1991 VALUATION ACTUARY
SYMPOSIUM PROCEEDINGS

SESSION 2

Developing Confidence in Results of Multiscenario Testing

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DESTROYING CONFIDENCE IN RESULTS OF MULTISCENARIO TESTING

MR. GORDON E. KLEIN: Cash-flow analysis, as it is currently discussed and practiced, is the use of a stochastic econometric model to test the adequacy of reserves on insurance products with interest-sensitive cash flows. A maximum acceptable probability of failure is selected, say x%. The cash-flow model is run a number of times, each time generating a possible level of surplus at the end of the projection period. An estimator of the x-th percentile of the probability distribution of final surplus is calculated. This process is repeated iteratively with the initial asset level varying until the x-th percentile of the distribution of the final surplus is 0. The conclusion drawn is that, with that level of initial assets, the probability is x% that the surplus will be negative on the block of business being tested. In other words, the probability of failure is x%, and the probability of success is (100-x)%. The title of this session is "Developing Confidence in Results of Multiscenario Testing." My part of the session is better entitled "Destroying Confidence in Results of Multiscenario Testing," at least multiscenario testing as it is currently practiced. I am going to discuss several theoretical problems with the bases of the current methodology of cash-flow analysis, or multiscenario testing. I hope this will help stimulate discussion among actuaries about the appropriateness of cash-flow analysis and the appropriateness of insurance companies selling the types of products whose emergence led to the development of cash-flow analysis.

Arbitrary Probability of Failure

Cash-flow analysis is used as a test of reserve adequacy. Let's look at a hypothetical example in order to see one of the shortcomings of this type of analysis. Let's say that an insurance company sells you, its only policyholder, a $1,000,000, one-year, nonrenewable term policy with no cash-surrender value. You are a 30-year old male, so the net single premium is around $1,688 using 1980 CSO and 5% interest. How much would you want the company to have in assets in order to assure you that the death benefit would be paid? Since you are the only policyholder of the company, the only way that you could be assured of payment of $1,000,000 upon your death is for the company to have $1,000,000 in assets backing up the policy now, for those assets to be liquid, and for them to be free from any risk of decreasing in value to less than $1,000,000. Assets of $1,500,000 in real estate or private placement bonds or junk bonds would probably not provide the kind of assurance that you want that the $1,000,000 death benefit would be paid, since these assets are either illiquid, subject to depreciation, or both.

Now, let's subject this example to traditional actuarial analysis. First of all, the actuarial present value of future benefits is $1,688. This is clearly not an adequate asset level for the company. If you died within the policy year, the company would be unable to meet its obligations. This inability of the company to meet its obligations, or failure of the company, is the type of situation that actuaries are supposed to be able to identify and quantify the risk of. The reason that the actuarial present value of the benefits is not an adequate amount of assets for the company to hold is that it is merely the expected value of the present value of the death benefit. The expected value is not meaningful in this case, since there is only one policyholder.

If we were to run a cash-flow analysis on this case, with an acceptable probability of failure of 1%, that is, a required probability of success of 99%, a funny thing would happen. With an initial asset of 0, the probability of failure is less than 1%. This is because the probability of death is only 1.73 per 1,000. So we find that any level of initial asset at all would be enough to pass this cash-flow analysis.

This raises a question which has not been prominently discussed concerning cash-flow analysis: What is the appropriate level at which to set the acceptable probability of failure? In a typical cash-flow analysis on single premium deferred annuities (SPDAs), the probability distribution function of the final surplus has a long tail to the left. The level of initial surplus required to achieve a particular desired probability of success is extremely sensitive to the probability chosen. If the appropriate probability of success is 99.9%, and the analysis is run using 99%, then the results are meaningless and misleading. In the example being discussed, if the desired probability of success is 99.9%, then the lowest initial asset that achieves this probability is around $980,000, assuming this asset is liquid and earns 5% interest with no risk of decreasing principal. Since the required probability of success is selected arbitrarily, and the resulting necessary initial asset is quite sensitive to it, cash-flow analysis is subject to manipulation. It is also likely that actuaries are setting the required probability of success too low. Thus, the first problem with the current methodology of cash-flow analysis is that it is extremely sensitive to the required probability of success, which is arbitrarily selected.

Given a required probability of success, the currently accepted methodology of cash-flow analysis introduces many simplifications which bias the results toward too low of a required initial asset. These biases will be dealt with in the remainder of this presentation.

Lack of Independence

If we change the example of a one-year term policy by adding another 999 identical policyholders, then we can use the normal approximation to the binomial distribution to calculate the amount of assets that must be held in order to achieve a desired probability of success. For example, if we want this probability to be 99%, then the company must have initial assets of about $4.7 million. This is about $4,700 per policy, and is 176% over the actuarial present value of the benefits, which is about $1.7 million, or $1,700 per policy. As the number of policyholders increases, the necessary percentage loading over the actuarial present value of benefits decreases.

It is important to note, however, that the use of the normal approximation is an application of the central limit theorem, and as such, its validity depends on the conditions of that theorem being met. One of these conditions is that the random variables be mutually stochastically independent. If the 1,000 policyholders are all subject to similar, nonindependent risks, then the central limit theorem does not apply, and the initial asset necessary to achieve a particular probability of failure is understated by the use of the normal approximation.

Life insurance insures against a risk, mortality, that is generally assumed to be independent among policyholders. While this is not strictly true, it is close enough for most if not all purposes. Earthquake insurance is generally recognized as covering nonindependent risks.

The probability that policyholder A has a claim, given that B does, is much higher than the unconditional probability that A has a claim. It should be obvious that the lapse or withdrawal probabilities for a number of SPDA policyholders are not from independent distributions. Given that policyholder A surrenders, the probability that B does is higher than the unconditional probability that B does. This is because B is subject to the same economic scenario and motivations that A is. If A surrendered due to unemployment, then it is quite possible that B will surrender for the same reason. More importantly, if A surrendered because he heard that B did, then C is much more likely to surrender upon hearing this than if A and B hadn't surrendered. Thus, SPDAs are more like earthquake insurance than life insurance. If a company sold only earthquake insurance, only in the San Francisco area, its assets would have to exceed its potential exposure in order for anyone to have confidence that its obligations would be met. Likewise, if a company issues largely GICs and SPDAs, then it should have marketable assets whose market value exceeds the cash-surrender value at all times. Otherwise, when it becomes known that this isn't the case, a run on the bank will ensue. A reserve methodology that ignores this, as cash-flow analysis generally does, will understate the initial asset required to achieve a particular desired probability of success.

More Lack of Independence

An area where stochastic independence is currently and faultily assumed is in the provision for default on junk bonds. Cash-flow analysis was invented to test for exposure to interest-rate fluctuations. Other sources of insolvency, such as bond defaults and depreciation of bonds and real estate are handled very poorly by cash-flow analysis. This is largely because they were not thought to be much of a problem when the methodology was first developed.

The usual procedure for recognizing the probability of default on junk bonds is to assume that a deterministic percentage of each bond defaults each year. The percentage varies by bond, based on the riskiness of the bond, but it is the same for a given bond every year. This is not, of course, how bond defaults work in real life. First of all, they are very low in many years, and very high in a smaller number of years. By assuming that they are some average figure every year, this element of variability is eliminated from the model. Second, defaults among bonds are highly correlated. The probability of default on bond A, given that bond B defaulted, is much higher than the unconditional probability that A defaults. By ignoring this, another source of variability in the distribution of surplus at the end of the projection period is ignored.

Any time that the variability of the distribution of final surplus is reduced, the initial asset required to achieve a desired probability of success is understated. This is true because we are estimating a percentile of the distribution that is in the left tail of the distribution. So far, we have seen three examples of simplification in the cash-flow analysis model that bias the results away from variability and toward an understatement of the required initial asset.

Replacement of Random Variables with Their Expected Values

Current cash-flow analysis methodology involves the use of randomly generated interest-rate paths. Several variables are dependent on the path that interest rates take. These include the prepayment rate on mortgages and mortgage-backed securities, the credited rate on interest-sensitive products, and the surrender rate on interest-sensitive products. These variables are generally represented in the model deterministically. For example, it might be assumed that mortgages with an 11% interest rate will be prepaid at a rate of 5% if they can be refinanced at 11% interest, while they will be prepaid at a rate of 15% if they can be refinanced at 9% interest. These rates are assumed to hold every time a stochastically generated interest-rate path leads to these interest rates.

A more theoretically correct approach would be to recognize that the prepayment rate on mortgages and the lapse rate on annuities are random variables which are conditional on the level of interest rates. The current approach is to replace these random variables with their conditional expected values. But the replacement of a random variable by its expected value always reduces the variability of the results. Cash-flow analysis is in fact an improvement over the prior methodology which used the expected value of the interest rate instead of the entire range of its probability distribution. However, the randomness of other variables still needs to be recognized. Once again, the failure to do this leads to an understatement of the level of assets required to achieve a given probability of success over the projection period.

Overly Simplistic Functions Relating Parameters

Much of the typical cash-flow model is based on simplistic assumptions as to the relationships among variables. For example, it is often assumed that the rate of mortgage prepayment is a quadratic function of the difference between the interest rate on the mortgage and the current interest rate on similar mortgages. A maximum prepayment rate is then established in recognition of the obvious fact that the rate can't actually be a quadratic.

