1988 VALUATION ACTUARY
SYMPOSIUM PROCEEDINGS

C-1 RISK AND INVESTMENT

MR. GREGORY D. JACOBS: The C-3 risk is the interest-rate risk, meaning interest rates move up and down. When they do, you might not have a product that is as profitable as you think, or your reserves might not be as adequate as you would hope. We're trying to analyze that. The key question is, how does one create the investment scenarios to which you expose your assets and liabilities?

A Treasury yield curve is the interest rate for a certain duration of an asset, specifically, a Treasury bill. An inverted curve says that higher interest rates are for short term, and the curve is tilted down. A normal yield curve says that short-term interest rates are lower than long-term rates.

The next step with the yield curve and the investment-scenario generation method is what the New York Insurance Department did to us: the famous "New York seven." In mathematical terms, we'll call it the deterministic way of generating scenarios, i.e., there is no random or stochastic process; you simply write down interest-rate yield curves to which you expose your assets. An example of a deterministic model is to pick today's yield curve, and in the pop-up scenario in which you move at least three points, you can either add three points to all yield curves and have a flat adjustment up, or you can adjust the slope. There are many ways you can do it.

What I'm going to talk about is the yield-curve universe transition probability approach, a stochastic process. In our firm this is the method with which we spend a little time generating yield curves for our clients.

Slide 1 shows what a yield curve looks like. The first thing we do is create a yield-curve universe. Very simply, we have five yield curves. The one-, five-, and ten-year rates are shown. For illustration, we're going to start at yield-curve three. Slide 2 shows an example of a transition probability matrix. The transition probability gives you the probability of moving from one yield curve to the next. This is saying that, starting at yield-curve three, we have a 50 percent chance of staying where we are. We have a 25 percent chance of moving down one yield curve and a 25 percent chance of moving up one yield curve. The computer goes through random numbers, i.e., it throws a four-sided die since we have units of four in that last transition probability. If a one comes up, we move from yield-curve three to two. If a three, or a two or a three, come up, we stay where we are. And if a four comes up, we move up one yield curve.

SLIDE 1

SIMPLE EXAMPLE

YIELD CURVE UNIVERSE

CURVE #	MATURITY 1 YEAR	5 YEAR	10 YEAR
1	3%	5 %	7%
2	5	6.5	8
3*	7	8	9
4	10	10	10
5	14	12	11

* STARTING YIELD CURVE

SLIDE 2

SIMPLE EXAMPLE (CONT.)

TRANSITION PROBABILITY

TO	MOVE FROM CURVE				
CURVE	1	2	3	4	5
1	75%	25%	-	-	-
2	25	50	25%	-	-
3	-	25	50	25%	-
4	-	-	25	50	25%
5	-	-	-	25	75

That's our 25-50-25 relationship.

The minute we move from one period yield curve to the next, then all the cash-flow mechanics take place, or as I call it, the interest-sensitive assumptions are triggered. New asset earnings rates are created because we're looking at a new yield curve. Corporates, Government National Mortgage Associations (GNMAs), Treasuries -- whatever is in our portfolio -- are generally tied to the Treasury yield curve, so the minute we're in a new yield curve, we have new assets from which to pick. We probably have a new investment strategy, depending on what the yield curve looks like. As interest rates change, market values change. Pause and prepayment occur. If interest rates are dropping, you're going to get a lot of prepayments in your GNMA portfolios. The same thing will happen on your bonds.

On the liability side, you're going to have a new inflation rate, probably tied to some sort of outside index like a Treasury bill. You're probably going to come up with the new interest-credited rate, a new market or a competition rate. Again, this is a key in the cash-flow modeling. Outside interest rates affect what goes on to the policyholder lapsation, so when interest rates change, market rates change what the consumer can do if his funds change. Therefore, the lapse rates are affected, which gets into the new lapse rates. You throw all these conditions into the computer model, and out comes new cash flows.

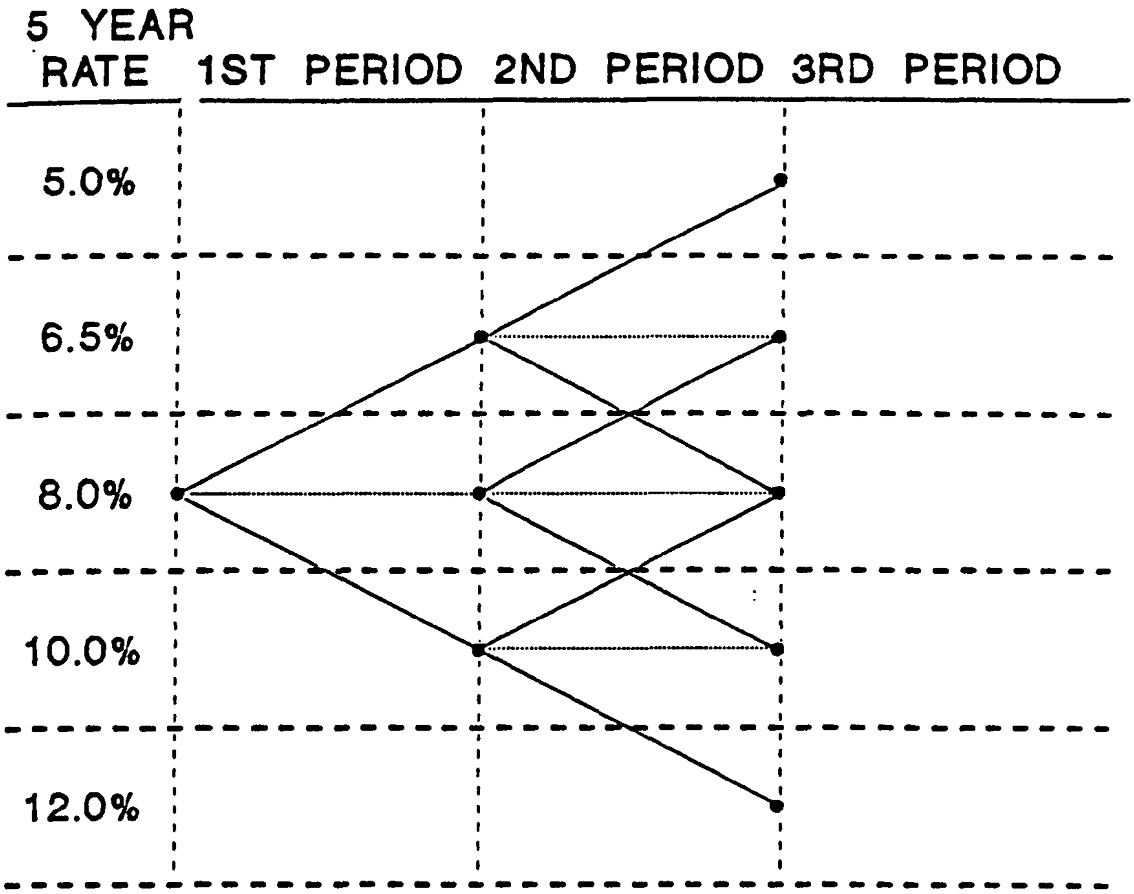
The next period, which is either a year or a quarter or a month, you throw the die again. You move from the ending curve from last month to the new curve this month, and you go through that whole last process again. You continue this for X years, twenty, thirty, forty -- however many you want. And that's one trial. Then you do that many times, twenty, thirty, fifty, a hundred, two hundred -- whatever number of trials you feel makes your results statistically significant. And now you have a stochastic process.

