

**1991 VALUATION ACTUARY  
SYMPOSIUM PROCEEDINGS**

**SESSION 12**

**Modeling: Basic Training**

**Meredith A. Ratajczak**

**Arnold Dicke**



## **MODELING: BASIC TRAINING**

**MS. MEREDITH A. RATAJCZAK:** I will be one of two speakers on the panel for this session. By way of introduction, I have been a consulting actuary with Milliman & Robertson for the last four years. Before that, I spent four formative years at Penn Mutual Life Insurance Company in Philadelphia. During my years with Milliman & Robertson, I have spent a substantial portion of my career on valuation-related activities -- primarily Regulation 126 testing for both large and small clients.

My colleague on the panel is Arnold Dicke. Arnold is currently vice president and actuary with Equitable Life Assurance Society in New York. Prior to joining the Equitable, Arnold was a consulting actuary with Tillinghast in New York. He too has been involved with valuation-related activities throughout his actuarial career.

This session is designed to be a how-to session on modeling assets and liabilities; consequently we will try to keep our discussion as basic as possible. Arnold and I are going to alternate our remarks. We will then turn the floor over to the audience for questions.

For all life insurance companies failing to meet certain selection criteria based on amount of invested assets, level of capital and surplus in relation to cash and invested assets, or amount of annuities, the inclusion of the following statement in the revision to the Standard Valuation Law means for the first time, qualified actuaries will need to consider all assets and all liabilities in their opinion that adequate provision is made for the company's future obligations and expenses. For most, this opinion will be based on an asset adequacy analysis:

Every Life Insurance Company, except as exempted by or pursuant to regulation shall also annually submit the opinions of the same qualified actuary that the reserves and related actuarial items held in support of the policies and contracts specified by the commissioner by regulation when considered in light of the assets held by the company make adequate provision for the company's obligations under such policies for the company's obligation under such policies and contracts.

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You as the qualified actuary or someone involved in the valuation process will probably find yourself spending much more of your time modeling and projecting your assets and liabilities. Since the opinion covers all of a company's business, we will be required to judge what type of testing should be done given the risks involved. The nature of the testing will depend on the nature of the risk. Materiality, complexity and time constraints will all weigh heavily on the qualified actuary's decision about the appropriate methods used to test for asset adequacy.

If your company is a typical multiline company, you can probably split your business into at least three categories.

In the first category, you assume a risk; however, experience will most certainly be less severe than that provided for in the reserves. For this business, asset adequacy may be determined through very simplistic comparisons of interest rate and valuation interest rate spreads and valuation mortality and morbidity in relation to actual experience. This category may include short-term health risks or other business where re-rating is available.

The second category would include business for which it is clear that cash-flow testing is necessary and appropriate. As a result of Regulation 126, we have been testing the adequacy of annuity and single-premium, interest-sensitive life insurance business for several years. Other interest-sensitive life business would certainly fall into this category.

The real dilemma for the valuation actuary lies in the third category or "all other" types of business. For these lines it is unclear what types of deviation from expected are adverse and consequently what type of testing is necessary and appropriate. The necessary testing is unclear because little valuation-actuary-type testing has been done on products other than interest-sensitive life insurance and annuities.

For products that fall into category three, multiscenario testing may be the only way to quantify the risks associated with the assets and liabilities.

Since the asset adequacy analysis covers all lines of business that may be aggregated in forming an opinion, surpluses in one line may be used to offset deficits in another. Consequently, the degree of specificity and refinements in your models may impact the overall adequacy of your company.

For the remainder of our session, Arnold and I are going to discuss the basics of asset and liability modeling. We will cover the basics in four distinct phases:

1. Development of the existing liability model,
2. Development of the existing asset model,
3. Development of the interest scenarios to be used or the present value model, and
4. Development of the corporate model. This phase would include such activities as determining the reinvestment strategy.

Since this is intended to be a how-to session, we will provide practical examples of what other companies are doing.

### **Development of the Existing Liability Model**

The first phase in the asset adequacy analysis involves modeling the business you currently have on the books as of the valuation date. New business does not enter into this analysis because you are opining on the adequacy of the assets and liabilities currently in the company's portfolio as of the valuation date.

Deciding what liabilities should be modeled for cash-flow testing is no easy process. All liabilities will need to be considered at least through some simplified process. It is not always clear what fits the cash-flow testing mold. Recent industry experience indicates what happens when the asset risks are not carefully weighed against the liability risks. A large

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asset risk may impair a company's ability to satisfy future obligations in lines not currently considered cash-flow-testing candidates potentially making all lines cash-flow-testing candidates.

Conservatism in reserves, few options granted to policyholders, and a large spread between the interest rate earned and that assumed in the reserves may give the qualified actuary enough comfort about the adequacy of assets without any detailed modeling.

For business that offers options to policyholders, such as surrenders, policy loans or long tail guarantees, a simplified demonstration of adequacy may not be rigorous enough to satisfy the qualified actuary that adequacy exists. One company's traditional product will not behave the same way as another due to interest-rate guarantees, dividend formulas and the availability of cash-out options.

As you can see, we can only offer examples of when cash-flow testing may be appropriate. You must consider the risks posed by the options granted to your policyholders. The nature of these risks will impact the nature of your testing.

On products, such as some health insurance and term products where the risks are almost entirely C-2 risk, your liability model may be very simple since interest-rate sensitivity will have a minor impact on your results. For products with interest-sensitive elements, those that offer options to the policyholder and for health insurance business which may generate large claim reserves, dynamic assumptions for lapses, policy persistency and interest-crediting strategy are necessary and appropriate to capture the true behavior of your business.

Simplistic testing methods will warrant simplified models. For many of you, the phrase *cash-flow testing* probably conjures up images of thousands of cells and complex projection methods. It has been our experience that more cells do not necessarily produce a better

model. As an example, deferred annuity business is frequently modeled in duration cells to capture surrender charge scales that are a function of duration. Life insurance on the other hand is typically modeled on an age and duration basis to capture mortality differences. The deferred annuity model may be simplified by collapsing the model into one representative duration. The life insurance model may be simplified by modeling several representative issue age/duration cells. Group business could be modeled as one aggregate cell with appropriate aggregate assumptions. For your model to be an appropriate representation of your liabilities, the major plans should be represented and the options granted to policyholders should be modeled. We have seen through recent research that overlooking the behavior of certain policy features in cash-flow testing may mean the difference between total company adequacy and deficiency.

Arnold will discuss model validation.

**MR. ARNOLD A. DICKE:** What is a "valid actuarial model" and how do you construct one?

Modeling is a process of representing reality, not reproducing it. The first step in creating such a representation is the grouping of data into "cells." The degree of grouping (i.e., the size of cells) is an important variable in this process. It is sometimes referred to as the granularity of the model.

The grouping of data into cells is done for a reason: to allow assumptions to be associated with each cell. A group is homogeneous if it is reasonable to associate all elements of the group with a single set of assumptions. Note that once assumptions are associated with cells, you really have a number of submodels: one for each cell.

*Validation* is the process of checking the model against reality. One asks: Can the model reproduce certain aspects of recent history reasonably well? Two kinds of validation are

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usually necessary: (1) validation of the model overall, and (2) validation of the cellular submodels.

*Granularity* – Insurance liabilities resulting from policies and contracts are grouped by examining their characteristics and developing rules that assign a given liability to a given class. The more characteristics that are recognized, the more granular the classification is said to be. For a given set of liabilities, the potential defining characteristics are examined, and those that are essential to reasonable grouping are identified. For example, for accumulation products, the list of potential defining characteristics would include:

- Contract structure, such as
  - Withdrawal provisions,
  - Bail-out provisions,
  - Transfer provisions, and
  - Employee withdrawal provisions,
- Crediting rate strategy,
- Current or historical rates credited,
- Guarantee periods and maturities, and
- Other characteristics.

*Homogeneity* – If the cells are reasonably chosen, homogeneity should take care of itself. However, it is necessary to check that each of the cells can actually be represented by a single set of assumptions regarding:

- Mortality or morbidity,
- Lapse and withdrawal,
- Premium persistency,
- Commissions and expenses,
- Investment income, and
- Other assumptions.

*Validation* -- Professionalism requires that any model used be grounded, to the extent possible, in reality, whether actual company experience, industry experience or experience from another source. For liabilities, validation usually takes the form of comparison of model value to actual value on a recent date for some or all of (1) Policy count, In-force face amount, and (3) R-reserves and/or account value.