Newton discovered that the distance traveled by a falling body is a quadratic in the variable time. Economists, actuaries, and other social scientists seem to be enchanted with the simplicity of Newton's discovery and expect that they will find equally simple relationships among the variables that they study. But a variable such as the prepayment rate on mortgages is far more complex than can be expressed by a quadratic function in one variable. For instance, the prepayment rate at a particular time obviously depends not only on the difference between the level of interest rates at that time and when the mortgage was initiated, but also on the condition of the economy. When people are unemployed, or the value of their houses has fallen, they can't refinance their mortgages, no matter how attractive the rates. Likewise, when the economy is booming, people will often sell their houses to relocate to better jobs, even though they had attractive interest rates on their mortgages.

Currently accepted methodology for cash-flow analysis ignores many relevant economic variables, such as the unemployment rate and the growth rate in gross national product. This is largely due to the desire for simplicity. But because the objective is to estimate a tail probability, this simplification again leads to an understatement of the initial asset necessary to achieve a given probability of success.

Distribution of Interest-Rate Changes

One of the most serious flaws in cash-flow analysis as it is generally practiced is the assumption that interest-rate changes are independent, lognormally distributed random variables. That they are not so distributed has been shown repeatedly in the financial literature over the last three decades. The tails of the actual distribution of interest-rate changes contain far greater probability than the tails of the lognormal distribution do. In other words, for example, jumps of say three standard deviations or more are far more common than would be expected if the underlying distribution were lognormal. This is the same phenomenon that is observed in the stock market with "crashes."

Benoit Mandelbrot was probably the first to notice this phenomenon. In 1963, he published an analysis of the relative changes in cotton prices which showed that their distribution was too "fat-tailed" to be lognormal. He proposed the Stable Paretian Distribution as an alternative to the normal. The Stable Paretian is actually a family of distributions that includes the normal as a special case. The non-normal members of the family are fatter-tailed than the normal.

I have recently submitted a paper to the *Transactions* which provides theoretical and empirical justification for the use of a non-normal Stable Paretian Distribution for the logarithm of the change in interest-rate relativities. Without going into as much detail as would be necessary to thoroughly discuss this issue, I will just illustrate the magnitude of the difference that results from using the normal distribution instead of the Stable Paretian. I ran two cash-flow analyses, identical in every respect except for the assumption of the distribution of changes in interest-rate relativities. The block of business consisted of $10,000,000 in SPDAs. Using the normal distribution for interest-rate changes, an initial asset of $10,650,000 was necessary to achieve a 99% probability of success. Using the Stable Paretian distribution, an initial asset of $15,250,000 was required to achieve the same 99% probability of success. In other words, currently accepted cash-flow analysis methodology revealed the need for an additional reserve of 6.5% over the book value of the liabilities, while the Stable Paretian methodology revealed the need for an additional reserve of 52.5%. The Stable Paretian distribution isn't a perfect model for interest-rate changes, but it is better than the normal, and this analysis reveals the extreme sensitivity of the results of a cash-flow analysis to the distribution chosen to model interest-rate changes.

Mean Reversion

An absurd notion that enters many cash-flow analyses is that of mean reversion. Before discussing it, though, we should look at a little background in order to discover why mean reversion has been considered necessary.

In a cash-flow analysis, it is assumed that each interest-rate change is independent from all the others and identically distributed with them. Combining this assumption with the assumption of lognormality, we arrive at the result that the variance of the change in interest-rate relativities over n periods increases without bound as n increases. This conclusion is the result of the assumption of independence. Because this result leads to very high interest rates, as well as negative ones, artificial bounds are placed on how high or low interest rates can go. In order to prevent the situation where the model gets stuck at the upper or lower bound, and as a result of some wishful thinking, some actuaries introduce mean reversion into the model.

Mean reversion is a technique where an interest rate is generated using the stochastic model, but is then adjusted toward some hypothetical mean. This can be accomplished either by adjusting the stochastically generated interest rate toward the alleged mean or by using a Markov chain of transition probabilities. It is alleged that when interest rates are higher or lower than this mean, they tend to fall or rise toward it. I am not aware of any statistical evidence supporting this technique. If there is any, I would like to know what the mean is to which interest rates are going to revert, so that I can take advantage of it. However, I think that I can illustrate the absurdity of mean reversion with an example.

Let's say that you are the actuary for the XYZ insurance company, and you are running a cash-flow analysis as of the end of 1967. Interest rates on long-term Treasury bonds have always been under 5%, except for the last three months. Consequently, you decide to start your model with a long-term Treasury rate of 5.36%, the rate in December, but to use mean reversion to always pull interest rates up or down toward some mean rate, say 4.5%. This sounds okay, but in the 24 years since the end of 1967, interest rates have not yet reverted to that mean, that is, they have always been over 5%. A more recent example would be the six years from November 1979 through November 1985, when you would have waited in vain for interest rates on 30-year Treasury bonds to revert to a mean of something under 10%.

Of course, mean reversion acts to constrain the range of results. In other words, it reduces the variability of the probability distribution of final surplus, and thereby introduces yet another downward bias into the estimate of the amount of assets required to achieve a given probability of success.

Misapplication of Central Limit Theorem

Once the parameters of the cash-flow analysis model are selected, and the required probability of success is selected, a number of stochastically generated scenarios are run. Each of these scenarios yields one possible level of surplus at the end of the projection period. If enough scenarios are run, the probability distribution of all possible final surpluses can be estimated from those that were actually obtained in the simulation. The x-th percentile of this estimated distribution is then used as an estimate of the value that will be exceeded in all but x% of the cases. If the process is repeated with various amounts of initial assets until the x-th percentile of the distribution of possible final surpluses is zero, then it is said that the probability of failure is x%, and that the probability of success is (100-x)%.

So far, we have discussed the arbitrariness of the selection of x, the acceptable probability of failure, and several theoretical problems along the way to getting a meaningful sample from the distribution of all possible final surpluses. The final area where current methodology fails is in the estimation of the distribution of all possible final surpluses given a small sample from the distribution.

This process of estimation of the distribution has historically been based on the naive assumption that all probability distributions are normal. Statistical tests such as the chi-squared test can be used to test the hypothesis that a sample comes from a normal distribution. In almost every cash-flow analysis that I've seen, the distribution of final surpluses is skewed to the left, and hence is not normal. For example, a case study was presented at the 1987 Valuation Actuary Symposium in which 40 scenarios were run. The sample mean of the 20th-year surplus for these 40 scenarios was around $90 million. The sample standard deviation was around $43 million. The highest surplus from the 40 scenarios was about $136 million, or about one standard deviation above the mean. The lowest surplus from the 40 scenarios was about negative $86 million, or around four standard deviations below the sample mean. It should be obvious, with no further statistical testing, that the underlying distribution is not normal. The normal distribution is symmetrical, which this distribution clearly isn't, and it is very rare to find an element from a normal random sample that is four standard deviations from the mean. Nevertheless, the presentation concluded that the probability of a negative 20th-year surplus was 1.7%, "based on normally distributed results."

The assumption that the distribution is normal, when it is actually skewed to the left, results in an overstated estimate of the x-th percentile where x is the acceptable probability of failure. The problem here is a misunderstanding or misapplication of the central limit theorem. The central limit theorem can be used to make probability statements concerning the mean of a non-normal distribution with a finite variance, given a random sample of the distribution. The central limit theorem does not state that every distribution is normal, if only you look at it long enough. It cannot be used to estimate the x-th percentile of a distribution, no matter how large a random sample is generated. Unless statistical testing reveals that the distribution of final surplus is normal, then the systematic overstatement of the x-th percentile of the distribution of final surpluses results in an understatement of the level of initial assets required to achieve the chosen required probability of success.

In the example just discussed, the only way to estimate the x-th percentile of the distribution is to run a very large number, n, of scenarios, order the results from smallest to largest, and use the (n)(x)/100th order statistic as the estimate. For example, if you want the first percentile, run 10,000 scenarios and use the 100th smallest one as the estimator.

Conclusion

In conclusion, currently accepted cash-flow analysis methodology is fundamentally flawed. First of all, an acceptable probability of failure is arbitrarily selected. Since the results of the analysis are highly sensitive to the level selected, there should be some reasonable basis for its selection. Second, several biases are introduced into the model, generally due to the desire for simplicity. Most or all of these biases result in understatement of the initial asset required to achieve the arbitrarily selected required probability of success. And finally, the distribution of all possible surpluses is assumed to be normal, generally with no justification, and generally introducing yet another bias away from conservatism.