Each move is independent of the previous move because of the randomness of throwing the die. Each move is also independent of the cash flows that were generated in the last move. Each trial is independent of the other trials. And in mathematical terms, this is a Markov chain stochastic process.

Going back to yield-curve three (Slide 3), the five-year rates are 5, 6.5, 8, 10 and 12 percent. We throw the die. The diagonal lines in Slide 2 show a 25 percent chance of moving from one point to the next. The horizontal lines indicate a 50 percent chance of moving. Throw the die, and we have a probability of going to either yield curve two, three or four. At that point, we throw the die again, and we get branches that go on out through time. Following one of those random branches through time is a trial.

SLIDE 3

SIMPLE EXAMPLE



MEAN 8.125% 8.1825%

Note: ············ = 50%
———— = 25%

You look at the mean of these. We're starting at 8 percent, and we go to 8 1/8 percent, then to 8.825 percent and so on. Even with a normal distribution of a transition probability, we have a trend in this sort of model. We're biased with an upward interest rate movement. It's extremely important that the individuals who set the yield curves and the transition probabilities understand the biases that they are accepting. They should be able to control those biases.

That's the science; now the art. The premise of this entire discussion is I don't believe there's a lot of science that we should go through in developing interest-rate scenarios. My investment professionals, friends and colleagues, don't believe for a minute that there's any statistical and empirical evidence that one can predict what interest rates are going to be. Given that basic premise, why do we spend a lot of analytical time trying to be extremely scientific in setting yield-curve scenarios and movement probabilities when our work is nothing more than a guess? Now, it can be an educated guess. We can base it on past history. We can do a lot of things to do as good a job as we can in setting the yield rates and the investment scenarios that we run. But it's my contention that we can't predict future interest rates.

Therefore we are going to change the course of this discussion from science to art. How does one develop yield-curve universes and transition probabilities? I came up with four ideas. One is, let the client do it. Pass the buck. To be perfectly honest, that's the one that I recommend. Two is the black box curves and probabilities. I will not emphasize this much as it gets into too much science. Three is where I think most of us feel comfortable: if you don't know what the future holds, you look to the past. Four, possibly the best, is guess. I have used all of these ideas in practice when I have done stochastic processes or investment-scenario, C-3 analysis for my clients.

Number one, let the client do it. This is from a consultant's point of view. The client should have the opportunity to establish his own biases with respect to trend, volatility, magnitude, and slope of the yield curve. All of those are impacted by how one puts together the Treasury curves and the transition probabilities. You have to have a bias.

Every day your investment officer does something with the assets. Every day your actuary, marketing people, president, and financial officers look at products' profitability when setting interest rates. These people have to have some knowledge of what the future is holding. Setting these yield curves and probabilities is where that knowledge comes to bear. It's a difficult discussion to take place because people have a hard time putting their biases on paper. I think this whole yield-curve universe probability transition matrix idea is premised on the fact that the client, or the person setting those yield curves, ought to have the right to establish his own biases with respect to trend, volatility, magnitude and yield-curve slope.

Number two, a client should not accept the black-box curves and probabilities. If the client doesn't know where they came from, why use them? That gets back to the basic premise that you have to understand the biases that are involved in the projection of the yield curves.

Number three, if the client can't make the decision, the consultants can't make it for him. We can't predict the future, but we can show him what's happened in the past. Think about ignoring interest rates in a pricing exercise. If we're pricing a term product and a client asks what mortality levels we expect, we can tell the client what he should expect, and we can show him industry experience and what other companies are doing. But we can't make the decision for the client as to what level of mortality he should be using in placing his term products.

Taking that analogy over to a C-3 analysis, taking the responsibility for setting the yield curves and transition probabilities is exactly the same sort of assumption. The bottom line is that we can consult, advise, suggest, question, challenge, but it's ultimately your decision. I don't think you should expect the consultant or this investment scenario method to have all the answers. There's no right answer. It's got to be a dialogue. It's got to be interactive.

A lot of companies have macroeconomic theory/forecasting available internally. I assume that some of the bigger New York companies have macroeconomists on staff, and they sit around all day thinking about what interest rates are going to be doing. This forecasting is also available through investment bankers, but beware of motives. If you talk to a certain investment banker about the future and get from him some ideas as to investment-rate movement, take it with a grain of salt because he's probably peddling a certain type of asset or security. Is it the investment banker's motive to sell, or is it their motive to predict interest rates and therefore recommend a sale?

