market value surplus.

In Exhibit 6-19, asset data is paired with the liability numbers just discussed. The example does not show any details of the asset portfolio. One puts a market value on the assets and calculates a duration index according to the methods discussed earlier. Exhibit 6-19 displays the snapshot report card. In this example, there is a positive surplus on a market value basis. It's quite small when compared in ratio form to either assets or liabilities and should not be quibbled about. Anybody who manages a SPDA portfolio and has true positive surplus ought to be fairly pleased.

It may also appear as if there is not much of a duration mismatch. After all, 4.1 years for the liabilities is pretty close to 5.2 years for the assets. But there is another number indicating the duration of surplus to be 32.5 years. How do we get 32.5 years from just a 1.1 year mismatch? What does the 32.5 year number mean anyway? Remember that we have defined duration to mean the interest-rate sensitivity of a stream of cash flows. So, duration of surplus measures how sensitive market value surplus is to changes in interest rates. What 32.5 years means is that market value surplus behaves in its price movement as if it were a little longer than a 30-year zero-coupon bond. If interest rates go up 100 basis points, market value surplus will decline by about 30 percent. That's not a lot in millions of dollars. Changes in actual dollar amounts are as important as percentage changes, and one shouldn't forget that. But duration does tell, on a percentage basis, what the sensitivity is.

How do we get to 32.5 years, given all the other numbers? Once again a valuable property of duration is that it's additive. In this case surplus equals assets minus liabilities. So, we get 32.5 by the following calculation:

$$(281.7 \times 5.2) - (270.8 \times 4.1) \div (281.7 - 270.8)$$

The duration of market value surplus is so large because there is a lot of leverage. Dollar surplus is small, and as a denominator in the calculation it magnifies the small difference in asset and liability durations. By this last example, I've shown the very few numbers needed to summarize exposure, in a SPDA portfolio, in relation to changes in interest rates. Since I developed a lot of material that is quite different from what many of you in the audience have seen before, there may be some questions.

FROM THE FLOOR: What is your view on reasonable, plausible and implausible choices for interest-rate volatility?

MR. TILLEY: That is a very good question. There are a few things you can use as a guide for making choices. One is to determine what volatility the market implies in the pricing of financial instruments, assets that are traded in the marketplace by the dealer banks. In other words, look at callable bonds, look at GNMAs, and you can determine how the market prices volatility. That's one useful guide, but as with spreads of corporate bonds to Treasury bills, situations sometimes occur in which the market acts in a strange fashion. There are no hard and fast scientific answers. There is a bit of art involved. In asset/liability planning, the collective judgment of managers can be used, if giving heavy credence to the views of investment officers about reasonable, long-term volatility. Another approach is to grade the volatility assumption down over the long run. Use a higher volatility early in the projection period and less later. Volatilities for short-term interest rates are much higher than for long-term interest rates. If you look at the longest part of the yield curve, volatilities on the order of 10 to 12 percent are reasonable choices, if interest rates are at 10 percent. A volatility of 10 to 12 percent implies a standard deviation of rate movements between 100 and 120 basis points. That may be a little low currently, but for a long-term view that may be a little high.

FROM THE FLOOR: Can taxes be incorporated into this methodology?

MR. TILLEY: Frankly, any analysis attempting to take full and proper account of taxes is extremely difficult. Items such as carry backs, carry forwards and whether you can just lift out one line of business from the whole company make taxes hard to incorporate into any model. If you make the same kind of simplifying assumptions that most people do, such as ignoring capital gains and losses, carry forwards, and carry backs, and deal only with single tax rates such as 36.8 percent, also forgetting the add-on tax for a mutual company, then you can use the same methodology to compute the indexes based on after-tax cash flows and after-tax interest rates. It can be done. However, when you start to put all the complexities of real-world taxes back into these calculations, it, like every other C-3 risk model I know of, does not do a very competent job. As was pointed out by another speaker at this symposium, it's not just a question of methodology. What tax law will we be dealing with over the long term? I never look at after-tax figures because of the uncertainty of the tax situation.

FROM THE FLOOR: How does your market value surplus definition square with Mr. Mateja's definition of cash flow surplus?

MR. TILLEY: Reasonably well. Under his definitions you get a different cash flow surplus number for every interest rate path. The reason he gets a different number for each path is essentially that his reinvestment strategy does not fully control interest-rate risk, in other words, it is not an immunizing strategy throughout the projection period. If he knew, for any particular scenario, what the fully immunizing dynamic strategy was, he would, in fact, find that all his cash flow surplus numbers discount to an unique number at the valuation date. The reason there's a dispersion to his cash flow surplus numbers has mostly to do with the choice of a static, nonimmunizing reinvestment strategy.

Notice in my lattice methodology I made no explicit reinvestment strategy assumption. In fact, though, you could determine it by looking at the solutions to the lattice. You would see that it is a dynamic reinvestment strategy throughout. Also, you would see that my market value surplus number is the amount that, if removed from (if MVS is positive) or if added to (if MVS is negative) initial funds, still ensures that exactly no assets are left over at the end of the projection period when all the remaining liabilities have been paid off. This is true path by path. You don't get positive residual amounts in some cases and negative in others, you get zero on every path. Given the interest-rate sensitivity assumptions, there is a unique market value surplus number.

FROM THE FLOOR: Do you really get the same market value surplus number for each path? Surely surplus does develop differently along each path. In some scenarios, lapses are high and the in-force declines rapidly. In others, lapses are low and there is a large amount of business in-force at the end of the projection period. How can the same MVS apply to each path?

MR. TILLEY: The answer is, yes, you do get different values for different paths. For a given set of assumptions — the interest-rate volatility represented by the lattice, the interest-rate sensitive cash flow assumptions regarding lapse rates, the interest-crediting rates — the methodology gives you a unique answer, the fair price of the lattice pattern of cash flows. This new methodology has eliminated some of the fuzziness in C-3 risk calculations. But it does not help you choose assumptions. If you make a different volatility assumption or if you make a different lapse-rate assumption, you will get different market value surplus answers, but at least you know your answer is consistent with appropriate financial theory. So using the new methodology, you can do good, clean sensitivity analysis.

FROM THE FLOOR: What is a proper immunizing condition? What is a proper relationship between duration of assets and liabilities? Is it proper to make the duration of the assets shorter than the duration of the liabilities?

MR. TILLEY: Well, it's a matter of what you are trying to immunize. If you are trying to immunize dollar surplus over some specified horizon, you can determine the immunizing condition to be that of maintaining equality between the ratio of asset to liability durations and the ratio of liabilities to assets on a market value basis. So, if market value surplus is positive, and you want to immunize dollar surplus, then assets should be an appropriate amount shorter in duration than liabilities. But, if you want to immunize the ratio on a market value basis of assets to liabilities or assets to surplus or liabilities to surplus, the proper immunizing condition, regardless of starting off with positive or negative surplus, is to maintain the duration of the assets as equal to the duration of the liabilities. That point is not well appreciated. Most people blindly assume that duration of assets equal to duration of liabilities is the proper immunizing condition. It is not necessarily. If you look back at the GIC example, to immunize that starting surplus of $2.1 million over the seven-year period, you should set the initial duration of surplus equal to seven years. That will immunize market value surplus over the seven-year period.