Chapter II

METHODS OF ANALYZING CASH FLOWS TO TEST FOR RESERVE ADEQUACY

Section 1: Introduction

In the final analysis, reserve certification is a subjective assessment. The valuation actuary, knowing everything he or she knows about the business and, especially, the sensitivities of both assets and liabilities to various interest rate conditions, certifies that the assets backing the reserves as stated in the insurer's annual statement can mature the company's contractual obligations at a very high probability level. Although the profession has not yet agreed as to whether this probability level should be 90, 95, or 99 percent, it seems clear that it is meant to be much higher than 50 percent, and that it cannot be 100 percent.

As developed in the next section, duration statistics, while useful, cannot alone provide this understanding of interest rate sensitivities that is so crucial. Instead, the valuation actuary will have to examine various interest rate scenarios and use computer models to simulate the behavior of assets and liabilities in the different scenarios. Such modeling can provide the valuation actuary with a good understanding of the conditions where losses may develop, as well as how severe the losses can be. The purpose of this chapter is to consider some basic tools for working with, understanding, and summarizing the data derived from such modeling.

Section 2: Basic Tools for Analyzing Cash Flows

Macaulay Duration:

Represent a valuable tool for summarizing the interest rate sensitivity of a stream of cash flows. Consequently, it is important for the valuation actuary to understand the meaning of duration, to be able to interpret duration statistics for assets and liabilities, and especially to understand the shortcomings of duration. In particular, it is critical to understand that it is generally insufficient to judge reserve adequacy solely using duration statistics.

Duration is the weighted average of the time to each future cash flow. The weights are the present value of the corresponding cash flows. In symbols,

$$\text{Duration (D)} = \sum_t t v^t CF_t / P, \text{ where}$$

$$P = \sum_t v^t CF_t.$$

Thus duration provides us with a sense of the average "length" of an asset or liability. The notation D or D_1 is used for this first moment of the present value function.

The second moment, or "spread" or "convexity," of the cash flows is defined as follows:

$$D_2 = \sum_t t^2 v^t CF_t / P.$$

Let the superscript A refer to the assets and liabilities, respectively. If at current interest rates, and assuming a flat yield curve, the following conditions are satisfied,

1. $P^A = P^L$,

2. $D^A = D^L$,

3. $D_2^A > D_2^L$,

then $P^A \geq P^L$ for all other interest rates (again, assuming a flat yield curve).

As described in Appendix 1, conditions 2 and 3 are related to calculus. In particular, if f(i) is defined as $P^A - P^L$, $D^A - D^L = 0$ is equivalent to f'(i) = 0, and $D_2^A - D_2^L = 0$ is equivalent to the second derivative of f(i) exceeding 0. Thus, with these conditions satisfied at i, f(i) represents the minimum point (or at least a local minima) of the present value function.

If, as is normally the case, P^A exceeds P^L, and conditions 2 and 3 are satisfied, then the ratio of surplus to liabilities—that is, $(P^A - P^L)/P^L$, is immunized. In other words, at all other interest rates, the surplus ratio is at least as great.

If one wants to immunize the dollar amount of surplus (that is, so that the dollar amount of surplus is at least as great in other interest environments), then the basic duration conditions are modified as follows:

$$2. \quad D^A = D^L \times P^L/P^A.$$

$$3. \quad D_2^A = D_2^L \times P^L/P^A.$$

A useful relationship that is used to derive these modifications of standard duration theory and is useful for computing durations for combinations of assets or liabilities is as follows. Let X and Y denote two separate assets. The duration of the combined asset, D^{X+Y}, is

$$D^{X+Y} = \left[D^X \times P^X/(P^X + P^Y)\right] + \left[D^Y \times P^Y/(P^X + P^Y)\right].$$

Appendix 1 contains several examples to illustrate these relationships.

Duration can also be used to approximate changes in present value for small changes in interest rates, using the following formula:

$$\Delta P \doteq - \frac{\Delta i}{(1+i)} \times D \times P.$$

In fact, this relationship can be used to compute duration. This is particularly useful when computing durations on a personal computer using spreadsheet packages such as LOTUS. For very small Δ i (for example, 0.000001), the following formula provides very good results:

$$D \doteq \frac{-\Delta P}{\Delta i} \times P (1 + i)$$

As noted earlier, there are a number of shortcomings of duration that make duration statistics of limited usefulness in actual practice. These shortcomings are as follows:

1. For immunization to be achieved with duration, the cash flows must be fixed, and not sensitive to interest rates. If the cash flows shift as interest rates shift, as is often the case, immunization cannot be ensured.

2. Although the present value and duration functions can be defined to reflect a given yield curve, immunization is only ensured for parallel shifts to the yield curve. In other words, if interest rates move so that the shape and slope of the interest rate yield curve are the same after as before, then immunization works. If the yield curve shifts, all bets are off. Generally, if the yield curve steepens, losses will develop.

3. Matching durations only ensures immunization (ignoring 1 and 2) at the point in time when durations are matched. However, as time passes, the duration characteristics of assets and liabilities may shift by different amounts, so that if interest rates shift at some later point, losses may occur.

4. When durations do not match at a particular interest rate, one may misinterpret the implications. For example, if D^A exceeds D^L (assets larger than liabilities), one would expect losses to develop if interest rates increased. This is generally true but is not necessarily so. (See the example in Appendix 1).

One partial solution for the third problem is to match cash flows precisely for the next n years, while also matching the duration statistics at present. Under this condition, it can be shown that immunization is still achieved (ignoring 1 and 2) for all interest rate shifts throughout the first n years. Furthermore, as new business is added, investments can be managed to preserve the cash flow matching over the next n years.

In general, Macaulay duration can be a useful tool for understanding the risk profile of assets and liabilities and for developing some feel for how asset and liability values will likely shift for different interest rate shifts. However, it is critical that the valuation actuary understand the limitations of Macaulay duration, and especially understand the fact that cash flow shifts and yield curve shifts can lead to reserve inadequacy despite the most ideal Macaulay duration relationships.

Option Theory Duration

It was noted previously that Macaulay duration cannot be counted on when cash flows themselves are sensitive to interest rates. A promising recent development that addresses this shortcoming is in response to this problem, an extension of duration using option pricing theory.[1] The basic idea is to view duration as an indicator of price sensitivity to interest rate change. Thus, we start with duration defined as:

$$\frac{\Delta P}{\Delta i} \; P \; (1+i)$$

Conceptually, we want to measure how price changes for a small change in interest rates, where the change reflects both the different discount rate and the

[1] An overview of the theory was presented by Joseph J. Buff. "Corporate Modeling and Forecasting: Practical Aspects for the Valuation Actuary." Panel discussion at the convention of the Society of Actuaries, Austin, May 1986.

assumed change in cash flows. The mechanics of the calculation involve an application of option pricing techniques to develop two "prices" of the interest sensitive asset (or liability) at a starting rate i and at i + Δi. The above formula is then used to compute duration.