In my opinion, cash-flow analysis in its current form is useless as a method of setting or checking the adequacy of reserves on interest-sensitive products. It might have some use in making decisions such as interest-crediting strategy or investment strategy. In order to improve the usefulness of cash-flow analysis, the problems that I have discussed need to be addressed. In addition, I think that a more appropriate decision rule should be used than the setting of an acceptable probability of failure. This decision rule ignores a large amount of the information generated by the cash-flow analysis. It only uses the signs of the possible final surpluses, not their magnitudes. A better decision rule would be to create a utility function assigning a utility value to each possible final surplus. Then the decision rule can be to maximize the expected utility at the end of the projection period. In this way, decisions are based on the entire distribution, not just on one point in the tail that is very difficult to estimate. In this light, serious thought should be given to how companies assign their utility functions. In particular, how appropriate is it for insurance companies to be risk neutral or risk seeking, as many seem to be these days? Wouldn't it be more appropriate to assign a large negative utility to any negative surplus, and not to assign as large a utility to very high positive surpluses? If this were done, insurance companies would return to their former status as conservative financial institutions and would likely leave the business of banking to the banks.

PRACTICAL ASPECTS OF MULTISCENARIO TESTING

MS. DONNA R. CLAIRE: A major question in multiscenario testing is how scenarios are enough to give a valuation actuary confidence in the resulting level of required reserves. I think this is somewhat akin to the question of how many angels fit on the head of a pin -- it is an interesting psychological question that does not yet have a good answer.

I have a simple answer to how many scenarios are enough -- the answer is one. The only problem is that one has to pick the scenario that is actually going to occur. Unfortunately, the possibility of that happening is very remote. Therefore we have to make do with the tools we have.

The number of scenarios I think are necessary depend on the use of the scenario testing. All of you are probably familiar with the "New York 7," which are about to become the "Valuation Actuary 7." These are the suggested minimum scenarios to be tested for regulatory requirements. These scenarios are simplistic, and can be said to be unrealistic. However, they do give a baseline picture as to what economic scenarios may cause trouble.

For internal management, a number of additional scenarios may be run, depending on what gives your internal management a comfort level with the results. Rating agencies are also interested in seeing the results of multiscenario testing. Typically, a company will not necessarily run additional scenarios for the rating agencies, but they will share with the rating agencies the results of multiscenario testing, whether it was done to meet regulatory requirements or for internal management.

The type of product will also effect the amount of multiscenario testing required. For example, I would recommend more multi-interest-rate scenario testing for a very interest-sensitive product, such as SPDAs, while less multi-interest-rate scenario testing may be needed for a block of whole life insurance which has variable policy-loan interest rates.

The size of the business is also relevant. Both the actuarial standards of practice and the model valuation law permit little testing on blocks of business considered *de minimis*.

There is a cost -- accuracy tradeoff. My guess is, however, that once both valuation and pricing actuaries use cash-flow testing regularly, virtually all blocks of business will be tested, since cash-flow testing does add to the accuracy of the pricing and reserving.

Considerations When Designing Scenarios

There are a number of items one must consider when designing interest-rate scenarios. The way I do it is start with the underlying yield curve on Treasury issues. This Treasury curve is typically the spot rates for the date I am starting the test, such as September 30 or December 31. From there, the relationships of other asset types that the company owns or expects to purchase is established. For example, noncallable investment-grade bonds may be modeled as 105% of the underlying Treasury rates plus 40 basis points.

There are other assets that do not have as good a fit to underlying Treasury rates. Examples of these are equity investments such as common stock and real estate. For these asset types, one may project expected earnings with little or no relationship to the underlying Treasuries.

Depending on the assets and liabilities being modeled, it may be appropriate to test other yield curves, such as inversions. For example, this would be important when one borrows if there is negative cash flow, so that the affect of inversions on portfolio rates can be seen.

As I stated before the number of curves depend upon the use of the testing. Option pricing and price-curve analysis can also be done to determine the expected surplus position. These can be better demonstrated with an example.

Results Using Model SPDA Portfolio

In order to more clearly show the results of different types of testing, I developed an example using a model SPDA portfolio (Chart 1). The liability product was $100 million of a vanilla SPDA issued in 1991. There are a couple of items to point out regarding the assets: For this model portfolio, I used an assumption of 50% investment-grade AA to A bonds, 50% low-quality bonds, with an average quality around BB. This may sound like a lot of low-quality assets. However, this portfolio, like many insurance companies' portfolios, had one-third of its assets as commercial mortgages. Commercial mortgages do not have a Standard & Poor's or Moody's rating, but one can get an idea of the equivalent rating on these assets by checking out the defaults on them. The ACLI does a quarterly review of defaults on mortgages. The last results were grim: Over 5% of the mortgages were delinquent. This sample portfolio also had some publicly traded junk and some private placements that were in NAIC Categories 3 and 4. A further description of the assets is given in my sample actuarial report, which is in Session 1.

CHART 1

Model SPDA Portfolio

Premiums:	$100 million in 1991
Surrender Charges:	7%, 6%, 5%, 4%, 3%, 2%, 1%, 0
Expenses:	$200 1st year, $50 thereafter
Commissions:	6%
Spread:	200 basis points
Assets:	50% Inv't grade, 50% low quality
Profit:	15% IRR, 2% of premiums

This model company also had a profit goal of 15% internal rate of return, or 2% of premiums. In order to meet this goal, the spread was established at 200 basis points. This spread reflects an additional margin of 50 basis points needed to cover expected additional defaults.

The first test I did was examine the results under the New York 7 (Chart 2). For those of you who have not filed under New York's Regulation 126 the New York 7 interest rates are level, pop-up, pop-down, gradually up, gradually down, cup and cap scenarios. The results of this model testing were probably as expected: The level scenario showed good results (Chart 3). In addition, in scenarios where interest rates went down, the assets were worth more, and the expected present value of surplus was higher. However, where the interest scenario varied or increased, the results were not as good. By the way, there is a

