MODELING THE PARTICIPATING WHOLE LIFE DIVISION

DR. ALLAN BRENDER: The participating division of the Solvent Stock Life Insurance Company began business twenty-five years ago, issuing its first policy in 1963. The division has only sold whole life insurance and continues to do so to this date. The product is standard. For purposes of the present model, all riders are ignored. A description of pricing, valuation and experience assumptions can be found on Pages 50 and 51 of the distributed handout materials. (See Appendix 1).

The dividend scale is calculated using a traditional three-factor formula. The interest and mortality components are calculated using tax-reserve factors; other approaches would use the cash values or statement-reserve factors. The expense component in this formula contains a somewhat arbitrary adjustment, a sort of balancing item, used to ensure that the dividend scale has an acceptable shape and level. It appears that this type of modification of the pure three-factor formula is quite common.

The policy-loan rate was assumed to have a maximum value of 6 percent for issues of 1968 and earlier, a maximum of 9 percent for issues of 1969 through 1974, and no maximum for issues of 1975 and later. This distinction in policy-loan rate is important in testing interest rate risk and is now recognized in the latest revision of the Canadian

Life and Health Insurance Association (CLHIA) minimum continuing capital and surplus requirement (m.c.c.s.r.) formula.

For purposes of the model, it was assumed policies are only issued at ages 25, 35, and 45. New business is assumed to be sold at these ages in the proportions 30:45:25. Since we are simulating a fictitious company, the initial position for the model, as of year-end 1987, was obtained by running the model for twenty-five years, beginning with some initial capital and no business in force. The resulting initial position shows $2.32 billion of insurance in force, 1987 premium income of $37.8 million, and actuarial reserves of $123 million dollars.

The participating division was modeled as a separate company. Therefore, the division manages its own assets. Assets are limited to cash, government bonds, industrial bonds carrying a rating of A, residential mortgages, and cash. For purposes of modeling, mortgages are restricted to a five-year term and are assumed to be amortized over twenty years -- the twenty-year amortization being a compromise between the more usual twenty-five-year initial term and the shorter terms found on renewals.

Commercial bonds are assumed callable at a premium any time after their fourth anniversary; similarly, mortgages also can be prepaid subject to an interest penalty. Calls and prepayments are governed by the difference between the interest rate stated in the contract and the prevailing current market interest rate. The standard investment policy calls for 10 percent of all investable cash to be placed in each of ten- and twenty-year government bonds, and five-, ten-, and twenty-year industrials for a total investment of 50 percent in bonds; 45 percent is placed in mortgages, and 5 percent is kept in cash and short-term Treasury bills. Again, in order to simplify the model, the division is assumed to invest under a buy and hold policy; assets are bought at issue at par and are held to maturity. There is no trading.

If, in any year, the division experiences a negative cash flow, the model assumes a one-year bank loan at the prime rate for the amount of the cash-flow deficiency. Such loans did occur during the historic period used to develop the initial 1987 position. Any model must allow for negative cash flow and make provision either for disinvestment or a loan.

Turning now to the operation of the model, I must explain the simplifications made in order to keep the programming manageable. The most important simplifying assumption is that the policy and calendar years coincide; in effect, the division only operates one day a year, January 1. While not terribly realistic, this assumption allows us to generate reasonable numbers and build a mode similar, in its sensitivity to changing scenarios, to a mode found in a real company. I am going to outline the activities of the company's employees on January 1. While this may be hard to believe, the sequence of activities is necessary in any model of this type. This sequence actually describes the detailed steps in the model.