Unfortunately, the calculation is fairly involved and requires some knowledge and experience with option pricing techniques. At this stage, it is probably beyond the capabilities of most valuation actuaries. This may change in the near future, if the concept proves valuable and if software packages become available.

While this measure appears to represent a substantial improvement over traditional duration, matching "option pricing" durations of assets and liabilities still does not assure immunization. Losses could develop if interest rates proved more volatile than assumed in the option pricing model and/or if the company does not continually rebalance the portfolios to maintain the duration match.

Scenario Testing

Given these shortcomings of traditional duration, and the difficulties of computing "option theory" duration, the valuation actuary will normally need to rely on modeling of assets and liabilities under various interest rate scenarios. The development of the scenarios will depend on the nature of the business. For example, if the valuation actuary finds that reserves are always adequate in decreasing scenarios, he will want to focus greater attention on increasing, and possibly up and down or down and up, scenarios. In general, enough scenarios must be examined to give the valuation actuary a thorough understanding of what type of conditions will cause reserves to be inadequate. This is obviously critical to provide the valuation actuary with a sound basis for reserve certification.

Basically, there are two ways to quantify the results of each scenario: the accumulation approach and the present value approach. Both approaches are discussed in greater detail and illustrated in Section 3.

Accumulation Approach

Under the accumulation approach, models are used to project assets and liabilities for n years, as well as to determine whether and how much assets are left at the point where the last liability benefit payment was made. If benefit payments stretch out for a very long time (for example, 30 or 40 years or longer), it is generally not necessary or desirable to run the model all the way to the last benefit payment date. In this case, one may limit the projection period to a shorter period, say, 20 years. One would then compute the asset market value and liability market value at the end of the projection period, assuming that the interest rate at that point under the scenario being considered remains level from that point forward, and still use a projection of asset and liability cash flows for the later years. Comparing asset and liability market values at the end point provides a measure of reserve adequacy.

Although these are termed economic tests, because they ignore statutory gains and losses throughout the projection period, one cannot ignore earnings computations altogether. At the very least, the model must develop year-by -year taxable income so that FIT can be computed for each year. FIT can be a critical cash flow requirement that should be considered together with the basic asset and liability cash flows.

Present Value Approach

The present value approach requires greater care in development than the accumulation approach, but it also provides more relevant data. In particular, if the discounting is done properly, one can determine for each scenario how much cash can be removed (or needs to be added) so that the remaining assets can precisely mature the liabilities (assuming experience unfolds exactly as assumed in the calculation with that scenario.) This amount of cash that can be removed has been defined as CFS, or cash flow-based surplus. Appendix 2 fully develops the theory of CFS, including specific guidance on its computation and thoughts on its use and meaning. The following is a relatively brief overview.

CFS can be determined by "brute force" by applying the accumulation model iteratively with different starting asset values, until the amount of initial assets that develops 0 ending year sufficiency/deficiency is determined.[2]

---

[2] The difference between actual initial assets and required assets is not CFS; since CFS is defined as "cash," this difference must be converted to market value.

A more refined and direct approach involves discounting the various cash flows. In discounting cash flows in nonlevel interest rate scenarios, it is critical that the discounting process recognize the interest rate scenario and the reinvestment assumptions. By definition, the present value of $1 n years from the present is that amount that, invested at present, will accumulate to $1 in n years. Obviously, this accumulation is a function of the assets actually invested in, of future reinvestment assumptions, and of the current and future interest rate assumptions. The process of developing such present values is described in Section 3 of this Chapter. However, Section 3 ignores the effects of FIT.

The treatment of FIT is an important consideration in discounting cash flows. We have found, largely through trial and error, that present values must be computed

- Using after-tax interest rates,
- Adjusting each asset and liability cash flow for its effect on taxes,
- Introducing as a cash flow the tax effects of non-cash flow items,
- For the asset cash flows, reflecting only the cash flows from the original assets (that is, ignoring cash flows from reinvestment).

This seems much more complicated than one would think necessary. In particular, one would think that present valuing asset cash flows less liability cash flows less FIT cash flows, using after-tax interest rates, would be sufficient to get the desired quantity. As developed in Appendix 2, this is not the case.

Instead, rather than reflect the FIT cash flow directly, it must be handled by tax-effecting various cash flow and non-cash flow items.[3]

Consider the example that is used in Appendix 3 involving a 4-year compound GIC with an interest guarantee of 13 percent, where new money rates remain level at 14 percent and where the initial asset is a 14 percent bond with annual coupons that mature in four years. The cash flows are as shown here:

Summary of Cash Flows
Annual Shareholder Dividend Paid

Year	Assets	Liability	FIT	Shareholder Dividend
1	$ 140	-	$ 3.68	$ 6.32
2	140	-	4.16	7.14
3	140	-	4.70	8.07
4	1,140	$1,630.47	5.31	9.12
PV-AFIT	$1,167.47	$1,161.53	$ 14.32	$ 24.59

Note that if we merely deduct the liability and FIT present values from the asset cash flow present value, we get the nonsensical answer of -$8.38. CFS, properly

3 The basic difference in the approaches is that the simple approach, in effect, reflects the FIT from investment income on assets derived from reinvestment activity. However, the present value of asset cash flows corresponding to the assets held on the valuation date should only reflect cash flows associated with those assets. Thus, another technique for properly computing CFS would be to take the present value of asset cash flows less liability and FIT cash flows, and then adjust for FIT effects from investment income on reinvested assets.

computed, equals the present value of shareholder dividends. Note that this relationship has intuitive appeal, since the cash flows that remain after all liabilities (including FIT payments) have been discharged would logically belong to shareholders.

As just discussed, one solution for computing CFS is to tax-effect the transactions that affect the tax liability. For example, the interest coupon of $140 is multiplied by the complement of the tax rate of 36.8 percent to yield a net aftertax cash flow of $88.48. On the liability side, by crediting interest at 13 percent, there is a reduction in the tax liability of 36.8 percent times the credited interest. Although this does not appear as a separate cash flow, it does offset the cash flow that would otherwise be paid on the asset side (that is, if the liability item did not exist).

The adjusted-for-tax cash flows and present values are thus as follows:

CFS Basis

Cash Flows Adjusted for Tax

Year	Assets	Liabilities
1	$ 88.48	$ (47.84)
2	88.48	(54.06)
3	88.48	(61.09)
4	1,000.00	1,630.47
PV-AFIT	$ 1,000.00	$ 975.41

Deducting $975.41 from $1,000.00 yields $24.59, which does equal the present value of shareholder dividends.

Alternatively, we can compute CFS as the present value of the actual pretax asset (corresponding to the initial asset) and liability cash flows less the excess of actual FIT cash flows over FIT on reinvested assets. This is shown here:

Year	NII from Reinvestment	X = 36.8%	AFIT=PV	
1	0.00	0.00	Assets	$ 1,167.47
2	18.20	6.70	- Liabilities	1,161.53
3	38.77	14.27	- FIT	14.32
4	62.01	22.82	+ FIT on reinvestment	32.97
PV-AFIT	89.59	32.97		$ 24.59

By whatever means one chooses to compute CFS, it should be clear that care is required in the computation. This is further illustrated in the following section.