CHART 2

New York 7 Scenarios

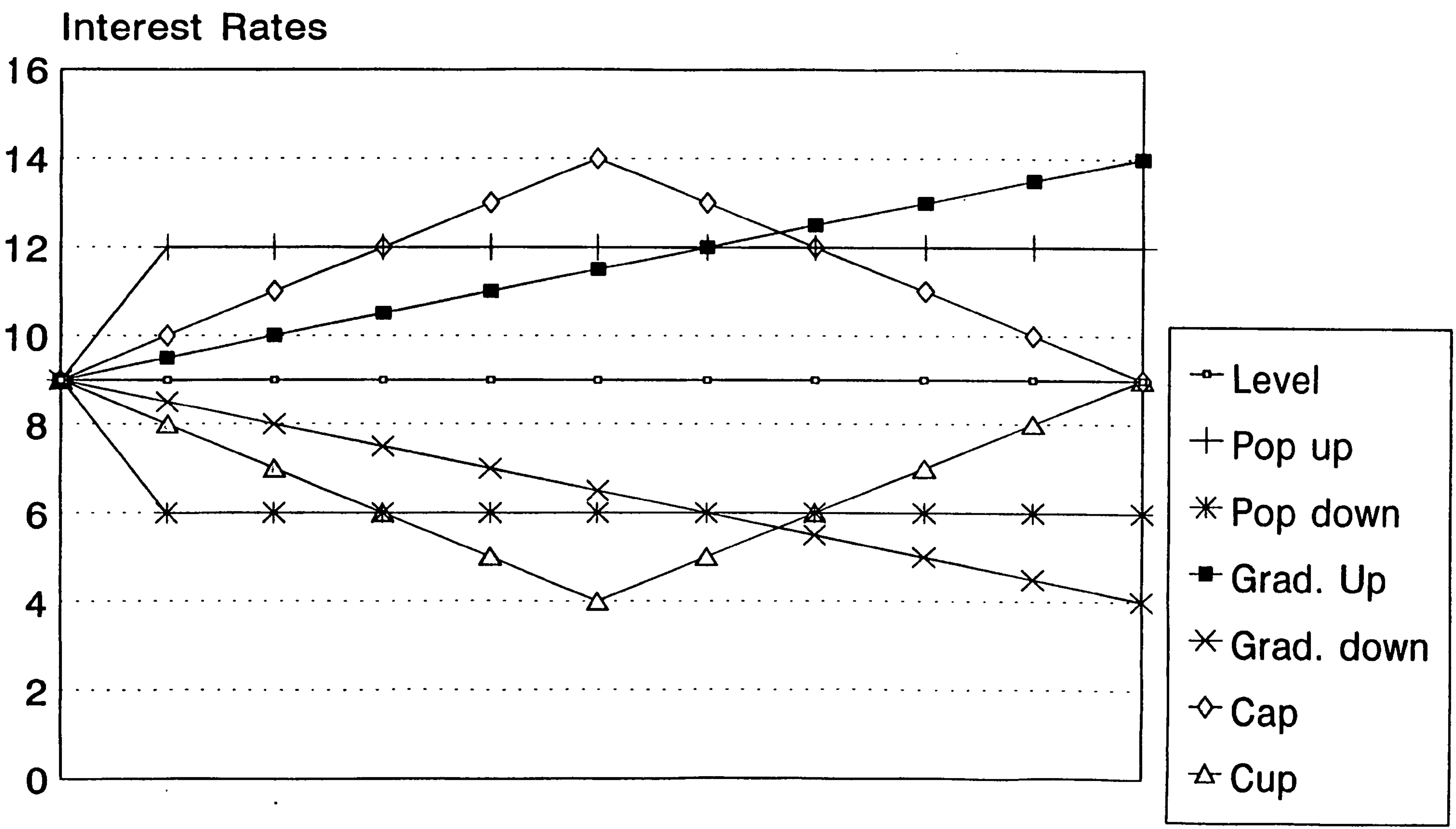
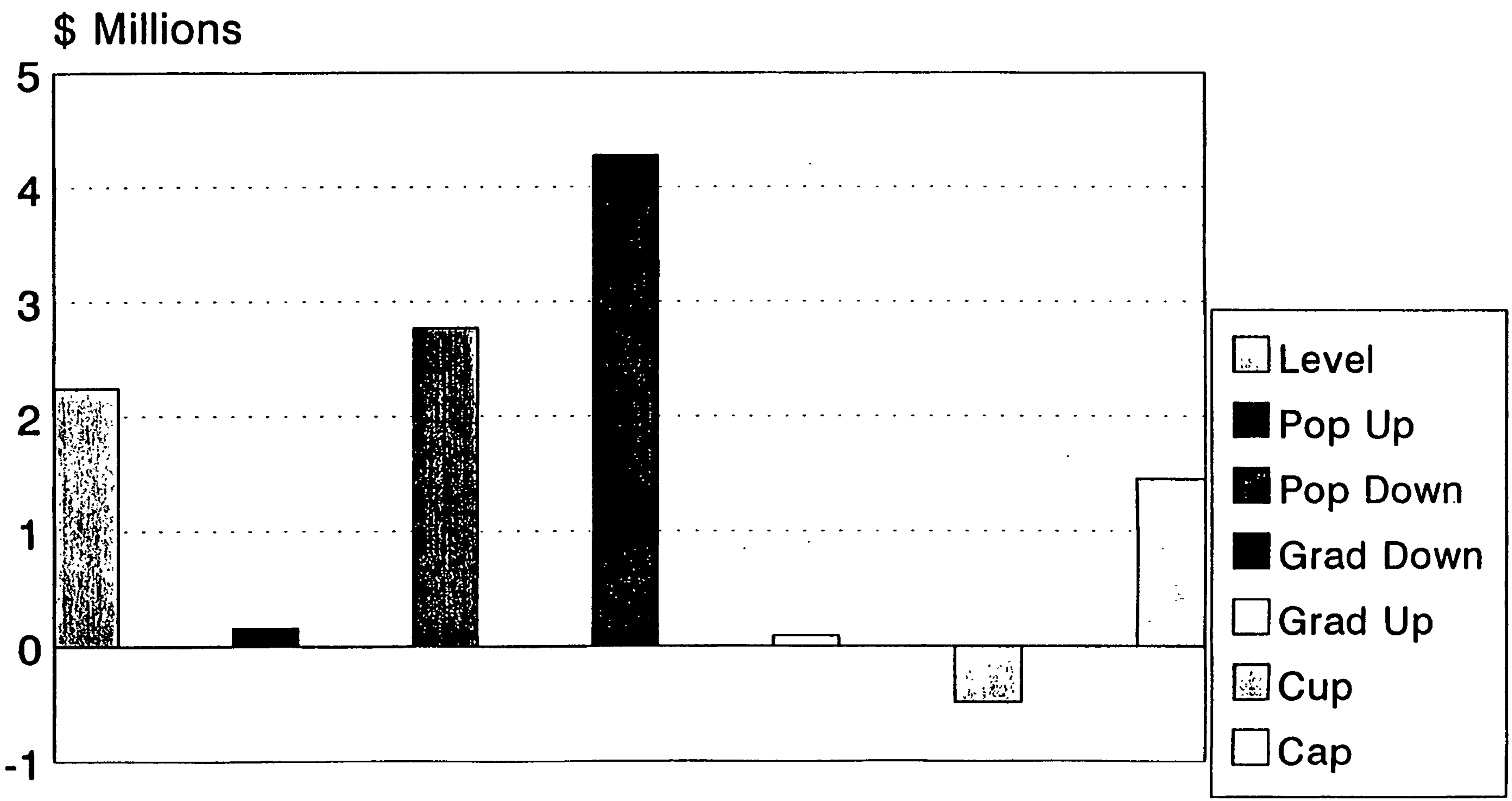


CHART 3

Model SPDA Results
Present Value of Tenth Year Market Value Surplus



question as to whether the reserve level should be sufficient to cover all 7 scenarios. On a companywide basis, the answer probably should be yes. However, I do not think it necessary for each product to pass this test on its own.

The next set of tests I did was to establish 99 random scenarios. If one is setting up random scenarios, it is important to set up reasonable parameters. For these tests, I started with the interest-rate curve on September 30, then assumed a lognormal distribution with a 16% volatility factor. This volatility factor has a significant impact on the results. I used 16%, which represented the type of volatility we had in the early to mid-1980s. The results are shown on the graph (Chart 4). It is interesting to compare these with the results under the New York 7 (Chart 5). In this particular instance, the New York 7 were probably a reasonable representation.

Is 99 scenarios enough? Too much? I am not sure. I am concentrating on the practical side of things, and even 99 scenarios take several hours to run. One interesting paper on this subject was written by James A. Tilley, called "An Actuarial Layman's Guide to Building Stochastic Interest Rate Scenarios." This paper was written for the International Actuarial Conference. He states that one can come up with reasonable results using stochastic interest-rate scenarios by using a model with stratified sampling and a limited number of paths. I just received a copy of this paper recently, so I have not yet tried to implement his method, but it does seem intuitively appealing. Gordon Klein will have more to say about the theoretically proper number of scenarios in his talk.

The next type of trial I did was option pricing. Option pricing is typically thought of as a single number, obtained from projecting interest rates on a binomial lattice. The problem is that one needs to project not only a single rate, but also a whole curve of interest rates, resulting in a cone of interest rates. Using a simplified option-pricing model, I came up with the option price of this block of business of $1.6 million. This was nice, but not particularly informative. It did not show where there was a possibility that interest rates would be inadequate. As with any other method of projecting rates, the most important