For some sorts of models, it may be necessary to validate other assumptions, such as premiums or deposits received (for a new line of business) or interest credited (for GICs). All these items are usually validated for each policy type. Table 1 displays a validation run adapted from an actual valuation project.

Another step in the validation process, sometimes taken for granted, is the validation at a higher level (line of business or even company level) of financial quantities such as expenses, commissions, dividends and loss ratios. Also, mortality rates and lapse rates need to be validated, either by a mortality study or by reproducing recent levels of benefit payments. This is the step that, in effect, validates the submodels discussed above.

Finally, it is important to note that validation typically refers to a point in time. As time goes on, the fact that the model matched actual experience on that one date is an increasingly less convincing proof of its continued applicability. Thus, for example, it is more important to validate a new mortality table than an old one.

**MS. RATAJCZAK:** You are all now experts in the art of existing liability model generation and validation. To test your model under alternative economic scenarios, you must literally quantify through assumptions and formulas the behavior of your business in changing economic environments. This implies that, in order to present a true picture of asset adequacy, formulas and assumptions should be dynamically modeled to capture the

**TABLE 1**

**Modeled vs. Actual In-force Amounts as of December 31, 1990.**

| Variable Life Segment                    | (\$ millions)  |                |                           |                 |                            |                |                             |              |  |
|--|----------------|----------------|---------------------------|-----------------|----------------------------|----------------|-----------------------------|--------------|--|
|  | Count          |                | Death Benefit<br>In force |                 | General Account<br>Reserve |                | Separate Account<br>Reserve |              |  |
|  | Model          | Actual         | Model                     | Actual          | Model                      | Actual         | Model                       | Actual       |  |
| 1. Variable Whole Life I                 | 39,740         | 39,740         | 1,839.9                   | 1,839.8         | 55.7                       | 55.7           | 181.6                       | 177.3        |  |
| 2. Variable Whole Life II                | 9,109          | 9,109          | 963.4                     | 962.9           | 5.5                        | 5.8            | 39.5                        | 40.0         |  |
| 3. Variable Single Px                    | 1,714          | 1,714          | 88.7                      | 88.7            | 6.4                        | 6.8            | 35.5                        | 39.8         |  |
| 4. ETI                                   | 5,625          | 5,625          | 275.3                     | 275.3           | 6.8                        | 6.8            | 0.0                         | 0.0          |  |
| 5. RPU                                   | 565            | 565            | 3.5                       | 3.5             | 0.8                        | 0.8            | 0.0                         | 0.0          |  |
| 6. Riders/Features                       |                |                |                           |                 | 11.0                       | 11.0           | 0.0                         | 0.0          |  |
| Segment Totals                           | 56,752         | 56,752         | 3,170.8                   | 3,170.2         | 86.2                       | 86.9           | 256.6                       | 257.0        |  |
| Ratio of Model to Actual                 | 100.00%        |                | 100.02%                   |                 | 99.24%                     |                | 99.84%                      |              |  |
| <b>Universal Life Segment</b>            |                |                |                           |                 |                            |                |                             |              |  |
| 1. Interest Sensitive Life I -- non ETI  | 228,178        | 228,178        | 15,513.5                  | 15,316.7        | 972.0                      | 972.4          | 0.0                         | 0.0          |  |
| 2. Interest Sensitive Life I -- ETI      | 19,890         | 19,890         | 1,143.1                   | 1,143.1         | 22.6                       | 22.6           | 0.0                         | 0.0          |  |
| 3. Interest Sensitive Life II -- non ETI | 44,404         | 44,551         | 3,497.2                   | 3,505.1         | 31.0                       | 30.7           | 0.0                         | 0.0          |  |
| 4. Interest Sensitive Life II -- ETI     | 643            | 643            | 54.4                      | 174.5           | 0.6                        | 0.6            | 0.0                         | 0.0          |  |
| 5. Universal Life                        | 31,729         | 31,729         | 4,392.8                   | 4,375.7         | 153.1                      | 150.5          | 0.0                         | 0.0          |  |
| 6. Variable Universal Life               | 125,572        | 125,572        | 29,458.6                  | 29,442.5        | 281.1                      | 278.9          | 641.6                       | 642.1        |  |
| 7. Riders/Features                       |                |                |                           |                 | 29.6                       | 29.6           | 0.0                         | 0.0          |  |
| Segment Totals                           | 450,416        | 450,563        | 54,059.6                  | 53,957.6        | 1,489.9                    | 1,485.2        | 641.6                       | 642.1        |  |
| Ratio of Model to Actual                 | 99.97%         |                | 100.19%                   |                 | 100.32%                    |                | 99.93%                      |              |  |
| <b>Single Premium Whole Life Segment</b> |                |                |                           |                 |                            |                |                             |              |  |
| 1. Single Px Whole Life                  | 40,103         | 40,103         | 3,256.0                   | 3,256.0         | 1,064.6                    | 1,064.9        | 0.0                         | 0.0          |  |
| Ratio of Model to Actual                 | 100.00%        |                | 100.00%                   |                 | 99.98%                     |                | N/A                         |              |  |
| <b>COMPANY TOTALS</b>                    | <b>547,271</b> | <b>547,418</b> | <b>60,486.4</b>           | <b>60,383.8</b> | <b>2,640.8</b>             | <b>2,636.9</b> | <b>898.3</b>                | <b>899.2</b> |  |
| Ratio of Model to Actual                 | 99.97%         |                | 100.17%                   |                 | 100.15%                    |                | 99.90%                      |              |  |

behavior of the business in response to external influences. We will show you some numerical examples that illustrate the importance of modeling using dynamic assumptions.

Liability assumption development can be viewed as a three-phase process. First you must establish your base or current level of experience. Second, you should review past experience in different economic environments to develop floors and ceilings for the assumption in question. Finally, using all the information you compiled from your review of current and past experience, quantify through dynamic formula the behavior of your liabilities in changing economic environments.

This is all very easy for us to say as outsiders looking in. Coming from people, who in the past have spent November through March assisting clients in their Regulation 126 filings, we know that developing assumptions that appropriately represent the behavior of the business will probably take far more time than the development of existing liability models.

Quantifying assumptions is easier said than done for a number of reasons. Depending on the size of your company, you may or may not do experience studies on a regular basis. You may not even have enough experience to make your studies credible. The type of studies you may be doing now may not be structured in such a way that they produce information in such detail that experience for your model cells may be gleaned.

In today's economic environment, there are also external influences that may be distorting your experience – external influences that may be totally unrelated to interest-rate change, such as shock lapses as an aftermath to the recent large insurance industry failures. These external influences further complicate this process.

We would like to add the additional point that you have not given much thought to defining the behavior of key assumptions. There are important assumptions that you actively manage on a month-to-month or even day-to-day basis, such as interest-crediting strategy.

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Just because interest-crediting strategy is actively managed does not mean it has been defined by a formula that can be used in your testing.

Some of the key parameters that should be reflected in your test of asset adequacy include lapses, mortality, morbidity, premium persistency, policy loan utilization, interest-crediting strategy, and the interplay of interest-rate changes on dividend scales and resulting lapses. The list should not be viewed as all inclusive, it is intended to list those liability assumptions that are highly sensitive to changes in the economic environment.

Let's look at some of these key assumptions in more detail. We all know that lapse experience can make or break a company. We have all witnessed the impact of a run on the bank on a company's ability to meet its future obligations. It goes without saying that for your liability model to be an appropriate representation of your business under differing economic environments, your lapse formula should be dynamic.

If you take ten Regulation 126 filings and place them side by side, you will probably see ten different lapse formulas being used. Upon review of these formulas you would see that there are some common characteristics (see Dynamic Lapse Formula). Most formulas have an underlying level of lapses that serve as the base from which experience is measured in a changing economic environment. This base may be level or may grade to some ultimate level. Your lapse formula will probably have a floor and a ceiling. The floor may be defined as your base level of lapses while the ceiling by definition would be 100%. The variation in your projected level of lapses may be expressed as some function of the spreads between market rates of interest and your current credited interest rates. The market rate is frequently defined as a function of some underlying index such as Treasury rates. Another component that may be reflected in your lapse formula is a surrender-charge component. Experience has shown that even a modest level of surrender charges is a deterrent to lapses.