Section 3: Discounting Cash Flows

As actuaries we have frequently used discounting and present values to the point where they have become second nature to us. Recent valuation developments have forced me to step back and examine discounting and the meaning of its results.

The valuation developments I refer to are those emanating from the New York Insurance Department in connection with GICs and annuities under the Dynamic Valuation Law. New York's version of the Dynamic Valuation Law permits the more favorable (higher) valuation interest rate if the actuary can demonstrate through a good and sufficient test (which takes into account the relationship between assets and liabilities) that the resulting reserves are adequate.

In order to demonstrate such adequacy, one approach would be to project along several possible future interest rate paths the cash flow of both the contract liabilities of the book of business in question and the assets that support the reserves being tested. The net cash flow streams for a given interest rate path could then be converted to a single value through discounting or accumulating. A positive value would demonstrate adequacy for that path; a negative value, inadequacy. (Let me duck the issue of the net book value of the business in question going negative somewhere in the path even though the net cash flow single value is positive.)

Under a level interest rate path assumption, traditional discounting and accumulating at the assumed, single interest rate are valid and produce meaningful results. But what about nonlevel interest rate paths? One has to question the meaning of the results in these cases if a single interest rate is used for discounting or accumulating. (More on this later.) One can also question the meaning of the results of an approach whereby the accumulation or discount factors are derived by simply "stringing together" the applicable new money interest rates of the interest rate path in question. (More on this, too, later.)

My thinking on discounting and accumulating under a nonlevel future interest rate assumption has led me to conclude that what is theoretically called for is a type of investment-year method that takes into account an assumed reinvestment strategy. (I am using the word reinvestment to apply to cash flow from "insurance operations" as well as "investment operations.") The assumed reinvestment strategy needs to be defined for each future year of a given interest rate path and represents the strategy that would actually be employed to handle the cash flow that would emerge each year, if in fact that interest rate path were to materialize. The reinvestment strategy applicable to any given year's cash flow could be a function of what has transpired to date, but, of course, we cannot peek ahead to get a glimpse of the future interest rates the path has in store. Thus, two interest rate paths that run identically through year m and diverge thereafter would have to have the same assumed reinvestment strategies through year m.

An assumed disinvestment strategy (for example, selling certain assets, borrowing short term) is also needed to deal with the situation of negative cash flow. For the remainder of this discussion, the disinvestment strategy is considered merely a part of the overall reinvestment strategy.

An example best illustrates how the investment year approach might operate in the accumulation or discounting process. Assume a simple book of business consisting of (1) a $1,000 deposit at time 0 that is guaranteed to be repaid with interest at the rate of 9 percent per annum at the end of 3 years and (2) a $1,000 bond, bought at par at time 0, paying annual coupons of $90 and maturing at the end of year 4.

Assume we are holding a reserve of $1,090 (equal to the accrued contract liability and supported by the $1,000 bond and first-year coupon) at the end of year 1, when interest rates are 10 percent, and that we want to perform with respect to that reserve a good and sufficient test that takes both asset and liability cash flows into account. Further assume that one future interest rate path to be tested has a 12 percent prevailing rate at the end of year 2, 14 percent at the end of year 3, and 16 percent at the end of year 4.

The assumed reinvestment/disinvestment strategy for this interest rate path is as follows. Each year any net positive cash flow is to be reinvested in annual coupon bonds at the prevailing rate to mature at the end of year 4 (assume a flat yield curve). Borrowing on mirror-image terms will be assumed to cover any negative cash flow, except that any investments existing at the end of year 4 will be sold at market value to minimize any borrowing at that point. The projected cash flow from assets existing as of the end of year 1 (before reinvestment of the first-year $90 coupon) and from contract liabilities and their net sum is as follows:

	Year-end Cash Flow			
	Year			
	1	2	3	4
Assets	$90	$90	$ 90.00	$ 1,090.00
Liabilities	0	0	(1,295.03)	-
Net (before reinvestment)	$90	$90	$ (1,205.03)	$ 1,090.00

The question now is how to discount or accumulate these net cash flows. Three possible approaches that use the investment year technique are (1) progressive accumulation, (2) accumulation factors, and (3) discount factors.

Progressive Accumulation Approach

The progressive accumulation approach is the most natural approach and presents the easiest way to understand the dynamics involved. The mechanics of this approach are illustrated here and are as follows. First, invest the (as yet uninvested) $90 net cash flow at the end of year 1 according to the assumed reinvestment strategy for year end 1. Next, modify the subsequent years' net cash flows to reflect the reinvestment. Repeat the process at the end of year 2 and subsequently at the end of year 3:

Year-end Cash Flow

	Year 1	Year 2	Year 3	Year 4
Assets	$90	$90	$ 90.00	$ 1,090.00
Liabilities	0	0	(1,295.03)	—
Net (before reinvestment)	$90	$90	$ (1,205.03)	$ 1,090.00
Year 1 reinvestment at 10%	→	9	9.00	99.00
		$99	$ (1,196.03)	$ 1,189.00
Year 2 reinvestment at 12%		→	11.88	110.88
			$ (1,184.15)	$ 1,299.88
Year 3 borrowing at 14%			→	(1,349.93)
				$ (50.05)

The end result shows that reserves of $1,090 as of year end 1, supported by the original asset and its first coupon, were deficient for this particular interest rate path by $50.05 (valued at the end of year 4).

Before moving to the second approach, it may be illuminating to see the earnings and balance sheet figures this example produces. They are as follows:

Year	1	2	3	4
Earnings				
Investment income	$ 90	$ 99.00	$ 110.88	$ 110.88
Interest credited	90	98.10	106.93	-
Debt interest paid	-	-	-	165.78
Net	$ 0	$ .90	$ 3.95	$ (54.90)
Balance Sheet				
Assets	$ 1,090	$ 1,189.00	$1,189.00	$ 0.00
Liabilities:				
Contract balance	1,090	1,188.10	0.00	0.00
Debt	-	-	1,184.15	(50.05)
	$ 0	$ .90	$ 4.85	$ (50.05)

Accumulation Factor Approach

Under the accumulation factor approach, accumulation factors are derived and applied directly to the projected net cash flow before reinvestment. The required accumulation factors are derived using the progressive accumulation approach to accumulate $1, separately from each year end to the end of year 4, taking into account the assumed reinvestment strategy. The following are the factors so derived and the accumulated value they produce when applied to the example:

Year End	Accumulation Factor		Net Cash Flow before Reinvestment		Accumulated Value
1	1.3397	x	$ 90	=	$ 120.57
2	1.2568	x	90	=	113.11
3	1.1400	x	(1,205.03)	=	(1,373.73)
4	1.0000	x	1,090	=	1,090.00
					$ (50.05)