CHART 4

Present Value of Tenth Year Market Value of Surplus

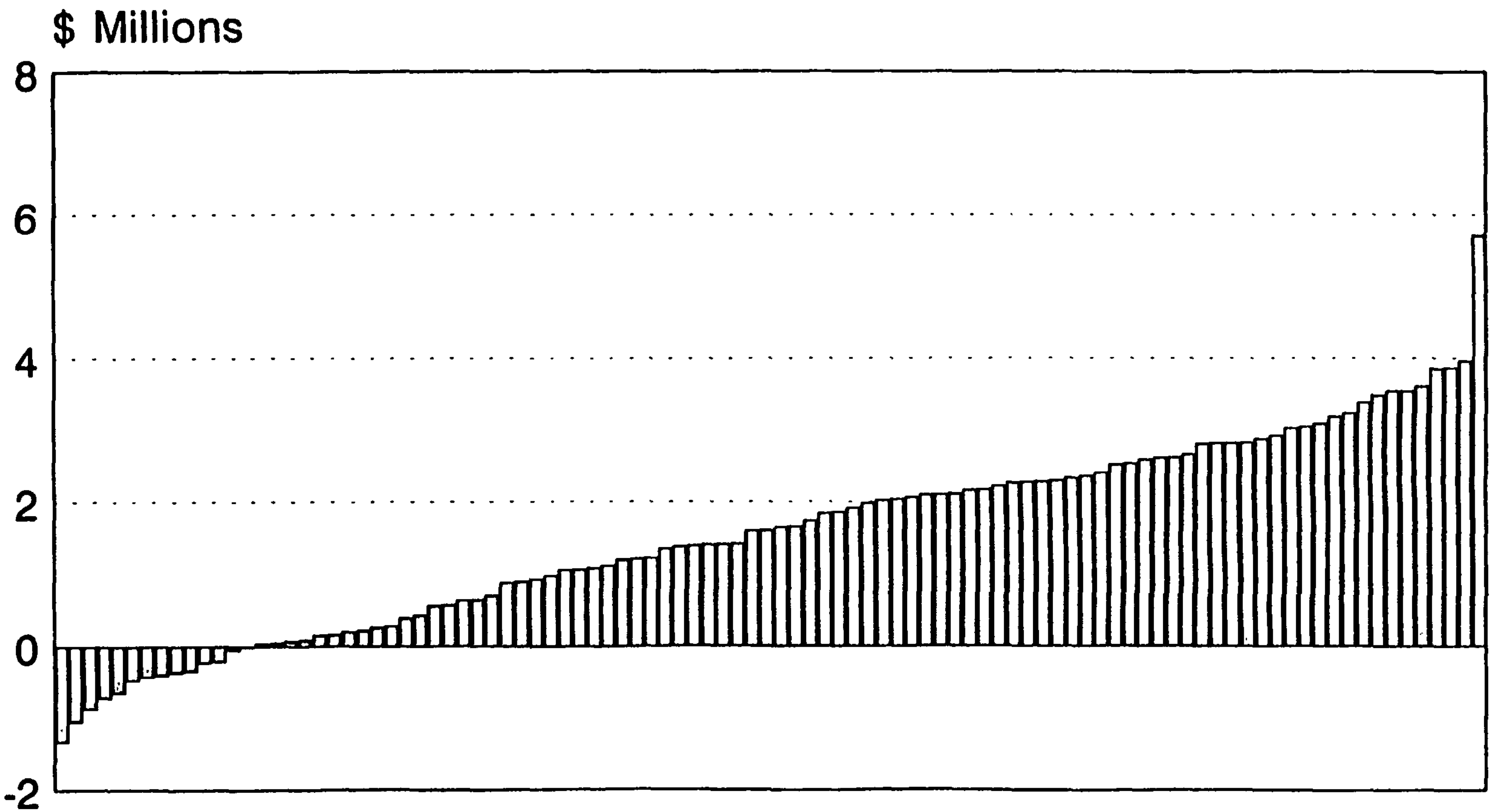
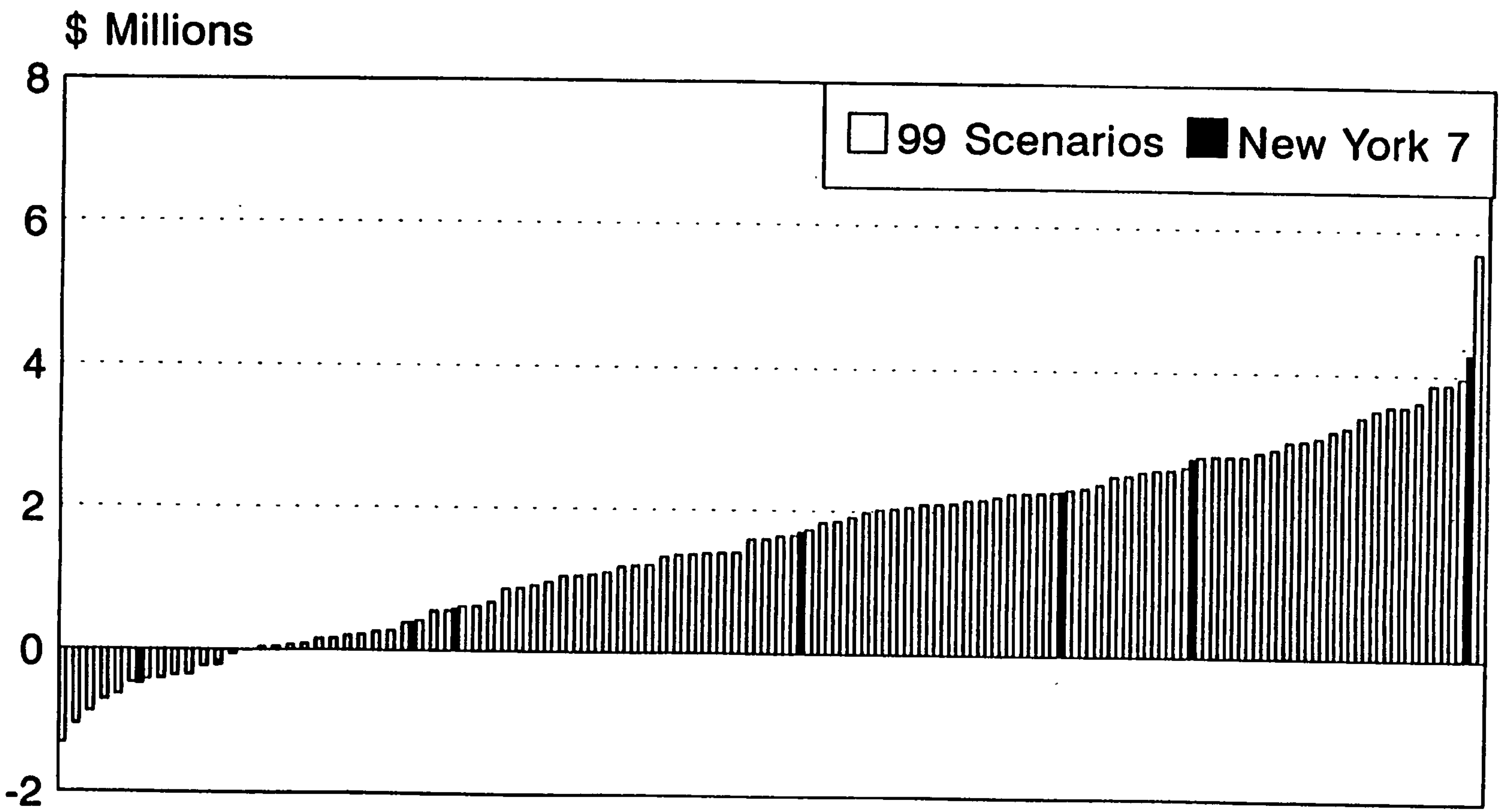


CHART 5

Present Value of Tenth Year Market Value of Surplus



thing to remember is GIGO -- garbage in, garbage out. Depending on the assumptions used in developing the option paths, the answer may or may not be meaningful.

Building on the option-pricing model, one can do price-curve analysis. For all of you who thought you have seen this before, yes, I am using the Chalke PTS software for the price-curve analysis model. This uses option-pricing techniques and can be used to determine what can happen if there were instantaneous shifts in the interest rate. It can be used by valuation actuaries to project some potential problems. The price-curve analysis for my sample portfolio shows that there may be a problem if interest rates fall more than 3% (Chart 6). This could be expected, since the assets may be prepaid or called, and the minimum interest-rate guarantees on the product will kick in. Option pricing, using the current technology, does add to the run time of the modeling. Determining the price curve for this portfolio ran for over two days.

Other Asset Concerns

The correct number of interest-rate scenarios will not be that useful if the underlying assumptions are not correct. For example, the projections I have shown so far assumed that the default rate similar to what has been experienced over the past 20 years, or about 2.5% for my lower quality assets. If, instead, the rate of defaults experienced in 1990 were to continue, the picture would be very different. This rate was almost double the average rate until then. I ran my New York 7 scenarios assuming that the 1990 rates would continue (Chart 7). This produced a much worse result. My conclusion was that this particular block of business was quite sensitive to default assumptions, and this business should therefore be tested for the sensitivity to default assumptions frequently.

Another area of concern recently has been liquidity, or the ability to withstand a "run on the bank." There are a number of valuation actuaries who have been asked by the management, and by some rating agencies, to establish the liquidity quotient for their business. In order to do this, cash and investment-grade bonds are treated as par assets.

