Article from:

The Actuary

WHAT ACTUARIES NEED TO KNOW ABOUT CONSTRUCTING MORTALITY TABLES

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Richard London's review of Robert Batten's *Mortality Table Construction* arrived as my actuarial students and I were completing our study of the material using the new text. I have reflected on it now for a few weeks and I find that I cannot agree with London's overall rating of the Batten text relative to the late Harry Gershenson's *Measurement of Mortality*, nor can I agree with the final statement in the Society of Actuaries' preface to the Batten text which claims, ". . . this work, which will be a valuable contribution to the education of future generations of actuaries."

From Gershenson to Batten

I prefer the text by Batten for the following reasons:

(a) He scrapped the tollroads. I have not covered that part of the Gershenson text for many years. In my opinion the number of car-miles travelled between interchanges on a road is not sufficiently easier to formulate than is the number of life-years lived between integers on a time axis to justify occupation of 10% of the exposition in a text at this level.

(b) He added Chapter One, Mathematical Foundations, which covers the calculus of the three usual mortality assumptions for one year intervals. I believe that this material is of value in its own right and for that reason have previously included it in the course. The four figures in chapter One have always been assigned as homework and discussed at the blackboard. I do agree with London that a detailed coverage is not necessary to an understanding of the remainder of the text but for me it fits here as well as in life contingencies.

(c) He added a chapter on tabulating rules. This concept is used without a definition in the Gershenson text. (I would like to see it more precisely defined as a function whose domain is a set of individual records and whose range is the set of positive integers. I find this is a helpful standard for a student to use for his tab rules).

(d) He added exposition on counterpart formulas including the algebraic proofs of such.

(e) His discussion on fiscal years is much better. In particular he points out that the first event considered, i.e. birth, does not follow the general definition for 'fiscal year of event.'

(f) He included the instructions for an Annual Study of Mortality by the Society of Actuaries.

I also have criticisms of the new book; however, only the first does not apply equally well to the Gershenson book.

(1) Some problems in Chapters Three and Four stated results and asked for the assumptions which were used. These problems tended to emphasize writing formulas by rote, because standard assumptions had to be made to reach textbook answers.

(2) Where an exercise included "stating all assumptions" the answer would state simply "Balducci hypothesis" for the shape of the mortality curve. I believe this assumption should be given explicitly by formulas.

(3) The text should have an index.

(4) Chapter Seven needs to be a more detailed and analytical discussion.

(5) Some exposition on the connection between this material and the multiple decrement material of life contingencies is needed.

A Fresh Approach

I cannot agree with the final statement in the Society of Actuaries' preface to the Batten text because I think the syllabus for "Principles Underlying the Construction of Mortality and other Tables" should be revised before another generation of actuaries prepares for Part V. In my opinion the current material spends too much time on the wrong problem using out of date methods.

I like to think of this part of an actuary's work in three steps. First is the mathematical step of selecting a family of models for the application. Second is the statistical step of selecting estimators for the parameters indexing the family. And third is the data processing step of calculating the values of the estimators. This section of the Part V syllabus is concerned with the second and third steps for the case when the family of models chosen is the traditional (perhaps multiple decrement) life table.

As written by Batten, the statistical step is covered in just five pages (pp. 16-20) and there it is done without any explicit coverage of the statistical basis and properties of the estimator. The basis for the estimator must be inferred from equation (2.1) which lays the (statistical) principle that the number of expected deaths less the number of expected deaths among those lives lost to the study should be equal to the number number of observed deaths. Next the Balducci assumption for the shape of the mortality curve over the estimation intervals is adopted ". . . primarily [due to] the ease of finding the q_x values . . .". I don't find this to be a convincing justification for use of the estimator. At best it serves only as a mnemonic for writing the estimator formulas.

I find it much more convincing to assume that the force of mortality is constant over the estimation interval and then to adopt the maximum likelihood estimator for this constant force, i.e. the ratio of the number of observed deaths to the total observed lifetime *lived* within the interval. The resulting estimator for q_x, one minus the antilog of the negative of the estimator of the force of mortality, would be the maximum likelihood estimator of q_x by the invariance principle.

A comparison of these and other estimators by simulation studies could be provided. A discussion of their statistical properties could be included. The problem of choosing the estimator for other decrements of interest — or in the presence of certain other decrements — or increments — should be discussed in the material: e.g. which are the suitable assumption and estimator in the study of remarriage rates?

Pages 21-210 of the Batten text deal with writing formulas for the data processing step which transforms individual record and valuation schedule data bases into values of the estimator. In this part of the material I believe that too much of the syllabus is expended in developing the techniques of writing the formulas. How one makes the choice between calculating seriatim or by grouping when the data base is a set of individual records needs discussion. The statistics of the grouping approximations could be developed.

In summary, my opinion is that Batten's text is the preferable one for the current syllabus; however, the Society should move quickly to alter the syllabus.　　□