Note that (1) not surprisingly, considering the conceptual equivalency with the progressive accumulation approach, this method produces the same $(50.05) accumulated value, and (2) use of the following accumulation factors would be inappropriate (that is, produce results of questionable meaning):

Year End	Accumulation Factor				Net Cash Flow before Reinvestment		Accumulated Value
1	(1.10)(1.12)(1.14)	or	1.4045	x	$ 90	=	$ 126.41
2	(1.12)(1.14)	or	1.2768	x	90	=	114.91
3	(1.14)	or	1.14	x	(1,205.03)	=	(1,373.73)
4	(1.00)	or	1.00	x	1,090	=	1,090.00
							$ (42.42)

Discount Factor Approach

Development of a discounted present value that has meaning in relation to the purpose of the testing requires some careful deliberation. The definition of present value that is needed must involve future accumulation values. The essence of the definition is that two things will be deemed equivalent today if they ultimately end up equivalent in the future. More specifically, the definition is as follows:

> The cash-equivalent present value of specified cash flow emerging in some future year is the amount of current cash needed to produce ultimately, over the interest rate path being tested, the same accumulated value that the specified cash flow in question would ultimately produce, provided that both the current cash and the specified cash flow are reinvested in accordance with the assumed reinvestment strategy for that interest rate path.

The cash-equivalent present value of an investment is simply the sum of the cash-equivalent present values of the cash flow expected to be generated by the investment over the given interest rate path.

Note that this definition of present value is a function of both future interest rates and the assumed reinvestment strategy. Note also that future interest rates can additionally affect the present value by affecting the expected cash flow of the investment (and also of the liability, for that matter).

By this definition, the discount factors can be derived as follows. (The "ultimate" value of $1 of cash flow emerging at each year end, including that at the end of year 1, was previously calculated.)

Year End	Ultimate Value of $1 Cash Flow Invested at Year End		$1 Current[a] Cash Ultimate Value of		Discount Factor		Net Cash Flow before Reinvestment		Present Values
1	$1.3397	/	$1.3397	=	1.0000	x	$ 90	=	$ 90.00
2	1.2568	/	1.3397	=	0.9381	x	90	=	84.43
3	1.1400	/	1.3397	=	0.8509	x	(1,205.03)	=	(1,025.36)
4	1.0000	/	1.3397	=	0.7464	x	1,090	=	813.58
									$ (37.35)

[a]At the end of year 1.

Note that (1) as appropriate, $(37.35) x 1.3397 = $(50.05), the accumulated value previously calculated, and (2) the following discount factors would be inappropriate:

Year End	Discount Factor				Net Cash Flow before Reinvestment		Present Value
1	1.00	or	1.0000	x	$ 90	=	$ 90.00
2	1/(1.10)	or	0.9091	x	90	=	81.82
3	1/(1.10)(1.12)	or	0.8117	x	(1,205.03)	=	(978.12)
4	1/(1.10)(1.12)(1.14)	or	0.7120	x	1,090	=	776.08
							$ (30.22)

Valuation Considerations

Given that the $1,090 reserve is inadequate in this example, how much additional reserve would be needed to be held in order to make the total reserve adequate for the sample interest rate path? The answer depends upon the specific (additional) assets that would be supporting the additional reserve. If cash is available and, in accordance with the year end 1 reinvestment strategy, a 3-year bond is purchased, then the amount of the additional reserve would be $37.35, supported by a 3-year bond with a book value (also a market value) of a like amount.

If more bonds identical to the existing $1,000 bond are available to support this book of business (for example, from investments implicitly supporting either surplus or redundant reserves of another book of business), then the additional reserve would be $37.35/0.97459, or $38.32. ($974.59 is the end of year 1 cash-equivalent present value of the existing bond, exclusive of the first-year coupon. Thus, $974.59 = (0.9381)($90) + (0.8509)($90) + (0.7464)($1,090). The amount of additional assets of this type would have a book (and par) value of

$38.32. Note that the market value of this additional asset would exceed $37.35 if such market value were determined by simply discounting the bond's cash flow at 10 percent.

Using the progressive accumulation approach, one can show the augmented reserve as of year end 1 to be just adequate for the sample interest rate path:

Year-end Cash Flow

	Year 1	Year 2	Year 3	Year 4
Original asset	$90	$ 90.00	$ 90.00	$ 1,090.00
Additional asset: $38.32	-	3.45	3.45	41.77
Liabilities: $1,000	0	0.00	1,295.03	-
Net (before reinvestment)	$90	$ 93.45	$ (1,201.58)	$ 1,131.77
Year 1 reinvestment at 10%	→	9.00	9.00	99.00
		$ 102.45	$ (1,192.58)	$ 1,230.77
Year 2 reinvestment at 12%		→	12.29	114.74
			$ (1,180.29)	$ 1,345.51
Year 3 borrowing at 14%			→	(1,345.53)
				(.02)
				(Rounding error)

As one more example, assume that 8 percent annual bonds previously purchased at par and maturing at the end of year 5 are available to support the additional reserve. If that type of asset is "assigned" to support the additional reserve, then pursuant to the reinvestment strategy, it will be sold at the end of year 4. Its market value at that time, taking the present value of its cash flow at 16 percent to be a close proxy, would be $931.03 per $1,000 par value. The bond's cash-equivalent present value would be $80 (0.9381 + 0.8509) + ($80 + $931.03)(0.7464) = $897.75. The amount of additional reserve and the amount of this bond necessary to support it would be $37.35 / 0.89775, or $41.60.

Checking this result through progressive accumulation yields the following:

Year-end Cash Flow

	Year 1	Year 2	Year 3	Year 4	Year 5
Original asset	$ 90	$ 90.33	$ 90.00	$ 1,090.00	$ -
Additional asset: $41.60	-	3.33	3.33	3.33	44.93
Liabilities	0	0.00	(1,295.03)	-	-
Net (before reinvestment)	$ 90	$ 93.33	$ (1,201.70)	$ 1,093.33	$44.93
Year 1 reinvestment at 10%		9.00	9.00	99.00	-
		$ 102.33	$ (1,192.70)	$ 1,192.33	$44.93
Year 2 reinvestment at 12%			12.28	114.61	-
			$ (1,180.42)	$ 1,306.94	$44.93
Year 3 borrowing at 14%				(1,345.68)	-
				$ (38.74)	$44.93
Year 4 asset sale at 16%				38.73	
				$ (.01)	
				(Rounding error)	

Comments

How does traditional discounting with a single level interest rate relate to the investment year approach previously defined and illustrated? There are at least three interpretations or uses of such traditional discounting, which are briefly discussed here. They all have traits that make them generally unsuitable for the reserve testing under consideration.