CHART 6

Price Curve Analysis -- SPDAS
Market Value of Surplus

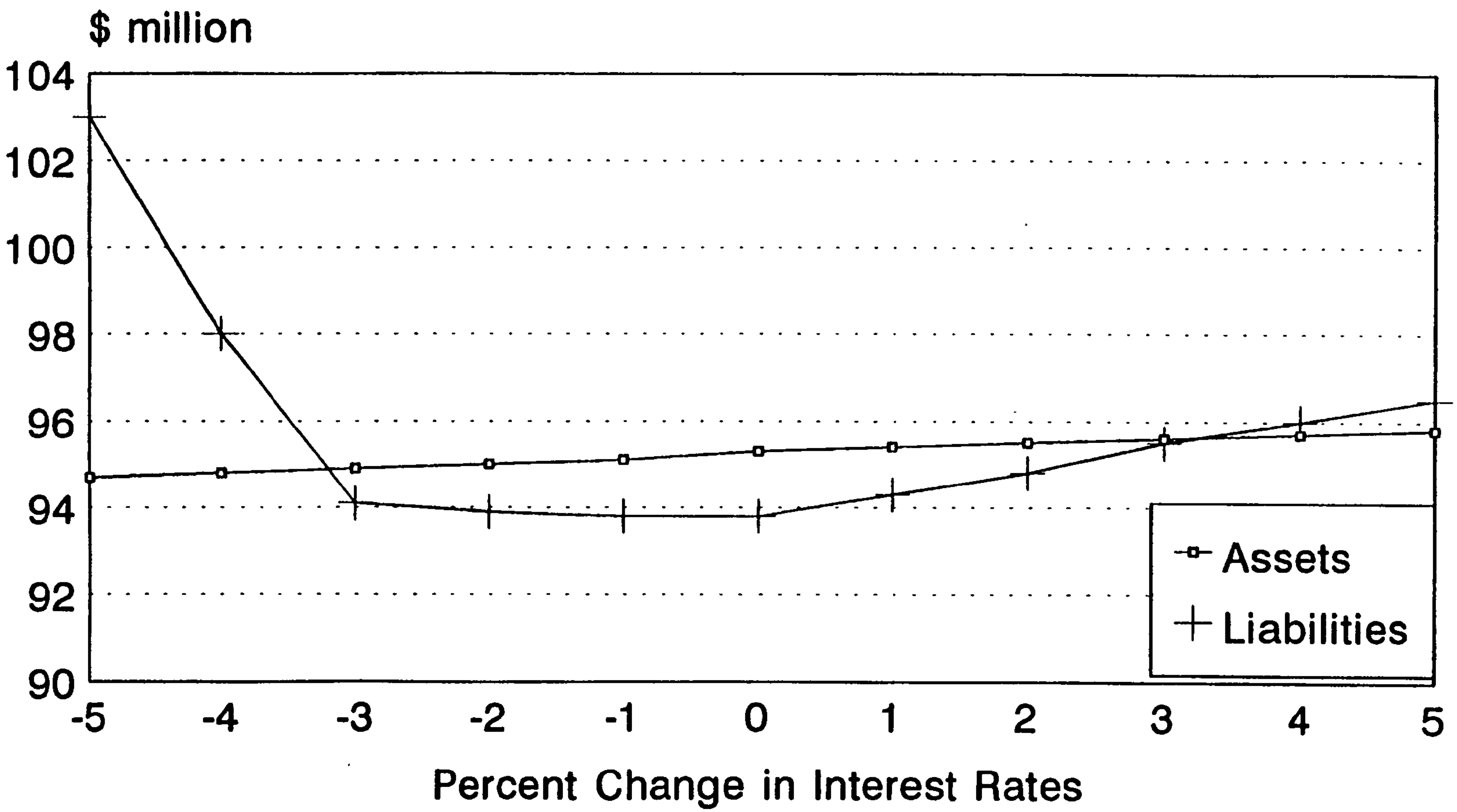
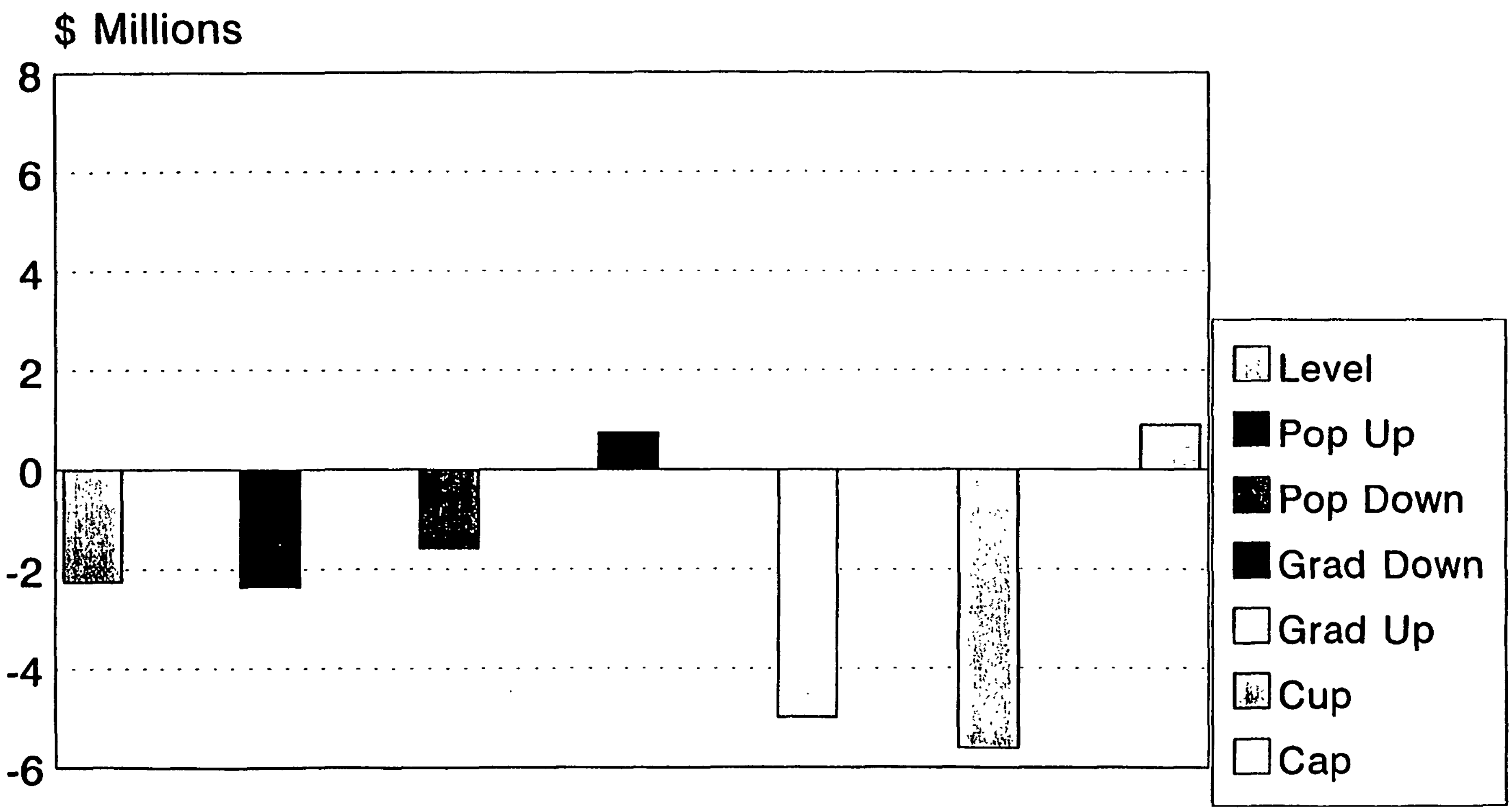


CHART 7

Model SPDA Results
Tenth Year Market Value Surplus Assuming Current Level of Defaults Continue



Private placements are typically given a discount for an illiquidity premium, and real estate may be treated as virtually zero market value in the event of a liquidity crunch. I think it is important for valuation actuaries to point out to management any potential problems in this area.

Another area of potential concern to valuation actuaries should be the concentration of risks. For example, if much of the company's money is tied up with a few real estate deals, management should be warned of a potential problem with diversity. Conversely, if a large chunk of liabilities is in GICs from a single employer, this should be looked at carefully by the valuation actuary to ensure that there was no problem in meeting maturity payments.

Historical Analysis

Historical analysis can be used by valuation actuaries to tie results together. There are several uses for historical analysis. One is to tie results of actual versus projected per quarter. I believe this is very important, as the cash-flow projections can become a management tool in order to predict future results. Jackie Keating will concentrate on this aspect.

A second tool is to learn by one's mistakes. Table 1 contains actual interest rates from the last 14 years. The one scenario I like to do to stress my testing is to see how the particular product design and investment strategy is expected to behave if the interest-rate climate of the early 1980s were to be repeated (Chart 8). Since much of management remembers the problems in that period, it is quite a useful test. For my particular portfolio, the expected surplus under the historical scenario is negative (Chart 9). It is also interesting to note that the results under this scenario is equal in severity to the worst scenario I tested when developing my 99 scenarios. I leave it up to the valuation actuary's judgment as to whether you think that the early 1980s is equal to the worst scenario you would want to test, or whether you feel additional tests would be necessary.