**Dynamic Lapse Formulas  
Simplified Examples**

- **Base + Multiplier x (MR-CR-Interest Margin)**
- **Base + Multiplier x (MR-CR) \* A + SC\*B**
- **Base + Percentage Decrease in Credited Rates from Period to Period**

Where MR is the Market Rate  
Where CR is the Credited Rate

These three examples of dynamic lapse formulas highlight the common characteristics. The first and second examples reflect the market rate/credited rate component. Formula three is one piece of a two-part formula. It is included in an analysis to reflect the behavior of policyholders who lapse when their interest rate is reduced. This formula may be used for business purchased by a sophisticated group of policyholders who are very sensitive to credited interest-rate changes.

I have taken a sample dynamic formula, which includes a market rate/credited rate component and a surrender charge component, and calculated some withdrawal rates (Table 2). Under this example, as the surrender charge decreases or the spread between credited and market rates widens, lapses markedly increase. The large interest spread could be indicative of the early 1980s where even hefty surrender charges could not stop the disintermediation out of annuities and life insurance business.

We have talked about the problems you may encounter using your persistency studies. If you have no experience from which to develop your lapse assumptions, you do have some alternatives available to you. Your annual statement may provide some aggregate information about your lapse experience from year to year. Large levels of reinsurance may make any comparison from year to year very difficult. The Life Insurance Marketing and

**TABLE 2**

**Sample Dynamic Withdrawal Assumptions**  
**Withdrawal Rate = 15% + 2x(MR-CR)\*2 - 3xSC**  
**Floor is 3%**

| Sample Rates     |           |                        |
|------------------|-----------|------------------------|
| <u>[MR - CR]</u> | <u>SC</u> | <u>Withdrawal Rate</u> |
| 1.00%            | 7.00%     | 3.00%                  |
| 3.00             | 7.00      | 12.00                  |
| 5.00             | 7.00      | 44.00                  |
| 1.00             | 0.00      | 17.00                  |
| 3.00             | 0.00      | 33.00                  |
| 5.00             | 0.00      | 65.00                  |

Research Association (LIMRA) compiles lapse information on a regular basis for some lines of business. The Society of Actuaries is in the process of looking at interest-sensitive lapses, so in the future the findings of this analysis may prove to be a valuable tool.

Most valuation actuary work done to date has dealt with life or annuity products where the investment aspects of the product overshadow the mortality element. The year 1992 will change this. For life and health insurance where re-rating is not an option to the company, making a statement with certainty without some form of testing that the assets when considered with the liabilities make adequate provision for future obligations and expenses will not be simple and straightforward. The AIDS risk and quantifying its impact on company solvency further complicates this process.

The complexity of your mortality and morbidity assumptions will go hand in hand with the complexity of your model. Aggregate models will warrant aggregate mortality and morbidity assumptions. Base experience can be gleaned from experience studies. If these are not available or if the data are not credible, there are numerous sources available to develop assumptions. To name a few, LIMRA and the Society are constantly looking at mortality

experience from year to year. Quantifying the AIDS risk is not a straightforward task. There is an interplay between the incidence of AIDS and lapses. In the course of merger and acquisition work we have found that more and more companies are maintaining separate claim data for AIDS and non-AIDS claims.

If your company's claim experience is not split by AIDS and non-AIDS claims, the Society study can be used as a source of incidence, death, and survival statistics. In situations where we have explicitly provided for the additional cost associated with AIDS that has not been provided for in the reserves, a company's AIDS claim experience was expressed as a scaling factor on the underlying mortality or morbidity rates.

Table 3 gives a page from the Report of the Society of Actuaries Committee on HIV Research, July 1989. This table is particularly useful in modeling since it is a select table, based on the assumption that HIV was known to be absent at time zero. These rates should be added to those of the base mortality table. However, the Task Force on the Financial Implications of AIDS recommends multiplying by 60% for use with typical insured mortality. Recent evidence seems to support an even smaller multiplier.

**MS. RATAJCZAK:** We included premium persistency and policy loan utilization on this list for the benefit of those who sell products with flexible premium or policy loan provisions. Premium persistency is a crucial pricing assumption. Maintaining premium persistency may mean the difference between aggregate profit and loss. A typical policy loan provision with a spread between the loan rate and the credited rate on loaned values, which is comparable to the underlying spread assumed in the crediting strategy, may require no special modeling. Preferred policy loan provisions that offer no spread between the loan rate and the credited rate are another story. Both premium persistency and policy loan utilization can be very volatile in a changing interest-rate environment. Premium suspension and loan utilization would tend to increase in a rising interest-rate environment.