Internal Rate of Return (IRR). If the market value of an investment is derived by straight discounting of the cash flow, then under this interpretation, the discount rate represents the internal rate of return of the investment. The critical word here is internal. Interest payments and principal repayments, after being made and discounted, are then considered to be outside the calculation. That is to say, the IRR interpretation is blind to whatever actual rates will prevail at the times the cash flow emerges and blind to what is done with the cash flow thereafter (that is, how it is reinvested). Under this interpretation, then, the traditional, single rate discount approach is generally inappropriate for our reserve testing purposes, since it ignores subsequent economic effects of emerging cash flow. (If you interpret IRR as implicitly assuming level future interest rates equal to the IRR, see "Level Future Interest Rates" later.

Return on Investment (ROI). The ROI interpretation of traditional discounting has been applied in the insurance world to evaluate the pricing of a product that initially creates a statutory surplus strain and subsequently produces a stream of statutory earnings.

The ROI is the discount rate that, when applied to the stream of earnings, produces a present value equal to the initial strain. The ROI is similar to an IRR in that it ignores subsequent reinvestment of the earnings and the longer-term economic effects of such reinvestments. The ROI differs fundamentally from the IRR as previously discussed in that it deals with surplus changes (strain and earnings) that are not generally synonymous with cash flow.

Thus, the ROI relates to the investor's (insurance company's) expectation of return on its venture (insurance product) and not to the yield on the assets involved in the product. As a result, this interpretation or use of traditional discounting is inappropriate for our reserve testing purposes.

Level Future Interest Rates. The key element of the third interpretation of single rate discounting is the implicit or explicit assumption of level interest rates throughout the entire future and a reinvestment strategy that requires only that emerging cash flow be reinvested at that rate. (The length of the reinvestment is immaterial.) The present value of the cash flow of an investment under this interpretation is identical to the cash-equivalent present value for a level interest rate path. Therefore, this interpretation of single rate discounting is applicable to our reserve testing, but only for that level interest rate path.

In testing reserves by the method described, the question arises as to how far the projection period should be taken. Although beyond the scope of this discussion, the question does raise a relevant point. Any assets or liabilities still remaining at the end of the projection period must be assigned an economic value, which would generally be different from book value. A market value approximated by the present value of the remaining cash flow, discounted by the assumed prevailing rate at the end point of the projection period, might be used. In so doing, it should be recognized that this is equivalent to extending the original projection period indefinitely with the assumption that (1) the "end-of-period" interest rate continues indefinitely and (2) subsequent reinvestment occurs at that rate.

Discounting, for example, end of year 4 cash flow by a factor of 1/(1.10)(1.12)(1.14) would generally produce a present value result of questionable meaning for adequacy testing purposes. It would have clear meaning if the reinvestment strategy always called for one-year investments (and borrowings) and the path's stated interest rates were one-year rates.

The example used some simplified assumptions regarding the assumed reinvestment strategy; that is, borrowing terms were the mirror image of reinvesting terms, and the yield curve was flat. If one or both of these assumptions did not hold, complications would arise in determining the amount of additional reserve needed if the reserve tested were shown to be deficient. In short, under those circumstances, if $1 of cash with an accumulation value of a is added to an asset/liability portfolio having a net cash-equivalent present value of P and an accumulation value of A, then the augmented portfolio may well have a present value and an accumulation value different from P + 1 and A + a, respectively. However, the progressive accumulation approach applied to the augmented portfolio avoids such complication.

Section 4: Testing Interest-Sensitive Cash Flows

4.1 Introduction

Valuation actuaries are concerned about the exposure of company surplus to the risk of interest rate volatility, called the C-3 risk. This risk is ever present, and it can be more difficult to quantify for assets and liabilities whose cash flows are interest-sensitive. This section will discuss an approach to measuring and managing that risk exposure based on market value accounting and option pricing. The section will also show that Macaulay duration may give misleading results for C-3 risk control under certain circumstances. The discussion will provide some self-contained theoretical development, but on occasion it will make assertions that are supported by the reading list given on page II-30 or by modeling work performed by the author.

Briefly, C-3 risk can be quantified by analyzing asset and liability market values. These market values can be studied even when the assets and liabilities are interest-sensitive, by viewing them as securities involving options and applying option pricing theory. C-3 risk exists because the asset and liability market values change when interest rates change. The relative sensitivity of market value to interest rate levels is measured by "duration." Duration calculations provide a quantitative basis for adjusting in-force assets and liabilities to immunize profitability against interest rate volatility or to anticipate particular interest rate movements.

4.2 Market Value Accounting

Statutory surplus is artificially stabilized by some aspects of book value accounting. Since surplus equals assets minus liabilities, risk assessment can focus on how asset and liability values may change. Since assets and liabilities both derive their values from the cash flows they generate, it is helpful to concentrate on these cash flows to measure risk. Estimating these cash flows and then calculating their present values leads to what has been called "market value accounting."

Here are some reasons to consider adding market value accounting to a company's management information system:

o Market value accounting gives information about risk exposure that is not provided by traditional statutory accounting.

- Market value accounting presents the real economic "worth" of the insurance operation and shows how that worth can and does change when the external environment changes. This information is valuable on a going-concern basis, as well as on a sale or liquidation basis, since an insurance company's vitality cannot be staked indefinitely on the benefits of future new business inflow.

- Market value accounting is a leading indicator for book accounting. That is, loss positions identified by market value accounting could eventually emerge on the statutory books. Present value calculations identify the magnitude of risk exposure or of economic loss.

- Market value accounting facilitates calculations that both measure risk exposure and suggest ways to control and reduce it. These calculations for measuring risk and reducing risk exposure will be elaborated on in this section.

Reinvestment risk and disintermediation risk are two sides to the problem of interest rate volatility. These risks manifest themselves in the day-to-day and year-by-year operations of an insurance company by making some of the critical business cash flows unpredictable. If asset and liability cash flows could remain closely matched in all interest rate environments, then C-3 risk would be minimized. But many assets, such as callable bonds or mortgage-backed securities, as well as many liabilities, such as universal life or single premium deferred annuities (SPDAs), have cash flows whose timing and magnitude are sensitive to future interest rates. When an insurer has interest-sensitive assets and/or liabilities on its books, cash flow matching becomes difficult. However, by studying present values of the cash flows, an approach to managing C-3 risk can be developed.

4.3 Duration

C-3 risk control is effective if profit is relatively insensitive to interest rate volatility. Valuation actuaries might want to review ways to "immunize" (stabilize) surplus against interest rate changes. There are at least two "target accounts" that a company might want to stabilize:

1. Dollar amount of surplus, on a market value basis.
2. Ratio of surplus to liabilities, on a market value basis.

In general, these two target accounts cannot be immunized simultaneously. I take these target accounts on a market value basis because, as I will show, market value accounting suggests effective hedging strategies, and it is the market values that respond most directly to this hedging. Some other target profit accounts can also be hedged using calculations beyond the scope of this discussion. ("Hedging" is an investment term synonymous in this context with immunizing or stabilizing).