The whole macroeconomic theory involves projection growth and gross national product, inflation monetary policy, etc. Here's some interesting comments from the *Handbook of Fixed Income Securities*. This book is a part of the reading list for the Certified Financial Analyst exams: "Serious studies generally indicate that short-term forecast of long-term interest rates contain little or no value added." And "The maximum in forecast is *caveat emptor*, meaning buyer beware." David Wolford, CFA, wrote this chapter on interest-rate forecasting.

When looking into the future, you look to the past first to have some comfort. As an example based on historical data, use the highest and the lowest Treasury curves experienced in the last few years to establish a yield curve universe. Second, use the standard deviations of interest rates during that time period to establish the volatility or the transition probability, the movements. In Attachment A we tracked Treasury interest rates all the way back into the 1960s. This is readily available information.

These are indisputable facts. During the 1980s Attachment B shows what the interest rate volatility looked like. The high interest rates were a high of almost 18 percent for the short term and a thirty-year Treasury bond was 15 percent for the long term. That was in October of 1981, during the big inversions. The low interest rates, which were probably six to nine months ago, were a low of 5.35 percent and high of 7.40 percent. The mean during that period is shown, and the standard deviation is shown. Interestingly enough, going back to the early 1960s (Attachment C), the period of the 1980s was the most volatile in that time frame. I imagine that going further back into the past from 1960, that 1980s volatility will still be the highest.

Taking the past eight years of information, my clients and I have set a yield curve. We arbitrarily set nineteen-year curves. In slide 4 the low yield curve was the lowest that you saw from the earlier graph. The highest yield curve was the high from the earlier graph. And the mean right in the middle of the yield curves is the mean from the earlier graph. Yield-curve twenty is where we were at the end of August 1988 or the average August 1988 Treasury yield curve.

ATTACHMENT A

EFFECTIVE YIELDS ON U.S. GOVERNMENT SECURITIES - TREASURIES

MONTH	YEAR	90 DAY	3 YEAR	10 YEAR	30 YEAR
JAN	1965	3.92	4.08	4.23	4.24
FEB	1965	3.92	4.06	4.23	4.22
MAR	1965	4.06	4.12	4.24	4.23
APR	1965	4.07	4.13	4.21	4.21
MAY	1965	4.01	4.13	4.23	4.21
JUN	1965	3.94	4.13	4.23	4.21
JUL	1965	3.91	4.04	4.23	4.21
AUG	1965	3.90	4.10	4.24	4.22
SEP	1965	4.01	4.22	4.30	4.30
OCT	1965	4.14	4.36	4.39	4.36
NOV	1965	4.17	4.42	4.43	4.37
DEC	1965	4.24	4.52	4.51	4.42
JAN	1966	4.60	5.00	4.64	4.52
FEB	1966	4.81	4.98	4.73	4.59
MAR	1966	4.82	5.09	4.97	4.81
APR	1966	4.67	4.85	4.66	4.63
MAY	1966	4.82	4.95	4.78	4.67
JUN	1966	4.80	5.15	4.81	4.74
JUL	1966	4.71	5.26	4.99	4.81
AUG	1966	4.91	5.33	5.06	4.83
SEP	1966	5.26	6.25	5.47	5.05
OCT	1966	5.51	5.47	4.91	4.80
NOV	1966	5.41	5.40	4.88	4.71
DEC	1966	5.35	5.47	5.07	4.84
JAN	1967	4.97	4.97	4.62	4.59
FEB	1967	4.65	4.66	4.55	4.47
MAR	1967	4.65	4.80	4.73	4.70
APR	1967	4.23	4.28	4.52	4.59
MAY	1967	3.79	4.48	4.74	4.77
JUN	1967	3.57	4.52	4.89	4.88
JUL	1967	3.92	5.12	5.25	5.09
AUG	1967	4.24	5.06	5.16	5.03
SEP	1967	4.51	5.39	5.26	5.13
OCT	1967	4.50	5.41	5.33	5.18
NOV	1967	4.70	5.65	5.65	5.45
DEC	1967	5.11	5.74	5.78	5.64
JAN	1968	5.20	5.82	5.72	5.58
FEB	1968	5.03	5.48	5.55	5.36
MAR	1968	5.20	5.59	5.55	5.38
APR	1968	5.34	5.73	5.75	5.57
MAY	1968	5.71	5.96	5.70	5.46
JUN	1968	5.88	5.98	5.71	5.48
JUL	1968	5.52	5.77	5.57	5.35
AUG	1968	5.33	5.37	5.28	5.17
SEP	1968	5.39	5.41	5.33	5.23
OCT	1968	5.33	5.39	5.39	5.34
NOV	1968	5.69	5.62	5.49	5.45
DEC	1968	5.72	5.66	5.63	5.65
JAN	1969	6.48	6.36	6.04	5.99
FEB	1969	6.43	6.27	6.09	6.09
MAR	1969	6.44	6.43	6.25	6.14
APR	1969	6.22	6.28	6.42	6.14
MAY	1969	6.08	6.25	6.16	5.94
JUN	1969	6.32	6.68	6.51	6.32
JUL	1969	6.32	7.22	6.55	6.27
AUG	1969	7.37	7.35	6.55	6.23
SEP	1969	7.20	7.36	6.68	6.25
OCT	1969	7.45	8.15	7.33	6.67
NOV	1969	7.29	7.41	6.92	6.50
DEC	1969	7.85	7.84	7.28	6.69