**TABLE 3**

**SOA Committee on HIV Research: Middle Scenario, Infected after 1988**

**MALE GENERAL POPULATION AIDS MORTALITY RATES PER THOUSAND LIVES:**

| CALENDAR YEAR | ATTAINED AGE IN 1986: |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |
|---------------|-----------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|               | 31                    | 32    | 33    | 34    | 35    | 36    | 37    | 38    | 39    | 40    | 41    | 42    | 43    | 44    | 45    | 46    | 47    | 48    | 49    |
| 1986          | 0.000                 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 1987          | 0.000                 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 1988          | 0.000                 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
|               | ATTAINED AGE IN 1989: |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |
|               | 34                    | 35    | 36    | 37    | 38    | 39    | 40    | 41    | 42    | 43    | 44    | 45    | 46    | 47    | 48    | 49    | 50    | 51    | 52    |
| 1989          | 0.000                 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 1990          | 0.002                 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.002 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 |
| 1991          | 0.013                 | 0.013 | 0.012 | 0.012 | 0.011 | 0.010 | 0.010 | 0.009 | 0.009 | 0.008 | 0.008 | 0.008 | 0.007 | 0.007 | 0.006 | 0.006 | 0.005 | 0.005 | 0.005 |
| 1992          | 0.036                 | 0.034 | 0.033 | 0.030 | 0.028 | 0.026 | 0.025 | 0.024 | 0.023 | 0.022 | 0.021 | 0.019 | 0.018 | 0.017 | 0.016 | 0.015 | 0.014 | 0.013 | 0.013 |
| 1993          | 0.069                 | 0.065 | 0.061 | 0.057 | 0.053 | 0.050 | 0.047 | 0.045 | 0.043 | 0.040 | 0.038 | 0.035 | 0.033 | 0.031 | 0.029 | 0.028 | 0.027 | 0.026 | 0.025 |
| 1994          | 0.108                 | 0.101 | 0.094 | 0.087 | 0.082 | 0.078 | 0.073 | 0.069 | 0.065 | 0.061 | 0.058 | 0.054 | 0.051 | 0.048 | 0.046 | 0.044 | 0.043 | 0.041 | 0.040 |
| 1995          | 0.148                 | 0.137 | 0.128 | 0.120 | 0.113 | 0.106 | 0.100 | 0.094 | 0.088 | 0.083 | 0.078 | 0.073 | 0.069 | 0.067 | 0.065 | 0.063 | 0.061 | 0.059 | 0.055 |
| 1996          | 0.187                 | 0.173 | 0.162 | 0.152 | 0.143 | 0.134 | 0.126 | 0.118 | 0.110 | 0.103 | 0.096 | 0.091 | 0.087 | 0.085 | 0.083 | 0.081 | 0.078 | 0.073 | 0.068 |
| 1997          | 0.220                 | 0.205 | 0.192 | 0.181 | 0.170 | 0.159 | 0.149 | 0.138 | 0.128 | 0.120 | 0.112 | 0.106 | 0.105 | 0.103 | 0.101 | 0.097 | 0.092 | 0.085 | 0.078 |
| 1998          | 0.248                 | 0.232 | 0.218 | 0.205 | 0.192 | 0.178 | 0.165 | 0.153 | 0.142 | 0.133 | 0.127 | 0.124 | 0.122 | 0.120 | 0.116 | 0.110 | 0.102 | 0.094 | 0.086 |
| 1999          | 0.269                 | 0.253 | 0.238 | 0.222 | 0.206 | 0.190 | 0.175 | 0.162 | 0.151 | 0.144 | 0.141 | 0.139 | 0.137 | 0.133 | 0.126 | 0.118 | 0.110 | 0.101 | 0.092 |
| 2000          | 0.284                 | 0.267 | 0.249 | 0.231 | 0.212 | 0.194 | 0.179 | 0.167 | 0.159 | 0.156 | 0.154 | 0.152 | 0.148 | 0.141 | 0.133 | 0.123 | 0.114 | 0.105 | 0.096 |
| 2001          | 0.294                 | 0.274 | 0.252 | 0.232 | 0.212 | 0.195 | 0.182 | 0.173 | 0.168 | 0.165 | 0.163 | 0.158 | 0.150 | 0.141 | 0.132 | 0.122 | 0.113 | 0.104 | 0.094 |
| 2002          | 0.294                 | 0.270 | 0.247 | 0.227 | 0.208 | 0.193 | 0.183 | 0.177 | 0.174 | 0.170 | 0.164 | 0.154 | 0.147 | 0.138 | 0.128 | 0.119 | 0.110 | 0.100 | 0.091 |
| 2003          | 0.283                 | 0.259 | 0.237 | 0.218 | 0.202 | 0.191 | 0.184 | 0.179 | 0.175 | 0.169 | 0.160 | 0.151 | 0.142 | 0.133 | 0.123 | 0.114 | 0.104 | 0.095 | 0.086 |
| 2004          | 0.267                 | 0.245 | 0.225 | 0.208 | 0.196 | 0.188 | 0.182 | 0.177 | 0.170 | 0.162 | 0.153 | 0.144 | 0.135 | 0.126 | 0.116 | 0.107 | 0.098 | 0.089 | 0.080 |
| 2005          | 0.249                 | 0.228 | 0.211 | 0.198 | 0.189 | 0.183 | 0.177 | 0.170 | 0.161 | 0.153 | 0.144 | 0.135 | 0.126 | 0.117 | 0.108 | 0.100 | 0.091 | 0.082 | 0.073 |
| 2006          | 0.231                 | 0.213 | 0.199 | 0.190 | 0.183 | 0.177 | 0.170 | 0.161 | 0.151 | 0.142 | 0.132 | 0.123 | 0.115 | 0.106 | 0.098 | 0.089 | 0.081 | 0.073 | 0.064 |
| 2007          | 0.212                 | 0.198 | 0.188 | 0.182 | 0.176 | 0.168 | 0.159 | 0.149 | 0.139 | 0.129 | 0.120 | 0.111 | 0.103 | 0.095 | 0.087 | 0.079 | 0.072 | 0.063 | 0.056 |
| 2008          | 0.195                 | 0.186 | 0.179 | 0.173 | 0.165 | 0.155 | 0.145 | 0.135 | 0.125 | 0.115 | 0.107 | 0.099 | 0.092 | 0.084 | 0.077 | 0.070 | 0.062 | 0.055 | 0.048 |
| 2009          | 0.182                 | 0.175 | 0.169 | 0.162 | 0.152 | 0.141 | 0.130 | 0.120 | 0.111 | 0.102 | 0.095 | 0.088 | 0.081 | 0.075 | 0.068 | 0.060 | 0.053 | 0.047 | 0.042 |
| 2010          | 0.171                 | 0.165 | 0.157 | 0.147 | 0.136 | 0.125 | 0.115 | 0.106 | 0.098 | 0.091 | 0.085 | 0.078 | 0.072 | 0.065 | 0.059 | 0.052 | 0.046 | 0.041 | 0.039 |
| 2011          | 0.164                 | 0.156 | 0.146 | 0.136 | 0.125 | 0.114 | 0.105 | 0.097 | 0.090 | 0.084 | 0.077 | 0.072 | 0.065 | 0.058 | 0.052 | 0.046 | 0.041 | 0.038 | 0.034 |
| 2012          | 0.155                 | 0.145 | 0.135 | 0.124 | 0.114 | 0.104 | 0.097 | 0.090 | 0.083 | 0.077 | 0.071 | 0.065 | 0.058 | 0.051 | 0.044 | 0.041 | 0.038 | 0.036 | 0.033 |
| 2013          | 0.144                 | 0.133 | 0.123 | 0.113 | 0.103 | 0.096 | 0.089 | 0.083 | 0.076 | 0.071 | 0.064 | 0.057 | 0.051 | 0.045 | 0.040 | 0.038 | 0.035 | 0.033 | 0.030 |
| 2014          | 0.132                 | 0.122 | 0.111 | 0.103 | 0.095 | 0.088 | 0.082 | 0.075 | 0.070 | 0.063 | 0.057 | 0.050 | 0.045 | 0.040 | 0.037 | 0.035 | 0.032 | 0.030 | 0.028 |
| 2015          | 0.120                 | 0.110 | 0.102 | 0.094 | 0.087 | 0.081 | 0.075 | 0.069 | 0.063 | 0.058 | 0.050 | 0.044 | 0.039 | 0.037 | 0.035 | 0.032 | 0.030 | 0.027 | 0.025 |
| 2016          | 0.109                 | 0.101 | 0.093 | 0.086 | 0.080 | 0.074 | 0.069 | 0.062 | 0.056 | 0.050 | 0.044 | 0.039 | 0.037 | 0.034 | 0.032 | 0.029 | 0.027 | 0.025 | 0.022 |
| 2017          | 0.100                 | 0.092 | 0.086 | 0.080 | 0.073 | 0.068 | 0.062 | 0.055 | 0.049 | 0.043 | 0.039 | 0.036 | 0.034 | 0.032 | 0.029 | 0.027 | 0.024 | 0.022 | 0.022 |
| 2018          | 0.092                 | 0.085 | 0.079 | 0.073 | 0.068 | 0.061 | 0.055 | 0.049 | 0.043 | 0.038 | 0.036 | 0.034 | 0.031 | 0.029 | 0.027 | 0.024 | 0.022 | 0.022 | 0.022 |
| 2019          | 0.085                 | 0.079 | 0.072 | 0.067 | 0.061 | 0.055 | 0.048 | 0.043 | 0.038 | 0.036 | 0.034 | 0.031 | 0.029 | 0.026 | 0.024 | 0.022 | 0.022 | 0.022 | 0.022 |

INFECTION SPREAD: 4,000,000 AT RISK, MIDDLE SCENARIO TO MATCH CDC  
 PROGRESSION RATES: WEIBULL, MEDIAN 10 YEARS, ALPHA 2.1  
 MORTALITY AFTER DIAGNOSIS: 40%-40%-35%-25%; CASES BEFORE 1986 HIGHER  
 AGE/SEX SPLIT: 90% MALE, DISTRIBUTE ALL CASES AMONG AGES 15-79  
 INCLUDED DEATHS: 100% OF INFECTIONS AFTER 1988

These features should be modeled dynamically, recognizing your companies experience in varying economic environments.