After reporting to work at 8:00 A.M. and spending an hour or so over coffee catching up with each others' lives over the past year, the employees begin the day at 9:00 by processing lapses on existing business. You recognize, of course, that 9:00 A.M. is the end of the previous policy year. By 9:30, the agency force has been heard from, and we set to processing new business. With our new fully automated underwriting system, this is done, and commissions and issue expenses are paid by 10:00 A.M., when we turn to the collection of annual premiums (we only have annual premium business) on all the old business. Within the following ninety minutes we pay all expenses, receive policy-loan repayments, and process new policy loans. Living in a slightly idealized world, the company has managed to insure only that subset of the general population which dies, if it dies at all, immediately after New Year's Eve celebrations. Therefore, by 11:30 A.M., we are ready to process the year's death claims. Having finished that grim task, we break for lunch, perhaps managing to see the tail end of the Rose Bowl Parade.

After an extended lunch, the employees resume business, more in the guise of finance than insurance people. At 1:30 P.M., we receive payments on asset maturities, bond calls, and mortgage prepayments. At 2:00, we start to open all those envelopes containing our investment income. Having received our income and calculated the net cash flow from the liability or product side, we put on our thinking caps and consider how to invest our net cash. At 3:00, we reach our brokers (they only work half an hour per year) and invest. By 3:30 the job is done, and we set to calculating income taxes and the new dividend scale and proceed to prepare the annual statement (a year in advance - Office of the Superintendent of Financial Institutions Canada (OSFI) loves us!). Being extremely efficient, we are done by 4:30 P.M. and adjourn to watch the Rose Bowl.

I want to focus for a moment on the last activity of the day, the preparation of the statement -- in particular, on the calculation of the actuarial reserves. My first point deals with the valuation system. Ever since we, the Committee on Solvency Standards, first described publicly the method presented at this symposium, I have heard actuaries question how they could use their current valuation systems in the solvency testing process. Those with seriatim systems, which do not make use of factors, have been particularly concerned, and rightly so. Quite frankly, in this modeling process, I firmly believe factors are necessary. When you consider that a single scenario requires at least five runs of the valuation system, that at least twenty to thirty scenarios will have to be run, and that most companies' valuation systems require many hours to run on a mainframe for a single run, the cost in computer time and dollars of using a seriatim-based valuation system seems prohibitive. In addition, use of such a system would require the creation of many phantom individual policies in order to be able to model the effect of new business over the projection period. It seems that simplified valuation models based on factors are the way to go.

My second point on valuation concerns the way policyholder dividends are taken into account in the reserves. Are they recognized implicitly or explicitly? The prevailing practice seems to be to use the implicit method. Indeed, this is the method used in the model to obtain our results.

A paper by Wayne Bergquist, written shortly after the introduction of the 1978 Canadian Valuation Method, shows that the implicit and explicit methods are equivalent. Now, Wayne is correct: the methods are equivalent, but only if a number of conditions are satisfied. Among these is the assumption that changes in experience are passed on to the policyholder immediately and in full through changes in the dividend scale. Equivalence also requires the dividend formula to reflect all the components used in the reserve. This is usually violated since most dividend formulas do not reflect experience with respect to the lapse assumption, payment of cash surrender values, and recovery of acquisition expenses. Similar remarks apply with respect to income taxes. Moreover, many dividend scales contain smoothing components and refer to cash value or tax reserves rather than policy reserves when calculating the interest and mortality components. In short, the conditions necessary for the equivalence of the implicit and explicit methods are not usually satisfied.

The model for the participating division can also be run using an explicit valuation method. Though we have not shown the results, I can tell you that the division is much more sensitive to changing scenarios under the explicit method than is revealed in the figures. The greater volatility under the explicit method arises from frequent and significant changes in reserves caused by changing experience and consequent revision of dividend scales. Notice that under the implicit method, there is no revaluation, since experience under all scenarios tested is never worse than the original valuation assumptions.

In studying the solvency position of a company with participating business, it seems to me that one of the more important questions to be investigated is: What are the financial effects on the company of a delay in reducing the dividend scale following a downturn in actual experience? Our recent experience with respect to the decrease in interest rates from their unusual highs in the early part of this decade demonstrates that such delays do occur, and the question is valid. Under the implicit method, our testing can only measure the effects of a higher than normal dividend payout during the projection years; since there is no revaluation, failure to pass on the change in full to policyholders beyond the projection years will not be captured. The implicit method is therefore not an adequate tool for the investigation of this question. It would seem that the explicit method should be preferred and perhaps required. This is a subject for further study.