Remember that the Macaulay duration of a bond is the weighted average of the time to each of the bond's cash flows, where the weights are the prices (present values) of the individual cash flows, each divided by the total price of the bond. Macaulay developed this measurement during the 1930s in order to approximate the change (ΔP) in the market value (P) of a bond, caused by a change in the yield to maturity of the bond (ΔI percent). (All of the equations given here will assume that ΔI is a change to continuously compounded interest rates, that is, to forces of interest.) So long as the cash flows of the bond in question are fixed and certain as to timing and amount, Macaulay duration (MD) satisfies this approximation:

(1) $$\Delta P = -MD \times P \times .01 \times \Delta I.$$

Thus, Macaulay duration gives a first-order approximation for the relative sensitivity of the price of a bond to interest rate changes.

Macaulay duration is not readily applicable to an asset whose cash flows depend on future interest rates. This is because it is unclear how the weighted average time-to-payment might be evaluated.

Notice that equation 1 can be rewritten as follows:

(2) $$MD = -(1/P) \times \Delta P/(.01 \times \Delta I)$$

so that Macaulay duration itself can be estimated by "working backwards" from information on how price changes when interest rates change. Some financial theorists in general define the "duration" (D) of a security whose price (P) is given as a function of interest rates (I) as follows:

(3A) $$D = -(1/P) \times dP/dI.$$

Here dP/dI is the derivative of the price function (P), based on a one-parameter model for the volatility of the continuously compounded interest rate yield curve. If I is given in terms of interest rates compounded M-thly, use this form of the equation instead:

(3B) $$D = (1 + I/M) \times -(1/P) \times dP/dI.$$

Note that this definition of duration does not refer to cash flow timing. The problem of computing duration according to this definition reduces to the problem of being able to "price" an interest-sensitive asset or liability.

In fact, the pricing of interest-sensitive cash flows—both assets and liabilities—has been addressed recently by financial theorists. Concepts from modern option pricing theory are directly applicable. Financial options are perhaps the prototypical assets with interest-sensitive cash flows. Research into option pricing theory has been active in recent years, and the results have been adapted to the context of insurance company assets and liabilities.

Once the necessary pricing has been accomplished, duration can be estimated using a numerical approximation ($\Delta P/.01 \times \Delta I$) to the derivative of the price function (dP/dI). This is done because pricing formulas for options on bonds often cannot be written out in closed form suitable for differentiation. (In practice, the term ΔI will be the percent value of the instantaneous shock to the yield curve, which is fed into the one-parameter volatility model being used.)

It can be shown that if the cash flows do not depend on future interest rates, then the price-sensitivity duration from equation 3A is identical to Macaulay duration.

Here is a brief reading list for valuation actuaries who want to learn more about pricing and duration for interest-sensitive cash flows, as well as about different models of interest rate volatility:

J. Buff, "Corporate Modeling and Forecasting: Practical Aspects for the Valuation Actuary." Panel discussion at the Society of Actuaries meeting, Boston, May 1986. RSA 12, no. 2 (also available from the author at his Yearbook address).

R. Clancy. "Options on Bonds and Applications to Product Pricing." TSA XXXVII, and Discussion by J. Tilley, P. Noris, J. Buff, and G. Lord.

D. Jacob, G. Lord, and J. Tilley. Pricing, Duration, and Convexity of a Stream of Interest-Sensitive Cash Flows. Morgan Stanley, 1986.

P. Milgrom. "Measuring the Interest Rate Risk." TSA XXXVII, and Discussion by J. Buff and G. Lord.

An interesting feature of modern option pricing theory, as discussed in these references, is that the option price does not depend on any particular interest rate forecast. Rather, the option price depends on the interest rate environment on the valuation date, as well as on an assumption about the degree of volatility of future interest rates. Consequently, price-sensitivity duration is not dependent on any particular interest rate scenario. This general result follows from arbitrage pricing theory, which is beyond the scope of this discussion. Further information can be found in the works by Clancy and by

Jacob, Lord, and Tilley. (Note that not all approaches to option pricing theory that have been published in the theoretical literature are based on the same precepts as the theories discussed in these two references.)

4.4 Usefuleness of Duration

The previous discussion showed how Macaulay duration can be extended into a general definition of "duration," which measures the sensitivity of an asset's or a liability's market value to changes in interest rates. Because this definition does not refer directly to cash flow timing, it can be applied to interest-sensitive cash flows. This section will show how calculating durations can provide a practical way to measure and manage C-3 risk.

Duration satisfies an "aggregation property." Suppose we have securities A and B with prices PA and PB and durations DA and DB, respectively. Let A + B be the security whose cash flows are the arithmetic sum of the cash flows of A and of B. Then the price of A + B is PA + PB, and its duration is given by

(4) $$D(A + B) = DA \times PA/(PA + PB) + DB \times PB / (PA + PB).$$

In other words, the duration of the sum is the weighted average of the durations of the addends, where the weights are the prices divided by the total price. Using this aggregation property, it is possible to determine the duration for an entire portfolio of different assets or liabilities once the prices and durations for each separate valuation cell are available. In fact, durations of separate product lines can be combined this way to obtain the duration of a financial variable on a companywide basis.

Financial ratios have durations. For instance, let A equal assets, L equal liabilities, and S equal surplus, all on a market value basis. Then some differential calculus will show that

(5A) $$D(A/L) = D(A) - D(L).$$

Using the aggregation property, one finds that

(5B) $$D(S) = (A/S) \times D(A) - (L/S) \times D(L).$$

Now we are ready to write down conditions on duration that approximately immunize the two target accounts mentioned above: the dollar amount of surplus S or the surplus ratio S/L. These conditions give relationships between the assets and the liabilities that must be satisfied for the given target account to be stabilized against future interest rate changes:

Target Account	Immunizing Condition
Dollar amount of surplus (S)	$D(A) = D(L) \times L/A$
Surplus ratio (S/L)	$D(A) = D(L)$

These conditions do immunize the target accounts, since by equations 5A and 5B they ensure that the duration of the target account is 0. By equation 3A, if the duration is 0, then dP/dI is 0, so at least approximately the target account will not change when interest rates change by relatively modest amounts.

Duration studies show management how to plan out attacks against its exposure to interest rate volatility. By structuring assets and liabilities properly, so that the appropriate immunizing condition is satisfied, the chosen target account can be rendered insensitive to interest rate volatility, at least approximately. If management prefers to bank on an interest rate forecast, leaving the target account unhedged in hopes of increasing profit should that forecast prove accurate, then duration calculations are helpful for properly arranging such an "interest rate bet."

These pricing and duration calculations can be used to establish a link between investment strategies and product design and pricing. This link, in turn, permits the optimizing of business policies and ensures more effective surplus management in the face of C-3 risk. An advantage of duration analysis is that, although the calculations are relatively technical, the output is concise and easily summarized.