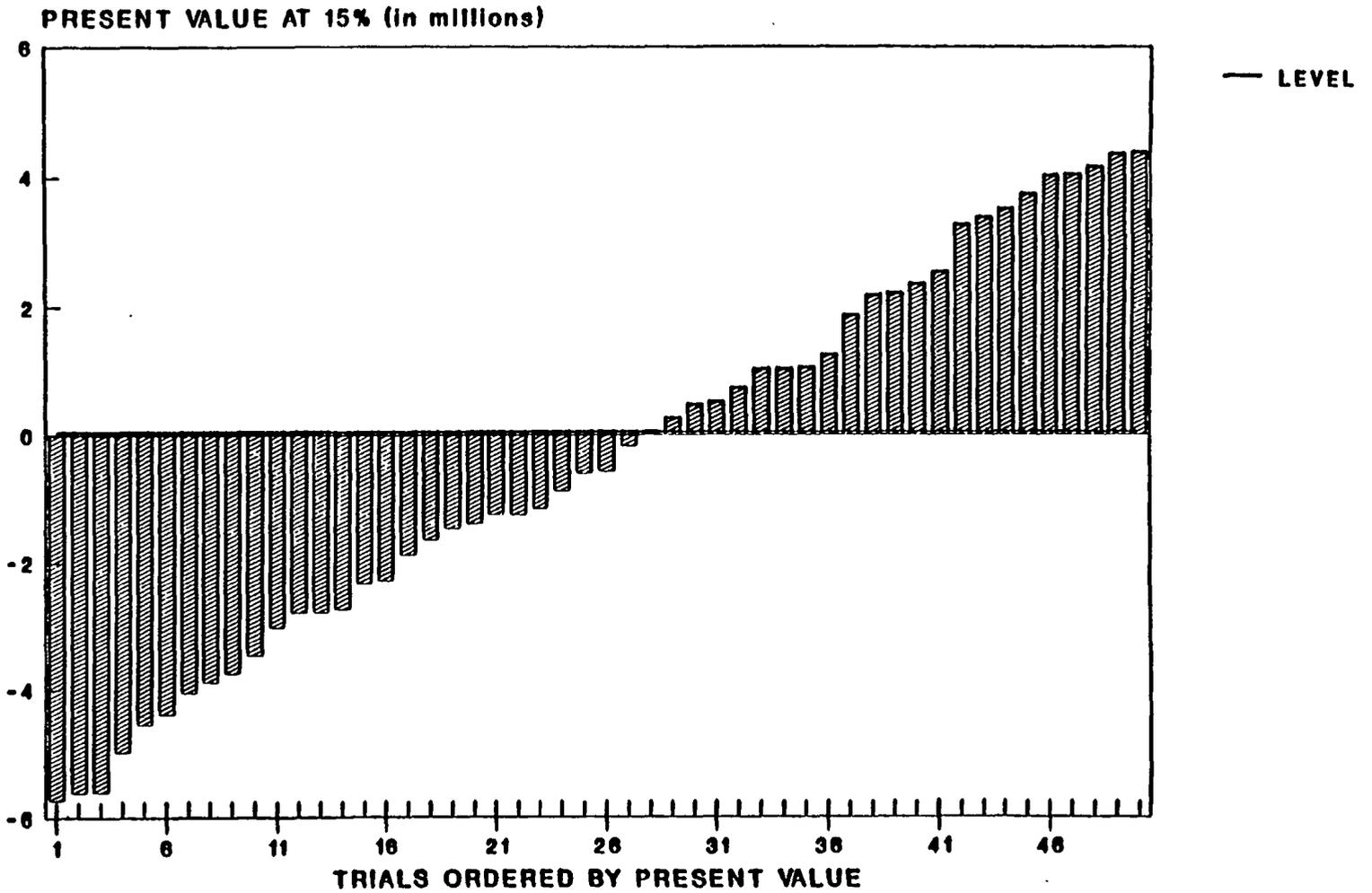
For many of you, traditional life insurance comprises a significant portion of your business. You must focus your thoughts on how to tackle the job of modeling participating life insurance if your company offers participating business. There is a school of thought in the actuarial community that participating life business does not fit the cash-flow-testing mold. In theory, deviations from expected for lapse, mortality, expense and interest should flow through as adjustments to the underlying dividend scale. Can you honestly say that for every dollar of additional expense, your company drops dividends one dollar? Probably not. Have you ever quantified policyholder lapse behavior immediately after you reduce your dividend scale?

Should you dynamically model your dividend scale? To the last question we can emphatically answer yes. Over the last few months we have been working on a research project dealing with cash-flow testing on a participating whole life product. To introduce interest-rate sensitivity into the lapse and dividend formula we modeled a 1980 CSO fixed premium whole life product like a universal life product (Chart 1). A combination of front-end loads, guaranteed mortality, and interest were used to build account values which mirrored 1980 CSO commissioners reserve valuation method (CRVM) reserves. Permanent surrender charges were used to produce the correct cash value/reserve relationship. Our intuitive sense told us that the dividend was an important policy feature, and how it was modeled would greatly impact the result of cash-flow analysis.

We looked at results for fifty randomly generated scenarios. In Chart 2, a static dividend scale was used. The worst case scenario for these fifty trials was approximately a \$6 million loss for \$25 million of life premium. Modest levels of interest-sensitive lapses are reflected in these results.

# CHART 1

## Sample Participating Whole Life Product Fixed Dividend Scale No Lapse Sensitivity



For the same 50 scenarios, we used a dynamic dividend formula that recognized changes in the earned rate from year to year (Chart 2). Our \$6 million loss worst-case scenario for the static scale has been reduced to approximately a \$2.6 million loss by the introduction of a modeling assumption that captures the expected behavior of a company in a changing interest-rate environment. It shows some \$3.4 million of deficit that will not use up surpluses that may be needed by other lines.

Two extremes have been presented to make a point. Dynamic assumptions are necessary and appropriate for modeling liability behavior in changing economic environments.

### **Development of the Existing Asset Model**

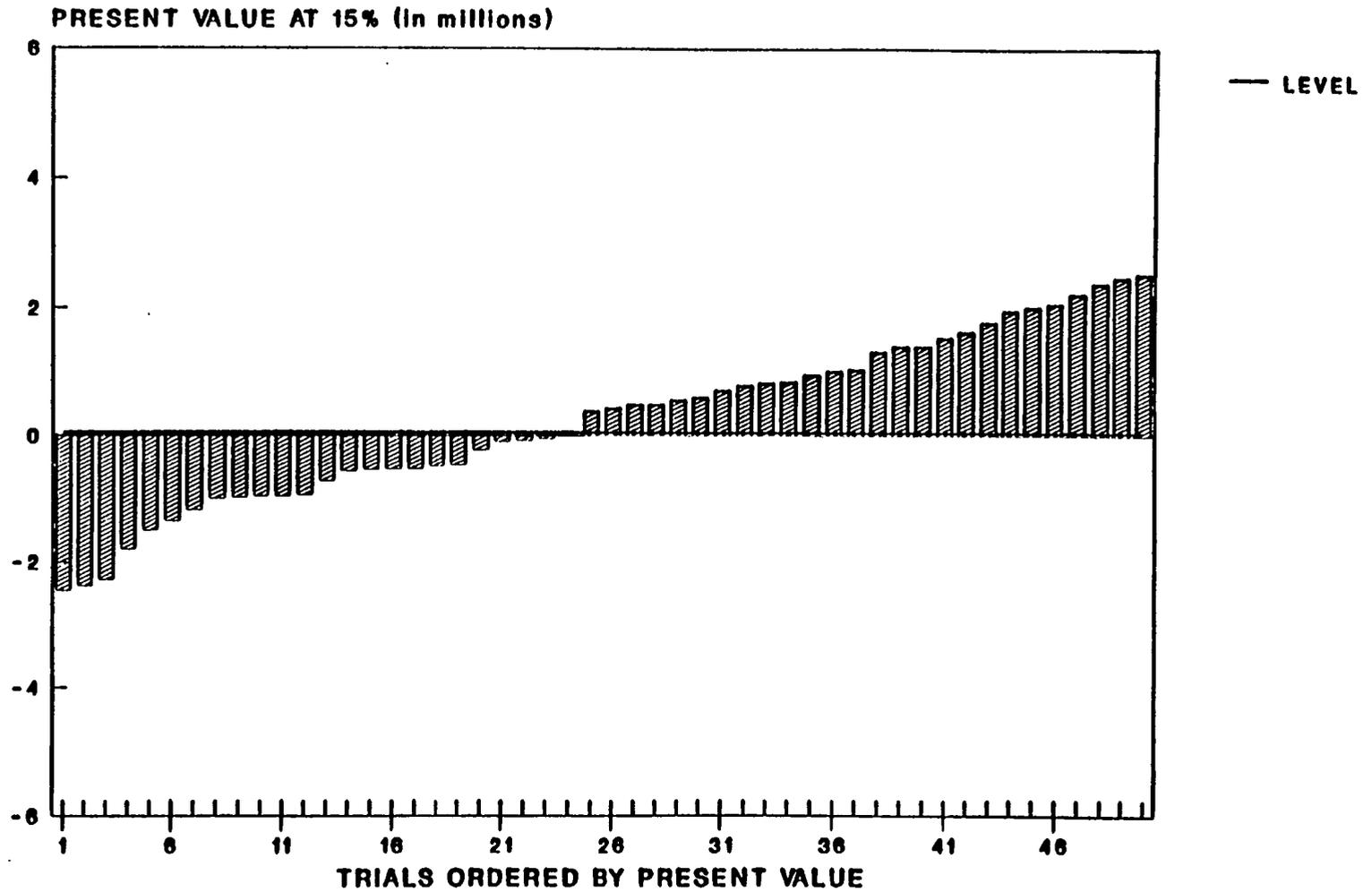
In the course of your day to day activities, if we took a count, we would probably find that most of you are more comfortable with the liability side of the balance sheet than the asset side of the balance sheet. Liability models are necessary for pricing and product development whereas simplistic asset assumptions, such as an assumed aggregate earned rate, are frequently used in pricing and product development.