The figures I developed were derived using an implicit valuation. Although there was no recalculation of reserve factors, the dividend scale was adjusted in projection years one, three, and five to reflect changes in experience.

I want to make one more observation on model building. A model will be used to simulate under many different scenarios. Some of these involve changes in experience factors such as mortality, lapse, or expense rates or changes in the external economic environment. Others involve changes in business policy such as marketing and sales, investment policy, or the dividend scale. Under all of these scenarios, we have to have the flexibility to change valuation assumptions dynamically as the simulation proceeds. For all this to work, the model must be designed with flexibility in mind so that a change in scenario assumptions is easily accomplished without major changes in the program. If you are going to create your own new simulation program to do solvency testing, it is important to build the required flexibility in at the lowest level. This is crucial. It is also the reason existing corporate models in many companies are inadequate for the task we are putting before you.

Turning now to the results of the simulations, let's first look at the initial position for the simulations: the balance sheet as of year-end 1987. The division has assets of $159 million, reserves of $123 million, other liabilities -- primarily the provision for policyholder dividends -- of $13.4 million, and total surplus of $22.1 million. Surplus is split into an investment valuation reserve (IVR) of $5.6 million, appropriations for cash surrender value (CSV) deficiencies and negative reserves of $7.9 million, and $8.6 million in unappropriated surplus.

Under the base scenario, which represents the best guess situation, earnings are healthy but decline over time from $957,000 in 1988 to $571,000 in 1992. The surplus position in 1992 is rather interesting; total surplus is then $18.2 million as compared to $22.1 million in 1987. This is split as (1) an IVR of $8.7 million, up from $5.6 million, (2) required appropriations for CSV deficiencies and negative reserves of $10.9 million as compared to $7.9 million, and (3) negative unappropriated surplus of $1.4 million. If this were a separate company, it would be in great difficulty by 1992 since unappropriated surplus would then be negative. However, under current legislation, the situation we have here is acceptable. The participating line can have negative unappropriated surplus as long as the company as a whole is solvent. However, there has been some speculation that, under the forthcoming new insurance legislation, solvency tests might be applied separately to each fund. In that case, the situation presented here would be unacceptable.

We should ask why unappropriated surplus becomes negative in 1992. Part of the reason is the relatively rapid growth in the IVR and required appropriations. Another cause of the decline is the size of the transfers to the shareholders' fund. The model assumes shareholders will take the maximum transfer allowed under law, in this case, 10 percent of all dividends distributed or, equivalently, one-ninth of all policyholder dividends. As it turns out, this amount, in each projection year, exceeds the division's net earnings; in fact, over the five-year projection period, the sum of total transfers is double the sum of net earnings, which is $4.1 million.

With respect to the m.c.c.s.r., the situation is not so bleak since the entire IVR and half the required appropriations, under the CLHIA rules, can be counted towards satisfaction of the test. In 1992 under the base scenario, total available (CLHIA) surplus exceeds requirements by 40.9 percent; I shall refer to this ratio of the excess of available surplus over formula-required surplus to required surplus as the free ratio.

Under the proposed federal standard, according to which required appropriations (other than the IVR) cannot be used, by 1992 the division only has slightly over 80 percent of required surplus, for a free surplus ratio of negative 20 percent, assuming the same surplus formula will apply. The lessons to be learned here are two:

1. You can't focus only on satisfying the surplus formula if at the same time you may be left with no unappropriated surplus.
2. There may be significant differences in your conclusions depending on whether you apply CLHIA rules or the proposed federal standard in determining available surplus.