As an example, let us discuss the surplus ratio S/L a bit more. This profit measure is of particular interest because it quantifies a profit margin that responds to changes in the magnitude of the liabilities. If management wanted this ratio, for some block of business, to be stable over time even though interest rates might change, then the company should arrange for the duration of assets to equal the duration of liabilities. This can be done in several ways:

- Make changes to the investments held by the block of business in order to alter the duration characteristics of the asset portfolio.

- Alter the product design, underwriting, or administration of the insurance policies in order to alter the duration characteristics of the liabilities.

In general, companies tend to make adjustments to the assets, since the liabilities are often a "given." Here are some ways in which asset durations can be adjusted:

- Use interest income, principal repayments, and cash flow from any new business to purchase longer or shorter assets.

- Exchange existing long duration assets for short duration ones, or vice versa.

- Use interest rate options or futures to alter the aggregate duration of assets, when regulations permit.

- Use interest rate swaps to shorten or lengthen the duration of the portfolio without having to sell off any existing assets.

Note that duration is itself a function of interest rates, and it also changes as time passes. Thus, a regimen for controlling C-3 risk by duration management works best when the asset and liability durations are periodically recomputed (say, quarterly) and adjustments made when needed to maintain the proper balance.

4.5 Effectiveness of Duration Matching

Now we have developed the theory behind an approach to managing interest rate risk exposure. The theory is complete and consistent. It would be interesting to test the effectiveness of the approach when exposed to the rigors of interest rate volatility. This was, in fact, done using traditional actuarial simulations modeling and some representative assumptions. A few typical SPDA products were analyzed, because duration analysis (as opposed to traditional simulations analysis) is more readily applied to single premium products. The products examined did not have market value adjustments, so the liabilities were truly interest-sensitive.

- The duration of the SPDA liability was computed using the Jacob/Lord/Tilley methodology (see earlier suggested readings) on the issue date of the product.

- Simulations of the SPDA were run across some sets of interest rate scenarios. The independent variable was the investment strategy. The effectiveness of different investment strategies for controlling C-3 risk was compared using the variance of the profitability outcomes across the scenarios, produced by each of the investment strategies. The lower the variance, the less investment risk for that investment strategy.

- The investment strategies studied were simple ones; in fact, each strategy was determined by specifying a bond term-to-maturity. In each run, bonds were never sold before maturity. All investing (or borrowing, when cash flow was negative) was done using bonds with the given term-to-maturity.

- The simulations showed that C-3 risk was minimized when the initial single premium deposit was invested in a bond whose duration matched that of the initial product liability. Bonds with durations shorter or longer than the liability's duration led to increased C-3 risk.

- Since the investment strategies were simple, and durations of assets and liabilities were not dynamically rematched over the life of the product, as they could be in practice, two variations of the testing were also performed. In the first variation, all borrowing or lending after issue of the liability was made in 3-month bonds. In the second variation, all borrowing or lending after issue was made in 10-year bonds. In these variations, the differences between the investment strategies were confined to the term of the bond bought with the initial deposit. The two variations capture extremes, making this overall testing process more robust.

- In both variations, it was also found that C-3 risk was minimized when the initial deposit was used to buy a bond whose duration matched the duration of the product liability.

This testing procedure validates the effectiveness of duration matching for hedging interest rate risk. It does so using traditional actuarial simulations, which are independent of the concept of duration and the methodology of the work by Jacob, Lord, and Tilley.

In practice, an insurer that makes use of duration analysis ought to test the sensitivity of the results to the input assumptions. Particularly important assumptions include interest rate volatility, policyholder lapsation, and the method by which the insurer will credit interest to policyholders over the lifetime of the in-force.

4.6 Pitfalls of Macaulay Duration

The discussion so far has shown that with a given set of assumptions, it is possible to price an interest-sensitive asset or liability and then find its duration. Very briefly, the methodology of Jacob, Lord, and Tilley proceeds as follows to determine an insurance product liability duration:

- A connected binomial lattice of interest rate yield curves is generated using today's interest rate environment as a starting point and applying an assumption about interest rate volatility.

- A set of paths through the lattice is chosen using variance-reducing statistical sampling techniques. These paths are really specially structured interest rate scenarios. The sampling is necessary to keep the volume of calculations within reasonable bounds, since the insurance cash flow projections will be dependent on each path.

- Using assumptions about the insurance product that are essentially the same ones needed for a traditional simulations study, periodic cash flows for the product are projected along each interest rate path.

- Using arbitrage pricing theory, the cash flow data are used to determine the price of the liability.

Since these calculations do produce a series of periodic cash flows for each set of interest rate scenarios, the data can be used to compute Macaulay durations for each scenario. One might naturally think that the Macaulay durations of these cash flows provide useful information about interest rate risk exposure. Can the duration of the liability be approximated by an average of the set of Macaulay durations?

The inaccuracy of Macaulay duration when cash flows depend on future interest rates was pointed out by Donald D. Cody, who showed algebraically that Macaulay duration does not immunize surplus when cash flows are interest-sensitive.[4]

It is important to stress that when cash flows are not a function of future interest rates, Macaulay duration is a very useful tool; in that case, it is identical to the price-sensitivity duration defined by equation 3A. Examples of insurance liabilities for which Macaulay duration is accurate and effective include some immediate annuities, structured settlements, GICs, and pension buyout products. Macaulay duration is also useful for assets that have no put or call (prepayment) features attached.

As part of the testing of the duration analysis concept described previously, the data were analyzed to compare some price-sensitivity durations to a set of Macaulay durations per scenario based on the same assumptions.

---

4 Donald D. Cody. "Contingency Surplus Needed for Risk of Change in Interest Rate Environment." RSA 7, no. 4 (1981).

These comparisons would put a numerical value, in the context of actual insurance business operations, on the inaccuracy of Macaulay duration. It was found that the simple average Macaulay duration exceeded the price-sensivity duration by several years for all of the SPDA cases tested. In almost every case tested, every one of the per-scenario Macaulay durations exceeded the SPDA price-sensitivity duration. The simulations testing previously described showed that if Macaulay durations had been used to develop investment strategies for these SPDAs, then C-3 risk would not have been effectively controlled.

The systematic error of Macaulay duration for SPDAs without market value adjustments results from the guaranteed cash value withdrawal privilege of this type of product. This can be exposed using the concept of a put option. A put option is the right to sell a specified bond for a price that is fixed in advance, regardless of the market value of the bond at the time the option is exercised. Thus, put options become more valuable when interest rates rise, since the less the bond costs, the more the put option is worth. Thus, a put option has a negative price-sensitivity duration: price rises when interest rates rise. But it is easy to see that any simulation model using a reasonable rule about timing and magnitude of the put option's payoff will give a set of cash flows that leads to a nonnegative Macaulay duration in every possible interest rate scenario. Thus, the Macaulay durations will always be nonnegative, although the correct price-sensitivity duration is negative. Any kind of weighted average of the Macaulay durations will not come close to the correct duration.