For the asset adequacy analysis, actuaries can choose practices that are deemed appropriate to them, given the composition of their portfolios. These practices should not be inconsistent with any guidelines supplied through standards, and they should provide results that do not distort total company adequacy.

When constructing an existing asset model, seriatim projection methods or some means of grouping by asset classes have been found to be acceptable projection methods at least in the valuation actuary work done to date for Regulation 126 testing. Materiality and asset complexity should be considered when constructing your models. As an example, let's assume your bond portfolio contains collateralized mortgage obligations (CMOs) comprising 1% of your total portfolio. In a situation such as this, the CMO could be modeled as a

## CHART 2

### Sample Participating Whole Life Product Calculated Dividend Scale No Lapse Sensitivity



## MODELING: BASIC TRAINING

bond with a maturity period equal to the average remaining life of the CMO. Simplistic assumptions could then be made for income and principal repayments. Simplifying assumptions are acceptable if they do not materially distort results. The methods you use to project your assets must be disclosed in your memorandum.

I will turn the floor over to Arnold for further discussion about existing asset models.

**MR. DICKE:** As was the case with liabilities, the concepts of granularity, homogeneity, and validation are good guides to the construction of models of existing assets. Note that the modeling of existing assets is treated independently from the assumptions regarding reinvestments. I prefer to think of the reinvestment strategy, the overall crediting strategy, the marketing strategy, the expense control strategy and the like as part of a "corporate" model. The corporate model, in this structure, is a collection of decision rules that reflect the manner in which management intends to run the company. It is helpful to distinguish between assumptions about decision rules (which may be changed at any time by management) and true actuarial assumptions (which are the parameters of a model that must be validated against actual results). Putting all the decision rules into the corporate model segment helps avoid confusion of these two kinds of assumptions.

*Homogeneity*, in the case of securities, means, not that yields are the same, but that they have the same relationship to underlying economic reality. For valuation models, underlying economic reality is usually represented by risk-free interest rates, i.e., Treasuries. The functional relationship might be represented as:

$$A X + B$$

where  $X$  is a Treasury of some duration. It is somewhat easier to group securities with the same  $A$  and different  $B$ 's than vice versa. The impact of the  $A$  may be too powerful to average.

## **1991 SYMPOSIUM FOR THE VALUATION ACTUARY**

In addition, it is necessary to assign to each cell a set of default rates, a set of rates of "extension" of balloon payment mortgages and a functional representation of any relevant prepayment options. While it may not be necessary to create separate cells for all varieties of prepayment options, it probably is necessary to separate noncallable bonds from callable bonds, etc.

One way to determine logical groupings for existing assets is to consider the list of data available on each security. Table 4 consists of several data request forms actually used in a valuation project. From examining the bond data request form, it is apparent, for example, that the criteria that are available for creating groups of bonds include:

- Public or private status,
- Years to maturity,
- Ratings,
- Call provisions, and
- Sinking fund provisions.

Of course, these data request items are merely representative. A request list should be developed that reflects the requirements of each specific project.

Although care should be used in modeling them, existing assets often have less impact on the results of a projection than might be supposed. In fact, the existing assets run off early in the projection. For this reason, it is not critical to the success of the model that all aspects of existing assets be faithfully reproduced.

**TABLE 4**

**Data Request Forms**

**Bond Data Request**

- |     |                   |                              |
|-----|-------------------|------------------------------|
| 1.  | CUSIP             |                              |
| 2.  | Type              | Treasury/GNMA/Public/Private |
| 3.  | Description       |                              |
| 4.  | US or Foreign     |                              |
| 5.  | Rating            | S&P or Moody's               |
| 6.  | MSVR Class        |                              |
| 7.  | Date of Purchase  |                              |
| 8.  | Purchase Price    |                              |
| 9.  | Coupon Rate       |                              |
| 10. | Coupon Mode       |                              |
| 11. | Book Value        |                              |
| 12. | Par Value         |                              |
| 13. | First Call Date   |                              |
| 14. | First Call Price  |                              |
| 15. | Second Call Date  |                              |
| 16. | Second Call Price |                              |

Etc.

**Mortgage Data Request**

- |    |                    |                             |
|----|--------------------|-----------------------------|
| 1. | Principal          |                             |
| 2. | Interest Rate      |                             |
| 3. | Mode               |                             |
| 4. | Maturity Date      |                             |
| 5. | Book Value         |                             |
| 6. | Call Date          |                             |
| 7. | Prepayment Premium | As percentage of book value |
| 8. | Balloon Date       |                             |

**ARMs only:**

- |     |                     |                              |
|-----|---------------------|------------------------------|
| 9.  | Index Maturity      | Avg maturity of the AR index |
| 10. | Multiplier          |                              |
| 11. | Spread              |                              |
| 12. | Max Incr per Period |                              |
| 13. | Max Decr per Period |                              |
| 14. | Absolute Max Rate   |                              |
| 15. | Absolute Min Rate   |                              |
| 16. | Next Reset Date     |                              |
| 17. | Renewal Reset Dates |                              |

TABLE 4 (cont.)

Data Request Forms

CMO Data Request

- |    |                                |                          |
|----|--------------------------------|--------------------------|
| 1. | CUSIP                          |                          |
| 2. | Type                           | IO, PO, Z-tranche, Other |
| 3. | Description                    |                          |
| 4. | Coupon Rate                    |                          |
| 5. | Coupon Mode                    |                          |
| 6. | Book Value                     |                          |
| 7. | Principal                      | Current face             |
| 8. | First Principal Repayment Date | (Expected)               |
| 9. | Last Principal Repayment Date  | (Expected)               |

Treatment of Sinking Funds

Break security with sinking fund into pieces with maturities on sinking fund payment dates.

Changes on Data Request:

Maturity Date

Par Value

Call Provisions

Equal to sinking fund payment

Including Book Value

However, one class of assets deserves special treatment: the assets utilized in the reinvestment strategy. As mentioned above, the reinvestment strategy itself is a set of decision rules and fits best in the corporate model segment. However, the programming that defines the cash flows that proceed from each class of asset is normally associated with the existing asset model. It is important that all relevant characteristics of the asset types that will be utilized for reinvestment be fully developed in the model.

MS. RATAJCZAK: Regardless of whether you are projecting your existing assets seriatim or through some form of grouping, you will need to project important asset features to appropriately present the behavior of your assets under different economic scenarios. Many of these features require dynamic formulas.

At one time, bonds were vanilla vehicles that provided their owner an income stream and some promise of principal repayment upon maturity. Bonds are not vanilla vehicles any longer, and the promise of future principal repayment is probably as good as the paper it is written on. If you are projecting the behavior of your bond portfolio for the first time, you will find a bevy of bond types. These would include callable, noncallable, deferred interest, floating rate or variable rate, junk bonds, sinking fund bonds, and convertible bonds and I am sure there are others that I have not included. Each of these types possess characteristics that behave differently in varying interest-rate environments.

To project the cash flows on these bonds, some of the key parameters include call triggers, sinking fund triggers, movement of yields and resulting market values as a result of interest-rate changes, and the hot-button default rates.

A bond will be called or a sinking fund payment made if the issuer feels it is in his or her economic best interest to do so. When interest rates go down, calls and the use of the sinking fund provision generally increase. In the Regulation 126 work we have done over the past few years, the call trigger and sinking fund trigger are modeled dynamically.

In Table 5 it is assumed that a bond will be called if the call market value which is calculated with a 1% spread in the yield rate exceeds the call price of the bond plus a transaction margin. The transaction margin is included to cover the cost to the issuer of calling the bond.

**TABLE 5**

**Sample Dynamic Call Trigger**  
**(Bonds are called if call market value exceeds the call price of a bond plus a transaction cost.)**

**Cash Flow from Calls**

| <u>Year</u> | <u>Level Scenario</u> | <u>Up 5% Scenario</u> | <u>Down 5% Scenario</u> |
|-------------|-----------------------|-----------------------|-------------------------|
| 1           | 112                   | 112                   | 116                     |
| 2           | 30                    | 29                    | 110                     |
| 3           | 14                    | 0                     | 128                     |
| 4           | 39                    | 0                     | 156                     |
| 5           | 15                    | 0                     | 76                      |

For a real portfolio of bonds, this formula produced this pattern of cash flows from calls under the level, up 5%, and down 5% interest scenarios. By ignoring the call features in your bonds, you would extremely understate your cash flows in a down environment. For fixed liability business, such as immediate annuities, this would tend to overstate the surplus development during the projection period.