Turning now to the results of testing the prescribed scenarios, we find that the experience of this line under unfavorable conditions is moderated by the ability to pass on some of the effects of that experience through reductions in dividends. Therefore, in scenario 1, which involves a steady increase in mortality, the free surplus ratio under the CLHIA definition only drops to 26.3 percent from 40.9 percent in the base; an increase in withdrawals, supposedly an unfavorable scenario, is actually beneficial as the 1992 ratio rises to 119 percent. Similarly, scenario 5, which tests falling interest rates, produces a final ratio of 47.9 percent, better than the base. The converse to this is scenario 4, an increase in interest rates, which produces a downturn and a free surplus ratio of just 6 percent, primarily due to increased dividends and higher income taxes. Also, in scenario 4, net income is actually negative in 1992; this is the only prescribed scenario under which this occurs.

Changes in the rate of new sales do not have a significant effect. For example, under scenario 7, which prescribes a doubling of the real rate of growth, the ratio drops from 40.9 to 36.6 percent. In this line, new business is assumed to grow at 6 percent; since 4.15 percent represents inflationary growth and a doubling of the real growth from 1.85 to 3.7 percent is not too significant. Since for this company the ninety-fifth percentile for 1988 claims is only at 112.5 percent of expected, the experience under scenario 8 is also not too bad; there is a decrease in earnings in 1988, but these quickly recover and the 1992 free surplus ratio ends up at 37 percent Scenario 9, which stipulates a doubling of asset default rates, produces the second worst result with the ratio dropping to 24 percent.

Of the additional scenarios which we chose to test, the division fared worst under number 11, which involves significant asset defaults; the ratio dropped to negative 2.9 percent. This is the only scenario under which the division fails the CLHIA test in 1992. We tested combinations of our interest patterns with a short investment strategy; the results were virtually the same as for the standard investment policy. Results for other scenarios tested are what one would expect in light of results on the prescribed scenarios.

So, we see that asset defaults and rising interest rates are the greatest of the problems studied for this line. Interestingly, in these situations, the dividend scale either does not respond or, in the case of rising interest rates, responds in a manner unfavorable to the company.

Finally, I would like to address a special topic: the determination of the ninety-fifth percentile for aggregate claims in the first year of the projection. Recall that the task is to find the value, L, such that the probability that total claims, C, for the year will not exceed L is 0.95. To do this, we have to find the distribution of aggregate claims. A Society of Actuaries study note on risk theory more than three or four years ago described this calculation as an impossible task involving untold number of convolutions, which would be guaranteed to tie up your mainframe for weeks. The calculation is now mathematically tractable -- it can be done very easily.

Details of what I am about to say can be found in Appendix 4 of the written material. That appendix is a guide as to how to proceed. It contains bibliographic references, which will lead you to the detailed technical explanations.

Let me remind you that aggregate claims, C, are a combination of N, the number of claims, and X, the amount of a single claim. Both of these are random variables. For N, we usually choose the Poisson distribution since it is easily derived from a set of postulates, which make sense in an insurance setting. However, other distributions may also be reasonable. The distribution for X, the claim-size distribution, is easily calculated for life insurance and does not have to be modeled by a standard distribution. It depends on the mortality table and the distribution of policy sizes.

I remind you that moments of the distribution of aggregate claims, C, can be calculated directly from those of N and X; for example, the mean or expected value of aggregate claims, C, is the product of the means of N and X. Therefore, even before we find the distribution for C, we can easily compute some important characteristics such as its mean, variance, skewness and kurtosis. The formulas are found in references cited in Appendix 4.

We should also observe that, as we increase the relative number of large policies in our portfolio, the tail of the secondary distribution will thicken. This will cause higher moments of the aggregate claims distribution to increase and the ninety-fifth percentile to move away from the mean. One reason why the ninety-fifth percentile in our study is so close to the mean is that we assumed our portfolio contained relatively few large policies.

The major new tool for the calculation of the distribution of C is a recursion formula by my colleague at the University of Waterloo, Professor Harry Panjer. This is much more efficient than the convolution method and is often quite tractable. There are a few technical difficulties, discussed in the appendix, but these are relatively easily overcome.