The withdrawal privilege of an SPDA liability (without a market value adjustment) behaves like a put option, because it gains in value when interest rates rise. Thus, in isolation, the withdrawal privilege has negative duration.

However, a simulations model of an SPDA product will produce per-scenario Macaulay durations that do not reflect this negative duration characteristic. By the aggregation property, the negative duration of the withdrawal privilege shortens the duration of the total SPDA liability. This is why the price sensitivity of an SPDA liability tends to be substantially shorter than indicated by Macaulay durations.

For representative callable bonds, testing has found that average Macaulay durations can differ from the price-sensitivity duration by more than 1 year. In general, the simple average Macaulay duration was found to be less than the price-sensitivity duration for the typical callable bonds the author analyzed.

Macaulay duration does not address the question of price-sensitivity, which is the key to interest rate risk control for products that cannot be cash flow matched. Matching average Macaulay durations might seem to match cash flows "on average" for interest-sensitive cash flows, but this does not eliminate C-3 risk. That is because, when the magnitudes of the cash flows vary with interest rates, a "timing match" does not ensure that asset flows will be adequate to cover liability flows. Matching price-sensitivity durations ensures that asset market values will be adequate to cover liability market values. Under the latter approach, assets can be liquidated as needed to cover whatever liability cash flows do occur, without impairing market value surplus.

4.7 Sample Applications of Duration

This discussion will present simple examples of how price-sensitivity duration can be used to estimate interest rate risk exposure. Suppose the insurer has a block of SPDAs backed by callable bonds. Let the ratio of assets to liabilities on a market value basis be 1.1 (that is, surplus is 10 percent of liabilities). We will assume the insurer wants to immunize the ratio of surplus to liabilities against C-3 risk. Suppose a reasonable set of assumptions produced a liability duration of 3 years. If the assets that backed this liability were also to have a duration of 3 years, then roughly speaking, the surplus ratio would remain at 10 percent if interest rates were to rise or fall. This is because, from equation 5, the duration of the financial ratio, assets divided by liabilities would be 0. Then the ratio of assets to liabilities would be approximately constant for modest interest rate changes, so the ratio of surplus to liabilities would also be stabilized.

Let us consider another example that illustrates the possible danger of using average Macaulay duration to estimate price-sensitivity duration. Suppose the SPDA liability again has a duration of 3 years, but the average Macaulay duration is 4 years. Suppose the assets are callable bonds with a duration of 5 years, but an average Macaulay duration of 4 years. (These errors are similar in magnitude and direction to those the author found in some test cases.)

Since the average Macaulay durations of the assets and liabilities are equal, the insurer might believe that the surplus ratio has been immunized. However, the duration of assets divided by liabilities is actually 2 years, not 0 years. Suppose interest rates rise by 1 percent.

We can now apply a general formula that estimates how a variable changes in value from P0 to P1 when it has a duration of D years, if interest rates rise by ΔI percent,(6)P1 = P0 x (1 - 0.01 x D x I).

This formula is very similar to equation 1. It follows from equation 3A by noting that (P1 - P0) /.01 xΔI is a numerical approximation to dP/dI.

The ratio of assets to liabilities has a duration of 2 years and starts out equaling 1.1; we have assumed interest rates rise 1 percent. Then, using equation 6, the ratio of assets to liabilities would become roughly 1.1 x (1 - 0.01 x 2 x 1), which is 1.078. Then the ratio of surplus to liabilities has dropped from 10 to 7.8 percent, which is a 22 percent reduction. Management would not have expected such a loss if it concluded from the match of average Macaulay durations that its surplus ratio was immunized.

Whenever a financial variable has a nonzero duration, then interest rate movements can cause gains as well as losses, of course, depending on whether the duration is positive or negative and on whether interest rates rise or fall. Using duration calculations and applying equation 6, management can estimate the effects of financial performance of different interest rate changes. By finding the effect on asset portfolio duration of a proposed investment strategy, the impact of that strategy on C-3 risk exposure can be reviewed. By finding the effect on liability portfolio duration of a proposed product design or administrative policy, again the impact on C-3 risk exposure can be reviewed.

Because duration itself is a function of interest rates (it changes as interest rates change), equation 6 becomes less accurate as ΔI becomes larger. Equation 6 can be extended to a better estimator of price change using the concept of "convexity," which is derived from the second derivative of the price function. Convexity is beyond the scope of this discussion. Interested readers can refer to the Milgrom paper and its discussion and to the work by Jacob, Lord, and Tilley.

4.8 Limitations of Duration Analysis

Duration analysis has some limitations of which the valuation actuary should be aware:

- Like any other actuarial model, duration analysis requires assumptions like interest rate volatility or policyholder lapsation, which will probably not be completely accurate over long projection periods.

- Duration calculations are more complex for recurring premium products like universal life, because the liability duration is sensitive to the future premium payment assumptions. That is, different future premium payment behavior by today's policyholders will have a big effect on today's liability duration. Simulations models get around this problem.

- Duration analysis does not directly address certain questions that are best studied with scenario-by-scenario simulations output. These include reviews of surplus needs under extreme future business conditions, especially with regard to risks other than C-3 risk. In addition, durations are always for existing in-force business as of a specified valuation date. They do not give much information about the possible effects of future new business.

- Durations for some classes of assets, such as stocks and real estate, are hard to define, because the price volatilities of these investments are not directly tied to interest rates the way they are for bonds and options on bonds. This is also partly true for high-yield (junk) bonds, whose price changes are not completely correlated with general interest rate changes.

4.9 Advantages of Duration Analysis

Duration analysis is a valuable adjunct to traditional simulations analysis for the following reasons:

- The results are independent of interest rate forecasting. They are not dependent on, or biased by, subjective views about future interest rate movements.

- The output summarizes results across many interest rate scenarios into a concise index of interest rate sensitivity. In contrast, traditional simulations studies sometimes produce large amounts of output that can be difficult and time consuming to interpret.

- Once a target account has been selected for immunization and assumptions chosen for the duration calculations, the output of the duration analysis gives direct and specific information on what to do to immunize effectively. This information is readily communicated within the insurance company.

- Duration targets are very flexible. For instance, a target asset duration can be achieved by any combination of investments whose aggregate duration equals the target. Management is free to pursue special market opportunities within very broad constraints. Regulatory restrictions, tax considerations, and transactions costs will not hobble the cost-effectiveness of a duration matching (or intentional mismatching) program.

4.10 Conclusion

This section has discussed a general technique for measuring and managing C-3 risk, called duration analysis. It allows management to set specific and concise targets for asset and liability portfolios, as well as to immunize profitability effectively against interest rate risk while permitting considerable flexibility in execution. Duration analysis cannot replace traditional simulations analysis but should be seen by the valuation actuary as a valuable adjunct. In particular, the valuation actuary should be aware of the pitfalls of Macaulay duration for interest-sensitive assets and liabilities, and he should make sure that any duration results he uses have a sound theoretical foundation.