ATTACHMENT A - Continued

MONTH	YEAR	90 DAY	3 YEAR	10 YEAR	30 YEAR
JAN	1970	8.37	8.39	7.54	6.91
FEB	1970	8.25	8.33	7.64	6.94
MAR	1970	7.14	7.30	6.94	6.50
APR	1970	6.62	7.15	7.01	6.55
MAY	1970	7.20	7.95	7.56	6.97
JUN	1970	7.20	7.94	7.87	7.37
JUL	1970	6.54	7.92	7.70	7.15
AUG	1970	6.62	7.61	7.33	6.86
SEP	1970	6.50	7.40	7.49	6.94
OCT	1970	5.99	7.04	7.24	6.75
NOV	1970	6.03	6.91	7.32	6.92
DEC	1970	5.28	5.62	6.51	6.32
JAN	1971	4.97	5.78	6.47	6.35
FEB	1971	4.22	5.30	6.08	6.04
MAR	1971	3.44	4.86	6.10	6.20
APR	1971	3.66	4.48	5.56	5.86
MAY	1971	4.08	5.73	6.04	6.07
JUN	1971	4.41	5.86	6.30	6.22
JUL	1971	5.22	6.52	6.50	6.40
AUG	1971	5.44	6.84	7.02	6.34
SEP	1971	4.40	5.85	6.33	6.01
OCT	1971	4.67	5.78	6.06	5.88
NOV	1971	4.39	5.26	5.91	5.81
DEC	1971	4.42	5.36	6.04	5.87
JAN	1972	3.68	5.06	6.05	5.89
FEB	1972	3.35	5.31	6.29	5.98
MAR	1972	3.49	5.33	6.27	5.94
APR	1972	3.87	5.83	6.30	6.02
MAY	1972	3.65	5.58	6.20	6.02
JUN	1972	3.86	5.41	6.18	5.86
JUL	1972	3.99	5.70	6.21	5.91
AUG	1972	3.78	5.73	6.30	5.81
SEP	1972	4.60	6.01	6.46	5.77
OCT	1972	4.64	6.03	6.68	5.91
NOV	1972	4.86	6.06	6.53	5.81
DEC	1972	4.98	5.95	6.42	5.66
JAN	1973	5.27	6.04	6.43	5.91
FEB	1973	5.73	6.42	6.60	7.01
MAR	1973	6.01	6.82	6.72	7.02
APR	1973	6.62	6.83	6.76	6.97
MAY	1973	6.42	6.79	6.76	7.00
JUN	1973	7.15	6.83	6.92	7.21
JUL	1973	7.83	7.02	6.92	7.28
AUG	1973	8.70	8.04	7.35	7.72
SEP	1973	8.96	7.48	7.15	7.45
OCT	1973	7.24	6.86	6.92	7.13
NOV	1973	7.64	6.83	6.78	7.40
DEC	1973	7.57	6.83	6.78	7.23
JAN	1974	7.73	6.83	6.92	7.40
FEB	1974	7.79	6.91	7.05	7.53
MAR	1974	7.73	6.95	7.00	7.62
APR	1974	8.69	7.90	7.23	7.95
MAY	1974	9.11	8.33	7.58	8.19
JUN	1974	8.44	8.17	7.56	8.23
JUL	1974	7.79	8.39	7.63	8.22
AUG	1974	8.27	8.76	7.85	8.58
SEP	1974	9.55	8.92	8.21	8.89
OCT	1974	6.38	8.17	7.64	8.63
NOV	1974	8.12	8.01	7.46	8.30
DEC	1974	7.74	7.50	7.23	8.18

ATTACHMENT A - Continued

MONTH	YEAR	90 DAY	3 YEAR	10 YEAR	30 YEAR
JAN	1975	7.35	7.36	7.15	8.11
FEB	1975	5.93	7.02	7.12	8.11
MAR	1975	5.54	6.61	7.01	7.98
APR	1975	5.70	7.08	7.11	8.37
MAY	1975	5.63	7.85	7.62	8.38
JUN	1975	5.33	7.30	7.52	8.51
JUL	1975	6.00	7.51	7.51	8.30
AUG	1975	6.43	7.97	7.62	8.43
SEP	1975	6.65	8.06	7.77	8.61
OCT	1975	6.81	8.42	8.26	8.73
NOV	1975	5.73	7.24	8.00	8.37
DEC	1975	5.68	7.56	8.04	8.50
JAN	1976	5.33	7.12	7.68	8.19
FEB	1976	4.80	7.00	7.52	8.17
MAR	1976	5.11	7.20	7.61	8.17
APR	1976	5.14	7.11	7.52	8.08
MAY	1976	5.08	7.07	7.56	8.16
JUN	1976	5.69	7.59	8.10	8.33
JUL	1976	5.57	7.33	8.01	8.21
AUG	1976	5.35	7.08	7.99	8.22
SEP	1976	5.27	6.81	7.79	8.05
OCT	1976	5.21	6.72	7.66	7.97
NOV	1976	5.04	6.42	7.54	7.95
DEC	1976	4.56	5.74	7.12	7.72
JAN	1977	4.80	6.32	7.34	7.43
FEB	1977	4.85	6.54	7.53	7.89
MAR	1977	4.77	6.58	7.60	7.95
APR	1977	4.71	6.42	7.51	7.89
MAY	1977	5.16	6.66	7.60	7.95
JUN	1977	5.22	6.49	7.41	7.79
JUL	1977	5.40	6.62	7.41	7.79
AUG	1977	5.72	6.91	7.54	7.83
SEP	1977	6.07	6.96	7.48	7.79
OCT	1977	6.45	7.32	7.66	7.92
NOV	1977	6.38	7.35	7.72	8.00
DEC	1977	6.35	7.43	7.84	8.10
JAN	1978	6.75	7.76	8.12	8.35
FEB	1978	6.76	7.82	8.19	8.42
MAR	1978	6.59	7.85	8.20	8.40
APR	1978	6.59	8.00	8.32	8.51
MAY	1978	6.72	8.23	8.52	8.61
JUN	1978	7.06	8.47	8.64	8.68
JUL	1978	7.37	8.72	8.83	8.84
AUG	1978	7.44	8.50	8.59	8.65
SEP	1978	8.29	8.59	8.60	8.65
OCT	1978	8.44	8.81	8.83	8.86
NOV	1978	9.16	9.24	9.00	8.94
DEC	1978	9.64	9.55	9.21	9.08
JAN	1979	9.95	9.73	9.31	9.14
FEB	1979	9.91	9.51	9.31	9.20
MAR	1979	10.09	9.60	9.33	9.23
APR	1979	10.07	9.65	9.39	9.29
MAY	1979	10.24	9.64	9.46	9.40
JUN	1979	9.62	9.15	9.11	9.12
JUL	1979	9.82	9.14	9.15	9.13
AUG	1979	10.14	9.35	9.23	9.18
SEP	1979	10.97	9.93	9.55	9.38
OCT	1979	12.60	11.25	10.57	10.09
NOV	1979	12.70	11.49	10.93	10.57
DEC	1979	12.99	11.00	10.66	10.38