Other techniques are also available and may be suitable for large portfolios. For example, we can make use of approximations to the distribution, but we should take care to use only approximations which are reasonably good. One method which is available when the skewness of the distribution of C is less than 2, is the "Normal Power" or NP method described in the beautiful book Risk Theory by Beard, Pentikainen, and Pesonen.

As a last resort, if the skewness is sufficiently close to zero and the kurtosis close to three, we could approximate by the Normal distribution and make use of readily available tables of that distribution. However, I must emphasize that the approximation is not valid unless these conditions are met, and it would not be correct to assume a Normal distribution without carrying out appropriate tests for these conditions.

In the case of the distributions used in our calculations for scenario 8, two distributions were calculated. The first was for all lines of individual business combined. The second was for group life. In both cases, skewness was less than 0.1, and the kurtosis was less than 3.1. Therefore, Normal approximations were in order and were used.

APPENDIX 1

PRODUCT DESCRIPTION

The product used in this model is a traditional whole life participating insurance. It is not the actual product of any particular company but was designed using design software of a Canadian life insurer.

There are not additional riders contained in this policy; in particular, there is no waiver of premium. Benefits are restricted to death benefits and cash surrender values.

The following rates describe most features of the policy. The average policy size is assumed to be $25,000. This amount is used to convert per policy expenses, as well as the policy fee, to amounts per thousand.

Premiums

ages	25	35	45
per thousand	$10.21	$15.14	$23.58

In addition, there is a policy fee of $13.50.

Expenses

commissions and overrides:

year 1	year 2	thereafter
160%	40%	0.4%

other acquisition expenses: $100 per policy

annual maintenance expenses: $25 per policy

Mortality

experience: 65% of CIA 1969-75 select and ultimate

dividend: 70% of CIA 1969-75 select and ultimate

valuation: 75% of CIA 1969-75 ultimate

Note: valuation here is by the implicit (recognition of dividends) method.

Lapse

for both experience and valuation, in the first 5 years the rates are 9%, 7%, 6.5%, 6%, 5.5%

thereafter, the rate is 5%

Interest

dividend: 1% below the previous year's earned rate

valuation: 4.5%

Dividend Formula

The dividend formulae is of the standard three-factor type. The interest and mortality components are based on tax reserves. The expense component contains an arbitrary adjustment used to control the overall level and shape of the dividend scale. It should be noted that the presence of this type of adjustment factor makes it unlikely that the implicit and explicit valuation methods will produce identical results.

APPENDIX 4

REMARKS ON CALCULATING THE 95TH PERCENTILE FOR AGGREGATE CLAIMS IN A SINGLE YEAR

The eighth Prescribed Scenario requires the actuary to project an insurer's experience under the assumption that in the first projection year the level of claims experienced is at the 95th percentile. This note discusses the meaning and determination of the 95th percentile.

The fundamental task is determine an amount S such that the probability that total claims in a single year exceed S is less than 5 percent (equivalently, the probability that total claims are less than S is at least 95 percent). In order to find S we will usually have to calculate the probability distribution of aggregate claims, or find a sufficiently accurate approximation to that distribution.

The distribution of aggregate claims depends on two auxiliary distributions: the distribution of claim frequencies, and the so-called secondary distribution or the distribution of the amount of a single claim given that a claim occurs. In particular, consider the following notation:

let p_n be the probability exactly n claims occur during the year, n = 0, 1, 2, 3, ...

let f(x) be the probability that if a claim occurs it is for amount x; here x will be taken as a discrete amount, usually in thousands of dollars.

let g(x) be the probability that total claims during the year equal x.

According to a well known formula, g(x) can be expressed as the sum over all possible values of n from 0 to infinity

$$g(x) = \Sigma\, p_n f^{*n}(x) \qquad (1)$$

where f^{*n} is the n-fold convolution of the distribution f. This is very difficult and tedious to calculate. Fortunately, there exist alternate computational methods as well as a number of approximations of varying degrees of accuracy.