CHART 8

Historical Interest Rates

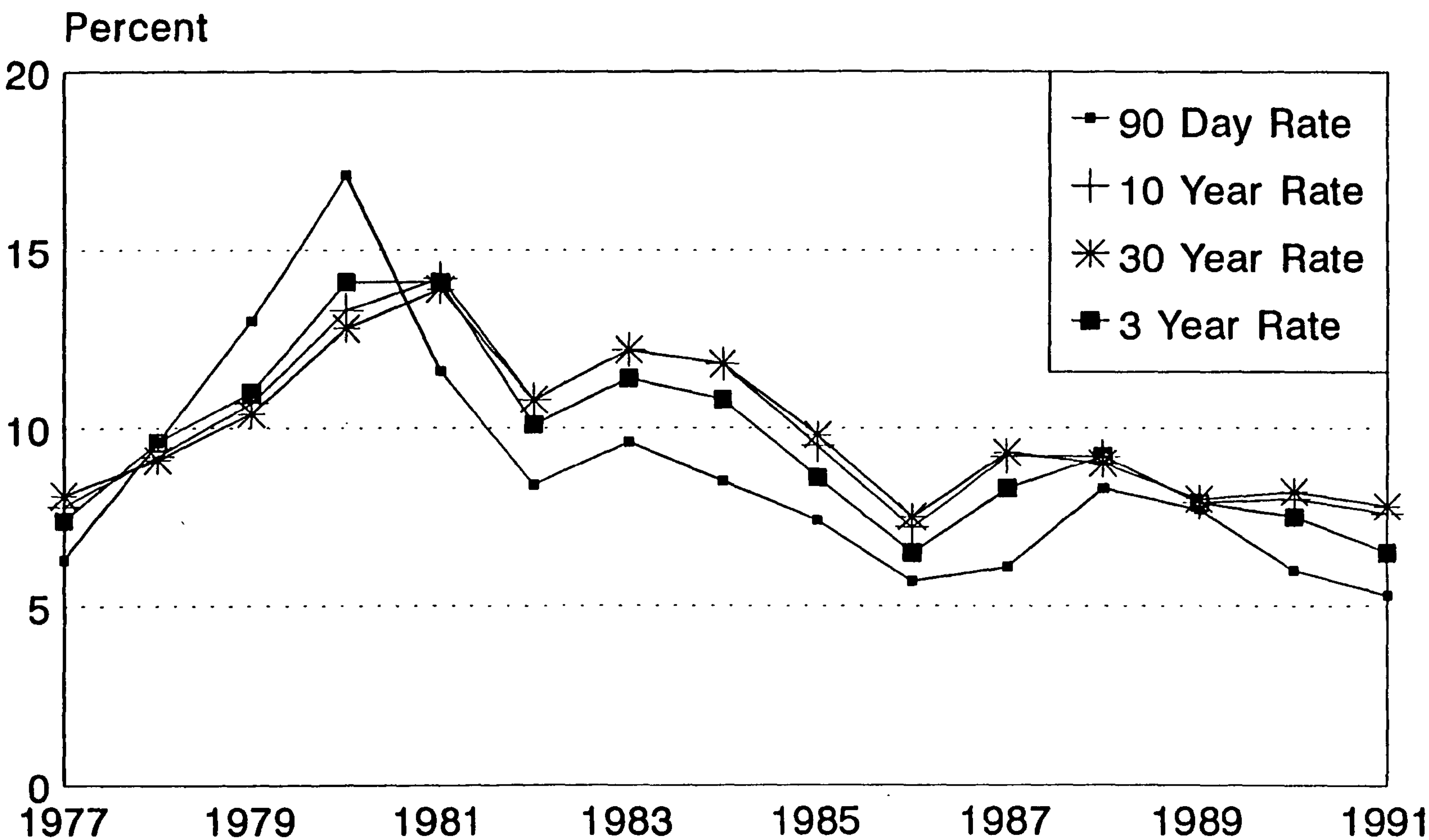


CHART 9

Model SPDA Results
Present Value of Tenth Year Market Value Surplus

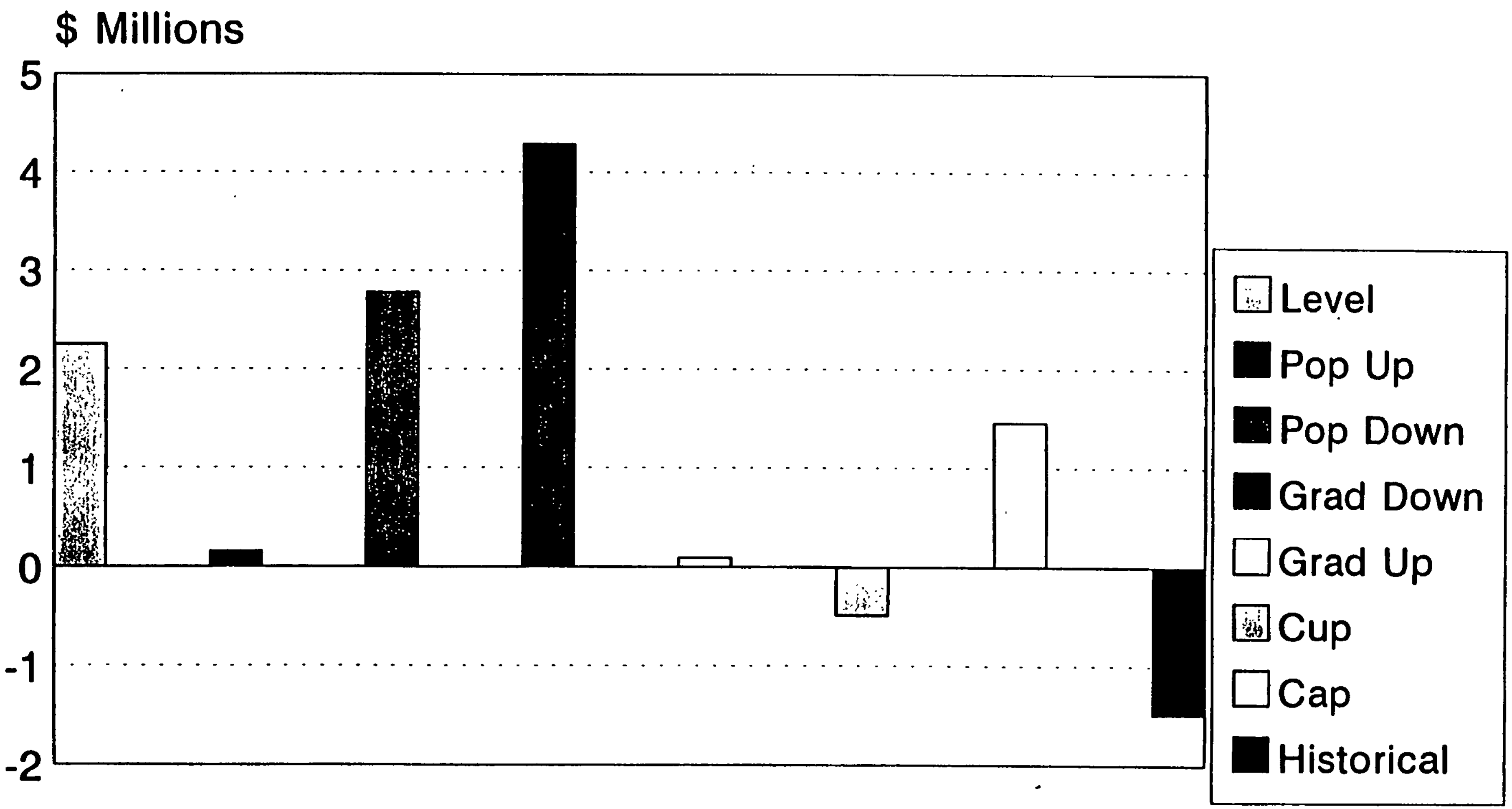


TABLE 1

Historical Interest Rates

Year	90-Day Rate	3-Year Rate	10-Year Rate	30-Year Rate
1977	6.3	7.4	7.8	8.1
1978	9.6	9.5	9.2	9.1
1979	13.0	11.0	10.7	10.4
1980	17.1	14.1	13.3	12.8
1981	11.6	14.1	14.2	13.9
1982	8.4	10.1	10.8	10.8
1983	9.6	11.4	12.2	12.2
1984	8.5	10.8	11.8	11.8
1985	7.4	8.6	9.5	9.8
1986	5.7	6.5	7.2	7.5
1987	6.1	8.3	9.2	9.3
1988	8.3	9.2	9.2	9.0
1989	7.7	7.9	7.9	8.0
1990	6.0	7.5	8.0	8.2
1991	5.3	6.6	7.6	7.8

December rates from Federal Reserve Statistical Release; except 1991 shows September rates.

Subjective Assumptions

This session was to concentrate on interest-rate scenarios. I would like to point out again the number of important assumptions that one must make when establishing the cash-flow model. Some of the subjective assumptions I made when setting up my cash-flow model are spelled out in the actuarial report which is in Session 1. I have a few comments on some of these assumptions:

Prepayment/Call -- Interest rates have fallen in the past year or so. Calls and prepayments may become a more important factor in projecting cash flows. One area I think valuation actuaries should look at is with CMOs. Many companies have invested heavily in CMOs in recent years. CMOs typically pay according to a projected schedule as long as the prepayments on the underlying mortgage pools are within certain parameters. We are probably getting close to interest rates where this may no longer be the case. A reasonable acceleration of prepayments on CMOs should be built into cash-flow modeling.

Lapse Sensitivity -- Parts of the insurance industry have again discovered that pricing lapse rates were not what can occur in all cases. These companies experienced runs because of concern about the insurance company's health. The early 1980s also showed runs could happen because of interest rates. It is important for the valuation actuaries not to discount these lessons, and to test the sensitivity of the business to runs on the company.

Dis- and Reinvestment Assumptions -- The cash-flow results are most likely sensitive to different dis- and reinvestment strategies. The base assumption should be tailored to the company's actual strategy, if known. If the company does not have a formal strategy, the valuation actuary can test several strategies in order to point out to management the pros and cons of various actions.

Mortality -- The C-2 risk does deserve some notice, especially since the entire company's liabilities are being certified to. Mortality variances can cause reserve deficiencies. This is probably more important for certain term products and business which is not underwritten.

Expenses -- Expense variations can cause reserves to be inadequate. For example, if one priced a product guaranteeing a spread of 100 basis points, but the expenses were 150 basis points, there would be a loss on the product which should be reserved for.

Cash-Flow Testing on Whole Life Insurance

There has been a question over the past several years as to whether traditional whole life insurance needed to be cash-flow tested. This question arose when Actuarial Standard of Practice (ASP) #14 was being written, and again when the model valuation law and regulation was being drafted. It was determined that the answer to this question was "yes."

I agree with this answer, for several reasons. One is that the company does have an exposure to changes in interest rates. For example, many dividend scales are set in the fall of one year, to be paid on policy anniversaries in the following year, which leaves the company exposed to changes in interest rates for at least one year. Another practical aspect in many companies is that dividend scales tend to be sticky down. In modeling, the valuation actuary should probably not assume that any interest-rate drops are immediately factored into the dividend scale. Also, with interest rates dropping, it is possible that the interest rates may fall below the reserve rates currently being used for some products, since this rate can be 5.5% or so.

Having said this, however, I do not think that whole life insurance needs to be tested under as many interest-rate scenarios as a block of more interest-sensitive business, such as SPDAs. In my actuarial report I did a sample block of whole life insurance. This report lists all the assumptions I used. The bottom line results show that the expected profit of this business does not vary dramatically. This is partly because of the model assumptions I used: No assets were callable for at least ten years, and all high quality assets were purchased. However, if you as a valuation actuary test your business and come up with similar results, it is possible that the business would not have to be tested for sensitivity of reserve adequacy to interest-rate shifts as often as other blocks of business.