ATTACHMENT A - Continued

MONTH	YEAR	90 DAY	3 YEAR	10 YEAR	30 YEAR
JAN	1980	12.94	11.18	11.09	10.88
FEB	1980	13.93	13.25	12.80	12.50
MAR	1980	16.67	14.54	13.16	12.72
APR	1980	14.33	12.38	11.80	11.73
MAY	1980	9.09	9.66	10.44	10.63
JUN	1980	7.43	9.12	10.02	10.05
JUL	1980	8.52	9.49	10.51	10.50
AUG	1980	9.70	10.91	11.41	11.30
SEP	1980	10.98	11.91	11.84	11.66
OCT	1980	12.51	12.37	12.10	11.93
NOV	1980	14.94	13.75	13.08	12.75
DEC	1980	17.10	14.12	13.25	12.78
JAN	1981	16.46	13.43	12.97	12.51
FEB	1981	16.18	14.12	13.63	13.21
MAR	1981	14.51	13.97	13.55	13.09
APR	1981	14.89	14.59	14.15	13.64
MAY	1981	17.98	15.65	14.60	14.06
JUN	1981	16.11	14.80	13.92	13.38
JUL	1981	16.37	15.72	14.79	14.05
AUG	1981	17.04	16.64	15.50	14.67
SEP	1981	16.08	16.88	15.91	15.21
OCT	1981	14.72	16.10	15.72	15.22
NOV	1981	11.64	13.54	13.84	13.80
DEC	1981	11.63	14.13	14.19	13.90
JAN	1982	13.26	15.18	15.12	14.73
FEB	1982	14.65	15.27	14.95	14.73
MAR	1982	13.72	14.63	14.34	13.99
APR	1982	13.75	14.68	14.35	13.82
MAY	1982	13.04	14.24	14.08	13.68
JUN	1982	13.48	15.00	14.81	14.40
JUL	1982	12.20	14.49	14.44	14.01
AUG	1982	9.20	13.02	13.49	13.18
SEP	1982	8.36	12.39	12.72	12.43
OCT	1982	8.13	10.90	11.21	11.48
NOV	1982	8.53	10.23	10.83	10.82
DEC	1982	8.38	10.12	10.82	10.82
JAN	1983	8.30	9.87	10.73	10.91
FEB	1983	8.57	10.16	11.01	11.18
MAR	1983	8.84	10.08	10.79	10.91
APR	1983	8.68	10.00	10.67	10.76
MAY	1983	8.66	9.89	10.65	10.81
JUN	1983	9.32	10.59	11.14	11.23
JUL	1983	9.64	11.20	11.70	11.73
AUG	1983	9.93	11.62	12.20	12.17
SEP	1983	9.56	11.38	11.99	11.97
OCT	1983	9.16	11.17	11.87	11.92
NOV	1983	9.29	11.26	12.03	12.10
DEC	1983	9.56	11.44	12.18	12.23
JAN	1984	9.44	11.23	12.01	12.10
FEB	1984	9.66	11.36	12.19	12.31
MAR	1984	10.14	11.93	12.70	12.76
APR	1984	10.33	12.34	13.03	13.05
MAY	1984	10.48	13.16	13.86	13.88
JUN	1984	10.53	13.61	14.02	13.89
JUL	1984	10.81	13.51	13.81	13.65
AUG	1984	11.20	12.89	13.12	12.93
SEP	1984	11.09	12.72	12.91	12.67
OCT	1984	10.38	12.20	12.53	12.34
NOV	1984	9.12	11.20	11.91	11.89
DEC	1984	8.52	10.84	11.83	11.85

ATTACHMENT A - Continued

MONTH	YEAR	90 DAY	3 YEAR	10 YEAR	30 YEAR
JAN	1985	8.19	10.70	11.70	11.78
FEB	1985	8.75	10.83	11.84	11.80
MAR	1985	9.02	11.36	12.21	12.16
APR	1985	8.40	10.77	11.76	11.80
MAY	1985	7.88	9.99	11.14	11.36
JUN	1985	7.30	9.26	10.42	10.72
JUL	1985	7.44	9.39	10.58	10.78
AUG	1985	7.51	9.53	10.60	10.84
SEP	1985	7.46	9.59	10.64	10.89
OCT	1985	7.53	9.46	10.50	10.78
NOV	1985	7.62	9.08	10.02	10.31
DEC	1985	7.43	8.58	9.47	9.77
JAN	1986	7.40	8.59	9.40	9.62
FEB	1986	7.39	8.26	8.89	9.13
MAR	1986	6.91	7.43	7.93	8.12
APR	1986	6.34	6.98	7.43	7.53
MAY	1986	6.40	7.40	7.86	7.66
JUN	1986	6.50	7.55	7.95	7.71
JUL	1986	6.10	6.98	7.43	7.40
AUG	1986	5.81	6.60	7.30	7.46
SEP	1986	5.40	6.73	7.59	7.77
OCT	1986	5.39	6.67	7.57	7.85
NOV	1986	5.57	6.56	7.38	7.66
DEC	1986	5.72	6.53	7.24	7.51
JAN	1987	5.68	6.51	7.21	7.53
FEB	1987	5.83	6.67	7.38	7.68
MAR	1987	5.80	6.69	7.38	7.69
APR	1987	6.03	7.45	8.18	8.42
MAY	1987	6.03	8.18	8.80	8.97
JUN	1987	5.95	7.97	8.58	8.75
JUL	1987	6.04	7.89	8.63	8.83
AUG	1987	6.29	8.19	8.95	9.17
SEP	1987	6.64	8.86	9.64	9.82
OCT	1987	6.72	8.94	9.75	9.84
NOV	1987	6.09	8.15	9.06	9.15
DEC	1987	6.07	8.30	9.19	9.33
JAN	1988	6.17	8.02	8.86	9.02
FEB	1988	5.95	7.52	8.38	8.61
MAR	1988	5.95	7.64	8.55	8.82
APR	1988	6.18	7.98	8.91	9.15
MAY	1988	6.55	8.41	9.30	9.44
JUN	1988	6.81	8.39	9.12	9.20
JUL	1988	7.06	8.62	9.27	9.35
AUG	1988	7.37	8.96	9.47	9.54