The popularity of below investment bonds in the 1980s as investment vehicles for insurance companies has added further complexity to projecting future cash flows. These bonds, especially in the current environment, do not behave in the same ways that investment grade bonds do. Lower grade bonds are called due to changes in the credit characteristics of the issuers rather than the result of interest-rate changes. In addition, the yields of below investment grade bonds are said to be more closely correlated with economic cycles and the stock market than they are with underlying Treasury rates.

The introduction of noticeable levels of below investment grade bonds in your portfolio, may indicate the need to project investment grade and noninvestment grade bonds under a different set of assumptions for calls, sinking funds, and yield changes. Two underlying yield curves would be used reflecting the spread between investment grade and below investment grade securities. Call logic for below investment grade bonds could be expressed as X% of all currently callable below investment grade bonds are called. Changes in the yield on below investment grade bonds could be expressed as some spread over Treasury rates plus a fixed component to recognize that below grade bonds do not move exactly in tandem with changes in the Treasury rates.

The past popularity of below investment grade bonds and the recent level of default losses have heightened policyholder, rating agency, and regulator awareness of the impact of default losses on insurance company solvency. In projecting future surplus levels, you may use an allocated share of the assets supporting the mandatory securities valuation reserve (MSVR) in your projections to provide for the risk of asset default. The importance of quantifying the financial impact of bond defaults on your portfolio will depend on the amount of below grade bonds in your portfolio. Moody's and Edward Altman have done recent studies on bond defaults which offer information that may be used to develop bond default assumptions. Using your company's actual experience in developing assumptions would be preferable to using industry studies.

The category of assets that includes mortgages and mortgage pass-through securities has also seen a recent decade of innovation. Mortgages have been packaged into CMOs to offer synthetic interest and principal repayment streams that more closely match the diverse liability side of the balance sheet. Mortgages have been stripped into IOs, or interest only, and POs, or principal only. This category of assets has also been under recent scrutiny as a result of several well-publicized failures. For those of you projecting the cash flows on your mortgage portfolios, we list prepayment rate and default rate as the key projection assumptions. The prepayment provisions in a commercial mortgage arrangement will be

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different than a government national mortgage association (GNMA) bond. Commercial mortgages would tend to offer more balloon payments instead of a somewhat smooth principal payment pattern.

These differences indicate that prepayment formulas would tend to be different for different types of mortgages and should be modeled using different formulas.

The prepayment formulas currently used in Regulation 126 testing have a common characteristic of prepayment rates being a function of some spread between the current yield and the actual underlying mortgage rate. Here are two examples. We used formula A below to project the cash flows for a block of GNMA's. As you would expect, there are significantly more prepayments in a down interest-rate environment when it is to the issuer's benefit to accelerate prepayments (Table 6). The volatility in these numbers once again shows the importance of reflecting all asset features, in your existing asset models.

### Sample Dynamic Prepayment Formulas

A.  $15 \times [\text{Rate on Mortgage} - \text{Current Yield}]$   
6% annual minimum  
48% annual maximum

B.  $5\% + (2 \times \text{SP}) + Z \times (4 \times \text{SP}^2)$

Where SP is spread between coupon and market rate

Where  $Z = 1$  if  $\text{SP} > 0$

Where  $Z = -1$  if  $\text{SP} < 0$

**TABLE 6**

**Mortgages and Mortgage Pass Throughs  
Sample Cash Flows From Formula A**

| <u>Year</u> | <u>Level<br/>Scenario</u> | <u>Up 5%<br/>Scenario</u> | <u>Down 5%<br/>Scenario</u> |
|-------------|---------------------------|---------------------------|-----------------------------|
| 1           | 5.5                       | 5.5                       | 5.8                         |
| 2           | 5.1                       | 5.1                       | 6.4                         |
| 3           | 4.8                       | 4.7                       | 11.0                        |
| 4           | 4.4                       | 4.4                       | 14.1                        |
| 5           | 4.1                       | 4.1                       | 14.1                        |

We are moving in to uncharted territory once we cover the first few categories of assets listed on the balance sheet. For any projection work you have done in the past, we would expect that the assets other than bonds and mortgages were assumed to back surplus or other lines not included in the projection. You can't use this excuse any longer to avoid projecting stocks, real estate and other invested assets. With the exception of preferred stock, which may be modeled like bonds on a seriatim basis, this class of assets may be impossible to model on a seriatim basis. There may be just too many unknowns to make seriatim valuation a worthwhile or meaningful exercise.

We have projected preferred stock as a long-term bond offering a set income stream. The key considerations for common stock are projected future capital gains and dividend income. We all know that stock prices are highly correlated to changes in skirt length and that the stock market always does better on Mondays. These fallacies point to the fact that the behavior of the stock market is sometime unexplainable, or if not unexplainable, then not quantifiable. If you must model your common or preferred stock portfolios, changes in stock indices could be used as a proxy for future capital appreciation. To project future income from your common stock portfolio, you could reflect some growth assumption based on portfolio experience or industry experience.

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You will need to consider the quality of your equity investments when formulating your default assumptions.

Other assets include real estate and Schedule BA assets. Many of the projection considerations are the same as those for common stock. To project future cash flow, you will be required to model future capital appreciation, projected capital infusions and income using your company's experience or some index which may serve as a proxy for the behavior of this class of assets. We have seen virtually no projections done which include this category of assets in the portfolio contributing to future cash flow. As this process matures, we may see certain standards set for projecting the cash flows of assets such as real estate and common stock.

Once you have developed your projection assumptions, you will be required to determine which assets support which lines of business if you are looking at adequacy on a line-of-business basis. You will be required to describe the methods you use for allocation, highlighting inconsistencies from year to year. The Actuarial Opinion and Memorandum Regulation indicates that assets designated to support specific reserves cannot support other reserves applying to segments if segmented. Pro rata allocation is also an acceptable method of designating assets to support specific lines of business. There appears to be nothing in the current regulation that precludes a notional allocation of assets to the lines of business they were notionally purchased to support. The key is your allocation method must be disclosed.

When the term *notional allocation* is used, it means selecting among all assets those assets that most closely match the cash-flow characteristics of your existing liabilities. Let's look at a very simplistic notional allocation. In this situation, all longer-term bonds and mortgages would be deemed to support longer-term liabilities. Long-term bonds and GNMA's would be modeled with structured settlement or immediate annuity business. For shorter-term, interest-sensitive business, a larger liquidity position may be held so a

combination of short-term and intermediate-term bonds and mortgages could be allocated. For products with little reinvestment risk such as health insurance or term insurance, the remaining short-term assets would be allocated.

### **Development of the Present Value Model**

**MR. DICKE:** So far, we have been discussing the modeling of a static situation: a block of in-force liabilities or a group of existing assets. To bring in dynamics, we have to follow the development of cash flows from these liabilities and assets into the future. And to do this, we have to focus on the influences that shape the future, both external and internal to the company.

The most powerful external influence is the economy. The influence of the economy enters an actuarial model through a *present value model*. Every actuarial model may be thought of as a model of actuarial risk (such as mortality or morbidity), together with a present value model. The present value model incorporates the actuary's view of future economic developments. It is important to note that the present value model is specific to a "person." Any model that produces a value must represent the point of view of an evaluator. In the case of an actuarial reserve calculation, the "person" is an amalgam of the valuation actuary and the state Insurance Department. That is, the actuary is responsible for the valuation, but must satisfy regulators that his or her view of the future is not overly rosy.

Present value models are usually presented in terms of future interest rates. Indeed, the simplest present value model is the set of discount functions that we all know how to derive from a single interest rate. This simple model is extraordinarily useful and powerful, as all actuaries know. It does, however, have its limitations. First, it can be dangerous to use if the cash flows extend beyond the duration of available assets. In such cases, a sequence of interest rates may be used. More important, such models are not capable of incorporating uncertainty. For this we need some form of stochastic present value model.

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Present value models may be thought of as falling into three categories: nonstochastic, stochastic, and "semistochastic."