Before proceeding to methods for the calculation of the distribution {g(x)}, let us first consider the frequency and secondary distributions $\{p_n\}$ and {f(x)} respectively. The traditional choice for the frequency distribution is the Poisson distribution given by

$$p_n = e^{-\lambda}\lambda^n / n! . \qquad (2)$$

Here λ is the expected number of claims in a year. λ will depend on the choice of mortality rates. If there is some uncertainty about these, this can be provided for by allowing itself to be random; if λ has a Gamma distribution, the resulting distribution turns out to be of the negative binomial family. Details can be found in section 2.9 of [1] or section 11.3 of [2].

The secondary distribution f(x) can be constructed as follows:

1) For each possible claim about x, let q(x) be the sum of all mortality probabilities for all policies in the portfolio with face amount x.

2. Let Q be the sum of all q(x)'s for all values of x.

3) Then set $f(x) = q(x) / Q$.

Generally the values x will be in units of $1,000 and all policies will be rounded to the nearest thousand. In order to speed up computation, it may be advisable to work in units of $5,000 or $10,000 instead.

The distribution of aggregate claims can now be calculated recursively using the recursion derived by H. Panjer:

$$g(x) = \sum (a + by/x) f(y) g(x-y) \qquad (3)$$

where for the Poisson distribution $a = 0$ and $b = \lambda$, while for the negative binomial distribution, $a = \beta / 1+\beta$, and $b = (r-1)a$ for appropriately chosen r and β. In the summation above, y runs over all values from 1 through x. Several comments are in order with respect to the use of the recursion (3):

i) The recursion requires an initial value g(0). This is equal to the probability p_0 that no claims occur. Unfortunately, this number can be so small that the computer treats it as being zero. The user should experiment with his/her computer to find the smallest non-zero number which the computer will accept (for many PC's this will be about exp(-700)). In the Poisson case, since we have $p_0 = e^{-\lambda}$ the distribution can be calculated initially by dividing by the smallest power of 2 necessary so that $\exp(-\lambda/2^n)$ is seen by the computer as non-zero. The resulting distribution can then be convoluted with itself, repeating this step n times to obtain the final result. A similar procedure can be carried out in the negative binomial case, replacing r by $r/2^n$; here, $p_0 = (1+\beta)^{-r}$.

ii) Since this calculation is recursive, the program will involve a loop and must contain some procedure to determine when the recursion comes to an end. It is suggested the stopping criterion be a combination of a condition limiting the maximum number of values of g(x) calculated, together with a condition that the sum of all values of g(x) calculated to date is sufficiently close to 1.

iii) If there is a smallest value k for which f(k) is non-zero and this value k is greater than 1, then values of g(x) can be calculated k at a time. If the recursion is programmed in APL this time-saving observation can be implemented by judicious use of matricies in the calculations.

Other Methods

While the Panjer recursion is recommended and is easily programmed, it may take a fair amount of time to compute. Several other approximations are available. In order to discuss these it should first be recalled that the moments of the distribution {g(x)} can be calculated directly from the moments of $\{p_n\}$ and those of {f(x)} without calculating the distribution {g(x)} directly (for details, refer to section 11.2 of [2]).

I. If the skewness of the distribution {g(x)} is not greater than 2, it may be appropriate to apply the normal power (NP) approximation to approximate {g(x)}. Refer to section 3.11 of [1] for further information.

II. If the skewness of {g(x)} is close to zero and the kurtosis is close to 3, then {g(x)} may be approximated by the Normal distributions and the 95th percentile is then located at the mean plus 1.65 standard deviations.

References

[1.] R.E. Beard, T. Pentikainen, E. Pesonen, *Risk Theory* Third Edition, Chapman and Hall, London and New York, 1984

[2.] N.L. Bowers et al, *Actuarial Mathematics*, Society of Actuaries, 1986

[3.] H.H. Panjer, *Recursive Evaluation of a Family of Compound Distributions*, ASTIN Bulletin 12 (1981) pp. 22-26