ATTACHMENT B

EFFECTIVE YIELDS ON U.S. GOVERNMENT SECURITIES - TREASURIES

FOR 1980 +
(THROUGH AUGUST 1988)

	90 DAY	3 YEAR	10 YEAR	30 YEAR
HIGH	17.98	16.88	15.91	15.22
LOW	5.39	6.51	7.21	7.40
MEAN	9.56	10.79	11.21	11.18
STD DEV	3.36	2.75	2.33	2.12

ATTACHMENT C

EFFECTIVE YIELDS ON U.S. GOVERNMENT SECURITIES - TREASURIES

STANDARD DEVIATION SUMMARY

(THROUGH AUGUST 1988)

PERIOD	90 DAY	3 YEAR	10 YEAR	30 YEAR
1965 +	3.04	2.86	2.80	2.76
1970 +	3.12	2.73	2.54	2.42
1975 +	3.23	2.70	2.41	2.16
1980 +	3.36	2.75	2.33	2.12
1985 +	0.91	1.26	1.38	1.36
1965 - 1969	1.03	1.00	0.81	0.71
1970 - 1974	1.76	1.09	0.59	0.87
1975 - 1979	2.20	1.29	0.91	0.64
1980 - 1984	2.92	1.98	1.50	1.30
1985 - 1988	0.91	1.26	1.38	1.36
1988	0.50	0.46	0.36	0.29
1987 - 1988	0.44	0.70	0.71	0.64
1986 - 1988	0.54	0.78	0.81	0.80
1985 - 1988	0.91	1.26	1.38	1.36
1984 - 1988	1.66	2.02	1.99	1.92
1983 - 1988	1.66	1.95	1.92	1.84
1982 - 1988	2.23	2.40	2.21	2.06
1981 - 1988	3.22	2.82	2.44	2.22
1980 - 1988	3.36	2.75	2.33	2.12
1979 - 1988	3.22	2.63	2.26	2.08
1978 - 1988	3.15	2.59	2.27	2.09
1977 - 1988	3.21	2.69	2.36	2.16
1976 - 1988	3.27	2.73	2.39	2.18
1975 - 1988	3.23	2.70	2.41	2.16
1974 - 1988	3.13	2.66	2.43	2.15
1973 - 1988	3.06	2.66	2.47	2.20
1972 - 1988	3.15	2.73	2.53	2.34
1971 - 1988	3.19	2.78	2.57	2.41
1970 - 1988	3.12	2.73	2.54	2.42
1969 - 1988	3.05	2.70	2.55	2.45
1968 - 1988	3.02	2.72	2.60	2.52
1967 - 1988	3.04	2.77	2.67	2.60
1966 - 1988	3.02	2.79	2.73	2.67
1965 - 1988	3.04	2.86	2.80	2.76

SLIDE 4

SET YIELD CURVE UNIVERSE

CURVE 3		90 DAY	3 YEAR	10 YEAR	30 YEAR
LOW	1	5.39%	6.15%	7.21%	4.39%
	2	.	.	.	.
	3	.	.	.	.
	.	.	.	.	.
	.	.	.	.	.
	.	.	.	.	.
MEAN	10	9.56	10.79	11.21	11.18
	.	.	.	.	.
	.	.	.	.	.
	.	.	.	.	.
	17	.	.	.	.
	18	.	.	.	.
HIGH	19	17.98	16.88	15.91	15.22
TODAY	20	7.37	8.96	9.47	9.54

INTERPOLATE INTERIM VALUES

To get the interim values, since there's no science involved here, we do is some sort of interpolation formula, to interpolate the mean and all yield curves. Again, there's no right answer.

The next thing we do is set the transition probabilities (Slide 5). This is a bigger graph than the earlier one because now we're dealing with twenty yield curves instead of five. The key to this graph is any of these vertical lines tell you just what the numbers are. The probability of staying in the same place is 36 percent. The probability of moving up or down one yield curve is 22.5 percent. The probability of moving up or down three yield curves is 8 percent. The probability of moving up or down four yield curves is 1.5 percent.

Taking the experience from the 1980s, we did two things. We created a distribution function that, under the standard deviations and the interest rates that were available or that existed during the 1980s, would emulate the standard deviations that were experienced. Therefore, if anybody were to call us to the stand in court and ask where we came up with these interest scenarios, we would feel comfortable that we could say honestly that the scenarios were based on and would replicate the last eight years of volatility in interest rates.