Conclusion

There is a lot of work for a valuation actuary to do in cash-flow testing, and the best teacher is to go through the process a number of times. However, if we do our jobs right, we can contribute to management's understanding of the business, and point out potential problems before a company becomes insolvent.

DEVELOPING CONFIDENCE IN RESULTS OF MULTISCENARIO TESTING

MS. JACQUELINE KEATING: I will focus my remarks on period-to-period reconciliation. I will discuss period-to-period reconciliation in two respects. First, with respect to the reconciliation of key assumptions with actual experience and second, with respect to the reconciliation of results from period to period.

So, I'll begin with a discussion of the period-to-period reconciliation of certain key assumptions. In order to have some confidence in the results of multiscenario testing, you must first develop some confidence in the assumptions underlying the projections. The idea I would like to convey is that developing confidence in certain key assumptions is an ongoing process.

I'd like to give you a specific example of some work we've done in trying to reconcile a key assumption with actual experience. The emphasis in this discussion should be placed on the process, rather than on the particular assumption I have chosen as an example.

The example I'll discuss pertains to some work we did several years ago for a client who had a large block of long-term single premium immediate annuities (SPIAs) and structured settlements that were subject to New York Regulation 126. Given the long-term nature of the liabilities, the prepayment assumptions on the assets backing these liabilities were crucial to the results obtained in declining interest scenarios. The assets backing these liabilities consisted of a block of government and corporate bonds, many with call options.

In order to have some confidence in the results of our cash-flow projections, we had to develop some confidence in our assumption concerning the level of calls. We had a formula to predict calls that we had developed several years prior that was based on the call experience of another company.

Chart 10 shows the formula we had developed to predict calls. The formula looks at the economics of the situation asking how much will it cost the bond issuer to refinance the issue and compares that cost with the cost to maintain the issue. The costs to refinance are the amount the issuer will pay to call the existing bond issue, which is the par value times (one plus the call premium), and there will be some costs to a new issue, the transaction cost. We express the transaction cost as a percentage of par value which varies for investment-grade and noninvestment-grade bonds.

CHART 10

Bond Call Options

Cost to Call
Par(1 + Call Premium) + Transaction Cost

Cost to Maintain
Call Market Value

Assume Bond is Called if:
Cost to Call < Cost to Maintain

So the cost to refinance a bond issue is the call price plus the transaction cost. The formula compares this cost with the cost to the issuer of maintaining the current issue. I have labeled this the call market value, and it is the present value of future coupon and principal payments on the bond, discounted at the current new-money rates, reflecting the remaining term to maturity of the bond.

We assume then that bond will be called if the cost to call is lower than the cost to maintain. In developing this formula and testing it against some call experience, we noticed an inertia in the marketplace. That is, you do not see an immediate increase in calls the minute the equation is true. To adjust for this we include a margin in developing the call market value by discounting at a rate that is 100 basis points above the current new-money rates.

We wanted to determine if this formula produced reasonable results for the particular company, recognizing this company's portfolio of corporate bonds.

In this example, we were doing the work during 1988, and the company's 1987 call experience was available. We wanted to see how well our formula reproduced the 1987 experience.

This is the process we set up to test the call assumptions. We began by creating an asset file that reflected the assets subject to call during 1987. This consisted of all assets still in force at year-end 1987 that had first call dates in 1987 or prior, and all assets that were actually called during 1987. We also checked to see if there was a material exposure on assets that were in force at the beginning of the year, but sold during the year.

Once this file was created, we ran it against our call formula and based the interest assumption on the actual interest rates during 1987. We then compared the actual 1987 calls with those projected under our call formula.

In our initial run-through of this process we determined that actual calls as provided by the company were approximately $76 million and projected calls were approximately $120 million, both measured in terms of book value. Our initial reaction in looking at these results was to surmise that our call formula was a bit heavy in projecting calls.

As we reviewed the results, we determined some interesting information:

1. Actual calls in 1987 were not $76 million, but $190 million. The initial 1987 call experience given to us was not accurate;
2. The call schedules provided by the company were incorrect in some cases; and
3. Some bonds were called that were not callable under the terms of the bond. These were instances where the company agreed to the call, but was not obligated to under the bond's terms.

Given these revised data, we did a subsequent analysis. This analysis showed $160 million of projected calls versus $190 million of actual calls.

We decided not to change the formula but to continue to review it against actual experience each year. Through this exercise, we were able to:

1. Review the actual experience numbers provided;
2. Check the accuracy of the asset data files provided;
3 Test the validity of a key assumption; and
4. Set up a procedure to review actual to expected results on an ongoing basis.

In running the cash-flow projections for each year-end or other period, we set up a process to capture the projected calls under each interest-rate scenario. Then in setting our assumptions for the next round of projections, we review the actual calls with the calls projected under the interest scenario that most closely followed the actual level of interest rates.

Table 2 shows the process which we had already set up for another company where we had been doing cash-flow projections for several years. First, we look at the prior year-end's cash-flow projections which summarized the projected calls for each scenario. Some of these are shown on the table. I have also listed the reduction in interest rates assumed during the first projection year for each scenario listed. For example, scenario 5 assumed a 50 basis point drop in interest rates during the first projection year, and the calls projected were $24 million.

We reviewed the actual calls during that year, which were about $22 million in an interest environment where interest rates had declined about 75 basis points.

TABLE 2

Analysis of Call Formula Based on Prior Projections

I. Projected

Scenario #	Reduction in Interest Rates (Basis Points)	Projected Calls (Millions)
1	0	$ 6
5	50	13
6	100	24
7	300	54

II. Actual

Reduction in Interest Rate:	75 Basis Points
Calls:	$22 Million

We compared the actual results with the results for Scenario 5 and 6. Scenario 5, with a 50 basis point drop in interest rates, had projected calls of $13 million. Scenario 6, with a 100 basis point drop in interest rates, had projected calls of $24 million. Given this information, we felt that the call formula was projecting a reasonable level of calls in this interest environment.

There are two comments I would like to make about this analysis. First, it is much easier to perform this analysis (where you have captured the projected amounts from prior projections) than to try to go back and recreate asset files as in my first example. Second, this analysis will not provide all the answers. It will not indicate, for example, how well our call formula works in an environment where interest rates go down 300 basis points.

Obviously, this type of analysis, or reconciliation of assumptions with experience, can be generalized to cover other assumptions as well. This procedure can be used to check the validity of lapse assumptions, prepayment assumptions, mortality assumptions as well as many others. The process requires that you set up a procedure to capture the actual and projected results pertaining to a particular assumption and that you use that information in refining your projections. Each year should provide additional information on which to base your cash-flow assumptions.

The second topic I would like to talk about in a little less detail than the first, is the reconciliation of results from one year to the next. After you've developed the results of your projections for a particular year-end or period, the logical step which most actuaries will take is to compare the current results with those from the prior year-end or period.

One way to approach this reconciliation is to complete the current period projection, compare it with the prior period results and justify any differences. We have found that this reconciliation can be very difficult in some cases. Your initial reaction might be that there should not be many changes from one period to the next and the results should not change materially. Our experience has been that in many cases there are numerous changes from one period to the next, and it is sometimes difficult to quantify the effect of those numerous changes particularly under varying interest scenarios, without actually running projections that focus on each specific change.

So my suggestion in this respect, is not to wait until you have this year's results in hand and then try and quantify the effect of changes from the prior year. Rather, you should set up a procedure to reflect the effect of changes individually, rather than on a combined basis. So, for example, if your current year's projections include several changes such as: (1) inclusion of new lines of business or products; (2) new default assumptions; (3) different starting interest rates; and (4) revised investment strategy, you would want to test the effect of these changes individually.

This procedure will serve two purposes. First, it will allow you some comfort that the results are consistent with prior years' results, subject to the specific changes, and second, it will increase your knowledge or feeling about how certain assumptions affect the business being tested.

Table 3 shows a chart listing potential changes from one period to the next. The idea is to quantify the effects of each significant change. There may be some overlap in the effect of various changes, but in this type of analysis you should be able to come close to reconciling the change in results from one period to the next. You will need to look at this type of analysis over several scenarios in order to be comfortable with the results. The effect of certain assumption changes, for example, will vary depending on interest scenario.

TABLE 3

Reconciliation of Period-to-Period Results

Changes from Prior Projections	Effect on Results
1. Changes in Asset Portfolio	
2. Changes in Liabilities	
3. Changes in Assumptions	
4. Changes in Starting Yield Rates	
Summary of Changes	

In conclusion, I would like to reiterate three thoughts:

1. The development of cash-flow projections should be viewed as an ongoing process, subject to frequent refinement. Each time you do one of these projections, it should be better than the last time;
2. There is a lot of information available as output from cash-flow projections. You need to develop a system that captures this information as well as the corresponding experience, and you need to use this information in refining your projections; and
3. Finally, if you wait until year-end to do your reconciliation of assumptions and results, it will be more difficult and less likely to be completed.