*Nonstochastic present value models* include single interest-rate models and "split-rate" models (10% for 15 years, then 5%). *Stochastic models* are of two basic types: *random scenario models* and *option pricing models*. We will return to these.

By "*semistochastic model*," we mean a model similar to the infamous "New York Seven." The New York Seven is, of course, a model in which seven future scenarios are singled out from among the infinite variety of possible scenarios and are assumed to provide a reasonable picture of the range of potential results. Another example of a semistochastic model is any model based on a set of "handmade" scenarios. Such models are often used by actuaries to test the results of a valuation. Semistochastic models are probably best thought of as sensitivity tests relative to nonstochastic models. In any case, they are not only powerful, but may well be sufficient to satisfy your responsibilities under the Standard Valuation Law. The main thing such models can't do is produce a single value. Generally speaking, the average of the results obtained under the New York Seven, or under any other set of handmade scenarios, is completely meaningless. In this sense, semistochastic models are less powerful than nonstochastic models.

Either a stochastic or a semistochastic model may be sufficient for the valuation actuary's purposes. However, in either case, the model must incorporate at least one more source of complexity: yield curves.

A *yield curve* is a plot of bond yield versus years to maturity. Nonlevel yield curves, i.e., situations in which long-term rates and short-term rates differ, are derived at least partly from uncertainty. Expectations regarding inflation, for example, are often cited to explain an excess of long-term over short-term interest rates.

Although data regarding the yields of coupon bonds are easier to procure, it is nevertheless best to base yield curves on *spot rates*. These are, in effect, zero-coupon bond rates. Coupon bond yield curves are inconvenient because their shapes depend upon the ratio of coupon to maturity payment.

One of the most important effects associated with a yield curve is *inversion*. An inversion occurs whenever short-term rates exceed long-term rates. Many of you (though by no means all!) are old enough to have been aware of the persistent inversions of the early 1980s. These inversions occurred simultaneously with the highest interest rates in U.S. history. However, inversions also occur in times when rates are more "normal." Treasury-rate inversions occur with surprising frequency. Most people are not aware of this situation since most people focus on yield curves for corporates rather than Treasuries. The risk premium received for corporates increases with duration, thus masking Treasury-bond inversions.

It is essential to represent yield curves in a present value model used for cash-flow testing purposes, so that the impact of inversions can be studied. But how is this done? One widely used approach is to model short-term and long-term rates separately and to form a yield curve by interpolation. Since yield curves are usually "humped," it is common to interpolate with two line segments. Such interpolation approaches are used both with semistochastic and with random scenario models.

Option pricing models incorporate yield curves automatically. In fact, in order to be valid, an option pricing model must be able to reproduce the actual current yield curve without introducing internal inconsistencies. This is beyond our scope this year. However, research is being undertaken that may lead to an option pricing model on which a small company actuary can depend to fulfill his or her valuation responsibilities. If this research is successful, the ability to use such models correctly (as opposed to the ability to construct such models) may be especially important for small company actuaries.

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Random scenario models are the stochastic models most commonly used by actuaries today. These models use a probability distribution to project interest rates from a fixed starting level into the future. A common probability distribution used for this purpose is the lognormal distribution. Actually, the use of this distribution only means that the logarithm of percentage changes in interest rates is expected to be normally distributed. There is literature purporting to show that historical Treasury rates really are lognormally distributed, but a recent article in the *Transactions* takes issue with this. In any case, a lognormal random scenario model is probably superior to most currently available alternatives.

The *Proceedings* of the 1987 Valuation Actuary Symposium contain a worked out example of the use of a lognormal bivariate model, in case you want to make your own model from first principles.<sup>1</sup> At least two available software packages have this feature; you might want to look into them. To use a lognormal model, there are a few quantities you need to understand. First, there is volatility. This is the degree to which interest rates change from period to period and is represented in the model by the standard deviation.

In a bivariate model, the degree of correlation must also be selected. Some models also allow the actuary to specify a level to which yields will tend in the long run. This feature, called reversion to the mean, is useful in preventing long-run dispersion of yields. However, it must be used with caution. Supplying any target yield curve that differs from the current yield curve builds a significant bias into the model.

In a present value model, perhaps even more than in the other models, it is essential for the actuary to ensure internal consistency. In particular, the model must reproduce the

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<sup>1</sup> The formulas in Slide 5, page 22, of the 1987 *Proceedings* should read as follows:  
 $i_{t+1} = i_t \exp(x - T_t^2/2)$ , etc. The author wishes to thank Mr. Alexander Scheitlin for pointing out this error.

current yield curve and must not build in opportunities for risk-free investment profits (called *arbitrage*). These kinds of inconsistencies can enter the model in subtle ways -- vigilance is essential! It is especially important to check for arbitrage possibilities if an optimization routine is in use. Such routines will inevitably search out internal inconsistencies and maximize their impact. Elimination of arbitrage opportunities is, of course, one of the first steps in constructing an option pricing model.

### Development of the Corporate Model

The corporate model may be thought of as a set of decision rules: crediting strategy, reinvestment strategy, reinsurance strategy, and shareholder dividend strategy.

Each of these strategies represents an area in which management is free to make decisions. The actuary's role is to reflect back the impact of alternate strategies.

These strategies are assumptions, but they differ from the actuarial assumptions employed elsewhere in the modeling process in that the strategies are not estimates of future uncontrollable events, but rather a description of choices to be made in the future by management. In fact, it is a reasonable modeling procedure to define decision rules that react to situations in the future. For example, a reinvestment strategy that allows for different choices in a time of inverted yield curves and in "normal" times is an eminently reasonable model. Unfortunately, there are instances of regulators disagreeing with this approach. If the modeling is being done to satisfy a regulatory requirement, it is obviously important to understand any special limitations that may apply.

In addition to decision rules, the corporate model usually reflects aspects of the modeled entity that can only be estimated in the aggregate. An example is federal income tax. Also, it may be useful to define aggregate measures of risk and return at the corporate level if the model is being used for strategic planning. Meredith will discuss the

determination of reinvestment strategy in more detail and will also discuss the use of assumptions.

**MS. RATAJCZAK:** You may rely on your investment department for the reinvestment assumptions that are used to test for asset adequacy. These assumptions take the form of types of assets, maturity periods, spread over some underlying yield curve, and asset quality. (The assumptions listed are for *types, periods, spreads, and quality.*) Some frequently used strategies include long-term bonds and mortgage-backed securities for single premium immediate annuities (SPIAs) and life insurance, and a combination of cash and intermediate bonds for deferred annuities or other lines where liquidity needs are greater and the duration of the liabilities are shorter.

Your reinvestment strategy should also address how current company policy would change in differing interest-rate environments. In a rising interest environment, the tendency may be to invest long term with the converse in a decreasing interest-rate or inverted interest-rate environment. We have seen a number of instances in recent valuation actuary work where there is a switch to short-term investing in a decreasing interest-rate environment.

In some scenarios and in some lines of business, negative cash flows are a real possibility. The most common method of handling negative cash flows is to assume that assets are sold to cover the shortfall. Assets can be sold on a pro rata basis or sold to minimize capital losses/maximize capital gains. This strategy necessitates monitoring the market value of your assets from period to period. Another alternative for handling negative cash flows is to assume the company borrows short term to cover the shortfall. Loan rates would be based on some underlying curve with some additional borrowing cost. This method has a caveat because in some scenarios those short-term loans become long-term loans that are never paid off. You may, in fact, benefit in these scenarios if you are not charging a realistic loan rate for long-term loans.

## **MODELING: BASIC TRAINING**

Contrary to New York Regulation 126, the Actuarial Opinion and Memorandum Regulation states that aggregation of results for all lines of business is an acceptable basis for forming the opinion that the liabilities, when considered with their supporting assets, make adequate provision for future obligations and expenses. Two methods can be used for aggregation. The first is aggregating liabilities and assets before testing for asset adequacy. This would be akin to a total company projection. The second method involves aggregating results of asset adequacy analyses for various lines of business. To use this method, the risks of aggregated lines must be mutually independent and the results must be developed under consistent interest scenarios. Since the regulation suggests that the qualified actuary must at least consider results under the seven interest scenarios used in Regulation 126 testing, you will be required to test even your simplistic models under these scenarios before you can aggregate your results.

As part of your memorandum, you are required to disclose which of these aggregation methods served as the basis for your opinion.