SLIDE 5

SET TRANSITION PROBABILITIES

ENDING CURVE	BEGINNING CURVE 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	5.0	5.0	5.0	1.5																
2	10.0	10.0	10.0	8.0	1.5															
3	25.0	25.0	25.0	22.5	8.0	1.5														1.5
4	25.0	25.0	25.0	36.0	22.5	8.0	1.5													8.0
5	20.0	20.0	20.0	22.5	36.0	22.5	8.0	1.5												22.5
6	10.0	10.0	10.0	8.0	22.5	36.0	22.5	8.0	1.5											36.0
7	5.0	5.0	5.0	1.5	8.0	22.5	36.0	22.5	8.0	1.5										22.5
8					1.5	8.0	22.5	36.0	22.5	8.0	1.5									8.0
9						1.5	8.0	22.5	36.0	22.5	8.0	1.5								1.5
10							1.5	8.0	22.5	36.0	22.5	8.0	1.5							
11								1.5	8.0	22.5	36.0	22.5	8.0	1.5						
12									1.5	8.0	22.5	36.0	22.5	8.0	1.5					
13										1.5	8.0	22.5	36.0	22.5	8.0	1.5	5.0	5.0	5.0	
14											1.5	8.0	22.5	36.0	22.5	8.0	10.0	10.0	10.0	
15												1.5	8.0	22.5	36.0	22.5	20.0	20.0	20.0	
16													1.5	8.0	22.5	36.0	25.0	25.0	25.0	
17														1.5	8.0	22.5	25.0	25.0	25.0	
18															1.5	8.0	10.0	10.0	10.0	
19																1.5	5.0	5.0	5.0	
20																				
	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100

We are doing a lot of work in placing single-premium deferred annuities. The person whom we're working for feels quite comfortable with his scenario, but he wants to do some stress tests. He wants to see how bad things have to get before the product isn't useful. We might run forty to fifty trials, sometimes a hundred. That's normal. I believe, to be statistically significant at the 90 percent tails, which is what's really important for the valuation actuary, you probably need to run two or three hundred trials.

How can we be comfortable at 90 percent when we're only maybe 75 percent comfortable with the other assumptions? What if we don't have any statistically credible evidence as to our lapse-rate formula or we're assuming an investment or a crediting strategy that the next management's going to change? Or as soon as there's a new synthetic security out there, how can we be 90 percent comfortable when there's so much other volatility associated with the projections we're doing?

There are no right answers, and there aren't any wrong ones either. I believe that establishing interest-rate scenarios should be viewed as an art and not a science. There certainly are some scientific exercises that we can do to justify the scenarios and rates that we're coming up with, but we place too much reliance on some of the scientific skills. There's a lot of art involved.

THE BETA DISTRIBUTION AND ITS APPLICATIONS

MR. IRWIN T. VANDERHOOF: In my background reading paper entitled *Bond Defaults*, which was included in the handouts, I refer to the BETA distribution and give an example of its application. The BETA distribution doesn't seem to be on anyone's top-ten list of distributions. It has been used in theoretical discussions. I believe there have been many references in the European actuarial literature. There may even be a recent reference in the North American literature.

All and all it must seem an unpromising topic. But this distribution may be a useful tool for actuaries, and I think the topic is interesting in itself.

I will discuss:

1. Why the BETA distribution has been genially ignored for so long and why it is due for a big jump in popularity now.
2. Why the BETA distribution should be of interest to actuaries and investment people concerned about the mathematical behavior of default statistics.

3. The actual mathematics of the BETA distribution.

4. Some ways in which the BETA distribution can be applied in actuarial work.

<u>Change in Popularity</u>

The BETA distribution has had little popularity among practical people because it has been difficult to work with. I believe that it will soon become more popular because it is now reasonably easy to work with.

The distribution has not changed, and the equations have not been simplified. The change has been in our ability to do calculations.

In a philosophical vein, I can argue that our scope of activities as applied mathematicians has not been keeping up with the changes in our computational ability. Personal computers (PCs) have vastly improved that ability. We are having difficulty in keeping up with the improvements in technique that are made possible by PCs. It is not just that we can do the old things better and faster. We can also do, at little cost, things that we used to find prohibitive in terms of time and money.

I'll give you a personal recollection. I wrote a study note for asset-liability matching in 1976. In that study note, I gave, as arguments against the use of duration matching, the facts that the calculation of durations was difficult and that the concept was relatively obscure. In a recent revision of that study note, I gave, as arguments in favor of the use of duration, the facts that the calculation was easy and that the concept was widely known. The calculation and the concept had not changed. Our technical competence had.

While we always talked about distributions when we studied statistics, we really only paid attention to the NORMAL distribution. While it has nice theoretical properties, it has an immense practical advantage. While the NORMAL distribution has two parameters, it is symmetrical, and the whole distribution can be put in a reduced form with the mean of 0 and a standard deviation of 1. Any other values can be translated into this form, and therefore, with one or two pages of tables, the student can do all sorts of practical calculations.

The BETA distribution, on the other hand, is not symmetrical, and any change in the parameters causes an incommensurate change in all the values of the distribution. You have to have a vast number of tables to be able to do anything, and you will spend plenty of time interpolating all the numbers to actually do any calculation. The only source I have seen for tables is Biometrika Tables for Statisticians, which is now dated 1962.

The change, at least for me, is that I was able to program my PC to do the monstrous calculations. As a result I can get a single value of the cumulative distribution function (c.d.f.) in a few seconds, and I can get 200 points of the c.d.f. (cumulative probability = .005, .01, .015, etc.) in about thirty minutes.

If, as I will argue momentarily, the BETA distribution should be part of your actuarial arsenal, the reason you haven't been using it is simply that it used to be hard to work with. That has changed.

Why BETA May Be Helpful

Let's start off with a little history and terminology of the BETA distribution.

Look at Euler's integral of the first kind.

(1) $B(x,y) = \int_0^1 t^{(x-1)} (1-t)^{(y-1)}\, dt.$

This related to the GAMMA function as follows:

(2) $B(x,y) = (\Gamma(x)\, \Gamma(y)) / (\Gamma(x + y)).$

Now we all remember that, if x is an integral number, $\Gamma(x) = (x-1)!$. Therefore, if both x and y are integral numbers,

(3) $B(x,y) = ((x-1)!\, (y-1)!) / ((x + y - 2)!)$

The reciprocal is then

(4) $1/(B(x,y)) = ((x + y - 2)!) / ((x-1)!\, (y-1)!)$

But what have we here? The right side looks like the number of different combinations of (x + y - 2) things, taken (x - 1) or (y - 1) at a time. The complete BETA function, which is the correct name for B(x,y) is then a continuous equivalent of the combinations we studied as an introduction to probability. Now let's think about the incomplete BETA function. This arises from the Euler integral when the upper limit of integration is not 1 but is less than 1.

If we now define the BETA distribution as:

(5) $I_x = (1/B(a,b)) \int_0^x t^{a-1} (1-t)^{b-1}\, dt$, we can develop the following:

(6) $\sum_{s=a}^{n} (n!/((n-s)!\ s!)\ q^s(1-q)^{n-s} = I_q\ (a,n-a+1).$

Or, in words, if there are **n** individuals with a probability of dying of **q**, then the probability of **a** or more dying is shown on the left side of the equation in the form of the binomial expansion and on the right side as the BETA distribution. The distribution is only defined for values of q from 0 to 1.

The reason that actuaries should be interested in the BETA distribution is that it is a continuous analogue for the binomial distribution, which underlies much of actuarial theory. If you think the difficulties in calculating the BETA are severe, think about the problems in calculating all the binomial terms for very large values of **n**.

The Mathematics of the BETA Distribution

I have already described the basic mathematical formulas underlying the BETA distribution. However, it is a continuous distribution and can be valued for other than integral values of the parameters. Since the values of the variable can run only between 0 and 1, the BETA distribution is suitable for any rate, which has that range -- mortality, default, lapse, etc.

The distribution has the same number of parameters (three) as the binomial expansion. However, since the distribution is continuous, there is not a parameter that can be related clearly to the total number of trials. The total number must be assumed to be infinite. The extra parameter can be related to the variance of the distribution, which is not fixed as is true for the binomial.

If we have a series of values for a rate, say the default rate on bonds, then we can calculate the mean and standard deviation for those values. If μ is the mean and σ is the standard deviation of those values, then the parameters of the BETA distribution are:

(7) $a = \mu\ (((\mu\ (1-\mu))/\sigma^2) - 1)$, and

(8) $b = (a/\mu)\ (1-\mu)$.

Given a set of values for the mean and standard deviation of the data, it is possible to calculate the probability that the observed value of q will be less than the value of q used in (6).

To do the actual calculations of the values of the distribution, I use both a Gauss-Laguerre formula and an approximation shown in the Dover book for small values for calculating the values of the needed GAMMA distribution. I use classical Gauss-Legendre formula parameters for the final calculation of the BETA distribution.

Application of the BETA Distribution for Bond Defaults

The real application of this mathematics is for defaults of book-value assets. We have been hypnotized in recent years by financial theory and its emphasis on market value. Most of our assets are not carried at market -- they are carried at book. I don't believe that this will change. The reason is simple. If everything is at market, the companies will have the opportunity periodically to display their insolvency to their customers. We got through the last several economic calamities because the public never knew how bad off we were. If market valuation makes us display our market value promptly to the public, we will not get through the next economic calamity. Considering what happened later, I would argue that the market was wrong and that the companies were not insolvent. If they really had been, they couldn't have survived.

Suffice it to say, I don't believe that book valuation is going to go away. If we are using a book-value base for valuation of assets, coupon and loss on default will retain their importance.

Using the default rate on junk as a random variable means that a series of assumptions are being made -- some weak, some strong, some heroic. Any use of default data on junk will require the same variety of assumptions. I believe that you make your choice and take your chances.

The reason for this situation is simple. There are many factors that should have an impact on the default rate of bonds of any rating class. Among those factors are chance distribution by rating class for each year, state of the economy, probability of change in rating class, and years since issue. I believe that all of these factors are important.

However, while there are a number of factors that are surely important, there are only eighteen years of experience covering about 250 defaults. It's nice to think that we will analyze the data one way and then another, but we are just kidding ourselves. That sort of thinking leads to an attempt to evaluate 251 parameters when there are only 250 defaults. While this sort of calculation can be done, it is not mathematics. It is not statistics. It is only arithmetic. The results cannot mean anything.

I'll go further. There can never be enough data for all these calculations to provide meaningful results because the world is always changing and new variables will enter into the formulations and become important while older ones vanish.

In econometrics we call this lack of data the identification problem. It is like the Heisenberg uncertainty principal in physics, or Gödel's incompleteness proofs in mathematics. We are simply saying that there will always be infinitely more equations to explain the data than there are data.

What are our choices? Professor Edward Altman of New York University has decided to look at the data from the point of view of a mortality table for bonds. There are nine classes of bonds and about ten years of experience being analyzed. That leaves less than three defaults per cell. Of course, there were very few defaults in the higher categories, but there were very few bonds in the lower grades. While I have heard some reservations about the techniques used to do the calculations, I don't view those reservations as being crucial. What concerns me is that, by using up all the information on the analysis by class and year, there is no meaningful information left to use for analyzing annual variability.

Another approach could be to concentrate on the explicit impact of the economy on default rates. I have not been able to find such a relationship. I am certain that there is an equation using economic variables that will work. However, when that is done, I don't believe that there will be information left in the data to support the random nature of the default phenomena.

There are other possibilities. The translation between the various rating classes is important. Industry is also important. The question is not whether these various factors are important, but rather upon which factor we should concentrate.

In the BETA distribution approach the concentration is clearly upon the random nature of the phenomena. Since I don't believe that we can usefully predict the economy, I don't think that using up the data to get correlations with other items we can't predict is a wise use of the scarce resource -- information. I believe that the best use of the data is to analyze them as random phenomena.

This approach should be of particular use to regulators because it provides an easy method of determining a level of risk for subinvestment-grade investments. I hope that the regulators will play fair if they accept this kind of approach. If this method were to be accepted, then the distribution should be recalculated periodically as the data progress. The regulators have a perfect right to determine the acceptable level of risk and the reserves thereby implied. If the regulators chose to change the parameters of the distribution and then insist upon some particular level of risk based on the "fudgedup" distribution, they will have replaced an intellectually defensible methodology with pure opinion.